

# SYDNEY BOYS HIGH SCHOOL <br> MoORE PARK, SURRY HILLS 

## Year 10

## Half Yearly Examination 2013

## Advanced Mathematics

## General Instructions

- Working time - 90 minutes
- Write using black or blue pen.
- Approved calculators may be used.
- All necessary working MUST be shown in every question if full marks are to be awarded.
- Marks may not be awarded for untidy or badly arranged work.
- If more space is required, clearly write the number of the QUESTION on one of the back pages and answer it there. Indicate that you have done so.
- All answers must be given in exact simplified form unless otherwise indicated.
- Clearly indicate your class by placing an X , next to your class

NAME:

| Class | Teacher |  |
| :---: | :--- | :--- |
| 10 A | Mr Boros |  |
| 10 B | Ms Ward |  |
| 10 C | Ms Miller/Mr Dunne |  |
| 10 D | Ms Evans/Ms Nesbitt |  |
| 10 E | Mr Hespe |  |
| 10 F | Mr Elliott |  |
| 10 G | Mr Gainford |  |


| Question | Mark |
| :---: | ---: |
| 1 | $/ 15$ |
| 2 | $/ 15$ |
| 3 | $/ 15$ |
| 4 | $/ 15$ |
| 5 | $/ 15$ |
| 6 | $/ 16$ |
| 7 | $/ 106$ |

## Section 1-15 Marks

1. 

a) Convert $4 \frac{2}{11}$ to a decimal.
b) Change $\frac{19}{20}$ to a percentage.
c) Round 0.030587 to 3 significant figures.
d) Write 0.725 as a simplified common fraction.
2. Convert 45 mL to litres and give the answer in Scientific Notation.
3. Simplify
a) $5 a-a$
b) $8 p q^{2}-6 p q+4 p^{2} q+q^{2} p-10 q p$
4. Write $6 \sqrt{50}$ as an entire surd
5. Calculate the value of $x^{2}-3 x y+y^{2}$ when $x=5$ and $y=-6$
6. If divers usually find 8 pearls in every 65 oysters,
a) What is the relative frequency of pearls?
b) How many pearls would you expect to find in 2000 oysters?
7. Solve
a) $x(x+3)=0$
b) $5(2 x-3)(x+6)=0$
8. Given that KY is parallel to HE , where L and H are the centres of the circles, find the value of the angle marked with the pro-numeral $i$


## Section 2-15 Marks

1. Two numbers are in the ratio $7: 11$. If their sum is 90 , what are the two numbers?
2. Simplify the following, writing the answer with a positive index:

$$
\frac{\left(2 d^{-2} c^{4}\right)^{2}}{30 d c^{5}}
$$

3. Simplify
a) $4 \sqrt{3} \times 7 \sqrt{12}$
b) $\sqrt{2}-\sqrt{18}+\sqrt{32} \div \sqrt{8}$
4. Solve the following quadratic equations.
a) $(x+2)^{2}=11$
b) $9 c^{2}=81$
c) $a^{2}+11 a+28=0$
5. Find the simple interest on $\$ 4500$ if it is invested at $8 \%$ p.a. for 9 months.
6. $\$ 10500$ is invested at $6 \%$ p.a. compound interest. How much does it grow to in 5 years?
7. A new car depreciates by $9.5 \%$ p.a. If it costs $\$ 24950$ new, what will it be worth in 3 years time?
8. A farmer fences his rectangular property as shown.


If a paddock is selected at random, what is the probability that
a) It has no fences on the boundary of the property?
b) It has one fence on the boundary of the property?

## Section 3-15 Marks

1. Andy borrowed $\$ 8000$ at $18 \%$ p.a. simple interest over 5 years to buy a motorbike. Find
a) The interest charged
b) The total amount repaid
c) The size of each monthly repayment if the total amount was repaid in equal instalments.
2. Solve the following quadratic equations by factorising
a) $4 b^{2}+21 b-18=0$
b) $5 x^{2}=13 x+6$
3. A survey was conducted on 100 students and the results were recorded as follows.

| Gender <br> Hair | Male | Female |
| :---: | :---: | :---: |
| Fair | 20 | 25 |
| Dark | 25 | 30 |

A student was selected at random. What is the probability that the student was
a) A girl?
(1)
b) A fair-haired boy?
c) Given that the student is a boy, what is the probability that he is dark-haired?
4. Solve by completing the square, leaving answers in simplified surd form
a) $x^{2}-6 x-11=0$
b) $3 x^{2}-12 x+7=0$

## Section 4-15 Marks

1. If $P(A)=\frac{3}{8}$, find $P\left(A^{\prime}\right)$
2. Chris chooses 2 marbles from a bag without replacement. The probability of him choosing a green marble is $\frac{1}{8}$ on the first selection. Given that he did choose a green marble on his first selection, then the probability that he chooses a green marble on the second selection is only $\frac{1}{15}$. How many marbles are in the bag?
3. Use the quadratic formula to solve, leaving your answer in exact form

$$
2 x^{2}-10 x+7=0
$$

4. If $\$ 25000$ is invested at $8 \%$ p.a. compounding monthly for 5 years, how much interest is earned?
5. How much must I invest now in a term deposit paying 9\%p.a. compounding monthly in order to have a million dollars in 30 years?
6. In a class of 32 students, 14 study Physics and 16 study Chemistry. 10 students study only Physics.
a) Display this information on a Venn Diagram and fill in the missing values.
(2)
b) Hence find the probability that a student in the class studies only Chemistry.
7. A home loan of $\$ 400000$ is paid off with monthly repayments of $\$ 2500$. If the interest rate is $5 \%$ p.a. reducible monthly, determine the amount owing after the third monthly payment.

## Section 5-15 Marks

1. A letter is chosen at random from the word ADVANCED. What is the probability that it is a) an $E$ ?
b) a consonant?
2. Westerley Bank offers the following investment plans:

Plan X: 5\% p.a. compounding monthly
Plan Y: 4.7\% p.a. compounding daily
John wants to invest $\$ 1360$ for 2 years. Which plan will give him more interest and by how much more?
3. If $\frac{p}{q}=\frac{2}{3}$, prove
a) $\frac{p-q}{q}=-\frac{1}{3}$
(1)
b) $\frac{p^{2}+q^{2}}{p^{2}-q^{2}}=-\frac{13}{5}$
4. If 3 coins are tossed, what is the probability of getting exactly 2 heads?
5. If n is a positive integer greater than 1 , write the following in decreasing order of magnitude: $\frac{1}{n}, \frac{1}{n+1}, \frac{1}{n-1}, \frac{n}{n-1}, \frac{n+1}{n-1}$
6. The Great Pyramid of Giza has a square base with sides of 230.4 m and it is 146.3 m high.

Calculate the angle, to the nearest minute, that one of the edges where 2 sides meet makes with the base.

## Section 6-15 Marks

1. Solve the following equation:

$$
\begin{equation*}
4(m-6)^{2}-12(m-6)=7 \tag{2}
\end{equation*}
$$

2. Solve $\frac{3 m+1}{3 m-1}-\frac{3 m-1}{3 m+1}=2$ (correct to 2 dp )
3. A bag has 1 yellow, 2 green and 3 black counters. Two counters are selected at random, and the first counter is replaced before the second one is selected. Find the probability of selecting
a) 2 green marbles
b) A green and a black marble
4. Consider the formula $R=r \sqrt{\frac{A+B}{A-B}}$. Make A the subject of the formula.
5. Solve $\frac{8}{x-1}-1=\frac{9}{x}$

## Section 7-16 Marks

1. Heather invested some money for one year. One third of it was invested at $14 \%$ p.a., one quarter of it was invested at $12 \%$ p.a. and the rest was invested at $10 \%$ p.a. What was the overall interest rate? (correct to 2 d.p.)
2. Solve the following equation

$$
\frac{5 x+5}{2 x-3}=\frac{3 x+5}{x+1}-\frac{2 x^{2}}{2 x^{2}-x-3}
$$

3. A box contains 7 blue and 3 yellow tickets. Two tickets are randomly selected, without replacement, from the box. Determine the probability that
a) both are blue
b) they are different colours.
4. The police are investigating the passing of counterfeit money last Saturday evening at 3 coffee bars: The Peacock, The Rajah and The Quest. There are 11 suspects and police have discovered the following facts:
i) 5 of the suspects visited The Peacock
ii) 4 remained in The Rajah all evening but 3 others looked in - these 3 visited all 3 of the coffee bars
iii) 6 visited The Quest
iv) 1 remained out of town all evening
a) Using P,Q and R for the three sets, draw a Venn Diagram to show all the known facts and fill in the missing values. Show your working for determining these values.
b) Given that the police then rule out the suspect who was out of town and that all the remaining suspects are equally likely to have committed the crime, what is the probability that the perpetrator visited only The Quest?
5. A wire 80 cm in length is cut into 2 parts and each part is bent to form a square. If the sum of the areas of the squares is $300 \mathrm{~cm}^{2}$, find the lengths of the sides of the two squares to the nearest cm

Section 1-15 Marks
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b) Change $\frac{19}{20}$ to a percentage.
c) Round 0.030587 to 3 significant figures.
d) Write 0.725 as a simplified common fraction.

2. Convert 45 mL to litres and give the answer in Scientific Notation.

3. Simplify
a) $5 a-a$
b) $8 p q^{2}-6 p q+4 p^{2} q+q^{2} p-10 q p$

$$
\begin{array}{r}
9 q p-16 p q+4 p^{2} q 4 p^{2} q+9 p q-16 p q \\
=4 q^{2}
\end{array}
$$

4. Write $6 \sqrt{50}$ as an entire surd
(1)
5. Calculate the value of $x^{2}-3 x y+y^{2}$ when $x=5$ and $y=-6$

$$
\begin{aligned}
& 25-3 \times 5 \times 6+6^{2} \\
= & 25+90+36=151
\end{aligned}
$$

6. If divers usually find 8 pearls in every 65 oysters,
a) What is the relative frequency of pearls?

$$
151
$$

$$
\frac{8}{65}
$$

b) How many pearls would you expect to find in 2000 oysters?

$$
2000 \times \frac{8}{65}=246.153 \ldots
$$

7. Solve
a) $x(x+3)=0$
(1)

$$
x=0,-3
$$

b) $5(2 x-3)(x+6)=0$

$$
x=\frac{3}{2},-6
$$

8. Given that KY is parallel to HE , where L and H are the centres of the circles, find the value of the angle marked with the pro-numeral $i$






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[^1]section 5

1. advance
a) $P(\operatorname{On} E)=\frac{1}{8}$
b) $P($ a constant $)=\frac{5}{8}$
2. Plan $x: 5 \%$ compounded monthly over 2 yrs

$$
\left.\begin{array}{rl}
\Rightarrow r & =\frac{5 \%}{12} \\
& =\frac{0.05}{12}
\end{array}\right\} \begin{aligned}
n=2 \times 12 \\
=24
\end{aligned} \begin{aligned}
\therefore A & =P(1+r)^{n} \\
& =\$ 1360\left(1+\frac{0.05}{12}\right)^{24} \\
& =\$ 1502.72 \\
\therefore I & =A-p \\
& =\$ 1502.72-\$ 1360 \\
& \$ 142.72
\end{aligned}
$$

Plan Y: $4.7 \%$ compounded dally over 2 years

$$
\left.\begin{array}{rl}
\Rightarrow r & =\frac{4.7 \%}{365} \\
& =\frac{0.047}{365}
\end{array}\right\} \begin{aligned}
n=2 \times 365 \\
=730
\end{aligned} \begin{aligned}
\therefore A & =\$ 1360\left(1+\frac{0.047}{365}\right)^{730} \\
& =\$ 1494.03 \\
\therefore I & =\$ 1494.03-\$ 1360 \\
& =\$ 34.03
\end{aligned}
$$

$\therefore$ Plan $X$ would give John $(142.72-134.03=\$ 8.69)$ more
3. $\frac{p}{a}=\frac{2}{3}$, prove
a)

$$
\begin{aligned}
& \frac{p-a}{q}=-\frac{1}{3} \\
& \therefore \begin{aligned}
L H S & =\frac{p}{q}-\frac{a}{q} \\
& =\frac{2}{3}-1 \\
& =-\frac{1}{3} \\
& =\text { RUS }
\end{aligned}
\end{aligned}
$$

$$
\text { b) } \begin{aligned}
& \frac{p^{2}}{p^{2}-q^{2}}=\frac{-13}{5} \\
& \therefore L H S=\left[\left(p^{2}+q^{2}\right) \times \frac{1}{q^{2}}\right] \\
& \div\left[\frac{\left.\left(p^{2}-q^{2}\right) \times \frac{1}{q^{2}}\right]}{\frac{p^{2}+q^{2}}{q^{2}}}\right. \\
&=\frac{p^{2}-q^{2}}{q^{2}} \\
&=\frac{\frac{p^{2}}{q^{2}}+\frac{q^{2}}{q^{2}}}{q^{2}}-\frac{q^{2}}{q^{2}} \\
&=\frac{\left(\frac{p}{q}\right)^{2}+1}{\left(\frac{p^{2}}{q}\right)^{2}-1} \\
&=\frac{\left(\frac{2}{3}\right)^{2}+1}{\left(\frac{2}{3}\right)^{-1}} \\
&=\frac{\frac{13}{9}}{-} \\
&= \frac{-5}{9} \\
&= \frac{R H}{5}
\end{aligned}
$$

4. $P($ exactly 2 heads $) \Rightarrow$

Con $1 \operatorname{con} 2 \operatorname{con} 3$


HHH HUT* HT HT T TH H* THE TH PT
$\therefore P($ exactly 2 heads $)=\frac{3}{8}$
5. Let $n>1($ ie $n=2$ )

$$
\frac{1}{n}=\frac{1}{2}, \frac{1}{n+1}=\frac{1}{3}, \frac{1}{n-1}=\frac{1}{1}
$$

$$
\frac{n}{n-1}=\frac{2}{1}, \frac{n+1}{n-1}=\frac{3}{1}
$$

$\rightarrow$ In decreasing order of magnitude $\therefore \frac{n+1}{n-1}, \frac{n}{n-1}, \frac{1}{n-1}, \frac{1}{n}, \frac{1}{n+1}$
6.


$$
\begin{aligned}
x^{2} & =230.4^{2}+230.4^{2} \\
x & =\sqrt{2 \times 230.4^{2}} \\
& =325.8348 \mathrm{~m}
\end{aligned}
$$

$325.8348 \div 2$ as the diagonal of a square bisect eachother.

$$
=162.9174 \mathrm{~m}
$$

Inside triangle:

$$
\begin{aligned}
& \frac{\theta}{162.9174} m \\
& \tan \theta=\frac{146.3}{162.9174} \\
& \theta=\tan ^{-1}\left(\frac{146.3}{162.9174}\right) \\
&=41^{\circ} 55^{1}
\end{aligned}
$$

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uo Hally Yearly 2013 Section 7
(1) midested morey for / year.
(4) $\frac{1}{3}$ morig a $14 \frac{1}{5}$ pa
(3) $\frac{1}{4}$ morey 1216 pa.
(c) rest $\left(\frac{5}{12}\right)$ a 106paa

Find dierall interest rate, $2 D P$
let us invest $\phi 120$. 7
(A) $\phi 40+\phi 5.60$; 50 \$ $\$ 120$ makes $\$ 1420$
(b) $\$ 30+\$ 3.60$
(c) $\phi 59+\phi 5 \quad \begin{aligned} & \frac{120}{120} \frac{11.83}{1} \%\end{aligned}$

$$
\begin{align*}
& \text { (2) } \frac{5 x+5}{2 x-3}=\frac{3 x+5}{x+1}-\frac{2 x^{2}}{2 x^{2}-x-3}  \tag{3}\\
& \frac{5 x+5}{2 x-3}=\frac{3 x+5}{x+1} \frac{2 x^{2}}{(2 x-3)(x+1)} \\
& \frac{(5 x+5)(x+1)}{(2 x-3)(3 x+5)(2 x-3)-2 x^{2}} \\
& 5 x^{2}+10 x+5=6 x^{2}+x-15-2 x^{2} \\
& 5 x^{2}+10 x+5=4 x^{2}+x-15 \\
& x^{2}+9 x+20=0
\end{align*}
$$

(3) TBlue, 3 yellow.

2 tickets no replacement

a) $P(B B)=\frac{7}{10} \times \frac{6}{9}=\frac{7}{15}$
b) $P(B Y)+P(Y B)=\frac{7}{10} \times \frac{3}{9}+\frac{3}{10} \times \frac{7}{9}=\frac{42}{90}=\frac{7}{15}$
(5)


$$
\begin{align*}
& 4 x+4 y=80 \Rightarrow x+y=20 \\
& x^{2}+y^{2}=300 \tag{2}
\end{align*}
$$

$$
\text { (1) } \quad y=20-x
$$

So

$$
\begin{aligned}
& x^{2}+(20-x)^{2}=300 \\
& x^{2}+400-40 x+x^{2}-300=0 \\
& 2 x^{2}-40 x+100=0 \\
& x^{2}-20 x+50=0 \\
& a=1, b=-20, c=50 \quad \text { So }
\end{aligned}
$$

$$
\text { so } x=\frac{20 \pm \sqrt{400-4 \times 1 \times 50}}{2}
$$

$$
x=\frac{20 \pm \sqrt{200}}{2}
$$

$$
x \div 2.9, x \div 17.1
$$

So 1 side could be 3 cm in Isquas
$4(a)$


$$
\begin{align*}
& x+y+p+3=5 \\
& \text { So } x+y+p=2 \tag{1}
\end{align*}
$$

and $\quad p+w+q+3=6$ +
So $p+w+q=3$
So. $x+y+w+7+p+9=10$ $\left.\frac{(11-}{\frac{1}{10} \text { suspects }}\right)$
now $\quad x+y+p=2$ in $x(y+y+7+p+q=10$

$$
\begin{array}{r}
2+w+7+q=10 \\
w+q=1
\end{array}
$$

and if $p+w+q=3$ in $x+y+w+7+p+q=10$

$$
\begin{align*}
& x+y+3+7=10 \\
& x+y=0 \\
& \Rightarrow x=0, y=0 \tag{3}
\end{align*}
$$

So $p=2(5$ suspects visited Peacock)
$\omega=0$ (4 suspect: visited Rajah.)
$q=1 \quad(6$ suspect's visited $Q$ lest $)$


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