



SYDNEY BOYS HIGH
MOORE PARK, SURRY HILLS

2014
YEAR 10 HALF-YEARLY
EXAMINATION

Advanced Mathematics

Directions to Candidates:

- Answer all questions in the spaces provided in this question booklet.
- All necessary working **MUST** be shown in every question if full marks are to be awarded.
- Marks may not be awarded for careless or badly arranged work.
- Use black or blue pen for written answers, but pencil for diagrams and graphs.

- If additional working space is needed, clearly write the **QUESTION** number on one of the back pages and answer it there. Indicate that you have done so.
- Answer in simplest exact form unless otherwise indicated.
- Board-approved calculators may be used.

Time allowed: 90 minutes

Examiner: Mr R. Boros

Name: _____

Your Mathematics Class (Tick the box)	
10Ma A Ms Kilmore	<input type="checkbox"/>
10Ma B Ms Chen	<input type="checkbox"/>
10Ma C Ms Millar	<input type="checkbox"/>
10Ma D Ms Likourezos/Nesbitt	<input type="checkbox"/>
10Ma E Mr Hespe	<input type="checkbox"/>
10Ma F Mr Choy	<input type="checkbox"/>
10Ma G Mr Fuller	<input type="checkbox"/>

Markers' Use Only	
Question 1	/25
Question 2	/22
Question 3	/16
Question 4	/22
Question 5	/16
Question 6	/19
Question 7	/16
Total	/136

Question 1 (25 marks)

Working and Answers

- (a) Approximate $384(1.065)^7$ correct to three decimal places. 1
- (b) Approximate 7.56×5.284 correct to four significant figures. 1
- (c) Evaluate $4\,893\,000 \times 58\,600$, giving your answer in scientific notation correct to four decimal places. 2
- (d) Express $\frac{2}{3}$ as a percentage of $\frac{5}{8}$. 2
- (e) A car is 4.2 m long and 1.5 m high. A model of the car is 70 mm long. What is the scale of this model car? 1
- (f) Transform $V = u + at$ to make t the subject 1
- (g) Expand and simplify 2
 - (i) $(5x - 12)(5x + 12)$,
 - (ii) $(x^2 + y^2)^2$.
- (h) Simplify $2x^{\frac{1}{3}} + 9x^{\frac{1}{4}} - 9x^{\frac{1}{3}} + 17x^{\frac{1}{4}}$. 2

(i) Express as a single fraction 2
 $\frac{x-3}{3} - \frac{2x-4}{5}$

(j) Fully factorise 3
 (i) $5x^2 - 125$,

(ii) $y(x-7) - 7(x-7)$

(k) Simplify $5\sqrt{27} + \sqrt{3} - 2\sqrt{54}$, 2
 leaving your answer in exact
 form.

(l) Express $\frac{3}{\sqrt{7}-2}$ as a fraction with 2
 a rational denominator.

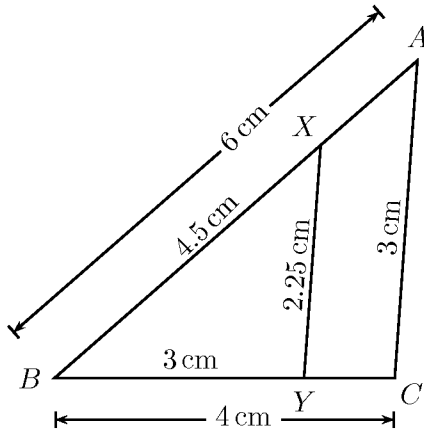
(m) $2\pi rh + 2\pi r^2$ is an expression for the 2
 of a
 Fill in the four missing words.

(n) If \$750 is invested at 1.75% simple 2
 interest per month for 2 years,
 how much interest would it have
 earned in total at the end of this
 time?

Question 2 (22 marks)

Working and Answers

(a)



(i) Prove that $\triangle ACB \parallel \triangle XYB$.

(ii) Why then is $XY \parallel AC$?

(b) $A(-2, 4)$ and $B(5, -3)$ are two points on the number plane.

(i) Find the distance AB in exact form.

(ii) Find the midpoint of AB .

(iii) Find the gradient of the interval AB .

(iv) Find the equation of AB , writing your answer in general form.

(c) Rewrite $\frac{a^{-4}}{3b^{-2}}$ without negative indices.

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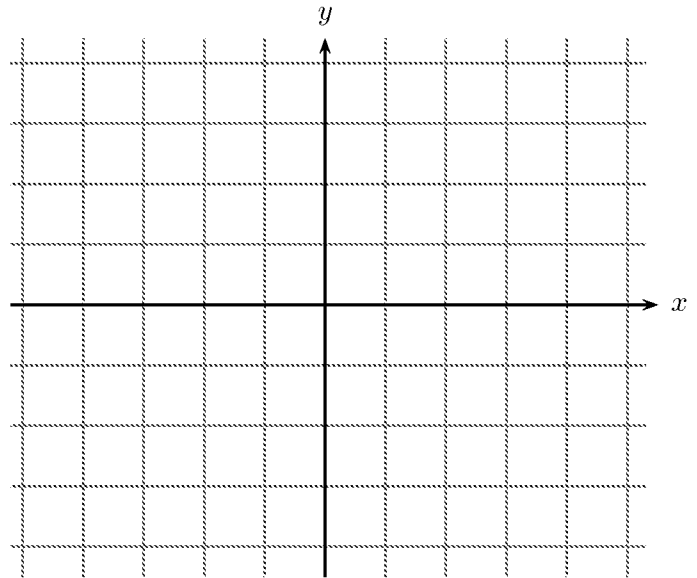
2

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1

- (d) Graph the region in the number plane which satisfies $3x - 4y + 12 > 0$.



2

- (e) In a spelling test, 10 year 7 students gained these marks
5, 7, 8, 10, 4, 6, 8, 8, 9, 6
Find the

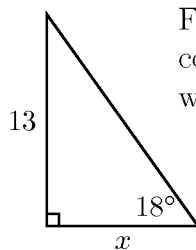
(i) mode,

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(ii) median.

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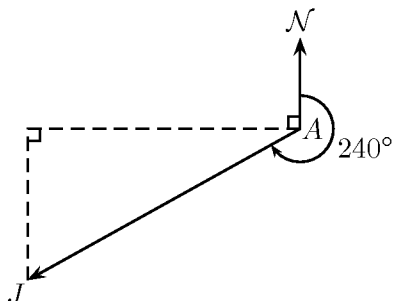
- (f) Find the value of x correct to the nearest whole number.



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- (g)



A jet J flew on a bearing of 240° T from an airport A until it was 155 km due west of the airport. Find (to 1 decimal place) the distance that J has flown.

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(h) Find the solution to the simultaneous equations
 $2x + y + 13 = 0$ and
 $3x - 2y + 9 = 0$.

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Question 3 (16 marks)

Working and Answers

- (a) The height of a plant in an experimental plot is increasing at the rate of 20% per week. On a certain date it was 25 cm high. Correct to 1 decimal place, calculate the plant's height after 8 weeks. 2

- (b) The value of a second hand car is \$9500 and it depreciates at the rate of 10% p.a. What is the value of the car after 4 years (correct to the nearest cent)? 2

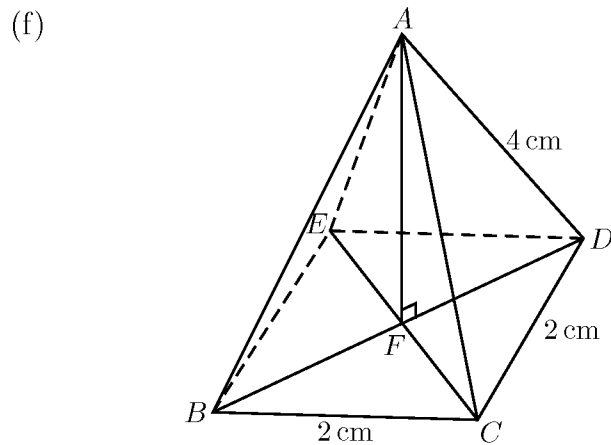
- (c) A shopkeeper wants to make a profit of 10% on an article of his stock. After selling it for \$53.50, he finds that he has only made a profit of 7% due to a miscalculation of the selling price. At what price should he have sold it? 2

- (d) From experience, it is known that 14% p.a. reducible is equivalent to a 9.92% p.a. flat (simple) rate over a period of 20 years, with payments made monthly. Using this information, if a couple borrowed \$60 000 at 14% p.a. reducible, find
 - (i) the total interest to be paid over the 20 years, 2

 - (ii) to the nearest dollar, the equal monthly payments to repay the loan, 2

 - (iii) what percentage of the total repayment of the loan the interest would be (answer correct to 1 decimal place). 2

- (e) A man invests a total of \$9000, 2
 partly at 3% p.a. and the rest at
 4% p.a., for only one year. If
 the total income from the invest-
 ments is \$295, how much money
 did he invest at each rate?



$ABCDE$ is a square based pyramid with $AD = 4$ cm. The base has sides 2 cm long. Leaving all answers in exact form, find

- (i) BD , 1

- (ii) the altitude AF 1

Question 4 (22 marks)

Working and Answers

(a) Solve the quadratic equations:

(i) $(2x - 1)(3x + 5) = 0$ 1

(ii) $x^2 = 5x$ 1

(iii) $(x + 1)^2 - 4 = 0$ 2

(b) What must be done to the following expressions in order to turn them into perfect squares?

(i) $x^2 + 7x$ 1

(ii) x^2 $x + 49$ 1

(c) Use the method of completing the square to solve $x^2 + 6x + 7 = 0$, giving roots in simplest surd form. Show ALL your steps. 3

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- (d) (i) Use the quadratic formula to solve $x^2 - 6x + 6 = 0$, calculating each root correct to 2 decimal places.

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- (ii) Hence solve: "A piece of wire 12 cm long is bent so as to form a rectangle of area 6 cm^2 . Find the sides of the rectangle."

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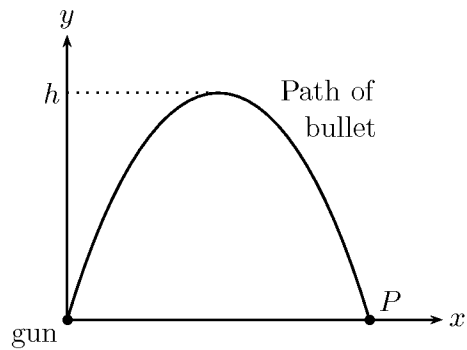
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- (e) A bullet fires out of a big gun so that its height, y metres, and its distance along the ground from the gun, x metres, are related by the rule $y = -2(x - 20)^2 + 800$.



- (i) The bullet lands at a point P . Find the distance P is away from the gun.

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- (ii) Find the maximum height of the bullet, h metres.

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- (iii) A rocket races after the bullet the instant the bullet leaves the gun. Its path has the rule $y = 60x$. Show (using algebra) at what height above the x -axis the rocket will hit the bullet.

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(f) Solve for x : $x^{-2} + x^{-1} - 6 = 0$.

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Question 5 (16 marks)

(a) In probability we often use the words “sample space.” What does this mean? 1

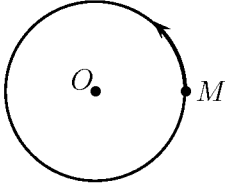
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(b) A game uses a 4-sided die in the shape of a triangular pyramid with two faces having **MOVE**, one face having **BUY**, and one face having **SELL**. I roll the die twice; find the probability that I roll a **BUY** and a **SELL** in any order. 2

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(c) A regular pack of cards is used. Find the probability of drawing out two Club cards, one at a time, without replacement of the first card. 2

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(d)  An electric train travels along a circular track of radius 7 m. At an instant, selected at random, the current is cut off and the train stopped. What is the probability (as a percentage to 1 decimal place) that the train stops within 2 metres of the terminus M ? 2

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(e) The numbers 1, 2, 3, 4 are marked on four marbles and placed in a bag. The marbles are selected at random, one after the other (and not replaced), to form a *two digit number*. Find

(i) P(that the number formed is greater than 23) 3

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(ii) P(that the number formed is prime) 2

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- (f) In a class of 30 students, 20 study history, 12 study geography, and 5 study neither. If a student is chosen at random, find $P(\text{they study history only or geography only})$.

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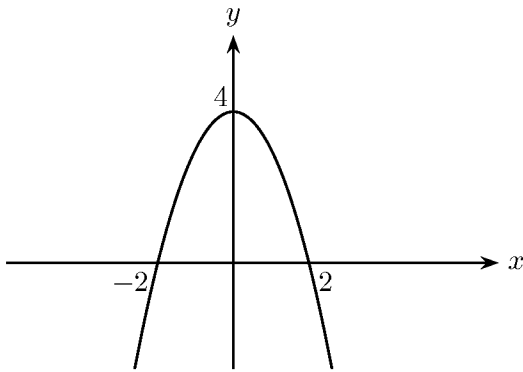
- (g) A lottery has 100 tickets, which are placed in a barrel. Three tickets are drawn, one at a time, from the barrel to decide the three prizes. If Ron has three tickets, find the probability that he wins none of the prizes.

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Question 6 (19 marks)

(a)



Circle the letter with the equation which matches the graph.

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(A) : $y = -(x - 2)^2$

(B) : $y = 4 - x^2$

(C) : $y = -(x + 4)^2$

(D) : $y = -(x + 2)^2 + 4$

(b) Given (A) : $y = (x + 4)^2$

(B) : $y = (x - 4)^2$

(C) : $y = x^2 + 4$

(D) : $y = 4x^2$

Circle the correct letter for the parabola which has $x = 4$ as its axis of symmetry.

1

(c) Which one of the following equations will have the steepest graph if all are drawn on the same pair of axes? Circle the correct letter.

(A) : $y = (x + 100)^2$

(B) : $y = 3(x + 10)^2$

(C) : $y = 10(x + 3)^2$

(D) : $y = x^2 + 300$

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(d) Find the vertex of the parabola $y = -(x + 1)^2 - 4$.

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(e) Find the minimum value of the expression $x^2 - 4x - 5$.

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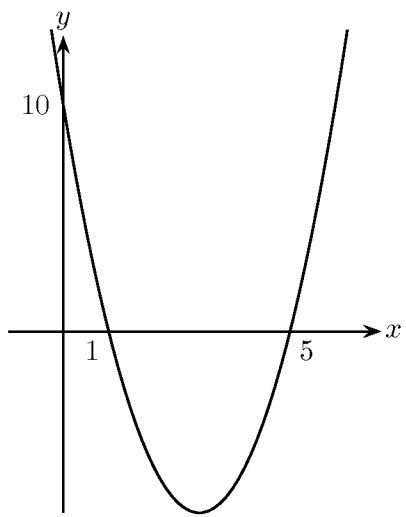
(f) Find the the equation of the quadratic when $y = (x + 3)^2$ is translated two units to the left.

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(g) Find the equation of this parabola.

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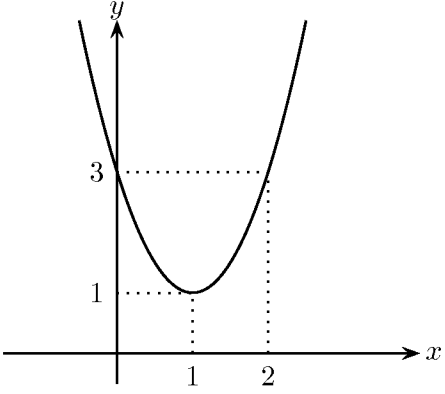
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(h) Find the equation of this parabola.

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(i) Given the circle equation $3 + x^2 + y^2 = 12$, find
(i) its centre,
(ii) its radius.

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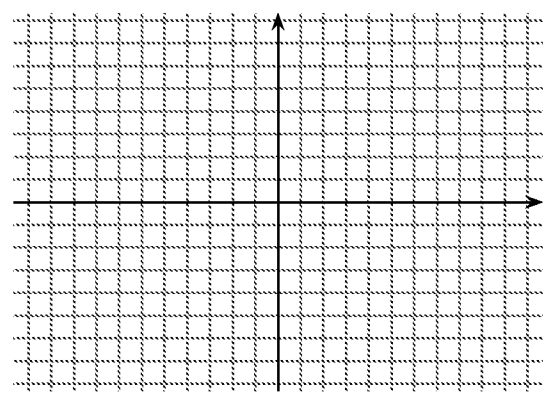
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(j) Accurately sketch the graph of $y = -\frac{6}{x}$, showing all essential features.

3



Question 7 (16 marks)

(a) (i) Prove that $\frac{a+b}{2} \geq \sqrt{ab}$ for all positive real values a and b by considering the perfect square $(\sqrt{a} - \sqrt{b})^2$. 2

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(ii) Hence prove that if a , b , and c are positive, then $(a+b)(b+c)(c+a) \geq 8abc$. 1

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(b) Numbers such as 8 310 256 947 can be written using *all* 10 digits. How many of these numbers are there if 0 cannot be the first digit? 2

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(c) Simplify $\sqrt{3+2\sqrt{2}} - \sqrt{3-2\sqrt{2}}$ by showing the answer is a rational number. Show all working. 3

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(d) A single bacteria divides into two every second. So one cell becomes two in the first second and in the next second two cells become four and so on.

(i) Write a rule connecting the
number of bacteria N after t
seconds.

1

(ii) How many bacteria will there
be after 10 seconds?
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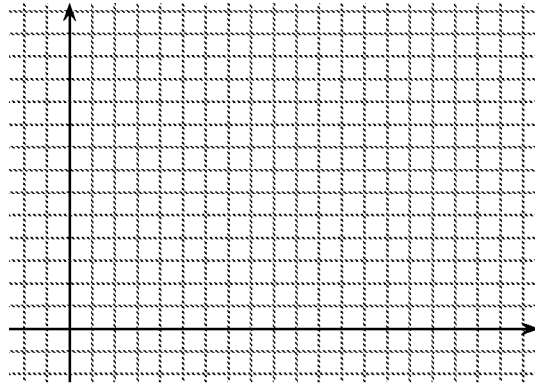
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(iii) How long does it take for the
population to exceed 10 000?
Round to the nearest second.

1

(iv) Graph your answer to 7(d)(i).

1



(e) Two mathematicians, X and Y , both independently try to solve a very hard mathematical problem. X is twice as likely to solve this problem as Y , and if the probability that the problem gets solved is $\frac{1}{2}$, find the probability that X solves the problem.

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End of Paper



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EXAMINATION

Advanced Mathematics Solutions

Question 1 (25 marks)

Working and Answers

(a) Approximate $384(1.065)^7$ correct to three decimal places.

Solution: 596.731

1

(b) Approximate 7.56×5.284 correct to four significant figures.

Solution: 39.95

1

(c) Evaluate $4\,893\,000 \times 58\,600$, giving your answer in scientific notation correct to four decimal places.

Solution: 2.8673×10^{11}

2

(d) Express $\frac{2}{3}$ as a percentage of $\frac{5}{8}$.

Solution: $\frac{2}{3} \div \frac{5}{8} \times 100\% = 106\frac{2}{3}\%$

2

(e) A car is 4.2 m long and 1.5 m high. A model of the car is 70 mm long. What is the scale of this model car?

Solution: $\frac{70}{4200} = \frac{1}{60}$

1

(f) Transform $V = u + at$ to make t the subject

Solution: $t = \frac{V - u}{a}$

1

(g) Expand and simplify

(i) $(5x - 12)(5x + 12)$,

Solution: $25x^2 - 144$

2

(ii) $(x^2 + y^2)^2$.

Solution: $x^4 + 2x^2y^2 + y^4$

(h) Simplify $2x^{\frac{1}{3}} + 9x^{\frac{1}{4}} - 9x^{\frac{1}{3}} + 17x^{\frac{1}{4}}$.

Solution: $-7x^{\frac{1}{3}} + 26x^{\frac{1}{4}}$

2

(i) Express as a single fraction $\frac{x-3}{3} - \frac{2x-4}{5}$.

Solution:

$$\frac{5(x-3) - 3(2x-4)}{3 \times 5} = \frac{5x - 15 - 6x + 12}{15},$$

$$= \frac{-x - 3}{15} \text{ or } -\frac{x+3}{15}$$

2

(j) Fully factorise

(i) $5x^2 - 125$,

Solution: $5(x^2 - 25) = 5(x+5)(x-5)$

3

(ii) $y(x-7) - 7(x-7)$.

Solution: $(x-7)(y-7)$

(k) Simplify $5\sqrt{27} + \sqrt{3} - 2\sqrt{54}$, leaving your answer in exact form.

Solution:
 $5\sqrt{9 \times 3} + \sqrt{3} - 2\sqrt{9 \times 6} = 16\sqrt{3} - 6\sqrt{6}$

2

- (l) Express $\frac{3}{\sqrt{7}-2}$ as a fraction with a rational denominator.

Solution:
$$\frac{3}{\sqrt{7}-2} \times \frac{\sqrt{7}+2}{\sqrt{7}+2} = \frac{3(\sqrt{7}+2)}{7-4},$$
$$= \sqrt{7}+2.$$

2

- (m) $2\pi rh + 2\pi r^2$ is an expression for the Surface Area ... of a ... Closed Cylinder.
Fill in the four missing words.

2

- (n) If \$750 is invested at 1.75% simple interest per month for 2 years, how much interest would it have earned in total at the end of this time?

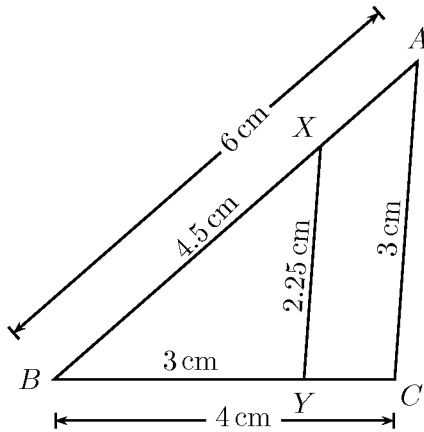
Solution:
$$\$750 \times \frac{1.75}{100} \times (2 \times 12) = \$315$$
 in interest.

2

Question 2 (22 marks)

Working and Answers

(a)



- (i) Prove that $\triangle ACB \parallel \triangle XYB$.
- (ii) Why then is $XY \parallel AC$?

Solution: (i) $\frac{BY}{BC} = \frac{3}{4}$,
 $\frac{XY}{AC} = \frac{2.25}{3} = \frac{3}{4}$,
 $\frac{BX}{BA} = \frac{4.5}{6} = \frac{3}{4}$,
 $\therefore \triangle ACB \parallel \triangle XYB$
 (all sides in same ratio).

(ii) $\angle BXY = \angle BAC$
 (corresp. angles in similar \triangle s),
 $\therefore XY \parallel AC$
 (corresp. angles are equal).

3

(b) $A(-2, 4)$ and $B(5, -3)$ are two points on the number plane.

- (i) Find the distance AB in exact form.
- (ii) Find the midpoint of AB .
- (iii) Find the gradient of the interval AB .
- (iv) Find the equation of AB , writing your answer in general form.

Solution: $\sqrt{(-2 - 5)^2 + (4 - -3)^2} = 7\sqrt{2}$.

2

Solution: $\left(\frac{-2 + 5}{2}, \frac{4 - 3}{2}\right) = (1.5, 0.5)$

2

Solution: $\frac{4 - -3}{-2 - 5} = \frac{7}{-7}$ i.e. -1 .

2

Solution: $y - 4 = -1(x + 2)$,
 $x + y - 2 = 0$.

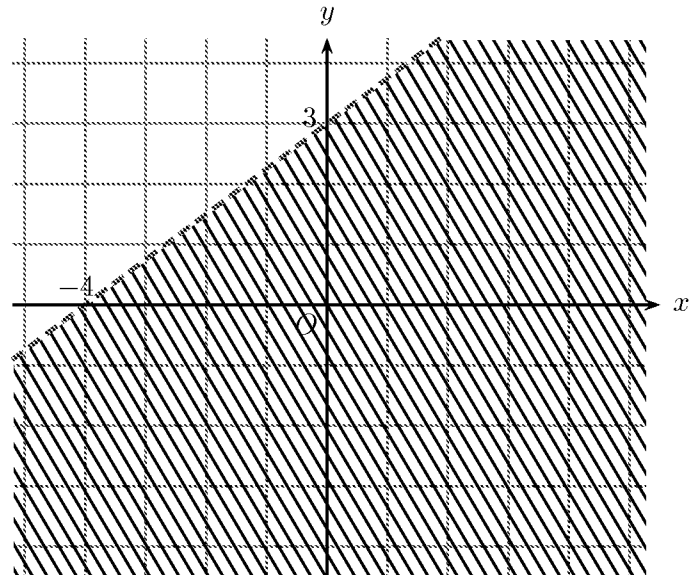
2

(c) Rewrite $\frac{a^{-4}}{3b^{-2}}$ without negative indices.

Solution: $\frac{b^2}{3a^4}$.

1

- (d) Graph the region in the number plane which satisfies $3x - 4y + 12 > 0$.



2

- (e) In a spelling test, 10 year 7 students gained these marks
5, 7, 8, 10, 4, 6, 8, 8, 9, 6

Find the

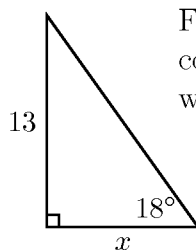
(i) mode,

(ii) median.

Solution: 8

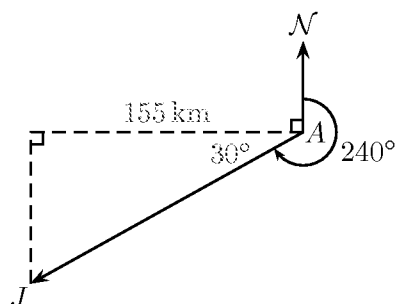
Solution: In order: 4, 5, 6, 6, 7, 8, 8, 8, 9, 10,
so the median is $\frac{7+8}{2} = 7\frac{1}{2}$.

- (f) Find the value of x correct to the nearest whole number.



Solution: $\tan 18^\circ = \frac{13}{x}$,
 $x = \frac{13}{\tan 18^\circ}$,
 ≈ 40 .

- (g)



A jet J flew on a bearing of 240° T from an airport A until it was 155 km due west of the airport. Find (to 1 decimal place) the distance that J has flown.

Solution: $\cos 30^\circ = \frac{155}{AJ}$,
 $AJ = \frac{155}{\cos 30^\circ}$,
 ≈ 178.9785834 by calculator.
So the jet flew 179.0 km.

2

2

2

(h) Find the solution to the simultaneous equations
 $2x + y + 13 = 0$ and
 $3x - 2y + 9 = 0$.

Solution:	$2x + y + 13 = 0 \dots\dots\dots$	$\boxed{1}$
	$3x - 2y + 9 = 0 \dots\dots\dots$	$\boxed{2}$
from $\boxed{1}$:		$y = -(2x + 13),$
sub. in $\boxed{2}$:	$3x + 2(2x + 13) + 9 = 0,$	
		$7x = -9 - 26,$
		$x = -5,$
sub. in $\boxed{1}$:	$-10 + y + 13 = 0,$	
		$y = -3.$

Question 3 (16 marks)

Working and Answers

- (a) The height of a plant in an experimental plot is increasing at the rate of 20% per week. On a certain date it was 25 cm high. Correct to 1 decimal place, calculate the plant's height after 8 weeks.

$$\text{Solution: } 25 \times 1.2^8 \approx 107.495, \\ \therefore 107.5 \text{ cm high.}$$

2

- (b) The value of a second hand car is \$9500 and it depreciates at the rate of 10% p.a. What is the value of the car after 4 years (correct to the nearest cent)?

$$\text{Solution: } \$9500(1 - 0.1)^4 = \$6232.95.$$

2

- (c) A shopkeeper wants to make a profit of 10% on an article of his stock. After selling it for \$53.50, he finds that he has only made a profit of 7% due to a miscalculation of the selling price. At what price should he have sold it?

$$\text{Solution: } \begin{aligned} \text{His cost} &= \frac{\$53.50}{1.07}, \\ &= \$50. \\ \text{Right price} &= \$50 \times 1.1, \\ &= \$55. \end{aligned}$$

2

- (d) From experience, it is known that 14% p.a. reducible is equivalent to a 9.92% p.a. flat (simple) rate over a period of 20 years, with payments made monthly. Using this information, if a couple borrowed \$60 000 at 14% p.a. reducible, find

- (i) the total interest to be paid over the 20 years,

$$\text{Solution: } \begin{aligned} \text{Interest} &= \$60\,000 \times \frac{9.92}{100} \times 20, \\ &= \$119\,040. \end{aligned}$$

2

- (ii) to the nearest dollar, the equal monthly payments to repay the loan,

$$\text{Solution: } \begin{aligned} \text{Payments} &= \frac{\$60\,000 + \$119\,040}{240}, \\ &= \$746. \end{aligned}$$

2

- (iii) what percentage of the total repayment of the loan the interest would be (answer correct to 1 decimal place).

$$\text{Solution: } \frac{\$119\,040}{\$60\,000 + \$119\,040} \times 100\% \approx 66.5\%.$$

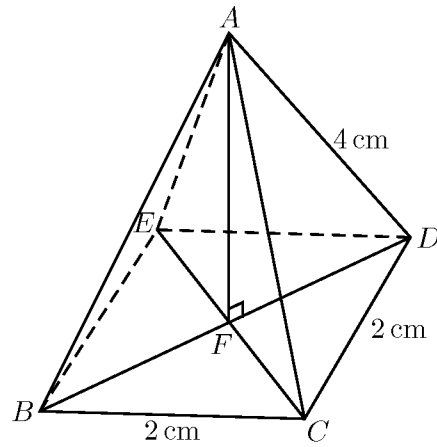
2

- (e) A man invests a total of \$9000, partly at 3% p.a. and the rest at 4% p.a., for only one year. If the total income from the investments is \$295, how much money did he invest at each rate?

$$\text{Solution: } \begin{aligned} &\text{Let the 3\% portion be } \$x. \\ 295 &= \left(x \times \frac{3}{100} + (9000 - x) \times \frac{4}{100} \right), \\ &= \frac{36\,000 - x}{100}, \\ x &= 36\,000 - 29\,500, \\ &= 6500. \\ \therefore &\$6500 \text{ at } 3\% \text{ and } \$2500 \text{ at } 4\%. \end{aligned}$$

2

(f)



$ABCDE$ is a square based pyramid with $AD = 4$ cm. The base has sides 2 cm long. Leaving all answers in exact form, find

(i) BD ,

Solution: $BD^2 = 2^2 + 2^2,$ $\therefore BD = 2\sqrt{2}$ cm.	1
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(ii) the altitude AF .

Solution: $4^2 = AF^2 + \left(\frac{BD}{2}\right)^2,$ $16 = AF^2 + 2,$ $AF = \sqrt{14}$ cm.	1
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Question 4 (22 marks)

Working and Answers

(a) Solve the quadratic equations:

(i) $(2x - 1)(3x + 5) = 0$

Solution: $x = \frac{1}{2}$ or $x = -\frac{5}{3}$.

1

(ii) $x^2 = 5x$

Solution: $x^2 - 5x = 0,$
 $x(x - 5) = 0,$
 $x = 0$ or $5.$

1

(iii) $(x + 1)^2 - 4 = 0$

Solution: $(x + 1)^2 = 4,$
 $x + 1 = \pm 2,$
 $x = -1 \pm 2,$
 $= 1$ or $-3.$

2

(b) What must be done to the following expressions in order to turn them into perfect squares?

(i) $x^2 + 7x + \left(\frac{7}{2}\right)^2$

Solution: *i.e.*, add $\frac{49}{4}$.

1

(ii) $x^2 \pm 2\sqrt{49}x + 49$

Solution: *i.e.*, insert $\pm 14.$

1

(c) Use the method of completing the square to solve $x^2 + 6x + 7 = 0$, giving roots in simplest surd form. Show ALL your steps.

Solution: $x^2 + 6x = -7,$
 $x^2 + 6x + 3^2 = -7 + 9,$
 $(x + 3)^2 = 2,$
 $x + 3 = \pm\sqrt{2},$
 $x = -3 \pm \sqrt{2}.$

3

(d) (i) Use the quadratic formula to solve $x^2 - 6x + 6 = 0$, calculating each root correct to 2 decimal places.

Solution: $x = \frac{6 \pm \sqrt{6^2 - 4 \times 6}}{2},$
 $= \frac{6 \pm \sqrt{12}}{2},$
 $= \frac{6 \pm 2\sqrt{3}}{2},$
 $= 3 \pm \sqrt{3},$
 ≈ 4.73 or $1.27.$

3

- (ii) Hence solve: "A piece of wire 12 cm long is bent so as to form a rectangle of area 6 cm². Find the sides of the rectangle." 2

Solution:

$$\begin{array}{|c|} \hline 6-x \\ \hline 6 \\ \hline \end{array} x$$

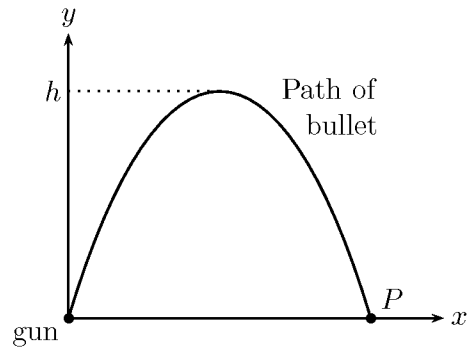
$$x(6-x) = 6,$$

$$6x - x^2 - 6 = 0,$$

$$\text{i.e., } x^2 - 6x + 6 = 0.$$

So the sides are 4.73 cm and 1.27 cm.

- (e) A bullet fires out of a big gun so that its height, y metres, and its distance along the ground from the gun, x metres, are related by the rule $y = -2(x - 20)^2 + 800$.



- (i) The bullet lands at a point P . Find the distance P is away from the gun.

Solution: At P , $y = 0$,
so $(x - 20)^2 = 400$,
 $x - 20 = \pm 20$,
 $x = 0$ or 40 .
 $\therefore P$ is 40 m from the gun.

- (ii) Find the maximum height of the bullet, h metres.

Solution: Maximum occurs when $x = 20$ m, and is 800 m.

- (iii) A rocket races after the bullet the instant the bullet leaves the gun. Its path has the rule $y = 60x$. Show (using algebra) at what height above the x -axis the rocket will hit the bullet.

Solution: $-2(x - 20)^2 + 800 = 60x$,
 $x^2 - 40x + 400 - 400 + 30x = 0$,
 $x^2 - 10x = 0$,
 $x(x - 10) = 0$,
 $x = 0, 10$.
And when $x = 10$, $y = 600$, so the height is 600 m.

- (f) Solve for x : $x^{-2} + x^{-1} - 6 = 0$.

Solution: First multiply through by x^2 ,
 $1 + x - 6x^2 = 0$,
i.e., $(1 + 3x)(1 - 2x) = 0$,
so $x = -\frac{1}{3}, \frac{1}{2}$.

Question 5 (16 marks)

- (a) In probability we often use the words “sample space.” What does this mean? 1

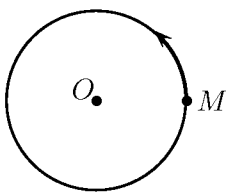
Solution: All the possibilities that can happen for the situation being considered.

- (b) A game uses a 4-sided die in the shape of a triangular pyramid with two faces having **MOVE**, one face having **BUY**, and one face having **SELL**. I roll the die twice; find the probability that I roll a **BUY** and a **SELL** in any order. 2

Solution: $P(\text{BUY then SELL}) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$,
 $P(\text{SELL then BUY}) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$,
 so $P(\text{either order}) = \frac{1}{16} + \frac{1}{16} = \frac{1}{8}$.

- (c) A regular pack of cards is used. Find the probability of drawing out two Club cards, one at a time, without replacement of the first card. 2

Solution: $P(\text{two Clubs}) = \frac{13}{52} \times \frac{12}{51}$,
 $= \frac{1}{17}$.

- (d)  An electric train travels along a circular track of radius 7 m. At an instant, selected at random, the current is cut off and the train stopped. What is the probability (as a percentage to 1 decimal place) that the train stops within 2 metres of the terminus *M*? 2

Solution: Circumference = $2 \times \pi \times 7$ m,
 so $P(\text{stopping within 2m}) = \frac{4}{14\pi}$,
 $\approx 9.1\%$.

- (e) The numbers 1, 2, 3, 4 are marked on four marbles and placed in a bag. The marbles are selected at random, one after the other (and not replaced), to form a *two digit number*. Find

- (i) $P(\text{that the number formed is greater than 23})$ 3

Solution: $P(24) = \frac{1}{4} \times \frac{1}{3}$,
 $P(3x \text{ or } 4x) = \frac{2}{4}$,
 so total probability = $\frac{1}{12} + \frac{1}{2}$,
 $= \frac{7}{12}$.

(ii) P(that the number formed is prime)

2

Solution: Possible primes ≤ 43 are 13, 23, 31, 41, 43.
So we have $\frac{1}{4} \times \frac{1}{3} \times 5 = \frac{5}{12}$.

(f) In a class of 30 students, 20 study history, 12 study geography, and 5 study neither. If a student is chosen at random, find P(they study history only or geography only).

2

Solution:

$$P(\mathbf{H} \text{ only or } \mathbf{G} \text{ only}) = \frac{13 + 5}{30},$$
$$= \frac{3}{5}.$$

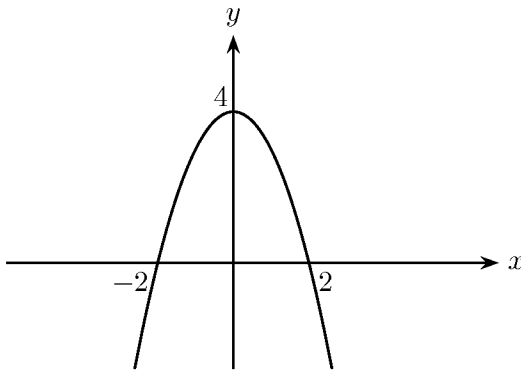
(g) A lottery has 100 tickets, which are placed in a barrel. Three tickets are drawn, one at a time, from the barrel to decide the three prizes. If Ron has three tickets, find the probability that he wins none of the prizes.

2

Solution:
$$P(\text{none}) = \frac{97}{100} \times \frac{96}{99} \times \frac{95}{98},$$
$$= \frac{7372}{8085}.$$

Question 6 (19 marks)

(a)



Circle the letter with the equation which matches the graph.

1

(A): $y = -(x - 2)^2$

(B): $y = 4 - x^2$

(C): $y = -(x + 4)^2$

(D): $y = -(x + 2)^2 + 4$

(b) Given (A): $y = (x + 4)^2$

(B): $y = (x - 4)^2$

(C): $y = x^2 + 4$

(D): $y = 4x^2$

Circle the correct letter for the parabola which has $x = 4$ as its axis of symmetry.

1

(c) Which one of the following equations will have the steepest graph if all are drawn on the same pair of axes?
Circle the correct letter.

(A): $y = (x + 100)^2$

(B): $y = 3(x + 10)^2$

(C): $y = 10(x + 3)^2$

(D): $y = x^2 + 300$

1

(d) Find the vertex of the parabola $y = -(x + 1)^2 - 4$.

Solution: $(-1, -4)$

2

(e) Find the minimum value of the expression $x^2 - 4x - 5$.

Solution: $x^2 - 4x + 4 - 5 - 4 = (x - 2)^2 - 9$,
So the minimum is -9 .

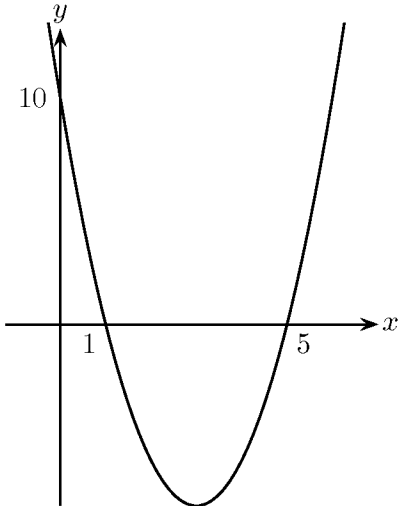
2

(f) Find the equation of the quadratic when $y = (x + 3)^2$ is translated two units to the left.

Solution: $y = (x + 3 + 2)^2$,
i.e., $y = (x + 5)^2$.

2

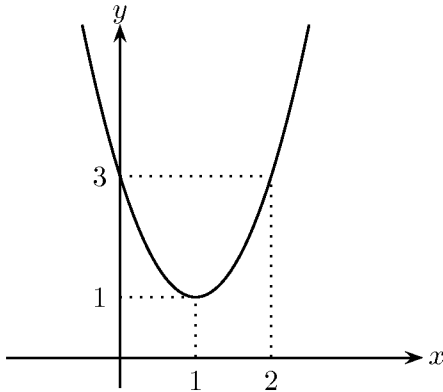
(g) Find the equation of this parabola.



Solution: $y = a(x - 1)(x - 5)$,
 now sub. $(0, 10)$:
 $10 = a(-1)(-5)$,
 $a = 1$,
 so $y = (x - 1)(x - 5)$.

2

(h) Find the equation of this parabola.



Solution: This looks like $y = ax(x - 2)$ raised up by 3, so substitute $(1, 1)$ in $y = ax(x - 2) + 3$:
 $1 = a \times 1 \times (1 - 2) + 3$,
 $= -a + 3$,
 $\therefore a = 2$.
 So $y = 2x(x - 2) + 3$,
 $= 2x^2 - 4x + 3$.

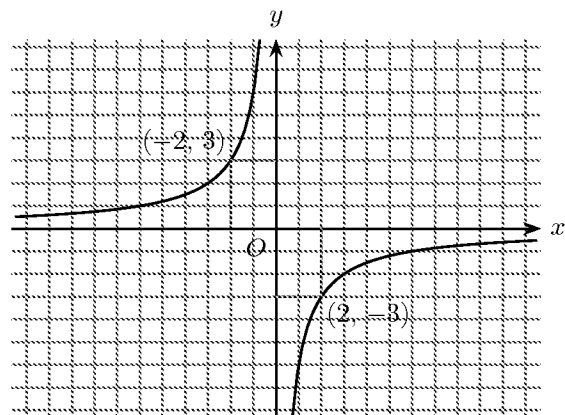
3

- (i) Given the circle equation $3 + x^2 + y^2 = 12$, find
 (i) its centre,
 (ii) its radius.

Solution: $x^2 + y^2 = 9$,
 (i) So centre = $(0, 0)$,
 (ii) and radius = 3.

2

- (j) Accurately sketch the graph of $y = -\frac{6}{x}$, showing all essential features.



3

Question 7 (16 marks)

- (a) (i) Prove that $\frac{a+b}{2} \geq \sqrt{ab}$ for all positive real values a and b by considering the perfect square $(\sqrt{a} - \sqrt{b})^2$. 2

Solution: $(\sqrt{a} - \sqrt{b})^2 \geq 0,$
 $a - 2\sqrt{ab} + b \geq 0,$
 $a + b \geq 2\sqrt{ab},$
 $\therefore \frac{a+b}{2} \geq \sqrt{ab}.$

- (ii) Hence prove that if $a, b,$ and c are positive, then $(a+b)(b+c)(c+a) \geq 8abc$. 1

Solution: Similarly, from above,
 $b + c \geq 2\sqrt{bc}$ and $c + a \geq 2\sqrt{ca}.$
 So $(a+b)(b+c)(c+a) \geq 2^3\sqrt{ab.bc.ca},$
 $\geq 8\sqrt{a^2.b^2.c^2},$
 $\geq 8abc.$

- (b) Numbers such as 8 310 256 947 can be written using *all* 10 digits. How many of these numbers are there if 0 cannot be the first digit? 2

Solution: $9 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 3\,265\,920.$

- (c) Simplify $\sqrt{3+2\sqrt{2}} - \sqrt{3-2\sqrt{2}}$ by showing the answer is a rational number. Show all working. 3

Solution: Put $y = \sqrt{3+2\sqrt{2}} - \sqrt{3-2\sqrt{2}},$
 where $y > 0,$ as $\sqrt{3+2\sqrt{2}} > \sqrt{3-2\sqrt{2}}.$
 Now $y^2 = 3 + 2\sqrt{2} - 2\sqrt{9-8} + 3 - 2\sqrt{2},$
 $= 6 - 2,$
 $= 4,$
 $\therefore y = 2,$ a rational number.

- (d) A single bacteria divides into two every second. So one cell becomes two in the first second and in the next second two cells become four and so on.

- (i) Write a rule connecting the number of bacteria N after t seconds. 1

Solution: $N = 2^t.$

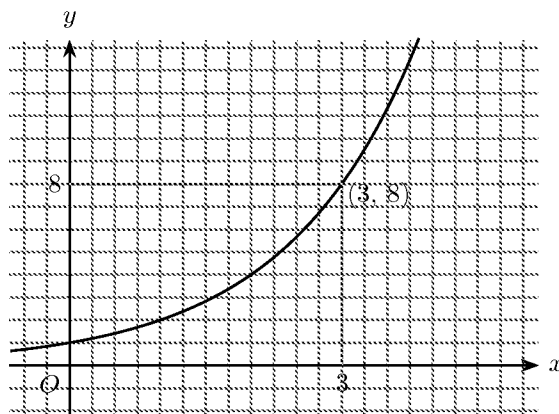
- (ii) How many bacteria will there be after 10 seconds? 1

Solution: $2^{10} = 1024.$

- (iii) How long does it take for the population to exceed 10 000? Round to the nearest second. 1

Solution: $2^{12} = 4096, 2^{14} = 16\,384,$
 $2^{13} = 8192,$ so it takes 13 seconds.

(iv) Graph your answer to 7(d)(i).



(e) Two mathematicians, X and Y , both independently try to solve a very hard mathematical problem. X is twice as likely to solve this problem as Y , and if the probability that the problem gets solved is $\frac{1}{2}$, find the probability that X solves the problem.

Solution:

$$\begin{aligned} \text{Let } P(X \text{ solves}) &= 2p, \\ P(Y \text{ solves}) &= p. \\ P(\text{at least one of them solves it}) &= \overline{P(\text{neither solves it})}, \\ &= 1 - (1-p)(1-2p), \\ &= 1 - (1-3p+2p^2), \\ &= 3p - 2p^2. \end{aligned}$$
$$\begin{aligned} \text{Now } 3p - 2p^2 &= \frac{1}{2}, \\ 4p^2 - 6p + 1 &= 0, \\ p &= \frac{6 \pm \sqrt{20}}{8}, \\ &= \frac{3 - \sqrt{5}}{4} \text{ as } 0 \leq p \leq 1. \end{aligned}$$
$$\begin{aligned} \text{So } P(X \text{ solves}) &= 2 \times \frac{3 - \sqrt{5}}{4}, \\ &= \frac{3 - \sqrt{5}}{2}. \end{aligned}$$

End of Paper