



SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

Year 10

Half Yearly Examination 2016

Stage 5.3 Mathematics

General Instructions

- Working time – 90 minutes
- Write using black or blue pen.
- Approved calculators may be used.
- All necessary working **MUST** be shown in every question if full marks are to be awarded.
- If more space is required, clearly write the number of the QUESTION on one of the back pages and answer it there. Indicate that you have done so.
- Clearly indicate your class by placing an X, next to your class

- All answers should be presented in simplest exact form, unless otherwise directed.
- Marks may not be awarded for untidy or badly arranged work.

Examiner: *A.M. Gainford*

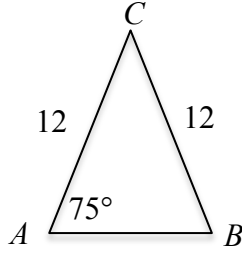
NAME:

Class	Teacher	
10 A	Ms Kilmore	
10 B	Mr Choy	
10 C	Ms Millar	
10 D	Mr Wang	
10 E	Mr Fuller	
10 F	Ms Ward	
10 G	Mr Parker	

Question	Mark
1	/15
2	/16
3	/15
4	/15
5	/14
6	/11
7	/14
Total	/100

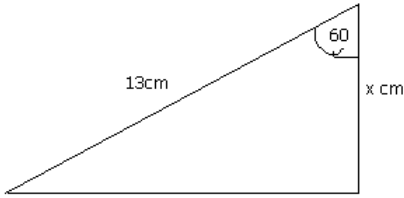
Question 1. (15 marks)

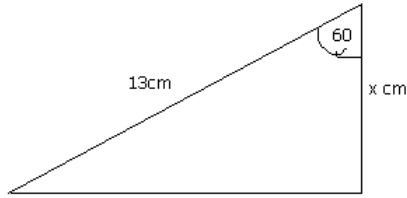
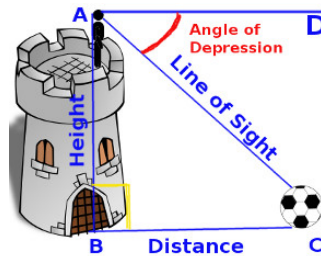
		Answers
(a) 1	Express $\frac{2}{11}$ as an exact decimal.	
(b) 1	Find $\sqrt[3]{\frac{27 \times 0.216}{8}}$ exactly.	
(c) 1	Simplify $\frac{(x^2y)^3}{x^3y}$	
(d) 1	Tom, Dick and Harry wish to share \$1200 in ratio 10:5:1. How much does Dick receive?	
(e) 1	A regular polygon has interior angles 156° . How many sides does it have?	
(f) 1	Express 2 576 798 in scientific notation, correct to 4 significant figures.	
(g) 1	Simplify $(4x - 6) - 3(x - 4)$	
(h) 1	Express $0.\dot{2}5\dot{2}$ as a common fraction in lowest terms.	

		Answers
(i)	Use your calculator to find the values of the following, correct to 4 decimal places:	
1	(i) $\cos 27^\circ$	(i)
1	(ii) $\tan 51^\circ 38'$	(ii)
(j)	Given that measurements are in centimetres, find the area of $\triangle ABC$ correct to 2 decimal places:	
1		
(k)	Solve $3x - 4 = 5 - x$.	
1		
(l)	Find the point intersection of the lines $x + 2y = 4$ and $x - y = -2$.	
1		
(m)	Simplify $\frac{3}{\sqrt{5}-2} + \frac{2}{\sqrt{5}+2}$.	
1		
(n)	Find where the line $2x + 4y = 9$ crosses the y -axis.	
1		

Question 2. (16 marks)

		Answers
(a) 2	A factory has a ratio of male to female workers of 5:4. If $\frac{1}{3}$ of the men, and $\frac{1}{4}$ of the women join the union, what fraction of the whole staff belong to the union?	
(b) 1	A shop buys a wide-screen television for \$800, and sells it for \$1500. Calculate the profit as a percentage of the selling price.	
(c) 1	Simplify $\left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right)\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right)$	
(d) 1	A triangular prism has a base of side 8 cm and altitude 9 cm. The height of the prism is 20 cm. Find its volume.	
(e) 2	Bill borrows \$1000 from a loan shark, and is charged 12% per month compound interest. How much does he owe after 3 months?	
(f) 1	Factorise completely: $a^2 - 4b^2 + a - 2b$	
(g) 1	Bill buys a new Apple computer for \$3500. For tax purposes it depreciates at 60% per annum. Find its worth after two years.	

<p>(h) 2</p>	<p>Fred, standing atop a small tower, sees a soccer ball at C. If AB is 12m, and BC is 10m, and $AD \parallel BC$ calculate the angle of depression, to the nearest degree.</p>	
<p>(i) 1</p>	<p>Solve $\frac{2x-1}{3} - \frac{x-2}{2} = 1$</p>	
<p>(j) 2</p>	<p>What is the equation of the line which has x and y intercepts at 4 and -3 respectively.</p>	
<p>(k) 1</p>	<p>Find the exact value of x:</p> 	
<p>(l) 1</p>	<p>A salesman earns \$46 commission for a particular sale, at a commission rate of 8%. What was the selling price of the item?</p>	

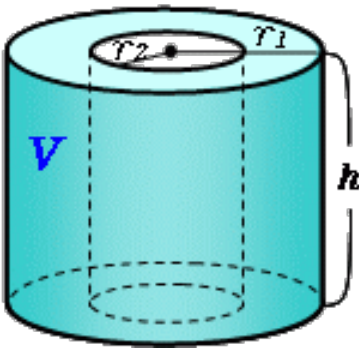


Question 3. (15 marks)

(a)	Solve the following quadratic equations:	
1	(i) $3x^2 = 15x$	(i)
1	(ii) $24x^2 = 46x - 10$	(ii)
(b)	If $x = 1$, find the value of:	
1	$3^x + 3^{1-x} + 3^{x-2}$	
(c)	A right circular cylinder has height 20 cm and base radius 4 cm.	
1	(i) Find the volume in cubic centimetres, in terms of π .	(i)
1	(ii) Find the total surface area in square centimetres in terms of π .	(ii)
(d)	Find, to the nearest degree, the smallest angle of the triangle ABC , where $a = 6$ cm, $b = 10$ cm, and $c = 8$ cm.	
1		

		Answers
(e)	<p>The weight loss (in kg) of 20 randomly selected people on a special diet over three weeks is: 8 5 9 6 6 7 4 5 5 6 8 6 7 7 7 6 6 4 5 5</p> <p>2 (i) Make a frequency distribution table of the data, including cumulative frequency.</p> <p>2 (ii) Calculate the mean, and median.</p>	<p>(i)</p> <p>(ii) Mean= Median=</p>
(f)	<p>Solve these quadratic equations by factorisation:</p> <p>1 (i) $2x^2 + 7x + 6 = 0$</p> <p>1 (ii) $6x^2 + 11x - 10 = 0$</p>	<p>(i)</p> <p>(ii)</p>
(g)	<p>Fred takes a home loan for \$240 000 over 20 years. He pays equal monthly instalments of \$1779.67.</p> <p>1 (i) Find the total amount repaid.</p> <p>1 (ii) Find the amount of interest paid.</p> <p>1 (iii) Use the simple interest formula to calculate the equivalent flat rate of interest per annum.</p>	<p>(i)</p> <p>(ii)</p> <p>(iii)</p>

Question 4. (15 marks)

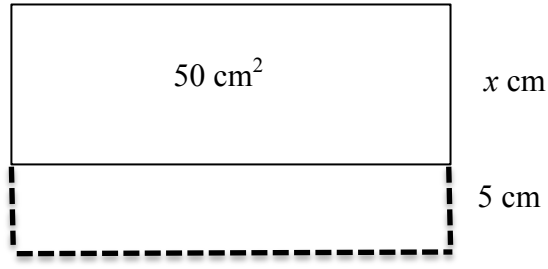
		Answers
<p>(a)</p> <p>2</p> <p>The hollow cylinder at right is made of “diecast”, of density 1.74 g / cm^3. The dimensions are: $h = 9 \text{ cm}$, $r_1 = 5 \text{ cm}$ and $r_2 = 2.5 \text{ cm}$ By first finding the volume, calculate the mass of the cylinder, in grams correct to 2 decimal places.</p>		<p>Volume:</p> <p>Mass:</p>
<p>(b)</p> <p>1</p> <p>(i) $x^2 + 4x + 1 = 0$</p> <p>1</p> <p>(ii) $x^2 - 3x - 5 = 0$</p>		
<p>(c)</p> <p>1</p> <p>(i) Find the total value of the investment at the end of five years.</p> <p>1</p> <p>(ii) State the amount of interest earned over the five years.</p> <p>1</p> <p>(iii) Find the simple interest rate which would have produced the same amount of interest.</p>		<p>(i)</p> <p>(ii)</p> <p>(iii)</p>

		Answers
(d)	For each equation, determine the discriminant, and state the number of distinct real solutions.	
2	(i) $x^2 + x - 6 = 0$	(i)
2	(ii) $x^2 - 3x + 9 = 0$	(ii)
2	(iii) $2x^2 - 12x + 18 = 0$	(iii)
(e)	Find a if $\sqrt{a} = \sqrt{27} + \sqrt{12}$.	
2		

Question 5. (14 marks)

		Answers				
(a) 2	If one letter is taken from each of the words 'BREAD' and 'BUTTER' at random. What is the probability that the two letters are the same?					
(b) 1 2	<p>A combination lock has a combination of 4 digits as illustrated below.</p> <div style="text-align: center;"> <table border="1" style="display: inline-table; border-collapse: collapse;"> <tr> <td style="width: 20px; height: 20px; text-align: center;">*</td> <td style="width: 20px; height: 20px; text-align: center;">*</td> <td style="width: 20px; height: 20px; text-align: center;">*</td> <td style="width: 20px; height: 20px; text-align: center;">*</td> </tr> </table> </div> <p>(i) How many combinations are possible?</p> <p>(ii) Alex has forgotten his combination but he remembers that:</p> <ul style="list-style-type: none"> ➤ the first number is odd. ➤ the second number is even and ➤ the third number is greater than 6. <p>What is the maximum number of combinations he may need to try before he could open the lock?</p>	*	*	*	*	<p>(i)</p> <p>(ii)</p>
*	*	*	*			

(c)



The rectangle above has area 50 cm^2 , and width $x \text{ cm}$.

1

(i) Find the length of the rectangle in terms of x .

(i)

2

(ii) The rectangle is extended by 5 cm to form a square. Form a quadratic equation and find the value of x .

(ii)

(d)

Researchers have developed a new test for the measles virus and have tested it on a group of patients for whom it is already known whether they have the disease or not. The results are shown in the table below.

	Positive Test Result	Negative Test Result	Totals
Patients with Measles	16	4	20
Patients without Measles	24	Value A	380
Totals		Value B	400

2

(i) Write down the missing values A and B at right.

(i)

A=

B=

1

(ii) What percentage of patients had the measles?

(ii)

2

(iii) What is the chance that a patient with measles will be correctly identified by this test?

(iii)

1

(iv) How many of the tests gave the wrong result?

(iv)

Question 6. (11 marks)

		Answers
(a)	Use an appropriate method to solve each of these equations:	
2	(i) $4x^2 - 9 = 0$	
2	(ii) $3x^2 - 2x - 1 = 0$	
2	(iii) $x^2 + 6x - 5 = 0$	
2	(iv) $\frac{x+1}{x} = x$	
(b)	A real estate agent charges the following scale of commission for properties sold: $2\frac{1}{2}\%$ of the first \$220 000 value 2% of the next \$100 000 value 1% thereafter	
1	(i) How much commission does an agent earn for selling a house for \$300 000?	(i)
2	(ii) An agent earned \$9900 commission. Find the price of the house he sold.	(ii)

Question 7. (14 marks)

		Answers
(a)	Solve these equations, each of which is reducible to a quadratic:	
2	(i) $x^4 - 13x^2 + 36 = 0$	(i)
2	(ii) $4^x - 9(2^x) + 8 = 0$	(ii)
(b)	A fixed term deposit of \$60 000 is established for ten years at $7\frac{1}{2}\%$ per annum compound interest, compounding annually. 2 Calculate the balance at the end of the ten years.	
(c)	The quadratic equation $x^2 + 4x - 1 = 0$ has two distinct solutions. Find:	
2	(i) The average of the two solutions.	(i)
2	(ii) The product of the two solutions.	(ii)

<p>(c)</p> <p>2</p> <p>2</p>	<p>A pack of 52 cards consists of four suits with 13 cards in each suit.</p> <p>(i) One card is drawn from the pack and kept on the table. A second card is drawn and placed beside it on the table. What is the probability that the second card is from a different suit to the first?</p> <p>(ii) The two cards are replaced and the pack shuffled. Four cards are chosen from the pack and placed side by side on the table. What is the probability that these four cards are all from different suits?</p>	<p>(i)</p> <p>(ii)</p>
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This is the end of the paper.



SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2016

Year 10 Half Yearly

Advanced Mathematics

Solutions

Question	Teacher
Q1	PSP
Q2	AW
Q3	AF
Q4	AYW
Q5	JM
Q6	EC
Q7	BK

Question 1

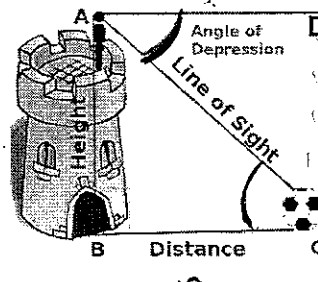
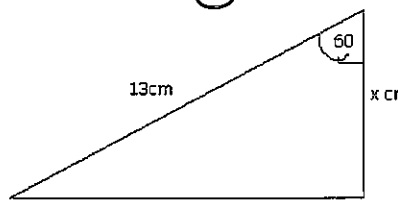
SOLUTIONS

(a)	Express $\frac{2}{11}$ as an exact decimal.	
1	$\frac{2}{11} = \frac{18}{99} = 0.\dot{1}8$	0.1 $\dot{8}$
(b)	Find $\sqrt[3]{\frac{27 \times 0.216}{8}}$ exactly.	
1	$\sqrt[3]{\frac{27 \times 0.216}{8}} = \sqrt[3]{0.729} = 0.9$	0.9
(c)	Simplify $\frac{(x^2y)^3}{x^3y}$	
1	$\frac{(x^2y)^3}{x^3y} = \frac{x^6y^3}{x^3y} = x^3y^2$	x^3y^2
(d)	Tom, Dick and Harry wish to share \$1200 in ratio 10 : 5 : 1. How much does Dick receive?	
1	Dick receives $\frac{5}{16}$ of \$1200 i.e. $\frac{5}{16} \times 1200 = 375$ of \$1200	\$375
(e)	A regular polygon has interior angles 156° . How many sides does it have?	
1	The exterior angles of a (convex) polygon is 360° . If the interior angle is 156° , the exterior angle is 24° . \therefore Number of sides = $360 \div 24 = 15$	15 sides
(f)	Express 2 576 798 in scientific notation, correct to 4 significant figures.	
1	$\underbrace{2\ 576\ 798}_{4\text{ sf}} \div 2\ 577\ 000 = 2.577 \times 10^6$	2.577×10^6
(g)	Simplify $(4x - 6) - 3(x - 4)$	
1	$(4x - 6) - 3(x - 4) = 4x - 6 - 3x + 12 = x + 6$	$x + 6$
(h)	Express $0.\dot{2}5\dot{2}$ as a common fraction in lowest terms.	
1	$0.\dot{2}5\dot{2} = \frac{252}{999} = \frac{28}{111}$. ALTERNATIVELY Let $x = 0.\dot{2}5\dot{2} \Rightarrow 1000x = 252.\dot{2}5\dot{2}$ On subtracting, $999x = 252$ and so $x = \frac{252}{999}$.	$\frac{28}{111}$

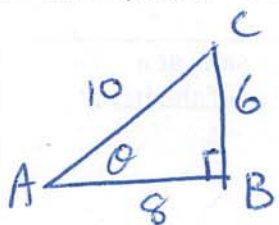
<p>(i)</p> <p>1</p> <p>(i) $\cos 27^\circ$</p> <p>1</p> <p>(ii) $\tan 51^\circ 38'$</p> <p>No half marks given for this question</p>	<p>Use your calculator to find the values of the following, correct to 4 decimal places:</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;">DEGREES mode</div>	<p>(i) 0.8910</p> <p>(ii) 1.2632</p>
<p>(j)</p>	<p>Deleted</p>	
<p>(k)</p> <p>1</p>	<p>Solve $3x - 4 = 5 - x$.</p> <p>$\therefore 4x = 9$ $\therefore x = 2.25$</p>	<p>$x = 2\frac{1}{4}, \frac{9}{4}, 2.25$</p>
<p>(l)</p> <p>1</p>	<p>Find the point intersection of the lines $x + 2y = 4$ and $x - y = -2$.</p> <p>$x + 2y = 4 \Rightarrow y = -\frac{1}{2}x + 2$ $x - y = -2 \Rightarrow y = x + 2$</p> <p>The point of intersection is at the y-intercept.</p>	<p>(0, 2)</p>
<p>(m)</p> <p>1</p>	<p>Simplify $\frac{3}{\sqrt{5}-2} + \frac{2}{\sqrt{5}+2}$.</p> $\frac{3}{\sqrt{5}-2} + \frac{2}{\sqrt{5}+2} = \frac{3(\sqrt{5}+2) + 2(\sqrt{5}-2)}{(\sqrt{5}-2)(\sqrt{5}+2)}$ $= \frac{3\sqrt{5} + 6 + 2\sqrt{5} - 4}{5-4}$ $= 5\sqrt{5} + 2$	<p>$5\sqrt{5} + 2$</p>
<p>(n)</p> <p>1</p>	<p>Find where the line $2x + 4y = 9$ crosses the y-axis.</p> <p>y-intercepts are when $x = 0$</p> <p>$\therefore 4y = 9$ $\therefore y = 2.25$</p>	<p>$y = 2\frac{1}{4}, \frac{9}{4}, 2.25$</p>

Question 2. (16 marks)

		Answers
(a) 2	<p>A factory has a ratio of male to female workers of 5:4. If $\frac{1}{3}$ of the men, and $\frac{1}{4}$ of the women join the union, what fraction of the whole staff belong to the union?</p> <p>Very poorly done.</p> <p>30% 2/3 10 men 6 women 16/34</p>	$\frac{8}{27}$
(b) 1	<p>A shop buys a wide-screen television for \$800, and sells it for \$1500. Calculate the profit as a percentage of the selling price.</p> <p>Reasonably well done.</p> <p>$\frac{700}{1500} \times 100$</p>	$46\frac{2}{3}\%$
(c) 1	<p>Simplify $\left(\frac{1-\sqrt{3}}{2}\right)\left(\frac{1+\sqrt{3}}{2}\right)$</p> <p>$= \frac{1}{4} + \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} - \frac{3}{4}$</p> <p>Poorly done.</p>	$-\frac{1}{2}$
(d) 1	<p>A triangular prism has a base of side 8 cm and altitude 9 cm. The height of the prism is 20 cm. Find its volume.</p> <p>Well done.</p> <p>$A_{\Delta} = \frac{1}{2}bh$ $= 36 \text{ cm}^2$</p> <p>$V = Ah$ $= 36(20)$</p>	720 cm^3
(e) 2	<p>Bill borrows \$1000 from a loan shark, and is charged 12% per month compound interest. How much does he owe after 3 months?</p> <p>Well done</p> <p>$A = P(1+r)^n$ $= 1000(1+0.12)^3$</p>	\$1404.93
(f) 1	<p>Factorise completely: $a^2 - 4b^2 + a - 2b$</p> <p>$(a+2b)(a-2b) + (a-2b)$</p> <p>$(a-2b)[a+2b+1]$</p> <p>Very poorly done</p>	
(g) 1	<p>Bill buys a new Apple computer for \$3500. For tax purposes it depreciates at 60% per annum. Find its worth after two years.</p> <p>Very well done.</p> <p>$A = P(1-r)^n$ $= 3500(0.4)^2$</p>	\$560

<p>(h)</p> <p>2</p>	<p>Fred, standing atop a small tower, sees a soccer ball at C. If AB is 12m, and BC is 10m, and $AD \parallel BC$ calculate the angle of depression, to the nearest degree.</p> <p>$\tan \theta = \frac{\text{OPP}}{\text{ADJ}}$ $= \frac{12}{10} \quad \theta = \tan^{-1}\left(\frac{12}{10}\right)$</p> 	<p>well done</p> <p>50°</p>
<p>(i)</p> <p>1</p>	<p>Solve $\frac{2x-1}{3} - \frac{x-2}{2} = 1$</p> <p>$(4x-2) - (3x-6) = 6$ $x+4 = 6$</p>	<p>Poorly done</p> <p>$x=2$</p>
<p>(j)</p> <p>2</p>	<p>What is the equation of the line which has x and y intercepts at 4 and -3 respectively.</p> <p>$x=4, y=0 \quad (4,0)$ $x=0, y=-3 \quad (0,-3)$</p> <p>$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$ $\frac{y-0}{x-4} = \frac{-3-0}{0-4}$</p>	<p>$y = \frac{3}{4}(x-4)$ $y = \frac{3x-12}{4}$ $y = \frac{3x}{4} - 3$</p>
<p>(k)</p> <p>1</p>	<p>Find the exact value of x:</p> <p>$\cos 60^\circ = \frac{x}{13}$ $\frac{1}{2} = \frac{x}{13}$</p>  <p>Generally well done</p>	<p>$3x - 4y - 12 = 0$</p> <p>$x = \frac{13}{2}$ $= 6\frac{1}{2}$</p>
<p>(l)</p> <p>1</p>	<p>A salesman earns \$46 commission for a particular sale, at a commission rate of 8%. What was the selling price of the item?</p> <p>$\\$46 = 8\%$ $5.75 = 1\%$</p> <p>Very well done</p>	<p>\$575</p>

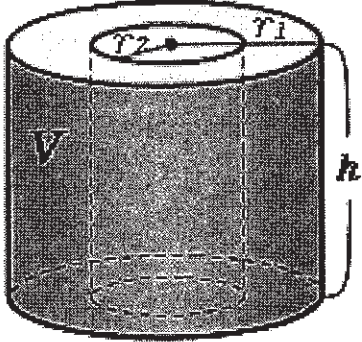
Question 3. (15 marks)

<p>(a)</p> <p>1</p> <p>1</p>	<p>Solve the following quadratic equations:</p> <p>(i) $3x^2 = 15x$</p> <p>(ii) $24x^2 = 46x - 10$</p> <p>$3x^2 - 15x = 0$ $3x(x - 5) = 0$</p> <p>$24x^2 - 46x + 10 = 0$ $12x^2 - 23x + 5 = 0$ $12x^2 - 20x - 3x + 5 = 0$ $4x(3x - 5) - (3x - 5) = 0$ $(3x - 5)(4x - 1) = 0$</p>	<p>(i) $x = 0, 5$</p> <p>(ii) $x = \frac{1}{4}, \frac{5}{3}$</p>
<p>(b)</p> <p>1</p>	<p>If $x = 1$, find the value of:</p> <p>$3^x + 3^{1-x} + 3^{x-2} = 3^1 + 3^{1-1} + 3^{1-2}$</p>	<p>$\frac{13}{3}$</p>
<p>(c)</p> <p>1</p> <p>1</p>	<p>A right circular cylinder has height 20 cm and base radius 4 cm.</p> <p>(i) Find the volume in cubic centimetres, in terms of π.</p> <p>(ii) Find the total surface area in square centimetres in terms of π.</p>	<p>(i) $V = \pi r^2 h$ $= \pi (4)^2 (20)$ $= 320\pi$</p> <p>(ii) $SA = 2\pi r^2 + 2\pi r h$ $= 2\pi (4)^2 + 2\pi (4)(20)$ $= 192\pi$</p>
<p>(d)</p> <p>1</p>	<p>Find, to the nearest degree, the smallest angle of the triangle ABC, where $a = 6$ cm, $b = 10$ cm, and $c = 8$ cm.</p>  <p>$\tan \theta = \frac{6}{8}$ $\theta = \tan^{-1}\left(\frac{3}{4}\right)$ $\approx 37^\circ$</p>	<p>37°</p>

		Answers																												
(e)	The weight loss (in kg) of 20 randomly selected people on a special diet over three weeks is: 8 5 9 6 6 7 4 5 5 6 8 6 7 7 7 6 6 4 5 5																													
2	(i) Make a frequency distribution table of the data, including cumulative frequency.	(i) <table border="1"> <thead> <tr> <th>x</th> <th>Tally</th> <th>f</th> <th>C.f</th> </tr> </thead> <tbody> <tr> <td>4</td> <td> </td> <td>2</td> <td>2</td> </tr> <tr> <td>5</td> <td> </td> <td>5</td> <td>7</td> </tr> <tr> <td>6</td> <td> </td> <td>6</td> <td>13</td> </tr> <tr> <td>7</td> <td> </td> <td>4</td> <td>17</td> </tr> <tr> <td>8</td> <td> </td> <td>2</td> <td>19</td> </tr> <tr> <td>9</td> <td> </td> <td>1</td> <td>20</td> </tr> </tbody> </table>	x	Tally	f	C.f	4		2	2	5		5	7	6		6	13	7		4	17	8		2	19	9		1	20
x	Tally	f	C.f																											
4		2	2																											
5		5	7																											
6		6	13																											
7		4	17																											
8		2	19																											
9		1	20																											
2	(ii) Calculate the mean, and median.	(ii) Mean = 6.1 Median = 6																												
(f)	Solve these quadratic equations by factorisation:																													
1	(i) $2x^2 + 7x + 6 = 0$ $\begin{array}{r} x \quad \quad 12 \\ + \quad \quad 7 \\ \hline \end{array}$	$2x^2 + 3x + 4x + 6 = 0$ $x(2x+3) + 2(2x+3) = 0$ $(2x+3)(x+2) = 0$	(i) $x = -\frac{3}{2}, -2$																											
1	(ii) $6x^2 + 11x - 10 = 0$ $\begin{array}{r} x \quad \quad -60 \\ + \quad \quad 11 \\ \hline \end{array}$	$6x^2 + 15x - 4x - 10 = 0$ $3x(2x+5) - 2(2x+5) = 0$ $(2x+5)(3x-2) = 0$	(ii) $x = -\frac{5}{2}, \frac{2}{3}$																											
(g)	Fred takes a home loan for \$240 000 over 20 years. He pays equal monthly instalments of \$1779.67.																													
1	(i) Find the total amount repaid.	$1779.67 \times 12 \times 20 =$	(i) \$427120.80																											
1	(ii) Find the amount of interest paid.	$427120.80 - 240000 =$	(ii) \$187120.80																											
1	(iii) Use the simple interest formula to calculate the equivalent flat rate of interest per annum.	$I = Prt$ $187120.80 = 240000r(20)$ $r = 0.0389835...$	(iii) 3.9%																											

$$r \approx 3.9\% \text{ p.a.}$$

Question 4. (15 marks)

		Answers
(a)	<p>The hollow cylinder at right is made of "diecast", of density 1.74 g/cm^3.</p> <p>The dimensions are: $h = 9 \text{ cm}$, $r_1 = 5 \text{ cm}$ and $r_2 = 2.5 \text{ cm}$</p> <p>By first finding the volume, calculate the mass of the cylinder, in grams correct to 2 decimal places.</p>  <p>1) $V = \pi h (R^2 - r^2)$ $= \pi \times 9 \times (5^2 - 2.5^2) = 168.75 \pi$</p> <p>ii) $\text{Mass} = \text{Density} \times \text{Volume}$ $= 1.74 \times \text{Volume}$</p>	<p>Volume: 530.14 cm^3 (2d.p.)</p> <p>Mass: 922.45 grams</p>
(b)	<p>Solve these equations by completing the square:</p> <p>1 (i) $x^2 + 4x + 1 = 0$ $x^2 + 4x + 2^2 = -1 + 2^2$ $(x + 2)^2 = 3$ $x + 2 = \pm \sqrt{3}$</p> <p>1 (ii) $x^2 - 3x - 5 = 0$ $x^2 - 3x + (1.5)^2 = 5 + (1.5)^2$ $(x - 1.5)^2 = 7.25$ $x - 1.5 = \pm \sqrt{7.25}$</p>	<p>1) $x = -2 \pm \sqrt{3}$</p> <p>1) $x = 1.5 \pm \sqrt{7.25}$ $= \frac{3}{2} \pm \sqrt{\frac{29}{4}}$ $= \frac{3 \pm \sqrt{29}}{2}$</p>
(c)	<p>Jason invested \$100 000 for five years at 7% p.a. compounding annually.</p> <p>1 (i) Find the total value of the investment at the end of five years. $\\$100000 (1.07)^5$</p> <p>1 (ii) State the amount of interest earned over the five years. $\\$140255.17 - \\100000</p> <p>1 (iii) Find the simple interest rate which would have produced the same amount of interest. $I = PRT$ $\\$40255.17 = \\$100000 \times R \times 5$</p>	<p>(i) $\\$140255.17$</p> <p>(ii) $\\$40255.17$</p> <p>(iii) $\approx 8.05\%$ (2d.p.)</p>

$$R = \frac{\$40255.17}{\$500000} \times 100\%$$

		Answers
(d)	For each equation, determine the discriminant, and state the number of distinct real solutions.	
2	(i) $x^2 + x - 6 = 0$ $\Delta = b^2 - 4ac$ $\Delta = 1 - 4 \times 1 \times -6$ $= 25$ (1)	(i) 2 solutions (1)
2	(ii) $x^2 - 3x + 9 = 0$ $\Delta = (-3)^2 - 4 \times 1 \times 9$ $= -27$ (1)	(ii) No ^{real} solution (1)
2	(iii) $2x^2 - 12x + 18 = 0$ $\Delta = (-12)^2 - 4 \times 2 \times 18$ $= 0$ (1)	(iii) 1 solution (1)
(e)	Find a if $\sqrt{a} = \sqrt{27} + \sqrt{12}$. RHS = $3\sqrt{3} + 2\sqrt{3}$ $= 5\sqrt{3}$ (1) $= \sqrt{75}$ (1)	$\therefore a = 75$ (1)

Notes

(i) (ii) & (iii) Students lost half a mark if they didn't leave their answer to 2 d.p. due to it is a money question.

(i) (iii) Students lost half a mark if they didn't write decimal places rounding.

Question 5

$$\begin{aligned} \text{(a)} \quad P(\text{two letters the same}) &= P(EE) + P(BB) + P(RR) \\ &= 3 \times \left(\frac{1}{5} \times \frac{1}{6} \right) \\ &= \frac{3}{30} \\ &= \frac{1}{10} \end{aligned} \quad [2]$$

$$\begin{aligned} \text{(b) i)} \quad 10^4 &= 10 \times 10 \times 10 \times 10 \\ &= 10\,000 \end{aligned} \quad [1]$$

$$\text{ii)} \quad \left\{ \begin{array}{l} \text{1st digit} \\ 1, 3, 5, 7, 9 \end{array} \right\} \left\{ \begin{array}{l} \text{2nd digit} \\ 0, 2, 4, 6, 8 \end{array} \right\} \left\{ \begin{array}{l} \text{3rd digit} \\ 7, 8, 9 \end{array} \right\} \left\{ \begin{array}{l} \text{4th digit} \\ 0-9 \end{array} \right\}$$

The maximum number of combinations possible

$$\begin{aligned} &= 5 \times 5 \times 3 \times 10 \\ &= 750 \end{aligned} \quad [2]$$

(Zero is an even number)

$$\begin{aligned} \text{(c) i)} \quad A &= x \times y \quad (\text{where } y \text{ is the length}) \\ 50 &= xy \\ \therefore y &= \frac{50}{x} \end{aligned} \quad [1]$$

$$\begin{aligned} \text{ii)} \quad x + 5 &= \frac{50}{x} \\ x(x + 5) &= 50 \\ x^2 + 5x - 50 &= 0 \\ (x + 10)(x - 5) &= 0 \\ x &= 5, -10 \end{aligned}$$

However $x > 0 \therefore x = 5$ [2]

(d) i) $A = 356$
 $B = 360$

[1]
[1]

ii) $\frac{20}{400} = 5\%$

[1]

iii) $\frac{16}{20} = \frac{4}{5}$

[2]

iv) $24 + 4 = 28$

[1]

Question (6)

$$(i) 4x^2 - 9 = 0$$

$$4x^2 = 9$$

$$x = \pm \frac{3}{2}$$

(2)

$$(ii) 3x^2 - 2x - 1 = 0$$

$$3x^2 - 3x + x - 1 = 0$$

$$3x(x-1) + (x-1) = 0$$

$$(3x+1)(x-1) = 0$$

$$\therefore x = -\frac{1}{3}, 1$$

(2)

$$(iii) x^2 + 6x - 5 = 0$$

$$x = \frac{-6 \pm \sqrt{36 + 20}}{2}$$

$$x = -3 \pm \sqrt{14}$$

$$x = \begin{cases} 0.742 \\ -6.742 \end{cases}$$

(2)

Solution (Yr 10)

$$(iv) x+1 = x^2$$

$$x = \frac{-1 \pm \sqrt{5}}{2}$$

$$x = \begin{cases} 0.62 \\ -1.62 \end{cases}$$

(2)

(b) (i)

$$220000 \times 0.025$$

$$= 5500$$

$$+ 80000 \times 0.02$$

$$= 1600$$

\therefore Commission

$$= 5500 + 1600$$

$$= \$7100$$

(1)

(ii)

$$0.025 \times 220000$$

$$= 5500$$

$$0.02 \times 100000$$

$$= 2000$$

$$9900 - 5500 - 2000$$

$$= 2400$$

$$2400 = 1\%$$

$$\therefore 240000 = 100\%$$

\therefore Price of House

$$240000 + 220000$$

$$+ 100000$$

$$= \$56000$$

(2)

Question 7. (14 marks)

		Answers
(a)	Solve these equations, each of which is reducible to a quadratic:	
2	(i) $x^4 - 13x^2 + 36 = 0$ $= (x^2 - 4)(x^2 - 9) = 0$ $x = \pm 2$ or $x = \pm 3$	(i) $x = \pm 2, \pm 3$ ✓✓
2	(ii) $4^x - 9(2^x) + 8 = 0$ $(2^x)^2 - 9(2^x) + 8 = 0$ $(2^x - 8)(2^x - 1) = 0 \Rightarrow 2^x = 8$ or $2^x = 1$	(ii) $x = 3$ or 0 ✓✓ (a) Part (i) generally done well but many students forgot about the negative solutions
(b)	A fixed term deposit of \$60 000 is established for ten years at $7\frac{1}{2}\%$ per annum compound interest, compounding annually.	More than half the students could not do part (ii)
2	Calculate the balance at the end of the ten years. $A = 60000(1 + 0.075)^{10}$ $= \$123661.89$	$\$123661.89$ ✓✓ (b) Nearly all students got this correct
(c)	The quadratic equation $x^2 + 4x - 1 = 0$ has two distinct solutions. Find:	
2	(i) The average of the two solutions. $x = \frac{-4 \pm \sqrt{20}}{2} = \frac{-4 \pm 2\sqrt{5}}{2} = -2 \pm \sqrt{5}$ $\frac{-2 + \sqrt{5} + -2 - \sqrt{5}}{2} = \frac{-4}{2} = -2$	(i) -2 ✓✓
2	(ii) The product of the two solutions. $(-2 + \sqrt{5})(-2 - \sqrt{5}) = 4 - 5 = -1$	(ii) -1 ✓✓

(c) Many mistakes here. In (i) some added and didn't divide by 2. Part (ii) was done poorly and often not attempted. Many students rounded their solutions for rather than leaving as a surd so that calculations were not exact.

(c)	A pack of 52 cards consists of four suits with 13 cards in each suit.	
2	(i) One card is drawn from the pack and kept on the table. A second card is drawn and placed beside it on the table. What is the probability that the second card is from a different suit to the first?	(i) $\frac{39}{51} = \frac{13}{17}$ ✓
2	(ii) The two cards are replaced and the pack shuffled. Four cards are chosen from the pack and placed side by side on the table. What is the probability that these four cards are all from different suits?	(ii) $1 \times \frac{39}{51} \times \frac{26}{50} \times \frac{13}{49}$ $= \frac{13182}{124950}$ $= \frac{2197}{20825}$ ✓

Many students did not realise that the probability of drawing the first card is $52/52$ which $= 1$.

Part (i) was done more successfully than part (ii). Many students did not use 39, 26 and 13 as the number of cards remaining in the different suits after each draw.

This is the end of the paper.