SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

Year 10

Half Yearly Examination 2012

5.3 Mathematics

General Instructions

- Working time – 90 minutes
- Write using black or blue pen.
- Approved calculators may be used.
- All necessary working MUST be shown in every question if full marks are to be awarded.
- If more space is required, clearly write the number of the QUESTION on one of the back pages and answer it there. Indicate that you have done so.
- Clearly indicate your class by placing an X, next to your class
- All answers should be presented in simplest exact form, unless otherwise directed.
- Marks may not be awarded for untidy or badly arranged work.

Examiner: *R.Elliott*

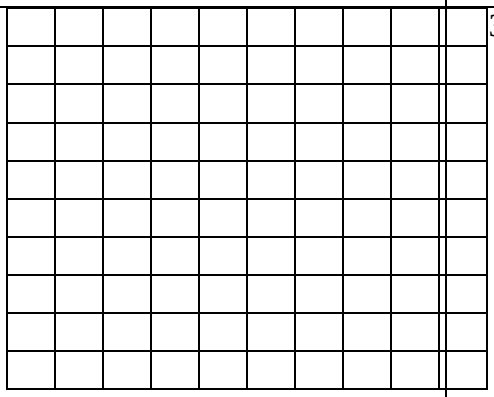
NAME: ……..

Class	Teacher	
10 A	Mr Fuller	
10 B	Mr Hespe	
10 C	Ms Chen	
10 D	Ms Nesbitt	
10 E	Ms Ward	
10 F	Mr Boros	
10 G	Mr McQuillan	

Question	Mark
1	**/19**
2	**/14**
3	**/15**
4	**/14**
5	**/14**
6	**/12**
7	**/14**
Total	**/102**

SECTION A

	QUESTION	ANSWER and WORKING	marks
1	Factorize (a) $8x^2y - 6x$ (b) $81 - x^2$		1 1
2	Simplify $\frac{12-3x}{6}$		1
3	Solve $\frac{x^2}{2} = x$		1
4	Solve and graph solution on a number line: $\frac{2x}{3} - 5 < 2$		2
5	Find correct to three significant figures $\frac{\sqrt{5}}{\sqrt{2} - 0.4^2}$		2
6	Write $\frac{5}{8}, 62\%, \sqrt{0.36}, \frac{1}{\sqrt{3}}$ in ascending order.		1
7	What is the gradient of the line joining (2,5) to (-1,11)		1
8	$x^2 - 5x + k$ is a perfect square. Find k.		1
9	A quadratic equation has solutions $x = 3$ and $x = -4$. Write it down in expanded form.		2
10	How many subsets are there of a set with three elements?		1
11	What is the percentage discount if a TV is bought for \$144 after receiving a \$16 discount?		1
12	A rhombus has diagonals of 15 cm and 12 cm.. Find the area.		1
13	Given $2\sqrt{5x} = \sqrt{y}$, find x in terms of y.		1
14	Find θ to the nearest minute.		2

	SECTION B		
1	Solve $x^2 - x - 2 = 0$ graphically by first drawing $y = x^2$		3
2	A number is selected at random from the numbers 1 to 60 inclusive. What is the probability that it contains at least one '3'.		2
3	The difference between a number and its reciprocal is $\frac{5}{6}$. Find the two possible values for the number. (Show all working).		3
4	For the points A(5,4), B(2,-3) and C(4,-3) (a) Find the midpoint D of BC (b) Find the length of AD (c) Find the area of triangle ABC		4
5	Factorize $p^3 + p^2 + p + 1$		2

	SECTION C	ANSWER and WORKING	marks
1	A line is drawn parallel to the line $y+2x=0$ through (-3,4) (a) Find the equation of this new line in general form. (b) What is the y intercept of this new line?		3
2	A normal coin is tossed 5 times. (a) What is the size of the sample space? (b) What is the probability of getting all tails? (c) What is the probability of getting at least one head?		3
3	The diagonal of a rectangle makes an angle of 30^0 with the longest side. What is the ratio of the length of the rectangle to its width?		2
4	Simplify $4^{3x-1} \div 8^{2x}$		2
5	Complete a table of values for the curve $y = x^2 - x - 6$ within the domain $-4 \le x \le 4$. Draw the graph of this curve, choosing a suitable y axis scale. Write the co-ordinates of it's vertex? For what values of x does $x^2 - x - 6 < 0$?	<table><tr><td>x</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr><tr><td>y</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr></table>	5

	SECTION D	ANSWER and WORKING	marks
1	A tent has a square base of area 16 m^2 and triangular sides. If the tent has a height of $2m$. at its vertex, find the area of one of its triangular sides. (To nearest cm^2.)		3
2	A horse trough is in the shape of a prism with a trapezium cross section. The parallel sides of the trapezium are 38 cm. and 28 cm. respectively. (a) If the trough is 2m. long find the surface area of metal needed to make it. (b)How many litres would it hold?		3
3	A man invested $20,000 at 8% PA Compound Interest for 4 years. (a) How much did this earn in interest? (b) What would the equivalent Simple Interest rate be to earn this amount?		4
4	In a year 1 class of 30 students, 18 study French, 17 study Art and 5 do neither. (a) Show this in a Venn diagram (b) A student is chosen at random. Find the probability that; (i) He does French but not Art (ii) He studies both subjects.		4

	SECTION E	ANSWER and WORKING	marks
1	Solve the equation $x^2 = 1 - 4x$ by the completing the squares method.		3
2	Given $(2\sqrt{3} - \sqrt{2})^2 = a + b\sqrt{c}$, find a, b and c,		3
3	In this diagram match each equation with the correct parabola.	(a) $y = x^2$ (b) $y = -x^2$ (c) $y = 3x^2$ (d) $y = -x^2 + 3$ (e) $y = x^2 + 3$	2
4	Given the formula Find the value of t if $s = 7, a = 6$ and $u = 1$		3
5	Show that the points A(-1,-3), B(3,6) and C(0,-1) are not collinear. Give full explanation.		3

	SECTION F	ANSWER and WORKING	marks
1	Write $(\sqrt{2}-\frac{1}{\sqrt{2}})^2$ as a surd with rational denominator.		3
2	Find the points where the parabola $y=x^2+3x+4$ cuts the line $y=5x+12$.		3
3	If a man wants to have \$10,000 in 5 years time and he can get 7% Compound Interest calculated annually, what must he invest?		3
4	An American says that he gets 40 miles per gallon of petrol from his car. If one gallon = 3.785 Litres and one mile = 1.61 km find this rate in litres per 100 kilometres.		3

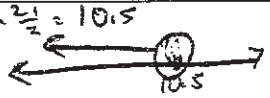
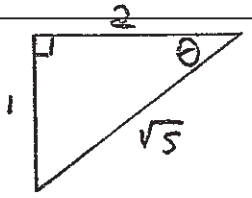
	SECTION G	ANSWER and WORKING	marks
1	A computer is now worth $320 after depreciating at 20% Per Annum for 3 years. Find its original value to the nearest dollar.		3
2	For what value of x does $y = x^2 + 2x - 8$ have a minimum value. What is this minimum value?		3
3	The product of two positive consecutive multiples of 3 is 378. Form an equation to show this information and hence find the two numbers.		3
4	"When High play St. Josephs in Basketball there are 3 possible results.; win, loss or draw. Therefore the probability, when High next plays St. Josephs, that High wins, is 1/3". Is this statement true? Justify your answer.		3
5	The digits 1,2,5 and 7 are used to form 24 different 3 digit numbers (each digit is used only once). If one number is selected at random what is the probability of it being even?		2

Use this space for working or rewriting answers

Use this space for working or rewriting answers

2012 YR10 Half Yearly

SECTION A

	QUESTION	ANSWER and WORKING	marks
1	Factorize (a) $8x^2y-6x$ (b) $81-x^2$	$2x(4xy-3)$ $(9-x)(9+x)$	1 1
2	Simplify $\frac{12-3x}{6}$	$\frac{4-x}{2}$	1
3	Solve $\frac{x^2}{2}=x$	$x=0, 2$	1
4	Solve and graph solution on a number line: $\frac{2x}{3}-5<2$ $\frac{2x}{3}<7$ $2x<21$	$x<\frac{21}{2}=10.5$ 	2
5	Find correct to three significant figures $\frac{\sqrt{5}}{\sqrt{2}-0.4^2}$	1.78	2
6	Write $\frac{5}{8}, 62\%, \sqrt{0.36}, \frac{1}{\sqrt{3}}$ in ascending order. 0.625 0.62 0.6 0.577	$\frac{1}{\sqrt{3}}, \sqrt{0.36}, 62\%, \frac{5}{8}$	1
7	What is the gradient of the line joining (2,5) to (-1,11)	$m=\frac{11-5}{-1-2}=-2.$	1
8	x^2-5x+k is a perfect square. Find k.	$\frac{25}{4}$ or 6.25	1
9	A quadratic equation has solutions $x=3$ and $x=-4$. Write it down in expanded form.	$(x-3)(x+4)=0$ $x^2+x-12=0$	2
10	How many subsets are there of a set with three elements?	8	1
11	What is the percentage discount if a TV is bought for \$144 after receiving a \$16 discount?	10%	1
12	A rhombus has diagonals of 15 cm and 12 cm.. Find the area.	$90\ cm^2$	1
13	Given $2\sqrt{5x}=\sqrt{y}$, find x in terms of y.	$\sqrt{20x}=\sqrt{y}$ $x=\frac{y}{20}$.	1
14	Find θ to the nearest minute. 	$\sin\theta=\frac{1}{\sqrt{5}}$ $\theta=26°34'$	2

Section B Yr 10 2012 Half Yearly. 11

(1) B

(2) 1 to 60 inclusive.

3 13 23 30 31 32 33 34 35 36 43 53.

$\frac{15}{60} = \frac{1}{4}$ (2)

(3) let the number be x.

$$x - \frac{1}{x} = \frac{5}{6}$$

$\times x$ $\quad x^2 - 1 = \frac{5x}{6}$

$\times 6$ $\quad 6x^2 - 6 = 5x$

$$6x^2 - 5x - 6 = 0$$

$$x = \frac{5 \pm \sqrt{25 - 4 \times 6 \times -6}}{12}$$

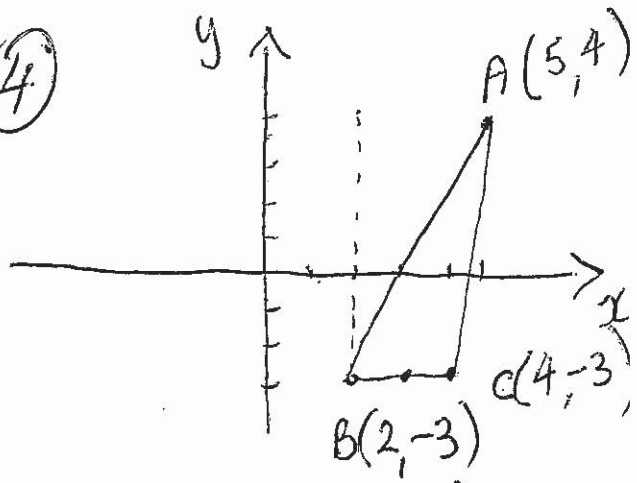
$$= \frac{5 \pm \sqrt{169}}{12}$$

$$= \frac{5 \pm 13}{12}$$

So $\frac{5-13}{12} = \frac{-8}{12} = -\frac{2}{3}$ //

$\frac{5+13}{12} = \frac{18}{12} = \frac{3}{2}$ // (3)

(4)



(a) $M\left(\frac{2+4}{2}, \frac{-3+-3}{2}\right)$

$= D(3, -3)$ (1)

(b) A(5,4) D(3,-3)

$$\sqrt{(3-5)^2 + (-3-4)^2}$$

$$= \sqrt{4 + 49}$$

$$= \sqrt{53}$$ (1) $(\doteq 7.28)$

(c) area $= \frac{b \times \perp h}{2}$

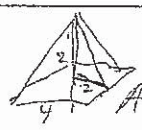
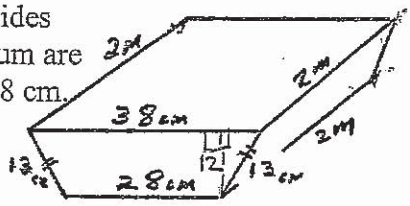
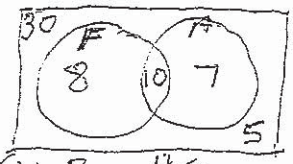
$= \frac{2 \times 7}{2} = 7\ u^2$ (2)

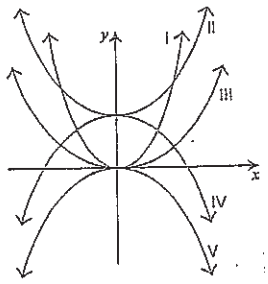
(5) $p^3 + p^2 + p + 1$

$p^2(p+1) + 1(p+1)$

$= (p^2+1)(p+1)$ (2)

	SECTION C	ANSWER and WORKING	marks
1	A line is drawn parallel to the line $y+2x=0$ through (-3,4) (a) Find the equation of this new line in general form. (b) What is the y intercept of this new line?	$y=-2x \quad m=-2$ $(-3,4)$ $y-4=-2(x+3)$ $y-4=-2x-6$ a) $2x+y+2=0$ b) $(0,-2) \quad y=-2x-2$	3
2	A normal coin is tossed 5 times. (a) What is the size of the sample space? (b) What is the probability of getting all tails? (c) What is the probability of getting at least one head?	$2^5=32$ $\left(\frac{1}{2}\right)^5=\frac{1}{32}$ $P(1-\text{no heads})=\frac{31}{32}$	3
3	The diagonal of a rectangle makes an angle of 30^0 with the longest side. What is the ratio of the length of the rectangle to its width?	$\sqrt{3}:1$ $\frac{1}{2}$	2
4	Simplify $4^{3x-1} \div 8^{2x}$	$2^{2(3x-1)} \div 2^{6x} = 2^{6x-2-6}$ $= 2^{-2} = \frac{1}{4}$	2
5	Complete a table of values for the curve $y=x^2-x-6$ within the domain $-4 \le x \le 4$. $(x-3)(x+2)$. Draw the graph of this curve, choosing a suitable y axis scale. Write the co-ordinates of it's vertex? $\frac{-b}{2a}=\frac{1}{2}$ For what values of x does $x^2-x-6<0$?	x: -4 -3 -2 -1 0 1 2 3 4 y: 14 6 0 -4 -6 -6 -4 0 6 $(\frac{1}{2}, -6.25)$ $-2<x<3$	5

	SECTION D	ANSWER and WORKING	marks
1	A tent has a square base of area 16 m^2 and triangular sides. If the tent has a height of $2m$. at its vertex, find the area of one of its triangular sides. (To nearest cm^2.)	 slant height $= \sqrt{8}$m $A = \frac{1}{2} \times 4 \times \sqrt{8}$ m^2 56568.5 cm^2 56569 cm^2 nearest cm^2	3
2	A horse trough is in the shape of a prism with a trapezium cross section. The parallel sides of the trapezium are 38 cm. and 28 cm. respectively.  (a) If the trough is 2m. long find the surface area of metal needed to make it. (b)How many litres would it hold?	(a) trap. $2 \times \frac{1}{2}(38+28)12 = 792$ base $28 \times 200 = 5600$ Sides $2 \times 200 \times 13 = 5200$ total 11592 cm^2 (b) $\frac{1}{2}(38+28)12 \times 200$ 79200 cm^3 79.2 L	3
3	A man invested \$20,000 at 8% PA Compound Interest for 4 years. (a) How much did this earn in interest? (b) What would the equivalent Simple Interest rate be to earn this amount?	$20000 \times 1.08^4 = 27209.779$ (a) Int $= 7209.79$ (b) $7209.79 \div 4 \div 20000 \times 100$ equiv rate. 9.01%	4
4	In a year 1 class of 30 students, 18 study French, 17 study Art and 5 do neither. (a) Show this in a Venn diagram (b) A student is chosen at random. Find the probability that; (i) He does French but not Art (ii) He studies both subjects.	 (i) $\frac{8}{30} = \frac{4}{15}$ (ii) $\frac{10}{30} = \frac{1}{3}$	4

	SECTION E	ANSWER and WORKING	marks
1	Solve the equation $x^2 = 1 - 4x$ by the completing the squares method.	$x^2 + 4x = 1$ $x^2 + 4x + 4 = 1 + 4$ $(x+2)^2 = 5$ $x + 2 = \pm\sqrt{5}$ $\boxed{x = -2 \pm \sqrt{5}}$	3
2	Given $(2\sqrt{3} - \sqrt{2})^2 = a + b\sqrt{c}$, find a, b and c,	$12 - 4\sqrt{6} + 2$ $= 14 - 4\sqrt{6}$ $\boxed{a = 14,\ b = -4,\ c = 6}$	3
3	In this diagram match each equation with the correct parabola. 	(a) $y = x^2$ III (b) $y = -x^2$ V (c) $y = 3x^2$ I (d) $y = -x^2 + 3$ IV (e) $y = x^2 + 3$ II	2
4	~~Given the formula~~ Find the value of t if $s = 7, a = 6$ and $u = 1$		3
5	Show that the points A(-1,-3), B(3,6) and C(0,-1) are not collinear. Give full explanation.	$m_{AB} = \frac{6+3}{3+1} = \frac{9}{4}$ $m_{BC} = \frac{6+1}{3-0} = \frac{7}{3}$ Since $m_{AB} \neq m_{BC}$ $\therefore$ A, B and C are not collinear.	3

J.C.

	SECTION F	ANSWER and WORKING	marks
1	Write $(\sqrt{2}-\frac{1}{\sqrt{2}})^2$ as a surd with rational denominator.	$\left(\frac{2-1}{\sqrt{2}}\right)^2 = \left(\frac{1}{\sqrt{2}}\right)^2$ $= \frac{1}{2}$	3
2	Find the points where the parabola $y = x^2 + 3x + 4$ cuts the line $y = 5x + 12$.	$x^2 + 3x + 4 = 5x + 12$ $x^2 - 2x - 8 = 0$ $(x-4)(x+2) = 0$ $x = -2, 4$ $y = -10 + 12 \ (x = -2)$ $= +2$ $y = 20 + 12 \ (x = 4)$ $= 32$ $\therefore$ Points $(-2, 2)$ $(4, 32)$	3
3	If a man wants to have \$10,000 in 5 years time and he can get 7% Compound Interest calculated annually, what must he invest?	$10000 = P(1+\frac{7}{100})^5$ $P = \frac{10000}{1.07^5}$ $= 7129.861795$ (calculator) $\therefore$ He must invest $\$7129.86$	3
4	An American says that he gets 40 miles per gallon of petrol from his car. If one gallon = 3.785 Litres and one mile = 1.61 km find this rate in litres per 100 kilometres.	$\frac{40 \text{ miles}}{1 \text{ gallon}} = \frac{64.4 \text{ km}}{3.785 \text{ L}}$ $\div 17.014531041 \text{ km/L}$ Now, this is $\frac{1}{17.01} \times 100 \div 5.877 \text{ L}/100 \text{ km}$ (3 dec. pl.)	3

	SECTION G	ANSWER and WORKING	marks
1	A computer is now worth \$320 after depreciating at 20% Per Annum for 3 years. Find its original value to the nearest dollar.	$320 = P(0.8)^3$ $P = \frac{320}{0.8^3}$ $P = \$625$ —	3
2	For what value of x does $y = x^2 + 2x - 8$ have a minimum value. What is this minimum value?	$y = x^2 + 2x + 1 - 9$ $y = (x+1)^2 - 9$ value of x is -1 min. value is -9	3
3	The product of two positive consecutive multiples of 3 is 378. Form an equation to show this information and hence find the two numbers.	let numbers be x & $x+3$ where x is a positive multiple of 3. $x(x+3) = 378$ $x^2 + 3x - 378 = 0$ $(x-18)(x+21) = 0$ $x = 18, -21$ since x is a positive multiple of 3. $x = 18$ & numbers are 18, 21	3
4	"When High play St. Josephs in Basketball there are 3 possible results.; win, loss or draw. Therefore the probability, when High next plays St. Josephs, that High wins, is 1/3". Is this statement true? Justify your answer.	There are 3 possible results but they are not equally likely. There are other factors like a teams ability etc	3
5	The digits 1,2,5 and 7 are used to form 24 different 3 digit numbers (each digit is used only once). If one number is selected at random what is the probability of it being even?	each 3 digit number is equally likely to end in a 1, 2, 5, 7 $\therefore$ probability is $\frac{1}{4}$.	2