SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

Year 10

Half Yearly Examination 2011

Mathematics

General Instructions

- Working time – 90 minutes
- Write using black or blue pen.
- Approved Calculators may be used.
- All necessary working MUST be shown in every question if full marks are to be awarded.
- Marks may not be awarded for untidy or badly arranged work.
- Write all answers in simplest exact form unless specified otherwise.
- If more space is required, clearly write the number of the QUESTION on one of the back pages and answer it there. Indicate that you have done so.
- Clearly indicate your class by placing an **X** in the space provided.

Examiner: *A. Fuller*

NAME:

Class	Teacher	
10MaA	Mr McQuillan	
10MaB	Mr Boros	
10MaC	Ms Ward	
10MaD	Ms Kilmore	
10MaE	Ms Evans	
10MaF	Mr Gainford	
10MaG	Mr Hespe	

Question	Mark
1	/15
2	/15
3	/15
4	/15
5	/15
6	/15
7	/15
Total	/105

Question One (15 marks)

(a)   Express $0 \cdot 5$ as a percentage.

(b)   Simplify the following fully:

(i) $\dfrac{a}{a+a+a}$

(ii) $\dfrac{a \times a \times a}{2 \times a}$

(iii) $\sqrt{5} \times \sqrt{5}$

(iv) -4^2

(v) $3\sqrt{3} \times 2\sqrt{6}$

(c)   State the number of significant figures in the following:

(i) $0 \cdot 0060$

(ii)   30000 (to nearest hundred)

(d)   Write $2 \cdot 15 \times 10^{-4}$ as a basic numeral.

(e)   Simplify the following ratios fully:

(i) $24 : 36 : 16$

(ii) $36 \, m^2 : 9 \, m^2$

(f)   Evaluate $\cos 42°$ correct to two decimal places.

(g)   What is the complementary angle of $75°$?

(h)   Describe the complementary event of drawing a game of soccer.

(i)   Write $\dfrac{1}{2\sqrt{3}}$ with a rational denominator.

Question Two (15 marks)

(a)   Find the acute angle α to the nearest minute if $\tan \alpha = \frac{3}{2}$.

(b)   Expand and simplify the following:

(i) $3(2x + 1) - 5x$

(ii) $(2x^2)^3$

(iii) $(3 + x^2)^2$

(iv) $(2 + \sqrt{3})(5 - 2\sqrt{3})$

(c)   Determine the linear relationship for the table of values.

x	1	2	3	4
y	4	1	-2	-5

(d)   Write an algebraic expression for "*m* less than the square of *p*".

(e)   "Sparkle" toothpaste comes in four different sizes. Which size is the best value?

75 g for $2.58

110 g for $3.98

150 g for $4.80

175 g for $5.70

(f)   Alice is paid $14.50 per hour for the first 36 hours she works in a week. She is paid time and a half for every extra hour worked. How much is Alice paid if she works 41 hours in one week?

(g)   What is the simple interest earned on $800 invested for 18 months at 8% per annum?

(h)   What is the compound interest earned on $800 invested for 18 months at 8% per annum compounded quarterly to the nearest cent?

(i)   Express $1 \cdot 0\dot{8}$ as a fraction.

Question Three (15 marks)

(a)   Factorise the following fully:

     (i) $6mn - 9m^2$

     (ii) $1 - 9m^2$

     (iii) $m^2 + 5m - 24$

     (iv) $2n^2 + n - 1$

     (v) $m(1 - m) - n(1 - n)$

(b)   Given *A* has coordinates (-3,5) and *B* has coordinates (-1,1). Find the:

     (i)   midpoint of the interval *AB*.

(ii)   distance AB.

(iii)   gradient of the line joining A and B.

(c)   An integer is selected at random from the integers 3 to 10 inclusive. What is the probability that the integer is:

(i)   divisible by 3

(ii)   divisible by 3 and 2

(iii)   divisible by 3 or 2

(d)   If the operation $\blacksquare$ is defined by $a \blacksquare b = \frac{1}{ab}$. Then find $a\blacksquare(b\blacksquare c)$.

(e) $x^2 + kx - 5 = 0$ has a solution $x = -1$. What is the value of k?

(f)   Write half of $1 \cdot 2 \times 10^{30}$ in scientific notation.

Question Four (15 marks)

(a)   A set of four numbers has a range of 4 and a mean of 4. None of the numbers is 4.
Write two sets of possible numbers.

(b)   A bookseller sells a book for $15.30 at a sale. This represents a loss of 10%. How
much did the bookseller lose on this book?

(c)   Solve the following:

(i) $(2 - x)(-3 + x) = 0$

(ii) $x^2 = 2x$

(iii) $(2x - 1)^2 = 9$

(iv) $2 - 3x < -10$

(d)   Write a simplified expression for the product of p factors each of which is m.

(e)   Solve $x + 2y = 1$ and $4x + 3y = 19$ simultaneously.

(f)   Some metal shapes are balanced on a set of scales.

One rectangle and three circles balance five squares.

Five squares and one circle balance two rectangles.

If I want to balance a rectangle with circles only, how many circles will I need?

(g)   A computer depreciates by 20% each year for five years. Calculate the percentage loss in value over five years to the nearest per cent.

Question Five (15 marks)

(a)   A die is made in the form of a tetrahedron (triangular pyramid). The faces are numbered 1,2,3 and 4. The die is rolled twice. The number on the face that the die lands on is recorded each roll. Find the probability that:

      (i)   the numbers 2 and 4 are recorded

      (ii)   the second number recorded is greater than the first number recorded

      (iii)   the sum of the two recorded numbers is 4.

(b)   Solve $x^2 - 3x - 1 = 0$ by completing the square.

(c)   Ben buys a stereo marked at $1100. He pays a $200 deposit and 24 equal monthly instalments of $51. Find the simple interest rate as a percentage correct to one decimal place.

(d)   Bert and Ernie put the same amount of money into different investments for one year. Bert's investments increased by 8%. Ernie's investments decreased by 7%. The difference in the value of their investments was $750 after one year. What was the original amount of money that each invested?

(e)   Determine a quadratic relationship in the form $y = x^2 + bx + c$ for the table of values.

x	0	1	2	3
y	7	4	3	4

(f)   If $\frac{a+3b}{a-b} = 3$. Evaluate $\frac{a}{b}$.

(g) $1200 is invested for two years at 10% per annum compounded annually. What is the amount of interest earned in the second year?

Question Six (15 marks)

(a)   The odometer on a car reads 73937 km. This is a palindromic number because it reads the same forwards as backwards. How many kilometres until the next palindromic number will appear on the odometer?

(b)   If the exterior angles of a triangle are in the ratio $4:5:6$, then what is the ratio of the interior angles?

(c)   Find the area of a triangle with vertices (1,2), (1,6) and (-2,k) where k is a constant.

(d)   Find the exact value of $\sin\theta$ where θ is acute and $\tan\theta = \frac{1}{\sqrt{2}}$.

(e)   The sum of a number and its reciprocal is $2\frac{1}{6}$. Find the number.

(f)   Samuel has two different pairs of socks loose in a drawer. He takes out 2 socks without looking at them. What is the probability that he selects a matching pair?

(g)   Find the equation of a line which passes through the point $(3,0)$ and is perpendicular to $x + 2y - 5 = 0$.

Question Seven (15 marks)

(a)
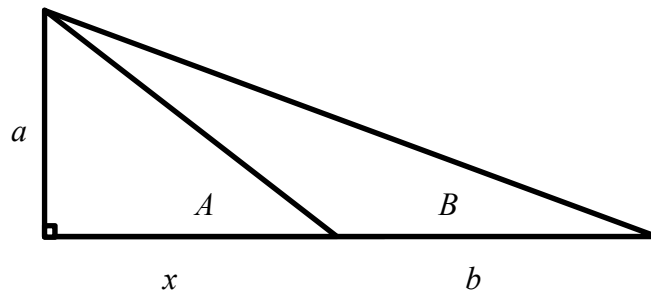
It is given that $\tan A - \tan B = 1$.

(i)   Prove that $x^2 + bx - ab = 0$.

(ii)   If $a = 3$ and $b = 6$.

   (α)   Find the value of x using the quadratic formula.

   (β)   Prove that $\tan A = \frac{\sqrt{3}+1}{2}$.

(b)   A class of 20 students was given a three question quiz. The results are listed below:

Question Number	Number of students who answered correctly
1	18
2	14
3	12

Determine the smallest possible number and the largest possible number of students that could have answered all three questions correctly and explain your answer.

(c)   A bag contains five balls, three red and two green. Tom and Jerry play a game in which they take turns to draw, at random, a ball from the bag and not replace it. The first to draw a green ball is the winner.

(i)   If Tom draws first:

(α)   Find the probability that Jerry wins with his first draw.

(β)   Find the probability that Tom wins.

(ii)   If they play a second game in which Jerry draws first, find the probability that Jerry wins at least one of the two games.

(d)   *PQRS* is a parallelogram. *PQ* is produced to *T* so that *QT=QR* and *PS* is produced to *U* so that *SU=PS*. It is now discovered that *T, R* and *U* are collinear.

   (i)   Draw a neat sketch showing the above information.

   (ii)   Prove that *PQRS* is a rhombus.

End of paper

Use this space if you wish to rewrite any answers

Clearly indicate the Question number

Question 1 (15marks)

  (a) Express $0{\cdot}5$ as a percentage. $\boxed{1}$

> Solution: 50%

  (b) Simplify the following fully:

    (i) $\dfrac{a}{a+a+a}$ $\boxed{1}$

> Solution: $\dfrac{a}{3a} = \dfrac{1}{3}$

    (ii) $\dfrac{a \times a \times a}{2 \times a}$ $\boxed{1}$

> Solution: $\dfrac{a^3}{2a} = \dfrac{a^2}{2}$

    (iii) $\sqrt{5} \times \sqrt{5}$ $\boxed{1}$

> Solution: 5

    (iv) -4^2 $\boxed{1}$

> Solution: -16

    (v) $3\sqrt{3} \times 2\sqrt{6}$ $\boxed{1}$

> Solution: $6\sqrt{18} = 18\sqrt{2}$

  (c) State the number of significant figures in the following:

    (i) $0{\cdot}0060$ $\boxed{1}$

> Solution: 2

    (ii) 30 000 (to nearest hundred) $\boxed{1}$

> Solution: 3

(d) Write $2 \cdot 15 \times 10^{-4}$ as a basic numeral. $\boxed{1}$

Solution: $0 \cdot 000\,215$

(e) Simplify the following ratios fully:
 (i) $24 : 36 : 16$ $\boxed{1}$

 Solution: $6 : 9 : 4$

 (ii) $36\,\text{m}^2 : 9\,\text{m}^2$ $\boxed{1}$

 Solution: $4 : 1$

(f) Evaluate $\cos 42°$ correct to two decimal places. $\boxed{1}$

Solution: $0 \cdot 74$

(g) What is the complementary angle of $75°$? $\boxed{1}$

Solution: $90° - 75° = 15°$

(h) Describe the complementary event of drawing a game of soccer. $\boxed{1}$

Solution: Winning or losing the game.

(i) Write $\dfrac{1}{2\sqrt{3}}$ with a rational denominator. $\boxed{1}$

Solution: $\dfrac{\sqrt{3}}{6}$

Question Two (15 marks)

(a) Find the acute angle α to the nearest minute if $\tan \alpha = \frac{3}{2}$.

$56°19'$

(b) Expand and simplify the following:

(i) $3(2x + 1) - 5x$

$= 6x + 3 - 5x \qquad = \qquad x + 3$

(ii) $(2x^2)^3$

$8x^6$

(iii) $(3 + x^2)^2$

$= 9 + 6x^2 + x^4 \qquad = \qquad x^4 + 6x^2 + 9$

(iv) $(2 + \sqrt{3})(5 - 2\sqrt{3})$

$= 10 - 4\sqrt{3} + 5\sqrt{3} - 6$

$= 4 + \sqrt{3}$

(c) Determine the linear relationship for the table of values.

x	1	2	3	4
y	4	1	-2	-5

$y = -3x + 7$

$y = mx + b \qquad 4 = m + b \qquad M = -3$

$-2 = 3m + b \qquad b = 7$

$\overline{-6 = 2m}$

②

(d)   Write an algebraic expression for "*m* less than the square of *p*".

$$p^2 - m$$

(e)   "Sparkle" toothpaste comes in four different sizes. Which size is the best value?

75 g for $2.58 $= 0.03544 \$/g$

110 g for $3.98 $= 0.0362$

150 g for $4.80 $= 0.032$

175 g for $5.70 $= 0.03257$

∴ 150g best value

②

(f)   Alice is paid $14.50 per hour for the first 36 hours she works in a week. She is paid time and a half for every extra hour worked. How much is Alice paid if she works 41 hours in one week?

$$36 \times \$14.50 + 5 \times 1.5 \times \$14.50 = \$630.75$$

(g)   What is the simple interest earned on $800 invested for 18 months at 8% per annum?

$$I = \frac{Pnr}{100} = \frac{\$800 \times 1.5 \times 8}{100} = \$96$$

(h)   What is the compound interest earned on $800 invested for 18 months at 8% per annum compounded quarterly to the nearest cent?

$$A = \$800\left(1 + \frac{2}{100}\right)^6 \quad \therefore I = A - \$800$$

$$= \$900.93 \qquad = \$100.93$$

②

(i)   Express $1.0\dot{8}$ as a fraction.

$$x = 1.0\dot{8}$$
$$10x = 10.\dot{8}$$
$$100x = 108.\dot{8}$$
$$90x = 98$$

$$x = \frac{98}{90}$$

$$= \frac{49}{45}$$

Question Three (15 marks)

(a)   Factorise the following fully:

(i) $6mn - 9m^2$

$$3m(2n - 3m)$$

(ii) $1 - 9m^2$

$$(1 + 3m)(1 - 3m)$$

(iii) $m^2 + 5m - 24$

$$(m + 8)(m - 3)$$

(iv) $2n^2 + n - 1$

$$\frac{(2n + 2)(2n - 1)}{2}$$

$$= (n + 1)(2n - 1)$$

(v) $m(1 - m) - n(1 - n)$

$$m - m^2 - n + n^2$$

$$m - n - m^2 + n^2$$

$$(m - n) - 1(m^2 - n^2)$$

$$= (m - n) - [(m + n)(m - n)]$$

$$= (m - n)(1 - (m + n))$$

$$= (m - n)(1 - m - n)$$

(b)   Given A has coordinates (-3,5) and B has coordinates (-1,1). Find the:

(i)   midpoint of the interval AB.

$$Mp = \left(\frac{-3-1}{2}, \frac{5+1}{2} \right) = (-2, 3)$$

(ii)   distance AB.

$$d = \sqrt{(-3+1)^2 + (5-1)^2}$$
$$= \sqrt{(-2)^2 + (4)^2}$$
$$= \sqrt{4 + 16} \qquad = \sqrt{20}.$$

(iii)   gradient of the line joining A and B.

$$m = \frac{5-1}{-3+1} \qquad = \frac{4}{-2}$$
$$= -2$$

#

(c)   An integer is selected at random from the integers 3 to 10 inclusive. What is the probability that the integer is: $3, 4, 5, 6, 7, 8, 9, 10 = 8 \text{ num}$

(i)   divisible by 3 $3/8$

(ii)   divisible by 3 and 2 $1/8$

(iii)   divisible by 3 or 2 $6/8$

(d)   If the operation ▦ is defined by $a \blacksquare b = \frac{1}{ab}$. Then find $a \blacksquare (b \blacksquare c)$.

$$a \, ▦ \left(\frac{1}{bc} \right)$$
$$= \frac{1}{\frac{a}{bc}} \qquad = \frac{bc}{a}$$

(e) $x^2 + kx - 5 = 0$ has a solution $x = -1$. What is the value of k?

$$1 - k - 5 = 0$$
$$k = -4.$$

(f)   Write half of $1 \cdot 2 \times 10^{30}$ in scientific notation.

$$6 \times 10^{29}$$

Q4.   Yr 10 exam Solutions (15)

(a) a, b, c, d $(a, b, c, d \neq 4)$

$$d - a = 4 \; (1) \Rightarrow d = a + 4$$
$$\bar{x} = 4 \Rightarrow \frac{a + b + c + d}{4} = 4$$
$$a + b + c + d = 16 \; (2)$$

$$\Rightarrow \frac{2a + 4 + b + c}{}$$
$$2a + b + c = 12.$$

$$a = 2, b = 3, c = 5$$

$$\Rightarrow \underline{2, 3, 5, 6} \quad \text{or} \quad \underline{1, 1, 5, 5} \quad (1)$$
$$\text{or} \underline{2, 2, 6, 6}$$
$$\text{or} \underline{1, 5, 5, 5}$$
$$\text{or} \underline{3, 3, 3, 7}$$

(b) $SP = 15.30$
    $Loss = 10\% \times CP$
    $\therefore SP. = \cdot 9 \times C.P.$
    $\cdot 9 \times CP = 15.30$
    $CP = \$17$

    $\therefore Loss = \underline{\$1.70}$ ✓ (1)

(c) (i) $(2 - x)(-3 + x) = 0$
    $\underline{x = 2 \text{ or } x = 3}$ ✓ (1)

(ii) $x^2 = 2x$
    $x^2 - 2x = 0$
    $x(x - 2) = 0$
    $\underline{x = 0 \text{ or } x = 2}$ ✓ (1)

(c) (iii) $(2x-1)^2 = 9$

$$2x - 1 = \pm 3$$
$$2x = 1 \pm 3$$
$$x = \frac{1 \pm 3}{2}$$
$$\underline{\underline{x = 2 \text{ or } -1}} \quad \checkmark\checkmark \quad ②$$

(iv) $2 - 3x < -10$

$$-3x < -12$$
$$3x > 12$$
$$\underline{\underline{x > 4}} \qquad \checkmark \qquad ①$$

(d) $m \times m \times m \times \ldots \times$ to p factors   ①

$$\underline{\underline{= m^p}} \quad \checkmark$$

(e) $x + 2y = 1$ ①
$4x + 3y = 19$ ②

$4 \times ① \Rightarrow 4x + 8y = 4$ ③

$③ ② \Rightarrow \quad 5y = -15 \qquad \checkmark \qquad ②$

$$\underline{y = -3}$$

Sub in ① $\quad x - 6 = 1 \quad \checkmark$

$$\underline{\underline{x = 7}}$$

Soln $(7, -3)$

(f)

$x \quad y$
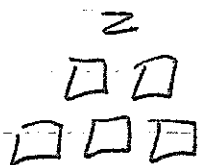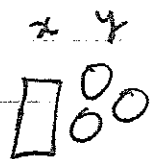
Let $x = $ no.of rectangles $y = $ no.of circles $z = $ no.of squares

$x + 3y = 5z \Rightarrow 5z - 3y = x$ ①

$5z + y = 2x$ ②

Find x in terms of y.

②

① − ② $\Rightarrow -4y = -x$

$$\underline{4y = x}$$

∴ $\underline{1 \text{ rectangle balances } 4 \text{ circles.}}$

(g) $n = 5 \quad r = \cdot 2$

$A = P(1 - r)^n$

$A = P(\cdot 8)^5$ ✓

3

Then $\%\ \text{Loss} = \dfrac{\text{loss}}{\text{Orig Amt}} \times 100\%$

$= \dfrac{P - P(\cdot 8)^5}{P} \times 100\%$

$= \dfrac{P(1 - \cdot 8^5)}{P} \times 100\%$

$\%\ \text{loss} \approx 67\%$ $\quad 0 \cdot 67232 \times 100$

Question Five (15 marks)

(a) A die is made in the form of a tetrahedron (triangular pyramid). The faces are numbered 1,2,3 and 4. The die is rolled twice. The number on the face that the die lands on is recorded each roll. Find the probability that:

(i) the numbers 2 and 4 are recorded

$\frac{2}{16} = \frac{1}{8}$ ①

(ii) the second number recorded is greater than the first number recorded

$\frac{6}{16} = \frac{3}{8}$ ①

(iii) the sum of the two recorded numbers is 4.

$\frac{3}{16}$ ①

(b) Solve $x^2 - 3x - 1 = 0$ by completing the square.

$x^2 - 3x = 1$

$x^2 - 3x + \left(\frac{3}{2}\right)^2 = 1 + \left(\frac{3}{2}\right)^2$

$\left(x - \frac{3}{2}\right)^2 = 1 + \frac{9}{4}$

$\left(x - \frac{3}{2}\right) = \pm\sqrt{\frac{13}{4}}$

$x = \frac{3}{2} \pm \sqrt{\frac{13}{4}} = \frac{3 \pm \sqrt{13}}{2}$

(c) Ben buys a stereo marked at $1100. He pays a $200 deposit and 24 equal monthly instalments of $51. Find the simple interest rate as a percentage correct to one decimal place.

Repay $= 24 \times 51 = 1224$

$I = PRT$

$324 = 900 \times R \times 2$

$R = \frac{324}{1800}$

Ans Simple interest rate 0.18

$= 18\%$ pa ②

(d) Bert and Ernie put the same amount of money into different investments for one year. Bert's investments increased by 8%. Ernie's investments decreased by 7%. The difference in the value of their investments was $750 after one year. What was the original amount of money that each invested?

$1.08x - 0.93x = 750$

$0.15x = 750$

$x = \$5000$ ②

(e) Determine a quadratic relationship in the form $y = x^2 + bx + c$ for the table of values.

x	0	1	2	3
y	7	4	3	4

$x=0$ $\quad y = x^2 + bx + C$ $\quad 0 + 0b + C = 7$ $\quad c = 7$

$x=1$ $\quad y = x^2 + bx + 7 = 4$ $\quad 1^2 + b + 7 = 4$ $\quad b = -4$

$y = x^2 - 4x + 7$ ②

(f) If $\frac{a+3b}{a-b} = 3$. Evaluate $\frac{a}{b}$.

$\frac{a+3b}{a-b} = 3$

$a + 3b = 3(a-b)$

$a + 3b = 3a - 3b$

$6b = 2a$

$\frac{a}{b} = \frac{3b}{b} = 3$ ②

(g) $1200 is invested for two years at 10% per annum compounded annually. What is the amount of interest earned in the second year?

$A_1 = 1200 \times 1.1 = 1320$

$A_2 = 1320 \times 1.1 = 1452$

$1452 - 1320 = \$132$ ②

29 ①

(6) 74947 = 1010 km

(a)   73937   74047 = 110   later

②   →km later

(b) $4x : 5x : 6x = 360$

$15x = 360$

$x = 24$

angles are $96°, 120°, 144°$

Interior angles are $84°, 60°, 36°$

$\div 12$. $7 : 5 : 3$ ②
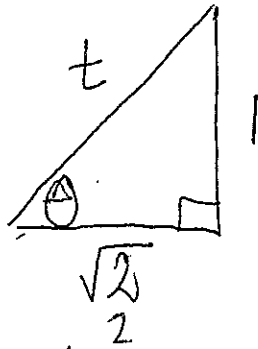
(c)
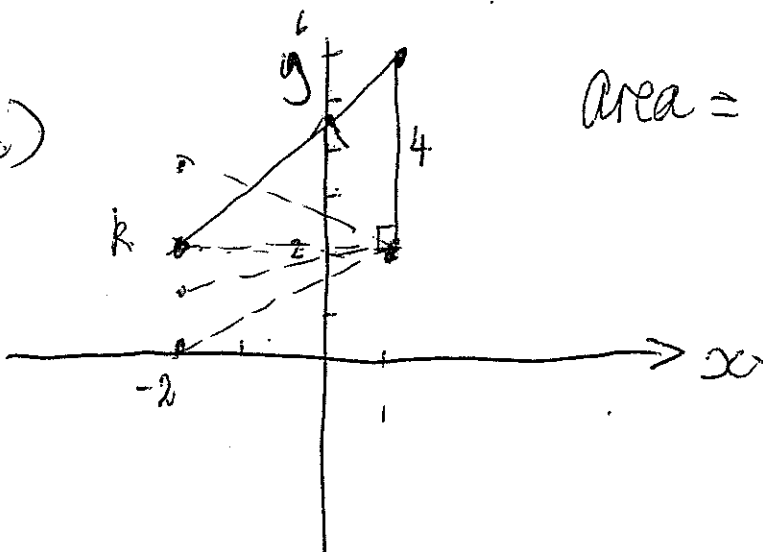
area $= \frac{1}{2} \times 3 \times 4$

$= 6 u^2$

②

6 (d) $\tan\theta = \frac{1}{\sqrt{2}}$

so $\sin\theta = \frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$.

$\sqrt{2}^2 + 1^2 = t^2$

$2 + 1 = t^2$

$t = \sqrt{3}$ ②

(e) $n + \frac{1}{n} = \frac{13}{6}$

$\frac{n^2 + 1}{n} = \frac{13}{6}$

$6n^2 + 6 = 13n$
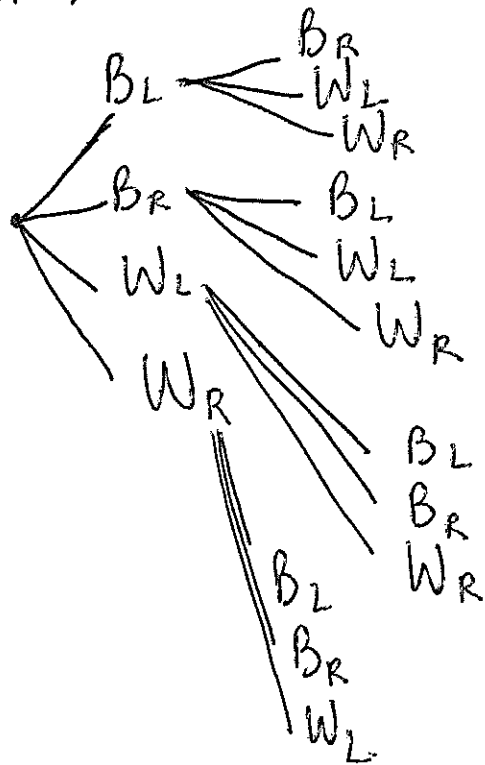
$6n^2 - 13n + 6 = 0$

$n = \frac{13 \pm \sqrt{169 - 4 \times 6 \times +6}}{12}$

$= \frac{13 \pm \sqrt{\text{ } 25}}{12}$

③

$\frac{13 \pm 5}{12} \Rightarrow \frac{8}{12} = \frac{2}{3}$ and $\frac{3}{2}$ number is $\frac{2}{3}$ or $\frac{3}{2}$

(f) let socks be B_L, B_R, W_L, W_R.

$B_L \quad B_R$

$B_R \quad B_L$

$W_L \quad W_R$

$W_R \quad W_L$

$\dfrac{2}{6} = \dfrac{1}{3}$

②

(g) $x + 2y - 5 = 0$

$2y = -1x + 5$

$y = -\dfrac{1}{2}x + \dfrac{5}{2}$

gradient is $-\dfrac{1}{2}$.

gradient $\perp$ is $m = 2$.

Point $(3, 0)$

$(y - y_1) = m(x - x_1)$

$y - 0 = 2(x - 3)$

$y = 2x - 6$
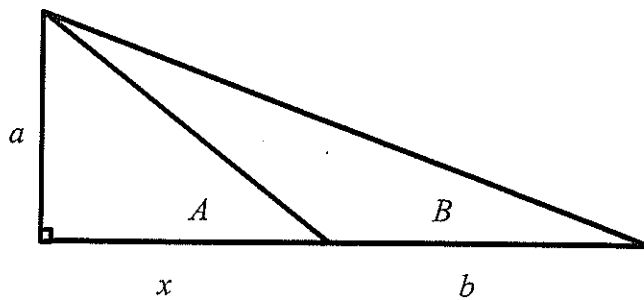
or $\quad 2x - y - 6 = 0 \; /\!/$ ②

$A \checkmark B \stackrel{\checkmark\checkmark\checkmark}{C D} = \checkmark E \checkmark C \checkmark$

Question Seven (15 marks)

(a)

It is given that $\tan A - \tan B = 1$.

(i)   Prove that $x^2 + bx - ab = 0$.

$$\tan A = \frac{a}{x} \qquad \tan B = \frac{a}{x+b}$$

So $\quad \dfrac{a}{x} - \dfrac{a}{x+b} = 1$

$$ax + ab - ax = x^2 + bx.$$

$$x^2 + bx - ab = 0.$$

(ii)   If $a = 3$ and $b = 6$.

   (α)   Find the value of x using the quadratic formula.

$$x^2 + 6x - 18 = 0$$

$$x = \frac{-6 \pm \sqrt{36 + 72}}{2.}$$

$$= -3 \pm 3\sqrt{3}$$

$$x = 3\sqrt{3} - 3$$

x is positive since it is a distance.

   (β)   Prove that $\tan A = \dfrac{\sqrt{3}+1}{2}$.

$$\tan A = \frac{3}{-3 + 3\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{\sqrt{3}+1}{2.}$$

(b)   A class of 20 students was given a three question quiz. The results are listed below:

Question Number	Number of students who answered correctly
1	18
2	14
3	12

Determine the smallest possible number and the largest possible number of students that could have answered all three questions correctly and explain your answer.

Largest is 12. If the 12 that answered Q3 correctly also answer Q1, Q2 correctly.

Smallest is 4. If the 12 that answered Q3 correctly were the six that got Q2 wrong and of the 6 that had it correct 2 of them got Q1 wrong. Then there are 4 left.

(c)   A bag contains five balls, three red and two green. Tom and Jerry play a game in which they take turns to draw, at random, a ball from the bag and not replace it. The first to draw a green ball is the winner.

(i)   If Tom draws first:

(α)   Find the probability that Jerry wins with his first draw.

$$\frac{3}{5} \times \frac{2}{4} = \frac{3}{10}.$$

(β)   Find the probability that Tom wins.

$$\frac{2}{5} + \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} = \frac{3}{5}.$$

(ii)   If they play a second game in which Jerry draws first, find the probability that Jerry wins at least one of the two games.

$$1 - \frac{2}{5} \times \frac{3}{5} = \frac{19}{25}.$$

(d)   *PQRS* is a parallelogram. *PQ* is produced to *T* so that *QT=QR* and *PS* is produced to *U* so that *SU=PS*. It is now discovered that *T, R* and *U* are collinear.

(i)   Draw a neat sketch showing the above information.
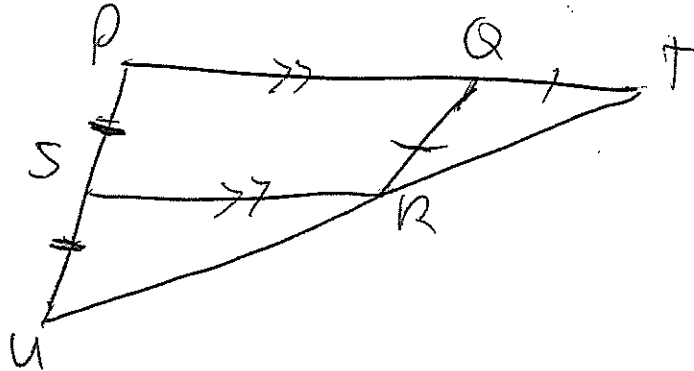
(ii)   Prove that *PQRS* is a rhombus.

Let $\angle T = x$.

$\angle QRT = x$;   (base $\angle$s isos $\triangle$).

$\angle PQR = 2x$ (ext $\angle$ of $\triangle$).

$\angle PSR = 2x$ (opp. $\angle$s parallelogram),

$\angle SRU = x$ (corres $\angle$s with $\angle T$).

$\angle SUR = x$.   (ext $\angle$ of $\triangle$).

$\therefore \triangle USR$ is isos   (base $\angle$s equal).

So $SU = SR$.

$PS = QR$ (opp side parallelogm)

$\therefore PQRS$ is a rhombus.

End of paper