

SYDNEY BOYS HIGH SCHOOL MOORE PARK, SURRY HILLS

Year 10 Half Yearly Examination 2015

Advanced Mathematics

General Instructions

- Reading Time 5 minutes
- Working time 90 minutes
- Write using black or blue pen.
- Approved calculators may be used.
- All necessary working MUST be shown in every question if full marks are to be awarded.
- Marks may not be awarded for untidy or badly arranged work.
- If more space is required, clearly write the number of the QUESTION on one of the back pages and answer it there. Indicate that you have done so.
- All answers must be given in exact simplified form unless otherwise indicated.
- Clearly indicate your class by placing an X, next to your class

NAME:

Class	Teacher
10 A	Mr Boros
10 B	Mr Hespe
10 C	Mr Dowdell/Ms Ward
10 D	Mr Parker
10 E	Ms Millar
10 F	Ms Chen/Mr Elliott
10 G	Mr Gainford

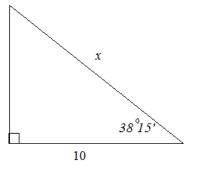
Question Mark /15 1 2 /15 3 /15 /15 4 /15 5 /15 6 /90 Total

Examiner: B. Kilmore

Section 1 (15 Marks)

- 1. Write 7.5% as a decimal.
- 2. Factorise fully : $m^3n^2 + m^2n^3$ (1m)
- 3. Calculate R to 1 decimal place , given $h = \frac{V}{\pi (R^2 r^2)}$, if V =100, h = 2.4 and *r* =3.7. (2m)

- 4. Factorise and simplify: $8pq^2 6pq + 4p^2q + q^2p 10qp$ (2m)
- 5. Calculate the simple interest on \$8000 for 9 months at 5% p.a. (1m)
- 6. How much will \$500 grow to in 3 years if it earns interest of 3% p.a. compounding annually? (1m)
- 7. Find the value of *x* correct to 1 decimal place (1m)



- 8. A card is drawn at random from a pack of 52 playing cards. Find the probability that the card will be a Queen or a heart. (1m)
- 9. Write as an entire surd: $6\sqrt{50}$ (1m)

(2m)

10. True or False?

a) All rectangles are similar.

- b) All rhombuses are parallelograms.
- 11. When playing darts the probabilities of Frank and Bill hitting a bullseye on the first throw are $\frac{1}{3}$ and $\frac{1}{5}$ respectively. What is the probability that Frank misses and Bill hits the bullseye on the first throw? (1m)

12. A family has three children. What is the probability they are all girls? (1m)

Section 2 (15 Marks)

1. Simplify and write with positive indices : $\frac{12x^2y^{-5}}{8x^{-6}y}$ (2m)

2. Simplify
$$\sqrt{2} - \sqrt{18} + \sqrt{32} \div \sqrt{8}$$
 (2m)

3. An integer is selected at random from the integers 3 to 20 inclusive.Find the probability that the integer is divisible by both 2 and 3. (1m)

4. Solve the following equations:

a)
$$9c^2 = 81$$
 (1m)

- **b)** $a^2 + 11a + 28 = 0$ (1m)
- c) $x^2 = 4x$ (1m)

5. The table below shows the results of a test designed to indicate if an animal is a carrier of a new virus.

Test Results			
	Positive	Negative	Total
Carrier		3	19
Non- Carrier	5		
Total		271	

a) Complete the table.

(1m)

- b) Find the probability that an animal carrying the virus was not detected by the test. (1m)
- c) If the testing showed that an animal did not carry the virus, what chance was there that it was wrong? Give your answer as an exact fraction. (1m)
- 6. Given that the volume of a cone is $V = \frac{1}{3}\pi r^2 h$ where r = radius and h = height, find a formula for the diameter of any cone, if you know its height and volume. (2m)

7. A new computer bought for \$2000 depreciates by 40% each year for 2 years. Calculate the percentage loss in value over two years. (2m)

Section 3 (15 Marks)

1. Solve using the quadratic formula: (Answer correct to 2 decimal places)

$$4a^2 - a - 1 = 0 \tag{2m}$$

2. Express in simplest form without negative indices: (1m)

$$\frac{a^{-1}+b^{-1}}{a+b}$$

3. What weekly repayments must be made on a lounge suite priced at \$1200 in order to pay it off in 6 months at 18% p.a. simple interest? (Use 6 months =26 weeks)

(2m)

4. \$3000 is invested for 5 years at 8%p.a. compounding monthly. Find the amount of interest it has earned at the end of the 5 years. (2m)

5. What principal must be invested to accumulate to \$15000 after 6 years at 17% p.a. compound interest? (2m)

6. Each year a printer depreciates by 25% of its value in that year.A new printer costs \$2400. By how much will it depreciate in the second year? (2m)

 7. Andrew and James put the same amount of money into different investments for one year. Andrew's investments increased by 8%. James' investments decreased by 7%. The difference in the value of their investments was \$750 after the one year. What was the original amount of money that each invested? (2m)

8. A bird cage contains 4 doves. Two are white and two are grey. If two doves escape, find the probability that of the two remaining, one is white and one is grey. (2m)

Section 4 (15 Marks)

2.

1. Solve by completing the square, leaving answers in exact form:

a)
$$x^2 - 6x - 15 = 0$$
 (2m)

b)
$$5x^2 - 9x + 2 = 0$$
 (3m)

a) Expand $(a-b)(a^2+ab+b^2)$ (1m)

b) Hence fully factorise
$$a^3 + \frac{a^2}{b} - b^3 - \frac{b^2}{a}$$
 (3m)

3. \$2500 accumulates to \$4400 after 4 years. What is the monthly compound interest rate correct to 1 decimal place?

(3m)

4. Twelve more than half a number is 21 less than the square of the number.(3m)

Section 5 (15 Marks)

1. Solve by completing the square, leaving your answer as a simplified surd:

 $4x^2 - ax - a^2 = 0$ (where *a* is a constant.) (3m)

2. Solve, leaving your answer in exact form:

$$\frac{1}{2-7x} + \frac{2}{2+7x} = -1$$
 (3m)

3. Simplify, writing your answer as a single surd: $\sqrt{7 + \sqrt{13}} - \sqrt{7 - \sqrt{13}}$ (3m)

4. 200 tickets are sold in a raffle in which 3 prizes are to be drawn by selecting 3 different tickets at random without replacement. Chris buys 3 tickets and wins one of the first two prizes drawn. What is the probability he will now win the 3rd prize? (3m)

5. The height, *h* metres, of a cricket ball after being struck by a batsman is given by the equation $h = 1 + x - \frac{x^2}{40}$ where *x* metres is the horizontal distance travelled by the ball from the bat. How far, to the nearest metre, does the ball travel before it hits the ground? (3m)

Section 6 (15 Marks)

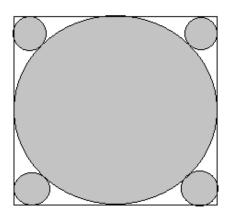
If an investment can be made at 18% p.a. compounding annually, 17% p.a. compounding quarterly, 16% p.a. compounding monthly or 15% p.a. compounding daily, which would give the greatest interest? Show all working. (3m)

Tasha has just bought a unit and has a mortgage of \$600000 at 4%p.a. compound interest reducible monthly. She has to make payments of \$2200 per month. How much does she owe after the third payment? (3m)

A cyclist travels at a certain speed to a town 30 km away, then reduces speed by 4 km/h and comes back. If the cyclist had travelled at a steady 8km/h for the whole trip, the total time would have been half an hour less. What was the initial speed of the cyclist? (3m)

4. The concentration of salt in sea water is 34 grams per litre. Given that $1 \text{ m}^3 = 1000 \text{ L}$, and that 1000 kg = 1 t, how many tonnes of salt are there in one cubic kilometre of seawater? (2m)

A circle is drawn inside a square with sides of 10cm so that it just touches all four sides of the square. Four small circles are placed in the corners so that they just touch the large circle and two sides of the square. Find the area of that part of the square that is not contained in circles. (4m)



Extra Working

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Extra Working



2015

Year 10 HALF YEARLY

Advanced Mathematics

Suggested Solutions

QUESTION	Marker
1	AMG
2	RE/JC
3	JM
4	TE
5	DH
6	RB

Section 1 (15 Marks)

- 1. Write 7.5% as a decimal.
- **2.** Factorise fully : $m^3n^2 + m^2n^3$
- $0.075^{(1m)}$ $M^{2}h^{2}(m+n)$ (1m)
- 3. Calculate R to 1 decimal place, given $h = \frac{V}{\pi (R^2 r^2)}$, if V = 100, h = 2.4 and
- r=3.7. $2.4 = \frac{1}{\pi} \frac{1}{(R^{2}-3.7)^{2}}$ $R^{2}-3\pi7^{2} = \frac{100}{7.4\pi}$ $R=\sqrt{13\cdot26+3.7^{2}}$ $= 5\cdot2.11$ $= 13\cdot26...$ 4. Factorise and simplify: $8pq^{2}-6pq+4p^{2}q+q^{2}p-10qp$ Pq(8q-6q+4p+q-10) = Pq(4p+qq-16) (2m)
- 5. Calculate the simple interest on \$8000 for 9 months at 5% p.a.

I= \$8000x3x0.05

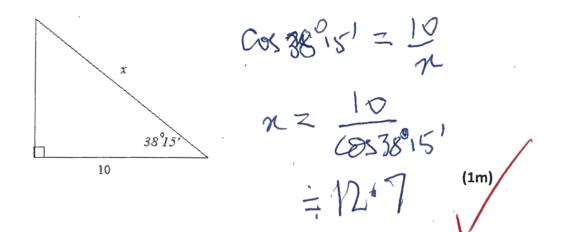
(1m)

= \$350

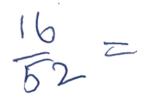
6. How much will \$500 grow to in 3 years if it earns interest of 3% p.a. compounding annually? $A_{2} = $500 (1 + \frac{3}{100})^{3} \doteq $546.36^{(1m)}$

Page **1** of **16**

7. Find the value of *x* correct to 1 decimal place



8. A card is drawn at random from a pack of 52 playing cards. Find the probability that the card will be a Queen or a heart. (1m)



(1m)

(2m)

9. Write as an entire surd: $6\sqrt{50}$

 $6\sqrt{50} = \sqrt{36750} = \sqrt{1800}$

10. True or False?

a) All rectangles are similar.

b) All rhombuses are parallelograms.

alse

11. When playing darts the probabilities of Frank and Bill hitting a bullseye on the first throw are $\frac{1}{3}$ and $\frac{1}{5}$ respectively. What is the probability that Frank misses and Bill hits the bullseye on the first throw? (1m)

2386 × 25

12. A family has three children. What is the probability they are all girls? (1m)

Marker's Comments

3, Candidates exhibited V. PI90V caba lation skitts, Many trought / Rm-r2 = R-r (say). A. Many do realise they should facturise fully. 6. "grow to" is not "grave by". 8. Many counted Q of Hearts Inice. 9. The term "entire sund" is poorly understood.

Section 2 (15 Marks)

1. Simplify and write with positive indices : $\frac{12x^2}{8x^2}$

(2m)

(2m)

2. Simplify
$$\sqrt{2} - \sqrt{18} + \sqrt{32} \div \sqrt{8}$$

= $\sqrt{2} - 3\sqrt{2} + 2$
= $2 - 2\sqrt{2}$

3. An integer is selected at random from the integers 3 to 20 inclusive. Find the probability that the integer is divisible by both 2 and 3. (1m)

$$n \{3, 4, \dots, 20\} = 18$$

$$n \{diminihle big 2, 3\} = n \{diminihle big 4\}$$

$$= n \{26, 12, 18\}$$

$$= 3$$

$$P = \frac{3}{18}$$

$$= \frac{1}{6}$$

3/2

Y

2

4. Solve the following equations:

a)
$$9c^2 = 81$$

 $c^2 = 9$
 $c = \pm 3$

b)
$$a^{2} + 11a + 28 = 0$$

 $(a + 7)(a + 4) = 0$
 $a = -4, -7$

c)
$$x^{2} = 4x$$

 $x^{2} - 4x = 0$
 $x(x - 4) = 0$
 $x = 0, 4$

Page **4** of **16**

(1m)

(1m)

(1m)

5. The table below shows the results of a test designed to indicate if an animal is a carrier of a new virus.

а

Tect Decult

Test Results			
a	Positive	Negative	Total
Carrier	16 -	3 5	19
Non- Carrier	5	268	273-
Total	21	271	292

a) Complete the table.

(1m)

- b) Find the probability that an animal carrying the virus was not detected by the test.
 (1m)
- c) If the testing showed that an animal did not carry the virus, what chance was there that it was wrong? Give your answer as an exact fraction. (1m)

311

6. Given that the volume of a cone is
$$V = \frac{1}{3}\pi r^2 h$$
 where $r =$ radius and $h =$ height, find

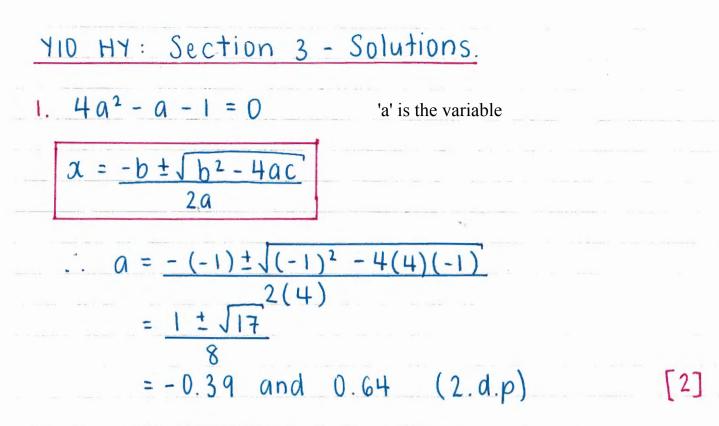
a formula for the diameter of any cone, if you know its height and volume.

(2m)

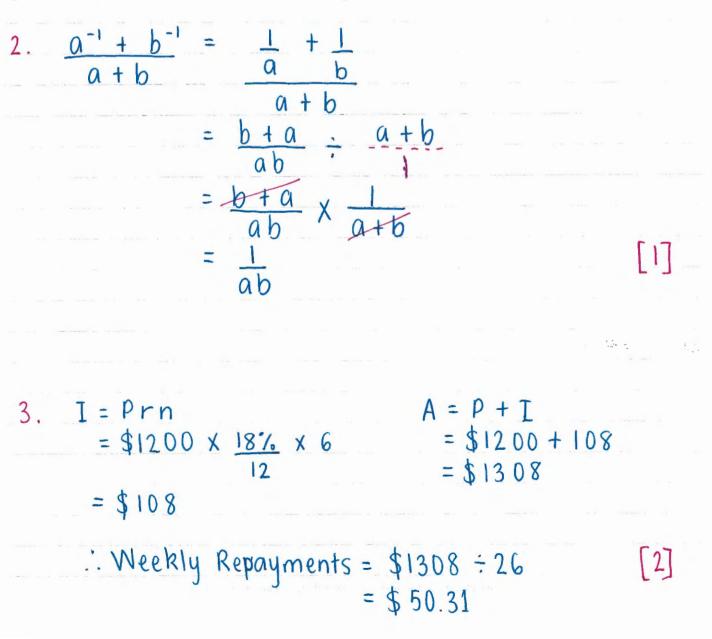
 $3V = \pi r^{2}h$ $r^{2} = \frac{3V}{\pi h}$ $r = \sqrt{\frac{3V}{\pi h}}$ $D = 2\sqrt{\frac{3Y}{\pi h}}$

7. A new computer bought for \$2000 depreciates by 40% each year for 2 years.
 Calculate the percentage loss in value over two years.
 (2m)

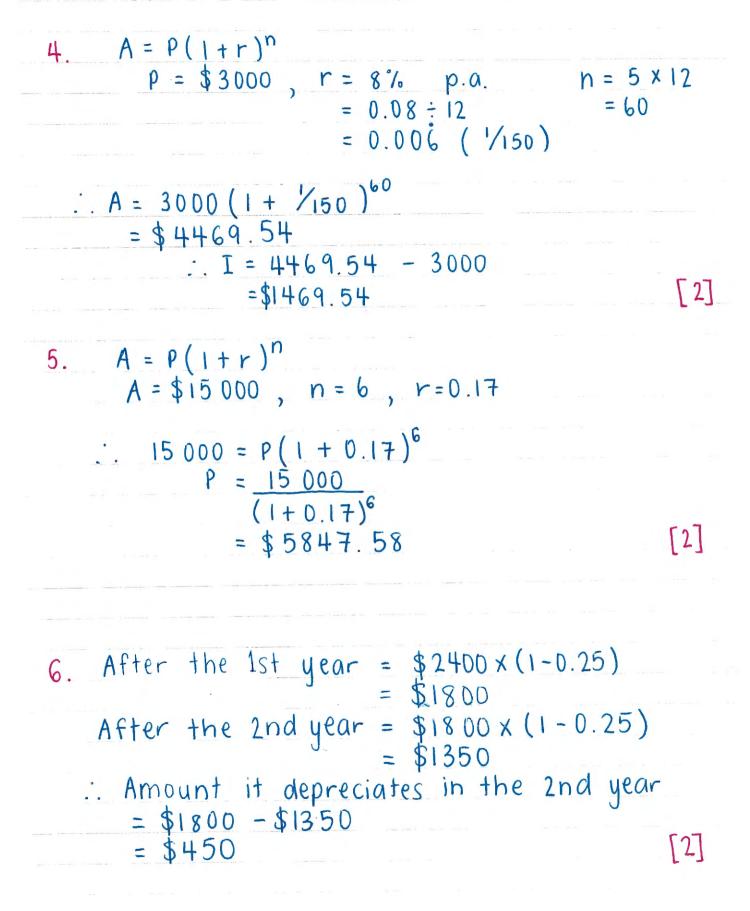
Value after $1 y ear = 0.6 \times 2000$ $= \frac{3}{200}$ Value after $2 y ears = 0.6 \times 1200$ = 720 = 720 $= 1280 \times 100$ Page 5 of 16 = 6100= 64%

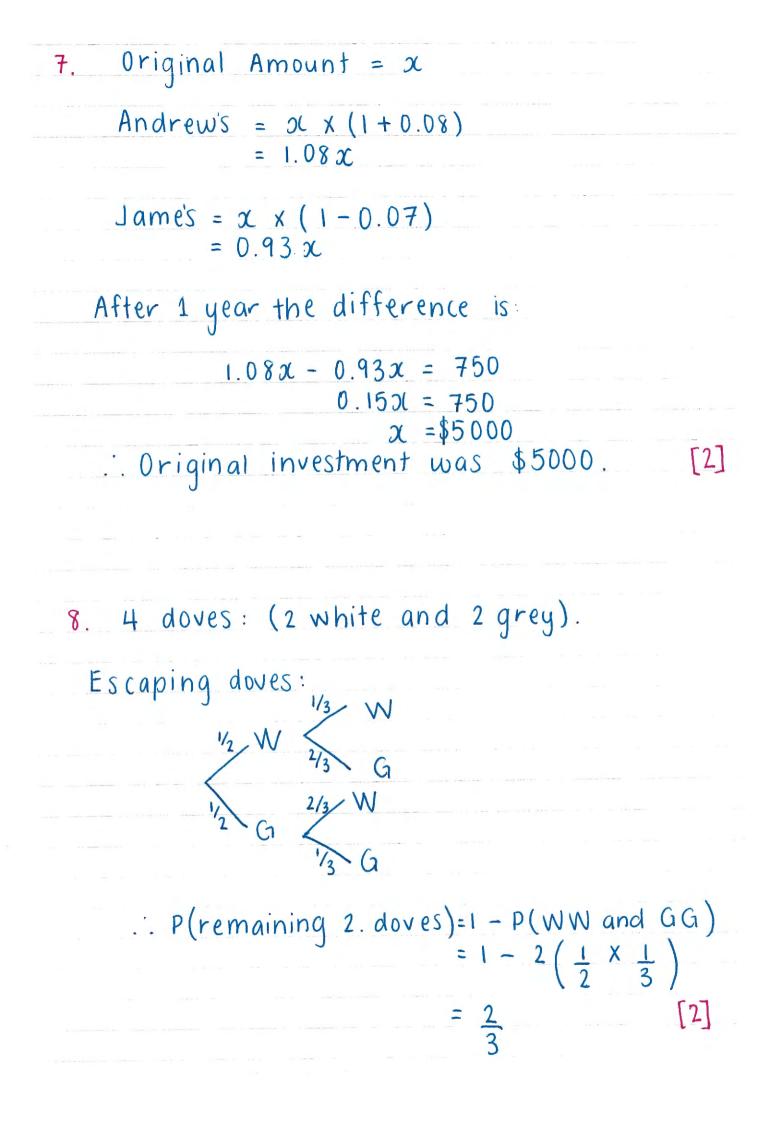


Make sure that you read the question as many students left the answer in exact from when the question clearly started 'correct to 2 decimal places'.



Most students forgot to clange the interest rate from per annum to monthly. (In questions 3 and 4)





Section 4 (15 Marks)

1. Solve by completing the square, leaving answers in exact form: a) $x^2 - 6x - 15 = 0$ (2m) $\begin{array}{c} x^{2} - 6x + (-3)^{2} = 15 + (-3)^{2} \\ (x - 3)^{2} = 15 + 9 \\ = 24 \\ x - 2 = \pm \sqrt{24} \end{array}$ $\alpha - 3 = \pm \sqrt{24}$ **b)** $5x^2 - 9x + 2 = 0$ $(x = \frac{9}{10})^2 = \frac{-2}{5} + \frac{8}{100}$ $\mathcal{X} = \frac{9}{10} = \frac{1}{2} \int \frac{41}{100}$ 2. a) Expand $(a-b)(a^2+ab+b^2)$ no izmarks for this question $= a^3 + a^3b + ab^3 - a^2b - ab^2 - b^3$ $= a^3 - b^3$

b) Hence fully factorise
$$a^3 + \frac{a^2}{b} - b^3 - \frac{b^2}{a}$$

 $a^3 - b^3 + a^2 - b^2$
 $b - a$
 $(a - b)(a^2 + ab + b^2) + a^3 - b^3$
 $a - b^3 - b^3$
 $a - b$

$$(a-b)(a^2 + ab + b^2)$$

$$\left[1 + \frac{1}{ab} \right]$$

$$-1 \quad \text{for eliminating fractic}$$

$$not \quad \text{factorising } a^3 - b^3$$

$$or \quad \text{combining into } 1$$

$$product \quad ot \quad \text{factors}$$

(3m)

3. \$2500 accumulates to \$4400 after 4 years. What is the monthly compound interest rate correct to 1 decimal place?

(3m) 4400 = 2500 (1+r)* $r = \frac{48}{25} \frac{44}{25} - 1$ $\frac{44}{25} = (1+r)^{48}$: r = 1.2% p.m. $48\sqrt{\frac{44}{25}} = (1+r)$ - I for year who ref to monthly, wrong no. of periods 4. Twelve more than half a number is 21 less than the square of the number. If the number is positive find the number. (3m) -1 for incorrect equation $\frac{x}{2} + 12 = x^2 - 21$ solved correctly arithmetic errors $x + 24 = 2x^2 - 42$ $2x^{2} - x - 66 = 0$ DC = 1+JI-4×2×-66 (only need positive answer) 2×2 = 1+ 529 4

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the number is 6

= 1+23 4

= 6

2015 Year 10 Mathematics Half-Yearly: Section 5 Solutions

1. Solve by completing the square, leaving your answer as a simplified surd: $4x^2 - ax - a^2 = 0$ (where a is a constant).

Solution:

$$x^{2} - \frac{ax}{4} + \left(-\frac{a}{8}\right)^{2} = \frac{a^{2}}{4} + \frac{a^{2}}{64}, \text{ or } 4\left(x^{2} - \frac{ax}{4} + \left(-\frac{a}{8}\right)^{2}\right) = a^{2} + \frac{a^{2}}{16},$$

$$\left(x - \frac{a}{8}\right)^{2} = a^{2}\left(\frac{17}{64}\right), \qquad 4\left(x - \frac{a}{8}\right)^{2} = \frac{17a^{2}}{16},$$

$$x - \frac{a}{8} = \frac{\pm a\sqrt{17}}{8}, \qquad \left(x - \frac{a}{8}\right)^{2} = \frac{17a^{2}}{64},$$

$$x = \frac{a(1 \pm \sqrt{17})}{8}, \qquad x - \frac{a}{8} = \frac{\pm\sqrt{17}a}{8},$$

$$x = \frac{a(1 \pm \sqrt{17})}{8}.$$

Comment: The most common error was failing to make the quadratic monic. Those who used the first method (division) were more likely to succeed than those who tried the second (factor) method.

A number of candidates took half the entire *x*-term instead of half the *coefficient* of the *x*-term.

Many candidates failed to simplify their result.

2. Solve, leaving your answer in exact form:

$$\frac{1}{2-7x} + \frac{2}{2+7x} = -1$$

Solution:
$$2 + 7x + 4 - 14x = 49x^2 - 4$$
,
 $49x^2 + 7x - 10 = 0$,
 $x = \frac{-7 \pm \sqrt{49 + 1960}}{98}$,
 $= \frac{-7 \pm 7\sqrt{41}}{98}$,
 $= \frac{-1 \pm \sqrt{41}}{14}$.

Comment: Most errors were the result of not first multiplying through by the common denominator. As above, many candidates failed to simplify their result.

3. Simplify, writing your answer as a single surd: $\sqrt{7 + \sqrt{13}} - \sqrt{7 - \sqrt{13}}$

Solution: Put
$$x = \sqrt{7 + \sqrt{13}} - \sqrt{7 - \sqrt{13}},$$

 $x^2 = 7 + \sqrt{13} + 7 - \sqrt{13} - 2\sqrt{49 - 13},$
 $= 14 - 12,$
So $x = \sqrt{2}$ as $\sqrt{7 + \sqrt{13}} > \sqrt{7 - \sqrt{13}}.$

3

3

Comment: Using a calculator to get an approximation and then just writing down the solution only gained one mark. A clear and fully worked solution was needed for full marks.

4. 200 tickets are sold in a raffle in which 3 prizes are to be drawn by selecting 3 different tickets at random without replacement. Chris buys 3 tickets and wins one of the first two prizes drawn. What is the probability he will now win the 3rd prize?

Solution: One of Chris's tickets taken out, two of the 200 taken out. \therefore Probability = $\frac{2}{198}$, $= \frac{1}{99}$. Comment: The majority of errors resulted from a failure to read the question,

ignoring the "now" in the last sentence. Those who made this mis-interpretation, *i.e.* assuming the examiner wanted the probability of winning only one of the first two prizes plus the third, were given one mark if they correctly calculated

 $\frac{3}{200} \times \frac{197}{199} \times \frac{2}{198} + \frac{197}{200} \times \frac{3}{199} \times \frac{2}{198} = \frac{197}{656\,700}.$

5. The height, h metres, of a cricket ball after being struck by a batsman is given by the equation $h = 1 + \frac{x^2}{40}$ where x metres is the horizontal distance travelled by the ball from the bat. How far, to the nearest metre, does the ball travel before it hits the ground?

Solution: h = 0 at ground level, $\therefore x^2 - 40x - 40 = 0$, $x = \frac{40 \pm \sqrt{1600 + 160}}{2}$, $= 20 \pm 2\sqrt{110}$, $\approx 40.976, -0.976$.

I.e. the ball travels about 41 m.

Comment: Candidates who did not realise that h = 0 at ground level, tended to get lost in the algebra.

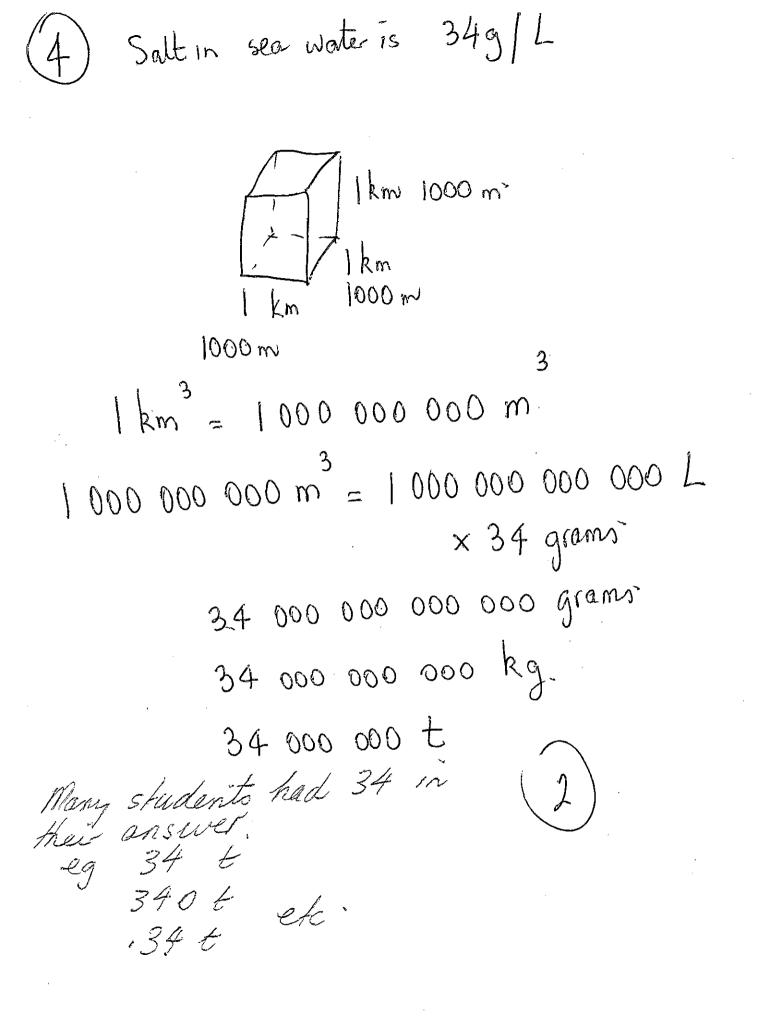
General Comment:	Legibility is a serious problem:
	If the marker cannot read the answer, no mark can be given.
	Many candidates seemed unable to read their own writing:
	e.g. 9 was sometimes confused with a and sometimes with 4.

3

YRIO Half Yearly exam 2015 Question 6 Drake \$1000 as an investment, I year 18 % pa: annually. A = 1000 (1+ 18) = \$1180 17% pa quarterly A= 1000 (1+ 17) 4 = \$1181.15 16] monthly $A = 1000 \left(1 + \frac{16}{12}\right)^{12} = \1172.27 $15\sqrt{7} daily A = 100(1+\frac{15}{365})^{365} = 1161.80 <u>n</u> pe 172 p.a. compounding quarterly (3) students dud not know where to start. Those that tried assigned a value eg \$1000 to work on 2) Mortgage \$ 600,000 4 3 p.a. compound interest reducible monthly. repayments \$2200 month. $\frac{42}{12} = \frac{1}{3}^{2}$ \$ 600,000 + \$2000 - \$2200 = \$ 599800 \$599,800 + \$1999.33 - \$2200 = \$599599.33\$599599.33 + \$1998.66 - \$2200 = \$599397.99Many students wanted a quick fix formula and not table form Impatient to get an answer (3)

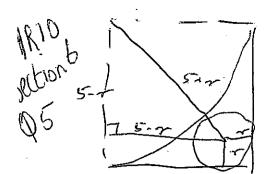
3
$$A \Rightarrow B$$
 30 km $V = \frac{4}{2}$
 $B \Rightarrow A$ 30 km $d = V.t$
let V be the speed from $A \neq B$.
 $halh$.
 $Then (V-4) km|h$ is the speed from $B \neq A$.
 $30 = Vt_1 \int f = 30$
 $30 = Vt_1 \int f = 30$
 $30 = Vt_1 \int f = 30$
 $4t = 30$
 $30 = (V-4)t_2 \int 30 = (V-4) \cdot \frac{1}{20}$
 $t_1 = \frac{30}{V}$
 $t_2 = \frac{30}{V-4}$
 $30 = (1+t_2, 4)$
 $4t = 30$
 $1+t_2 = \frac{30}{V} + \frac{30}{V-4} = t_1 + t_2, 4$
 $4t = \frac{30}{V} + \frac{30}{V-4} = t_1 + t_2, 4$
 $4t = \frac{30}{V} + \frac{30}{V-4} = \frac{1}{V-4}$
 $t_1 + t_2 = \frac{60}{8} + \frac{1}{2} = 8$
 $\frac{30(V-4) + 30V}{VV-4} = \frac{8}{1}$

Students had $\frac{30}{8} = 3\frac{3}{4}hn$ A-> B trouble putting all the data into 33 ho B-7A equation form and "establishing as link to them solve a quadratic total 6 hrs + 90 mm This 30 mins So took 8 hours' $\frac{30}{V} + \frac{30}{V-4} = \frac{8}{1}$ 30(v-4) + 30v = 8v(v-4)30V-120+30V = 8V-32V $60V - 120 = 8V^2 - 32V$ $8v^2 - 92v + 120 = 0$ 92 ± 18464-4×8×120 initial speed was initial speed was 16 = 92 ± 14624 16 $= 92 \pm 68$ 16 $= \frac{92+68}{16}, \frac{92-68}{16}$ 12 = 10



)0 anea sipuae 10×10 = 100 u² 10 10 10 TIR = 2STIUradius lag circle R=5 length diagonal square is $\sqrt{10^2 + 10^2}$ $= \sqrt{200}$ $= 10 \sqrt{2}.$ diameter big arde is 10. Squarea = 100 cm² Big Circle ara = 25TT am2 $5 + r + \sqrt{2}r = 5\sqrt{2}$ 7(1+52)= 52-5 $r = \frac{5}{1+\sqrt{2}}$ 0.8579 Small circle areas= 4TT 2 cm2 Residue = 100-25TT-4TT-2 = 12.2122 cm 2

Bodly answered.



2(5-1)2 = (5-1) $1 + 2n^2 = 25 \pm 10r + n$ 50-30=1900-100 15 ± 10,52 = 29.14, 0.8579 Butris · ~ = 0.8579 (from John Jian in 10 MaB). Hence thes rest of the question can be obtained.