SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

Year 10

Half-Yearly Examination 2010

Mathematics

NAME:

Examiner: *E. Choy*

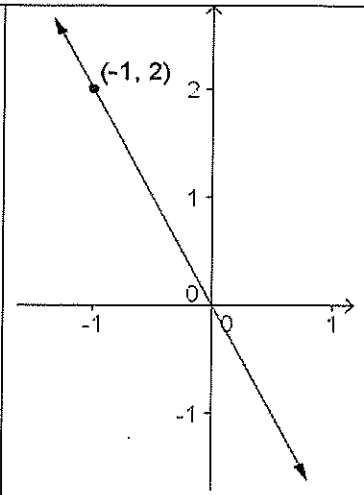
General Instructions

- Working time – 90 minutes
- Write using black or blue pen.
- *Approved* calculators may be used.
- All necessary working should be shown in every question if full marks are to be awarded.
- Marks may not be awarded for messy or badly arranged work.
- If more space is required, clearly write the number and the SECTION on the back page and answer it there. Indicate that you have done so.
- Clearly indicate your class by placing an **X**, next to your class

Class	Teacher	
10 A	Mr Fuller	
10 B	Ms Nesbitt	
10 C	Ms Ward	
10 D	Ms Roessler	
10 E	Mr McQuillan	
10 F	Mr Boros	
10 G	Mr Hespe	

Section	Mark
1	/15
2	/10
3	/13
4	/10
5	/11
6	/10
7	/11
Total	/80

Question One			
(a)	Expand and simplify $(x-5)(x+3)$.		**1**
(b)	Find the midpoint of the interval joining P(10, 7) and Q(4, 1).		**1**
(c)	Find the simple interest earned if \$4000 is invested for 5 years at a rate of 6% p.a..		**1**
(d)	What is the probability of choosing a king from a standard deck of 52 playing cards.		**1**
(e)	Solve the quadratic equation $x^2 = 81$.		**1**
(f)	Write down the gradient of the line $5y = x+10$.		**1**
(g)	Write 3% p.a. as a monthly rate.		**1**
(h)	Write down the solutions of the quadratic equation $3x(x-3)=0$.		**1**
(i)	Write down the equation of the vertical line passing through the point (5, –5).		**1**

(j)	 What is the gradient of the given line?		1
(k)	What number must be added to $x^2 + 7x$ in order to create a perfect square?		1
(l)	To what amount will \$4000 grow over 6 years if it is invested at 8% p.a. compound interest? (Give your answer to the nearest cent.)		1
(m)	Find in general form, the equation of the line with gradient –3 passing through the point (5, –6).		1
(n)	Bob was asked to solve the quadratic equation $x^2 = x$. He divided both sides by x to get the solution $x = 1$. Is Bob's solution completely correct? Explain your answer.		1
(o)	Construct a quadratic equation whose solutions are $x = -2$ and $x = 5$. Give your answer in the form $x^2 + bx + c = 0$.		1

Question Two

(a)	Charlie took out a loan of \$5500 over 4 years at a simple interest rate of 8% per annum. (i) Calculate the amount of interest charged. (ii) Find the total amount to be repaid. (iii) Hence find the size of each equal monthly repayment.		**6**
(b)	A car is depreciating at a rate 10% per annum. Find its value two years ago if it is now worth \$38028 (give your answer correct to the nearest dollar).		**2**
(c)	What is the gradient of line $x + y + 2 = 0$?		**1**
(d)	Find the length of the interval joining (4, –1) and (–4, 5).		**1**

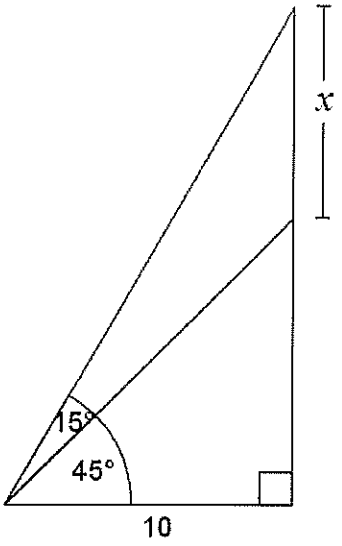
Question Three

(a)	(i) Find the x-intercepts of the line $x+2y=4$.		**3**
	(ii) Sketch the line $x+2y=4$.		
(b)	Solve each of the following quadratic equations by the method specified: (i) $x^2-7x-18=0$ by factorising,		**5**
	(ii) $x^2-x-1=0$ by the quadratic formula,		
	(iii) $x^2-6x+2=0$ by completing the square.		
(c)	On the same number plane, graph the equations indicating their point of intersection: $x=4$, $x+y=4$		**2**

(d)	(i) Show that the equation $\frac{20}{x} = 2x + 3$ can be written in the form $2x^2 + 3x - 20 = 0$.		3
	(ii) Hence solve the equation $\frac{20}{x} = 2x + 3$.		

Question Four			
(a)	Four cards marked 2, 4, 6 and 8 are placed in a hat. One card is drawn out and put on the table. Another card is then drawn out of the hat and put on the table to the right of the first card to form a two-digit number. (i) Draw a tree diagram to show all the possible outcomes. (ii) What is the probability of drawing a two-digit number divisible by 3?		**6**
(b)	Fifty two tagged fish were released into a dam known to contain fish. Later a sample of thirty fish was melted from this dam, of which eight were found to be tagged. Estimate the total number of fish in the dam just before the sample of thirty were removed.		**4**

Question Five

(a)	Two regular 6-sided dice are thrown and the numbers 1 to 6 on each die are equally likely to appear. Using a dot diagram, or otherwise, find the probability that a sum of 8 or greater is thrown.		**2**
(b)	 Show that $x = 10\left(\sqrt{3}-1\right)$.		**3**
(c)	Write (i) $\sqrt{x\sqrt{x\sqrt{x}}}$ in index form. Simplify (ii) $\dfrac{2^{n+3}-2^{n+1}}{2^{n+2}}$		**3**
(d)	Solve $4^{3x} = 8^{x-2}$		**2**

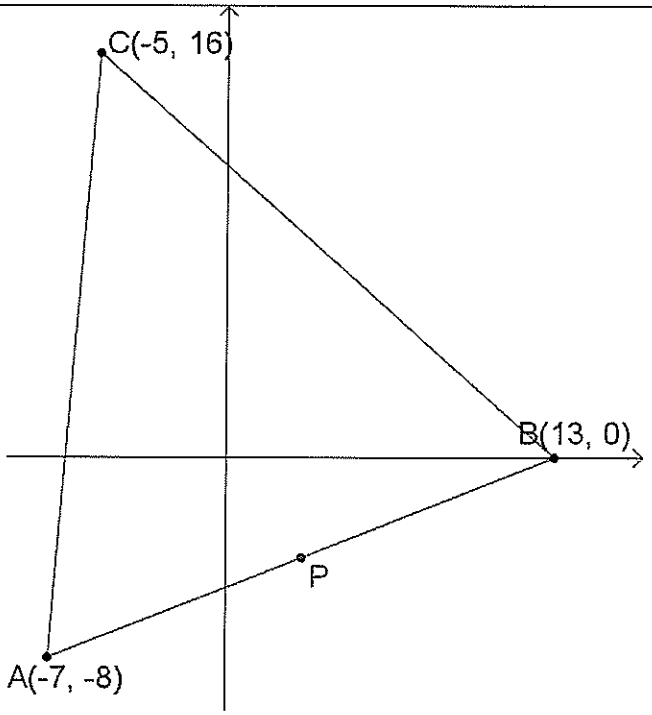
Question Six

A "A line with gradient -1 is not as steep as a line with gradient 1 because $-1 < 1$."

Is this statement true of false? Justify your answer using a diagram.

2

B



In the diagram, A(–7, –8), B(13, 0) and C(–5, 16) are the vertices of ΔABC and P is the mid-point of AB.

(i) Show that P has coordinates (3, –4).

(ii) Show that AB has gradient $\frac{2}{5}$.

(iii) Hence find, in general form, the equation of the perpendicular bisector of AB. (Hint: The perpendicular bisector of an interval is the line which passes through its mid-point at $90°$.)

4

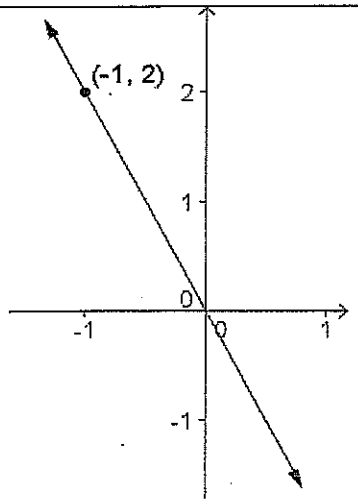
	(iv) Show C lies on the perpendicular bisector found in part (iii). (v) Show that ΔABC is isosceles.		**4**

Question Seven

(a)	Consider the quadratic equation $x^2 - 2ax + 4 = 0$ where a is a constant. (i) Show that $x = a + \sqrt{a^2 - 4}$ or $x = a - \sqrt{a^2 - 4}$. (ii) For what values of a does the equation has no solution? (iii) If the equation has 2 solutions, prove their product does not depend on the value of a.		**5**
(b)	If $\frac{x}{y} = \frac{a}{b}$, prove that $\frac{a-b}{a+b} = \frac{x-y}{x+y}$.		**3**

(c)	The points A, B and C are collinear where $A(2a, a^2)$, $B\left(-\frac{2}{a}, \frac{1}{a^2}\right)$ and $C(x,-1)$. Find x in terms of a. (give x in simplest form).		**3**

Question One			
(a)	Expand and simplify $(x-5)(x+3)$.	$x^2+3x-5x-15$ $=x^2-2x-15$	1
(b)	Find the midpoint of the interval joining P(10, 7) and Q(4, 1).	$\left(\frac{10+4}{2}, \frac{7+1}{2}\right)=(7, 4)$	1
(c)	Find the simple interest earned if \$4000 is invested for 5 years at a rate of 6% p.a..	$\$4000 \times \frac{6}{100} \times 5 = \$1200$	1
(d)	What is the probability of choosing a king from a standard deck of 52 playing cards.	$\frac{4}{52} = \frac{1}{13}$	1
(e)	Solve the quadratic equation $x^2=81$.	$x = \pm 9$	1
(f)	Write down the gradient of the line $5y=x+10$. $y = \frac{x}{5} + 2$	$\frac{1}{5}$	1
(g)	Write 3% p.a. as a monthly rate.	$\frac{3}{12}\% = \frac{1}{4}\%$ (or 0.25%)	1
(h)	Write down the solutions of the quadratic equation $3x(x-3)=0$.	$x=0, 3$	1
(i)	Write down the equation of the vertical line passing through the point (5, –5).	$x=5$	1

(j)	 What is the gradient of the given line?	$\frac{2-0}{-1-0}$ -2	1
(k)	What number must be added to x^2+7x in order to create a perfect square?	$\left(\frac{7}{2}\right)^2 = \frac{49}{4}$ $\left(= 12\frac{1}{4}\right)$ or	1
(l)	To what amount will \$4000 grow over 6 years if it is invested at 8% p.a. compound interest? (Give your answer to the nearest cent.)	$\$4000\left(1+\frac{8}{100}\right)^6 = \$6347.50$	1
(m)	Find in general form, the equation of the line with gradient –3 passing through the point (5, –6).	$y+6=-3(x-5)$ $y+6=-3x+15$ $3x+y-9=0$	1
(n)	Bob was asked to solve the quadratic equation $x^2=x$. He divided both sides by x to get the solution $x=1$. Is Bob's solution completely correct? Explain your answer. Alternative explanation: Never divide by a pronumeral if it might be zero and, in this case, zero works.	No, $x=0$ is also a solution. $x^2-x=0$ $x(x-1)=0$ $\therefore x=0, 1$	1
(o)	Construct a quadratic equation whose solutions are $x=-2$ and $x=5$. Give your answer in the form $x^2+bx+c=0$.	$(x+2)(x-5)=0$ $x^2-5x+2x-10=0$ $x^2-3x-10=0$	1

/10

Question Two

(a) Charlie took out a loan of \$5500 over 4 years at a simple interest rate of 8% per annum. **6**

(i) Calculate the amount of interest charged.

$S.I = 5500 \times \frac{8}{100} \times 4$

$= \$1760$ (2)

(ii) Find the total amount to be repaid.

$\$5500 + \$1760$

$= \$7260$ (2)

(iii) Hence find the size of each equal monthly repayment.

$\frac{\$7260}{48} = \$151.25$ (2)

(b) A car is depreciating at a rate 10% per annum. Find its value two years ago if it is now worth \$38028 (give your answer correct to the nearest dollar). **2**

$A = P\left(1 - \frac{r}{100}\right)^n$

$38028 = P\left(1 - \frac{10}{100}\right)^2$

$P = \frac{38028}{.9^2} = \$46948$ (2)

(c) What is the gradient of line $x + y + 2 = 0$? **1**

$x + y + 2 = 0$

$y = -1x - 2$

gradient is -1 (1)

$x_1 \; y_1$

(d) Find the length of the interval joining (4, –1) and (–4, 5). **1**

$x_2 \; y_2$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(-4-4)^2 + (5-1)^2}$

$= \sqrt{64 + 36}$

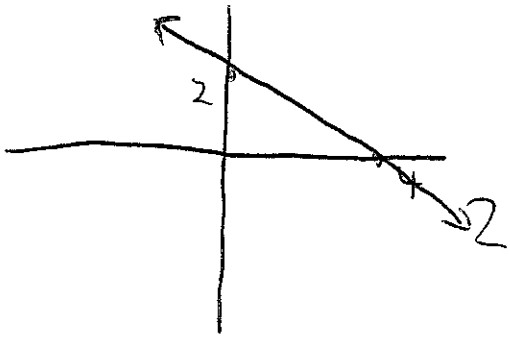
$= \sqrt{100} = 10$ (1)

Question Three

(a) (i) Find the x-intercepts of the line $x+2y=4$. **3**

$x=4$. 1

(ii) Sketch the line $x+2y=4$.



(b) Solve each of the following quadratic equations by the method specified: **5**

(i) $x^2-7x-18=0$ by factorising,

(i) $(x-9)(x+2)=0$

$x=9, -2$. 1

(ii) $x^2-x-1=0$ by the quadratic formula,

(ii) $x=\dfrac{1\pm\sqrt{1+4}}{2}$.

$=\dfrac{1\pm\sqrt{5}}{2}\approx 1.618$ / -0.618 2

(iii) $x^2-6x+2=0$ by completing the square.

(iii) $(x-3)^2=7$.

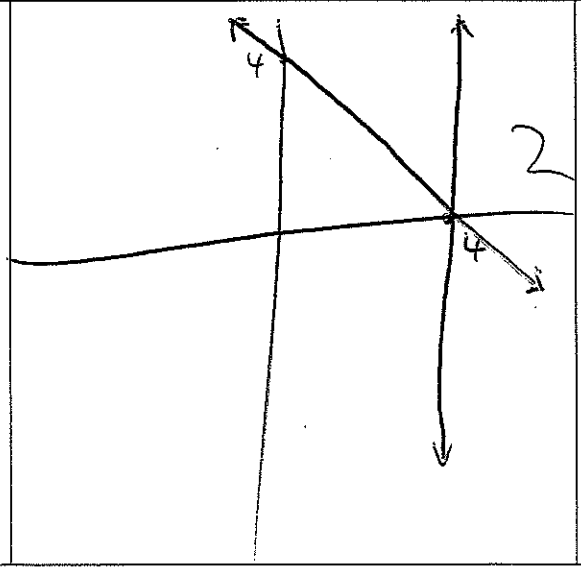
$x=3\pm\sqrt{7}$. 2

5.65, 0.35

(c) On the same number plane, graph the equations indicating their point of intersection: **2**

$x=4$,

$x+y=4$



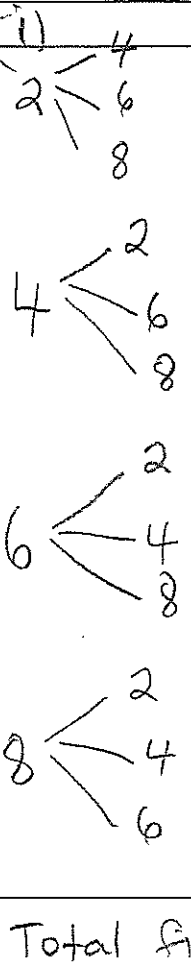
(d)	(i) Show that the equation $\frac{20}{x} = 2x+3$ can be written in the form $2x^2+3x-20=0$. (ii) Hence solve the equation $\frac{20}{x} = 2x+3$.	(i) $20 = 2x^2+3x.$ $2x^2+3x-20=0.$ (ii) $\frac{(2x+8)(2x-5)}{2}=0$ $(x+4)(2x-5)=0$ $x=-4, \frac{5}{2}.$	3

Question Four

(a) Four cards marked 2, 4, 6 and 8 are placed in a hat. One card is drawn out and put on the table. Another card is then drawn out of the hat and put on the table to the right of the first card to form a two-digit number. **6**

(i) Draw a tree diagram to show all the possible outcomes.

(i)



(ii) What is the probability of drawing a two-digit number divisible by 3?

$$P = \frac{4}{12} = \frac{1}{3}$$

(b) Fifty two tagged fish were released into a dam known to contain fish. Later a sample of thirty fish was melted from this dam, of which eight were found to be tagged. Estimate the total number of fish in the dam just before the sample of thirty were removed. **4**

Total fish = $52 + x$

$$\frac{8}{30} = \frac{52}{52+x}$$

$$8x = 1144$$

$$x = 143$$

Total fish = 195

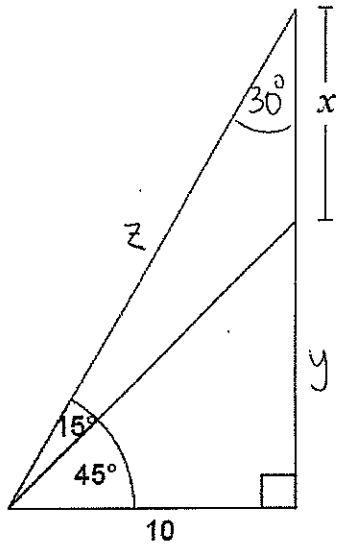
Question Five

(a) Two regular 6-sided dice are thrown and the numbers 1 to 6 on each die are equally likely to appear. Using a dot diagram, or otherwise, find the probability that a sum of 8 or greater is thrown. **2**

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$P(\text{sum} \geq 8) = \frac{15}{36} = \frac{5}{12}$

(b) **3**



Show that $x = 10(\sqrt{3} - 1)$.

$\tan 45° = \frac{y}{10}$

$y = 10 \tan 45°$

$y = 10$

$\sin 30° = \frac{10}{z}$

$z = \frac{10}{\sin 30°}$

$z = 20$

$(x+y)^2 = 20^2 - 10^2$

$(x+y)^2 = 300$

$x + y = \sqrt{300}$

$x = \sqrt{300} - y$

$x = 10\sqrt{3} - 10$

$x = 10(\sqrt{3} - 1)$ //

(c) **3**

Write (i) $\sqrt{x\sqrt{x\sqrt{x}}}$ in index form.

(i) $\sqrt{x\sqrt{x\sqrt{x}}} = (x(x(x^{\frac{1}{2}}))^{\frac{1}{2}})^{\frac{1}{2}}$

$= x^{\frac{1}{2}}(x(x^{\frac{1}{2}}))^{\frac{1}{4}}$

$= x^{\frac{1}{2}} x^{\frac{1}{4}} (x^{\frac{1}{2}})^{\frac{1}{4}}$

$= x^{\frac{1}{2}} x^{\frac{1}{4}} x^{\frac{1}{8}}$

$= x^{\frac{7}{8}}$

Simplify

(ii) $\frac{2^{n+3} - 2^{n+1}}{2^{n+2}}$

(ii) $\frac{2^{n+3}}{2^{n+2}} - \frac{2^{n+1}}{2^{n+2}}$

$= 2 - 2^{-1}$

$= 2 - \frac{1}{2}$

$= 1\frac{1}{2}$

(d) Solve $4^{3x} = 8^{x-2}$ **2**

$4^{3x} = 8^{x-2}$

$2^{6x} = 2^{3x-6}$

$6x = 3x - 6$

$3x = -6$

$x = -2$

Question Six

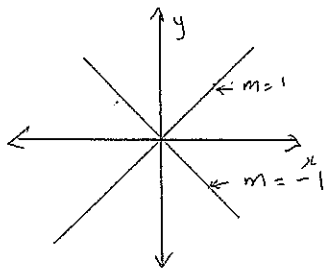
A (2 marks)

"A line with gradient -1 is not as steep as a line with gradient 1 because $-1 < 1$."

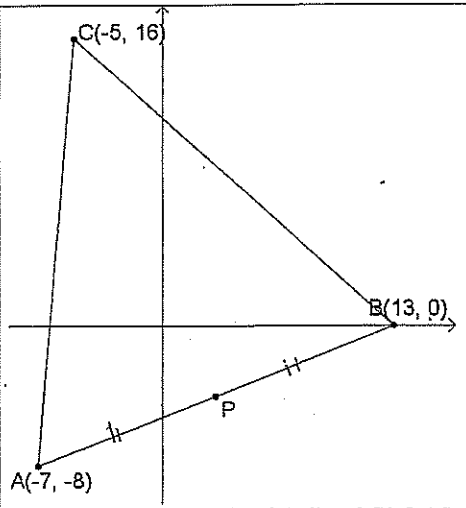
Is this statement true of false? Justify your answer using a diagram.

The sign of the gradient does not affect the steepness

A line with gradient -1 is as steep as a line with gradient 1



B (4 marks)



In the diagram, A(–7, –8), B(13, 0) and C(–5, 16) are the vertices of ΔABC and P is the mid-point of AB.

(i) Show that P has coordinates (3, –4).

(ii) Show that AB has gradient $\frac{2}{5}$.

(iii) Hence find, in general form, the equation of the perpendicular bisector of AB. (Hint: The perpendicular bisector of an interval is the line which passes through its mid-point at 90°.)

(i) $\left(\frac{-7+13}{2}, \frac{-8+0}{2}\right)$

$= (3, -4)$

(ii) $\frac{y-y_1}{x-x_1} = \frac{0--8}{13--7}$

$= \frac{8}{20} = \frac{2}{5}$

(iii) $y - y_1 = m(x - x_1)$

$m = -\frac{5}{2}$

$y + 4 = -\frac{5}{2}(x - 3)$

$2y + 8 = -5x + 15$

$5x + 2y - 7 = 0$

(iv) Show C lies on the perpendicular bisector found in part (iii). (4 marks)

(v) Show that ΔABC is isosceles.

(iv) $5x + 2y - 7 = 0$

$5(-5) + 2(16) - 7 = 0$

$(-5, 16)$ is on bisector

(v) $CA = CB$

$\sqrt{(13 - -5)^2 + (-16)^2} = \sqrt{580}$

$\sqrt{(-7+5)^2 + (-8-16)^2} = \sqrt{580}$

$AC = BC$

ΔABC is isosceles

Question Seven

(a) Consider the quadratic equation $x^2 - 2ax + 4 = 0$ where a is a constant. **5**

(i) Show that $x = a + \sqrt{a^2 - 4}$ or $x = a - \sqrt{a^2 - 4}$.

$$x^2 - 2ax + 4 = 0$$
$$x^2 - 2ax + a^2 = a^2 - 4$$
$$(x - a)^2 = a^2 - 4$$
$$x - a = \pm\sqrt{a^2 - 4}$$
$$x = a \pm \sqrt{a^2 - 4}$$

(ii) For what values of a does the equation has no solution?

$$a^2 - 4 < 0$$
$$a^2 < 4$$
$$-2 < a < 2$$

(iii) If the equation has 2 solutions, prove their product does not depend on the value of a.

$$\left(a + \sqrt{a^2-4}\right)\left(a - \sqrt{a^2-4}\right)$$
$$= a^2 - \left(\sqrt{a^2-4}\right)^2$$
$$= a^2 - (a^2 - 4)$$
$$= 4$$ which doesn't depend on the value of a.

(b) If $\frac{x}{y} = \frac{a}{b}$, prove that $\frac{a-b}{a+b} = \frac{x-y}{x+y}$. **3**

$$LHS = \frac{a-b}{a+b}$$
$$= \frac{b\left(\frac{a}{b} - 1\right)}{b\left(\frac{a}{b} + 1\right)}$$ but $\frac{x}{y} = \frac{a}{b}$
$$= \frac{\frac{x}{y} - 1}{\frac{x}{y} + 1} \times \frac{y}{y}$$
$$= \frac{x - y}{x + y}$$
$$= RHS$$
$$\therefore \frac{a-b}{a+b} = \frac{x-y}{x+y} \text{ if } \frac{x}{y} = \frac{a}{b}$$

(c)	The points A, B and C are collinear where $A(2a, a^2)$, $B\left(-\frac{2}{a}, \frac{1}{a^2}\right)$ and $C(x,-1)$. Find x in terms of a. (give x in simplest form). $m = \frac{y_2 - y_1}{x_2 - x_1}$	$m_{AB} = \frac{a^2 - \frac{1}{a^2}}{2a + \frac{2}{a}} \times \frac{a^2}{a^2}$ $= \frac{a^4 - 1}{2a^3 + 2a}$ $= \frac{(a^2-1)(a^2+1)}{2a(a^2+1)}$ $= \frac{a^2-1}{2a}$ $m_{AC} = \frac{a^2+1}{2a-x}$ since A, B and C are collinear $m_{AB} = m_{AC}$	3

$$\frac{a^2-1}{2a} = \frac{a^2+1}{2a-x}$$

$$2a^3 - a^2x - 2a + x = 2a^3 + 2a$$

$$x(1 - a^2) = 4a$$

$$x = \frac{4a}{1-a^2} \quad \text{OR} \quad \frac{-4a}{a^2-1}$$