



SYDNEY BOYS HIGH
MOORE PARK, SURRY HILLS

2008
YEAR 10 HALF-YEARLY
EXAMINATION

Advanced Mathematics

Directions to Candidates:

- Answer all questions in the spaces provided in this question booklet.
- Full marks may not be awarded for careless or badly arranged work.
- Use black or blue pen for written answers, but pencil for diagrams and graphs.

- If additional working space is needed, use the spare pages at the end of the booklet. Show clearly which question you are continuing.
- Board-approved calculators may be used.

Time allowed: 90 minutes
Examiner: Mr P. Bigelow

Name: _____

Your Mathematics Class (Tick the box)	
10MaA Mr Fuller	<input type="checkbox"/>
10MaB Mr McQuillan	<input type="checkbox"/>
10MaC Mr Choy	<input type="checkbox"/>
10MaD Ms Ward	<input type="checkbox"/>
10MaE Ms Nesbitt	<input type="checkbox"/>
10MaF Mr Boros	<input type="checkbox"/>

Markers' Use Only	
Question 1	/16
Question 2	/14
Question 3	/15
Question 4	/15
Question 5	/14
Question 6	/21
Total	/95

Question 1 (16 marks)

Answers

Marks

- (a) If $a = 6$, $b = 7$, and $c = -5$, evaluate

1

$$\frac{a-c}{b-a}$$

$$\frac{6 - (-5)}{7 - 6} = \frac{11}{1} = 11$$

- (b) Simplify $\frac{2x-8}{4}$.

1

$$\frac{2x-8}{4} = \frac{x-2}{2}$$

- (c) Factorise

3

- (i) $4a + 6a^2b$

$$2a(2 + 3ab)$$

- (ii) $x^4 - 16$

$$(x^2)^2 - 4^2 = (x^2 - 4)(x^2 + 4)$$

- (d) Expand and simplify

2

- (i) $(3a - 1)^2$

$$9a^2 - 3a - 3a + 1 = 9a^2 - 6a + 1$$

- (ii) $(\sqrt{5} - 3)(2 - 3\sqrt{5})$

$$2\sqrt{5} - 15 - 6 + 9\sqrt{5} = 11\sqrt{5} - 21$$

- (e) Solve then graph solutions on a number line

2

$$5 - \frac{2a}{3} < 6$$

(f) Express with a rational denominator
(in simplest form)

3

(i) $\frac{5}{3\sqrt{10}}$

(ii) $\frac{\sqrt{5} - 1}{\sqrt{5} + 1}$

(g) Write down the gradient of
(i) the interval joining $(4, -1)$ and $(2, -5)$,

2

(ii) the line $4x + 3y - 12 = 0$.

$-\frac{4}{3}$

(h) Solve the pair of simultaneous equations
(show all working)

2

$$4x - y = 30 \dots\dots\dots \boxed{1}$$

$$3x + 2y = 17 \dots\dots\dots \boxed{2}$$

Question 2 (14 marks)**Answers****Marks**

(a) Evaluate $\sin 27^\circ$ (correct to 3 decimal places).

1

(b) Solve the following quadratic equations
(by factorisation):

4

(i) $x^2 - 6x = 0$

(ii) $3x^2 - 17x - 6 = 0$

~ 181
 $3x^2 - 18x + 1x - 6 = 0 \quad (3x+1)(x-6)$
 $3x(x-6) + 1(x-6) = 0$

(c) Write down the value of "a" in order to complete the square.

1

$$x^2 - 16x + a$$

64

(d) Find x to the nearest minute given that $\cos x = 0.629$, and that x is acute.

1

(e) Find the simple interest on \$4800 over 3 years at $7\frac{1}{2}\%$ p.a. interest.

1

(f) Find the length of the interval joining $(4, -3)$ and $(8, -5)$. Leave your answer in simplest surd form.

2

(g) Sketch on separate number planes (indicate at least two points on each).

4

(i) $y = 2^x$

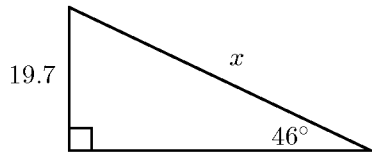
(ii) $xy = -1$

(iii) $x^2 + y^2 = 16$

(iv) $y = (x - 4)^2$

Question 3 (15 marks)

(a)



Find x correct to one decimal place.
Diagram not to scale.

2

(b) Solve using the quadratic formula (leave answer in simplest surd form).

4

(i) $x^2 = 5 - x$

$$x^2 + x - 5$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \times 1 \times -5}}{2}$$

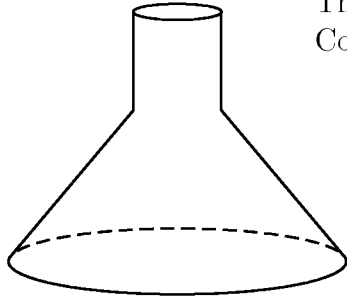
$$x = \frac{-1 \pm \sqrt{21}}{2}$$

(ii) $3a^2 + 4a - 2 = 0$

(c) Find the amount to which \$100 000 grows if it is invested at 9% p.a. for 9 years, compounded monthly.

2

(d)

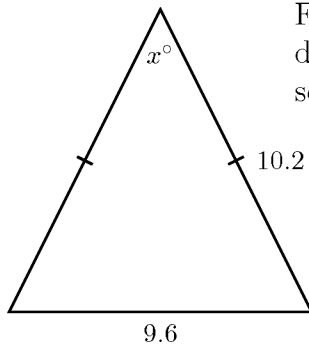


This container is filled with water at a steady rate.
Complete the graph to indicate the rise in water level.



1

(e)



Find x to the nearest degree. Diagram not to scale.

2

(f) Three coins are tossed. What is the probability of getting
(i) only one head?

2

(ii) at least 2 heads?

(g) Find the points of intersection of the graph of $y = x^2$ and the graph of $y = 2x + 3$. Use an algebraic method.

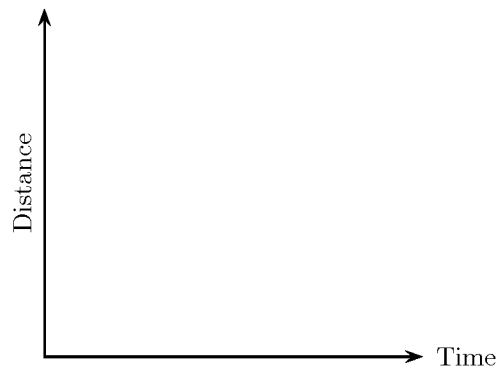
2

Question 4 (15 marks)

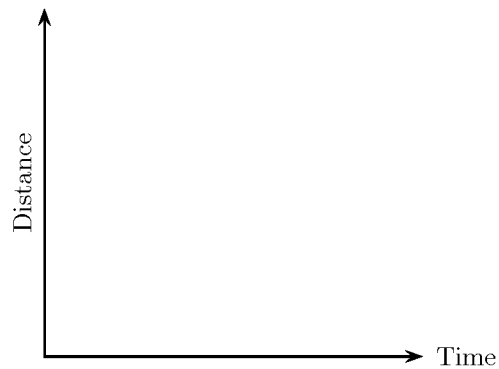
(a) Draw distance-time graphs to represent the following situations.

2

(i) Sharon accelerates from the traffic lights until she reaches 60 km/h , then maintains this speed.



(ii) Fred is driving at a steady speed and applies the brakes to slowly come to a stop.

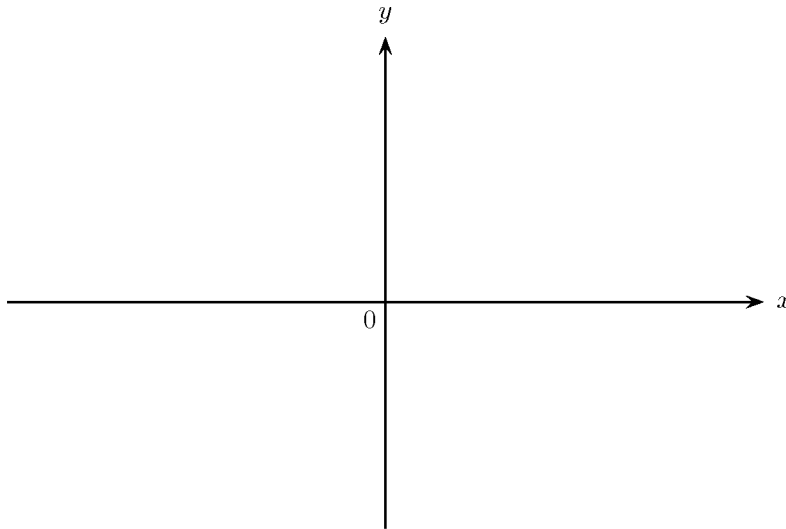


(b) A builder receives a trade discount of 15% off all supplies from his local hardware store. If he pays cash he gets a further discount of 4% of the purchase price. If the builder buys \$2870 worth of goods at the hardware store, how much is actually paid for the supplies?

2

- (c) Sketch $y = 2x^3 + 1$ (indicate at least 2 points).

2



- (d) Solve $2x^2 = 10x + 1$ by the method of “completion of squares.”

3

- (e) Three cards labelled 1, 2, 3, & 4 are placed in a hat. A card is withdrawn without replacement, and a second card is withdrawn.

4

What is the probability that

- (i) The second card is a smaller number than the first?

- (ii) the sum of the two numbers exceeds 5?

- (f) Johan bought a car four years ago and it is now worth \$14 162. If the rate of depreciation is 19% per year, find the original value of the car.

2

Question 5 (14 marks)

(a) Simplify $\frac{x-1}{4} - \frac{x-2}{3}$.

2

(b) Of 32 students in a class, 8 play Rugby, 23 play Cricket, and 5 play both sports. One student is selected at random. What is the probability that

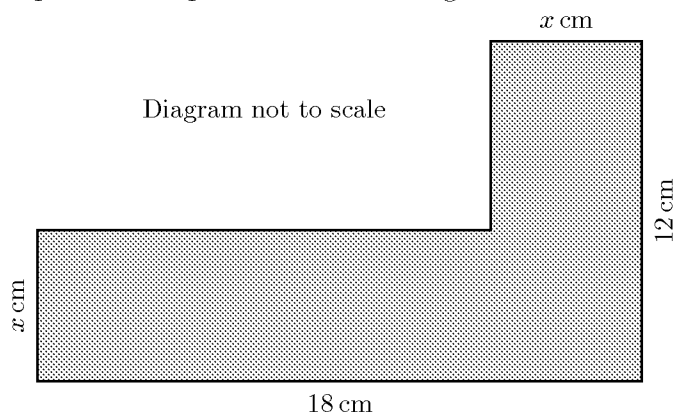
3

(i) he plays only Rugby?

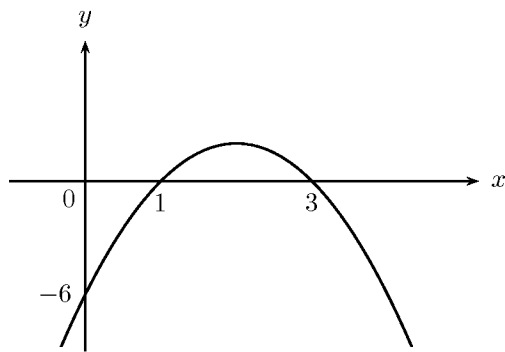
(ii) he plays neither sport?

(c) The shaded area is 200 cm^2 . Find x by forming a quadratic equation then solving.

3



(d)



Find the equation of the parabola.

2

(e) If $A(-1, 3)$, $B(2, 4)$ and $C(t, -1)$ are collinear, find t .

2

(f) A student wishes to use a measure of central tendency to analyse how she has performed in ten English tests. Her marks were

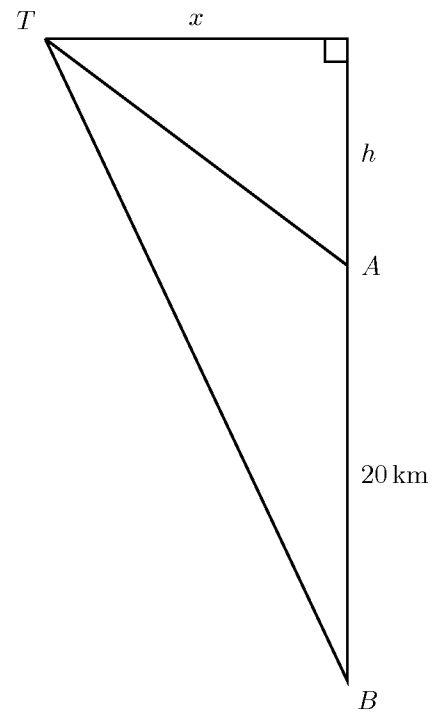
2

58, 57, 56, 55, 84, 98, 57, 54, 58, 57

Which should she use: mean, mode, or median?
Give reasons for your answer.

Question 6 (21 marks)

- (a) From B the bearing of a tower T is 330° . From A , which is 20 km north of B , the bearing of the tower is 290° .
- (i) Find x in kilometres correct to 2 decimal places.



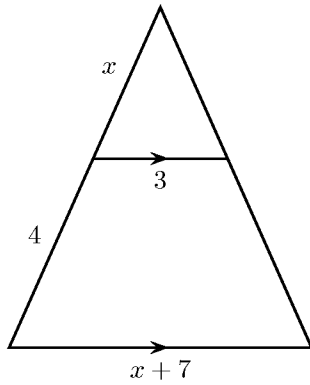
4

- (ii) Find the distance of the tower from A in kilometres correct to 2 decimal places.

- (b) What single percentage discount would be equivalent to successive discounts of 25% and 5%?

2

(c)



Using similar triangles or otherwise, form an equation and then solve to find x .

3

(d) Without using a calculator, find the exact value of

3

$$\sqrt{3 - 2\sqrt{2}} + \sqrt{3 + 2\sqrt{2}}.$$

(Working must be shown to obtain full marks.)

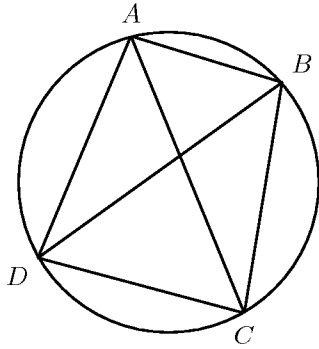
(e) (i) Show that $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$.

4

(ii) Given that $x + y = 1$ and that $x^3 + y^3 = 19$, using part (i) or otherwise, find the value of xy .

(iii) Hence find the value of $x^2 + y^2$.

(f) (i)



The diagram illustrates that if 4 points are marked on a circle, 6 chords may be drawn joining the points.

You are given the identity

$$1 + 2 + 3 + 4 + \dots + n = \frac{n(n+1)}{2}$$

How many chords may be drawn using

α) 5 points

(you must explain how you arrived at your answer)

β) 10 points

(you must explain how you arrived at your answer)

γ) n points

(you must explain how you arrived at your answer)

(ii) In the preliminary round of a state-wide basketball competition each team played each other team once. If 91 games were played, how many teams were entered? (All working must be shown.)

3

2

End of Paper

Extra working page

Extra working page

YR10 2008 Half Yearly

1) a) $\frac{6-5}{7-6} = \frac{1}{1}$ ①

b) $\frac{x(x-4)}{2} = \frac{x-4}{2}$ or $\frac{x-2}{2} \cdot \frac{2}{1}$ ①

c) (i) $2a(2+3ab)$ ①

(ii) $x^4 - 16 = (x^2 - 4)(x^2 + 4)$
 $= (x-2)(x+2)(x^2 + 4)$ ②

d) (i) $(3a-1)(3a-1) = 9a^2 - 6a + 1$ ①

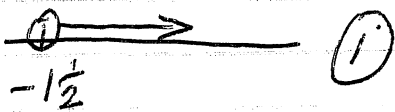
(ii) $(\sqrt{5}-3)(2-3\sqrt{5}) = 2\sqrt{5} - 15 - 6 + 9\sqrt{5} = 11\sqrt{5} - 21$ ①

e) $5 - \frac{2a}{3} < 6$

$-\frac{2a}{3} < 1$

$-2a < 3$

$a > -\frac{3}{2}$ ①



f) (i) $\frac{5}{3\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} = \frac{5\sqrt{10}}{30} = \frac{\sqrt{10}}{6}$ ①

(ii) $\frac{(\sqrt{5}-1)}{(\sqrt{5}+1)} \times \frac{(\sqrt{5}-1)}{(\sqrt{5}-1)}$

$= \frac{(\sqrt{5}-1)^2}{4}$ ②

$\frac{5-2\sqrt{5}+1}{4} = \frac{6-2\sqrt{5}}{4} = \frac{3-\sqrt{5}}{2}$

g) (i) $(4, -1)$ and $(2, -5)$

$m = \frac{-5 - (-1)}{2 - 4} = \frac{-4}{-2} = 2$ ①

(ii) $4x + 3y - 12 = 0$
 $3y = -4x + 12$
 $y = -\frac{4}{3}x + 4$

$m = -\frac{4}{3}$ ①

1) $4x - y = 30$ — ①
 $3x + 2y = 17$ — ②

① $\times 2$
 $8x - 2y = 60$ +
 $3x + 2y = 17$

$11x = 77$

$x = 7$

so $28 - y = 30$ ②

$y = -2$ (7, -2)

Question 2 (14 marks)

Answers

Marks

- (a) Evaluate $\sin 27^\circ$ (correct to 3 decimal places).

1

$$0.454.$$

- (b) Solve the following quadratic equations (by factorisation):

4

(i) $x^2 - 6x = 0$

$$x = 0, 6.$$

(ii) $3x^2 - 17x - 6 = 0$

$$\frac{(3x-18)(3x+1)}{3} = 0.$$

$$(x-6)(3x+1) = 0$$

$$x = 6, -\frac{1}{3}.$$

- (c) Write down the value of "a" in order to complete the square.

1

$$x^2 - 16x + a$$

$$a = 64.$$

- (d) Find x to the nearest minute given that $\cos x = 0.629$, and that x is acute.

1

$$51^\circ 1'$$

- (e) Find the simple interest on \$4800 over 3 years at $7\frac{1}{2}\%$ p.a. interest.

1

$$\$1080$$

- (f) Find the length of the interval joining $(4, -3)$ and $(8, -5)$. Leave your answer in simplest surd form.

2

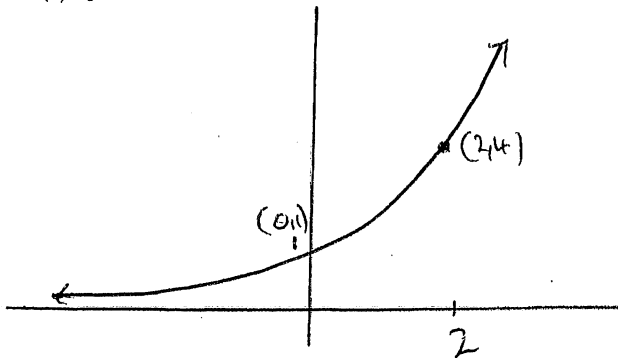
$$d^2 = (8-4)^2 + (-3+5)^2$$

$$= 16 + 4$$

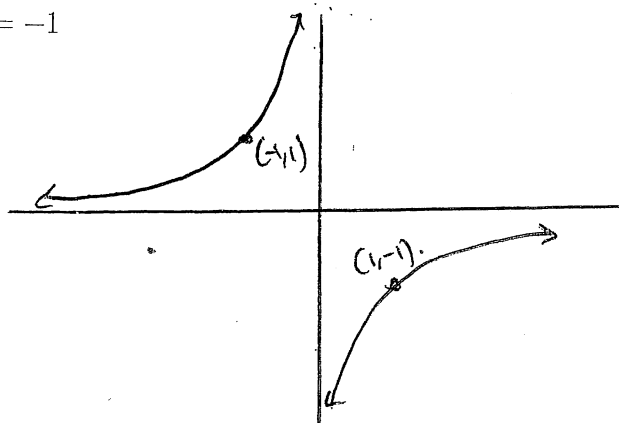
$$d = 2\sqrt{5}$$

(g) Sketch on separate number planes (indicate at least two points on each).

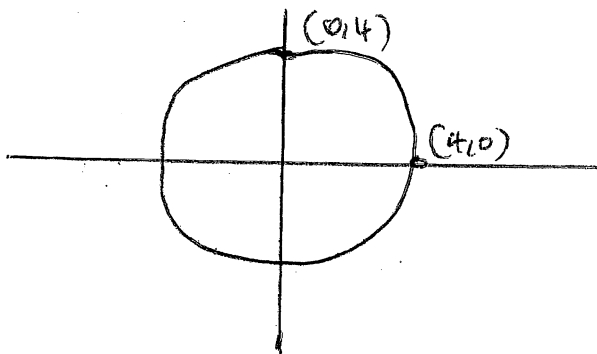
(i) $y = 2^x$



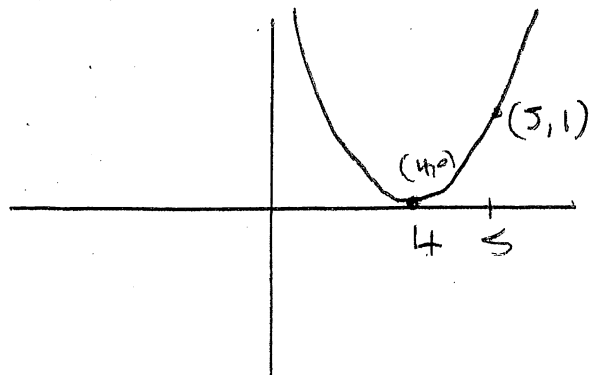
(ii) $xy = -1$



(iii) $x^2 + y^2 = 16$



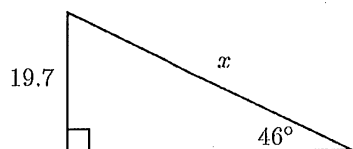
(iv) $y = (x - 4)^2$



Question 3 (15 marks)

Marks

(a)



Find x correct to one decimal place.
Diagram not to scale.

$$\sin 46^\circ = \frac{19.7}{x} \quad [2]$$

$$x = \frac{19.7}{\sin 46^\circ} = 27.4 \text{ (1.d.p.)}$$

(b) Solve using the quadratic formula (leave answer in simplest surd form).

[4]

(i) $x^2 = 5 - x$

$$x^2 + x - 5 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{21}}{2}$$

$$x = \frac{-1 \pm \sqrt{1^2 - (4)(1)(-5)}}{2}$$

(ii) $3a^2 + 4a - 2 = 0$

$$x = \frac{-4 \pm \sqrt{16 - 4(3)(-2)}}{6}$$

$$= \frac{-4 \pm \sqrt{40}}{6}$$

$$\rightarrow x = \frac{-4 \pm 2\sqrt{10}}{6} = \frac{-2 \pm \sqrt{10}}{3}$$

(c) Find the amount to which \$100 000 grows if it is invested at 9% p.a. for 9 years, compounded monthly.

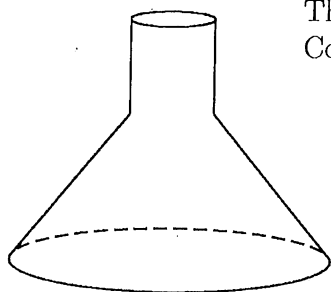
[2]

$$A = P(1+r)^n$$

$$A = 100,000 \left(1 + \frac{0.09}{12}\right)^{9 \times 12}$$

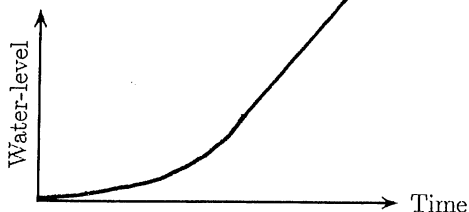
$$= 224,112.42 \text{ (nearest cent)}$$

(d)

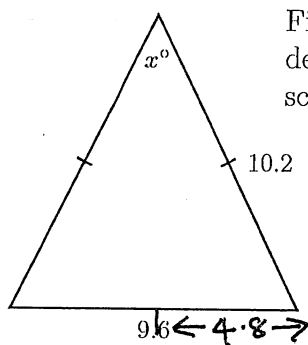


This container is filled with water at a steady rate. Complete the graph to indicate the rise in water level.

1



(e)



Find x to the nearest degree. Diagram not to scale.

2

$$\sin\left(\frac{x}{2}\right) = \frac{4.8}{10.2} = 0.470588$$

$$\frac{x}{2} = 28.07248694$$

$$x = 56^\circ \text{ (nearest degree)}$$

(f) Three coins are tossed. What is the probability of getting

2

(i) only one head?

$$\begin{array}{l} \text{HTT} \\ \text{THT} \\ \text{TTH} \end{array} = \frac{3}{8}$$

(ii) at least 2 heads?

$$\begin{array}{l} \text{HHH} \\ \text{HHT} \\ \text{HTH} \\ \text{T HH} \end{array} = \frac{4}{8}$$

(g) Find the points of intersection of the graph of $y = x^2$ and the graph of $y = 2x + 3$. Use an algebraic method.

2

$$x^2 = 2x + 3$$

$$\text{when } x=3 \text{ } y=9$$

$$x^2 - 2x - 3 = 0$$

$$(3, 9)$$

$$(x-3)(x+1) = 0$$

$$\text{when } x=-1 \text{ } y=1$$

$$x = 3 \text{ or } -1$$

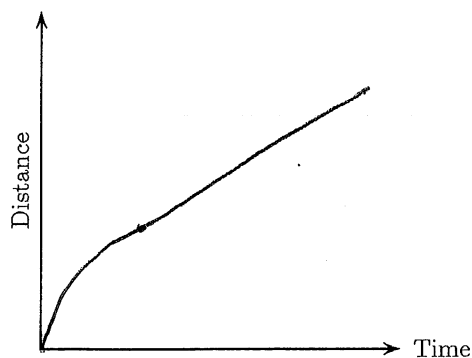
$$(-1, 1)$$

Question 4 (15 marks)

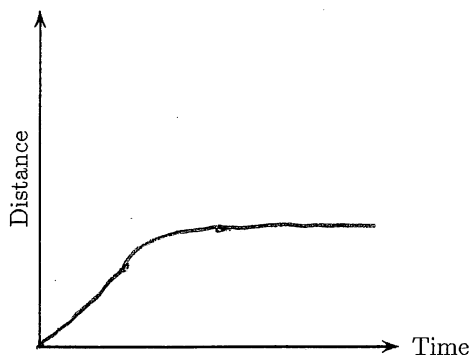
(a) Draw distance-time graphs to represent the following situations.

2

(i) Sharon accelerates from the traffic lights until she reaches 60 km/h , then maintains this speed.



(ii) Fred is driving at a steady speed and applies the brakes to slowly come to a stop.



(b) A builder receives a trade discount of 15% off all supplies from his local hardware store. If he pays cash he gets a further discount of 4% of the purchase price. If the builder buys \$2870 worth of goods at the hardware store, how much is actually paid for the supplies?

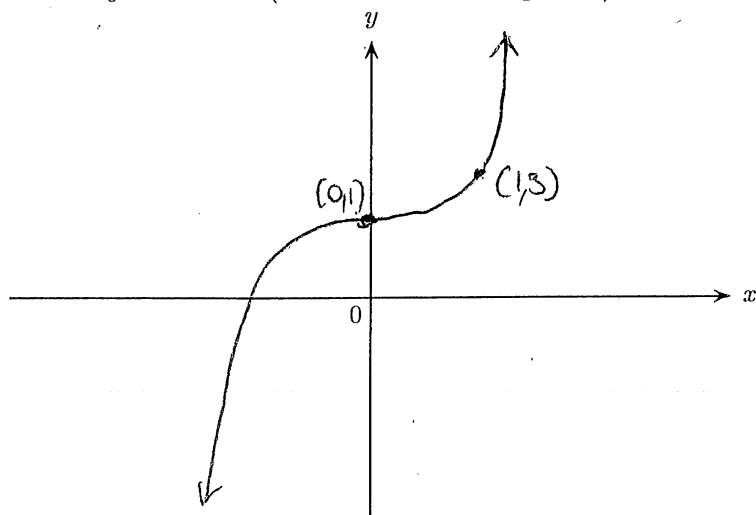
2

\$2439.50 without cash

\$2341.92 with cash

(c) Sketch $y = 2x^3 + 1$ (indicate at least 2 points).

2



(d) Solve $2x^2 = 10x + 1$ by the method of "completion of squares."

3

$$2x^2 - 10x - 1 = 0$$

$$x^2 - 5x - \frac{1}{2} = 0$$

$$\left(x - \frac{5}{2}\right)^2 - \frac{25}{4} - \frac{1}{2} = 0$$

$$x - \frac{5}{2} = \pm \sqrt{\frac{27}{4}}$$

$$x = \frac{5}{2} \pm \frac{3\sqrt{3}}{2}$$

$$x = \frac{5 \pm 3\sqrt{3}}{2}$$

(e) Three cards labelled 1, 2, 3, & 4 are placed in a hat. A card is withdrawn without replacement, and a second card is withdrawn.

4

What is the probability that

(i) The second card is a smaller number than the first?

$$\frac{1}{2}$$

(ii) the sum of the two numbers exceeds 5?

$$\frac{2}{12} = \frac{1}{6}$$

(f) Johan bought a car four years ago and it is now worth \$14 162. If the rate of depreciation is 19% per year, find the original value of the car.

2

$$P = \frac{14162}{\left(1 - \frac{19}{100}\right)^4} = \$32899.14$$

Question 5 (14 marks)

Marks

(a) Simplify $\frac{x-1}{4} - \frac{x-2}{3}$.

2

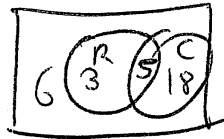
$$\frac{3x-3-4x+8}{12} = \frac{5-x}{12}$$

(b) Of 32 students in a class, 8 play Rugby, 23 play Cricket, and 5 play both sports. One student is selected at random. What is the probability that

3

(i) he plays only Rugby?

$$\frac{3}{32}$$

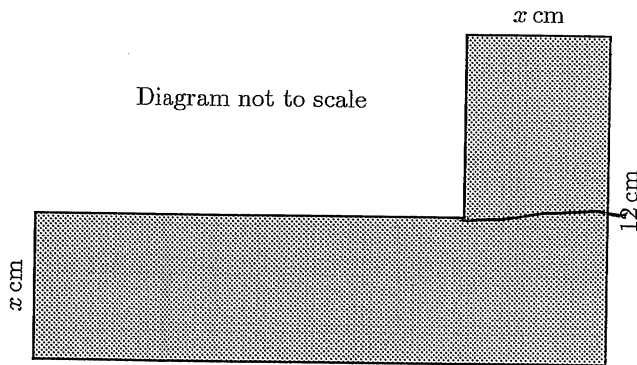


(ii) he plays neither sport?

$$\frac{6}{32} = \frac{3}{16}$$

(c) The shaded area is 200 cm^2 . Find x by forming a quadratic equation then solving.

3



$$18x + x(12-x) = 200$$

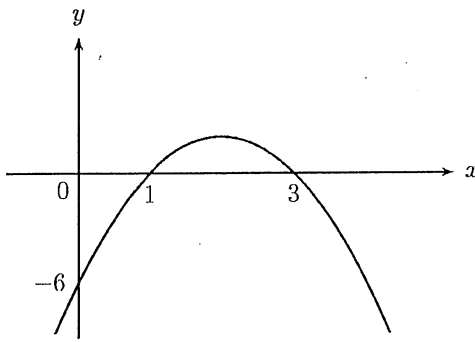
$$x^2 - 30x + 200 = 0$$

$$x = 10, 20$$

$$x \neq 20$$

$$\therefore x = 10$$

(d)



Find the equation of the parabola.

2

$$y = -2(x-1)(x-3)$$

$$y = -2x^2 + 8x - 6.$$

(e) If $A(-1, 3)$, $B(2, 4)$ and $C(t, -1)$ are collinear, find t .

2

$$m_{AB} = \frac{4-3}{2-(-1)} = \frac{1}{3}.$$

$$m_{BC} = \frac{4+1}{2-t} = \frac{1}{3}.$$

$$15 = 2-t$$

$$t = -13$$

(f) A student wishes to use a measure of central tendency to analyse how she has performed in ten English tests. Her marks were

2

58, 57, 56, 55, 84, 98, 57, 54, 58, 57

Which should she use: mean, mode, or median?
Give reasons for your answer.

Mean. The mean will take the two outstanding scores into account.

Question 6 (21 marks)

- (a) From B the bearing of a tower T is 330° . From A , which is 20 km north of B , the bearing of the tower is 290° .

- (i) Find x in kilometres correct to 2 decimal places.

$$\tan 70 = \frac{x}{h}$$

$$h = \frac{x}{\tan 70} \quad (1)$$

$$\tan 30 = \frac{x}{20+h}$$

$$(20+h) \tan 30 = x \quad (2)$$

sub (1) into (2)

$$\left(20 + \frac{x}{\tan 70}\right) \tan 30 = x$$

$$20 \tan 70 \tan 30 + x \tan 30 = x \tan 70$$

- (ii) Find the distance of the tower from A in kilometres correct to 2 decimal places.

$$x(\tan 70 - \tan 30) = 20 \tan 70 \tan 30$$

$$x = \frac{20 \tan 70 \tan 30}{\tan 70 - \tan 30}$$

$$x \approx 14.62 \text{ km}$$

$$\sin 70 = \frac{x}{a}$$

$$a = \frac{x}{\sin 70}$$

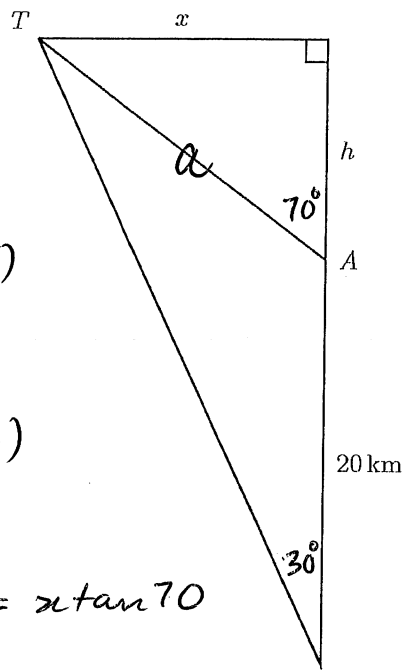
$$a = \frac{14.62}{\sin 70}$$

$$a \approx 15.56 \text{ km}$$

- (b) What single percentage discount would be equivalent to successive discounts of 25% and 5%?

$$0.75 \times 0.95 = 0.7125$$

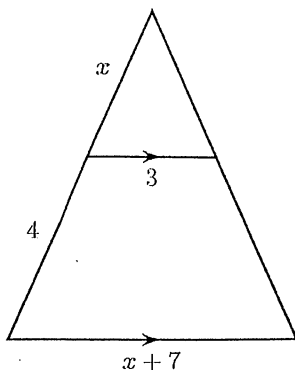
which is a single discount of 28.75% .



4

2

(c)



Using similar triangles or otherwise, form an equation and then solve to find x .

3

$$\frac{x}{x+4} = \frac{3}{x+7}$$

$$x(x+7) = 3(x+4)$$

$$x^2 + 7x = 3x + 12$$

$$x^2 + 4x - 12 = 0$$

$$(x+6)(x-2) = 0$$

$$x = 2, -6$$

$x = 2$, since x is a distance

(d) Without using a calculator, find the exact value of

3

$$\sqrt{3 - 2\sqrt{2}} + \sqrt{3 + 2\sqrt{2}}$$

(Working must be shown to obtain full marks.)

$$\text{let } x = \sqrt{3 - 2\sqrt{2}} + \sqrt{3 + 2\sqrt{2}}$$

$$x^2 = 3 - 2\sqrt{2} + 2\sqrt{(3 - 2\sqrt{2})(3 + 2\sqrt{2})} + 3 + 2\sqrt{2}$$

$$x^2 = 6 + 2\sqrt{9 - 8}$$

$$x^2 = 8$$

$$\therefore x = 2\sqrt{2}$$

(e) (i) Show that $(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$.

4

$$\begin{aligned} \text{LHS} &= (x+y)(x+y)^2 \\ &= (x+y)(x^2 + 2xy + y^2) \\ &= x^3 + 2x^2y + 2xy^2 + x^2y + 2xy^2 + y^3 \\ &= x^3 + 3x^2y + 3xy^2 + y^3 \\ &= \text{RHS} \end{aligned}$$

(ii) Given that $x+y=1$ and that $x^3+y^3=19$, using part (i) or otherwise, find the value of xy .

$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

$$(x+y)^3 = x^3 + y^3 + 3xy(x+y)$$

$$(1)^3 = 19 + 3xy$$

$$3xy = -18$$

$$xy = -6$$

(iii) Hence find the value of $x^2 + y^2$.

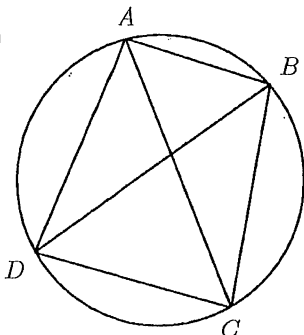
$$(x+y)^2 = x^2 + 2xy + y^2$$

$$(x+y)^2 = x^2 + y^2 + 2xy$$

$$(1)^2 = x^2 + y^2 + 2(-6)$$

$$x^2 + y^2 = 13$$

(f) (i)



The diagram illustrates that if 4 points are marked on a circle, 6 chords may be drawn joining the points.

3

You are given the identity

$$1 + 2 + 3 + 4 + \dots + n = \frac{n(n+1)}{2}$$

How many chords may be drawn using

α) 5 points

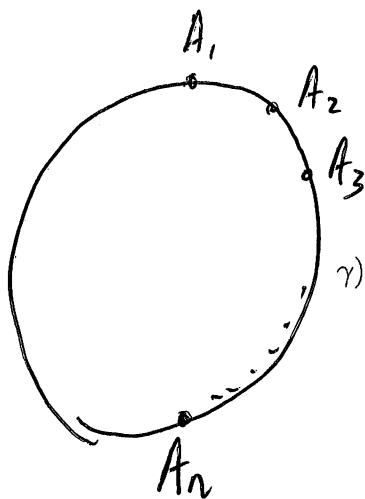
(you must explain how you arrived at your answer)

10

β) 10 points

(you must explain how you arrived at your answer)

45



γ) n points

(you must explain how you arrived at your answer)

From A_1 , you can draw $(n-1)$ points A_2, A_3, \dots, A_n
 A_2 you can draw $(n-2)$ points A_3, \dots, A_n since A_1, A_2 has already been counted
 \vdots
 A_n you can't draw any chords that haven't already been drawn.
 $= (n-1) + (n-2) + \dots + 3 + 2 + 1 + 0 = \frac{(n-1)n}{2}$

(ii) In the preliminary round of a state-wide basketball competition each team played each other team once. If 91 games were played, how many teams were entered? (All working must be shown.)

2

If there are x teams

$$(x-1) + (x-2) + \dots + 3 + 2 + 1 + 0 = 91 \quad (\text{just like (i)})$$

$$\frac{(x-1)x}{2} = 91$$

$$x^2 - x = 182$$

End of Paper

$$x^2 - x - 182 = 0$$

$$(x-14)(x+13) = 0$$

$$x = -13, 14$$

there must be 14 teams.