Sydney Boys High School

MOORE PARK, SURRY HILLS

YEAR 10 ADVANCED MATHEMATICS

Half Yearly Examination 2017

General Instructions:

Time Allowed: 90 minutes

- All questions may be attempted.
- **Marks may be deducted for careless or badly arranged work.**
- All working and answers are to be written in this test booklet.
- If you wish to rewrite an answer, draw a line through your faulty answer and rewrite your answer on the back page of this booklet. **Show the number and part of the answer being rewritten.**
- Leave your answers in the simplest exact form, unless otherwise stated.
- Board approved calculators may be used.
- Clearly indicate your class by placing an ✕ next to your class.

Examiner: JM

Name: ______________________________

Class	Teacher	
10**A**	Ms Kilmore	
10**B**	Ms Chan	
10**C**	Ms Evans and Mr Elliott	
10**P**	Ms Ward	
10**L**	Mr Choy	
10**U**	Mr Wang	
10**S**	Mr Gainford	

Question	Marks
1	/ 16
2	/ 16
3	/ 16
4	/ 17
5	/ 15
6	/ 15
7	/ 14
Total	/ 109

Question 1 (16 Marks)

(a) Calculate 68% of $\frac{14}{25}$, leaving your answer as a decimal. **1**

(b) Express your answer to $2017\times(28)^4$ in scientific notation correct to 4 significant figures. **2**

(c) Expand and simplify:

(i) $8x+7-3x$ **1**

(ii) $3(4x-5)+12$ **1**

(iii) $(5-x)(5+x)$ **1**

(iv) $(3x-7)(2x+5)$ **2**

(d) Find the value of k if, $7\sqrt{3}=\sqrt{k}$. **1**

(e) Simplify:

(i) $2\sqrt{3}-5\sqrt{2}+3\sqrt{3}$ **1**

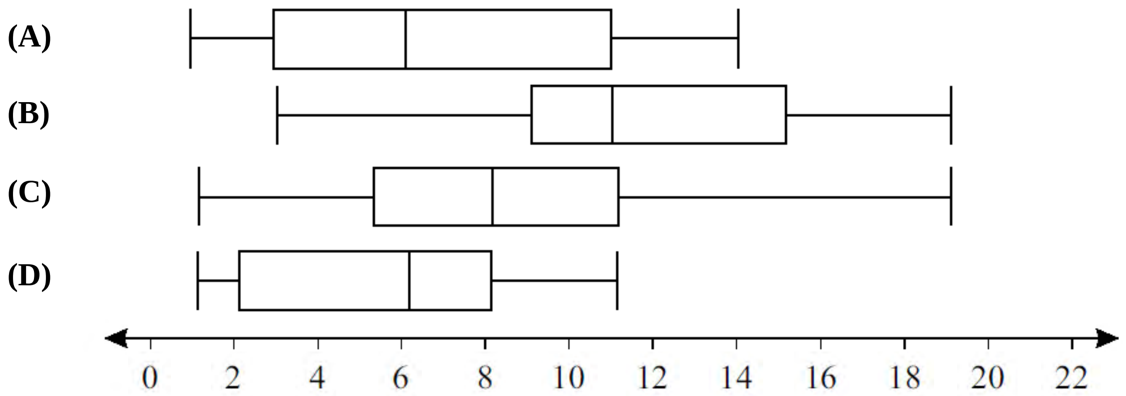
(ii) $3\sqrt{2}\times\sqrt{6}$ **1**

(iii) $\dfrac{4\sqrt{20}}{2\sqrt{12}}$ **1**

(iv) $2\sqrt{2}\left(\sqrt{3}-5\right)$ **1**

(f) At Rhonda High School, 36 students are on either the baseball team, the hockey teams, or both. If there are 25 students on the baseball team and 19 students on the hockey team, how many students play both sports? **2**

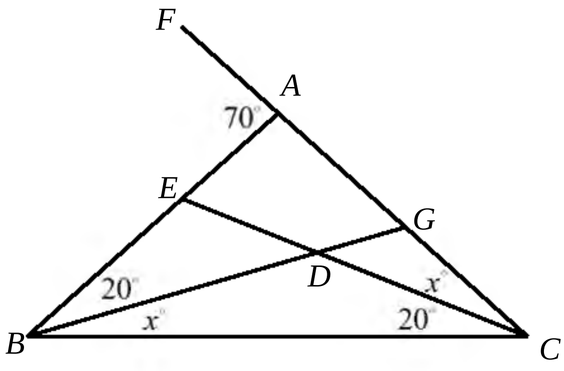
(g) Which of the data sets displayed in the following box-and-whisker plots has the largest interquartile range? **1**



Question 2 (16 Marks)

(a) Simplify $\sqrt{72}+\sqrt{18}$. **2**

(b) Find the value of x, giving reasons? **2**



(c) If $\frac{3}{4}$ of the boys in a junior class use a backpack, and $\frac{2}{5}$ are not in full uniform, find the probability that a boy chosen by lot would be in uniform and using a backpack. **3**

(d) The mean of three numbers, x, y and z is x. What is the mean of y and z? **2**

(e) David is paid at these rates:

Weekday rate	$18.00 per hour
Saturday rate	Time-and-a-half
Sunday rate	Double time

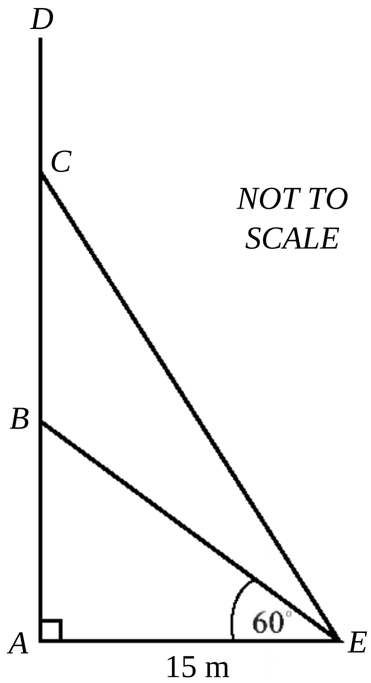
His time sheet for last week is:

	Start	*Finish*	*Unpaid break*
Friday	9.00 am	1.30 pm	30 minutes
Saturday	9.00 am	4.00 pm	1 hour
Sunday	8.00 am	2.00 pm	1 hour

(i) Calculate David's gross pay for last week. **2**

(ii) David decides not to work on Saturdays. He wants to keep his weekly gross pay the same. How many extra hours at the weekday rate must he work? **1**

(f) The diagram shows a radio mast *AD* with two of its supporting wires, *BE* and *CE*. The point *B* is the bisector of *A* and *C*.



(i) Calculate the height *AB* in metres, leaving your answer in exact form. **2**

(ii) Calculate the distance *CE* in metres, correct to one decimal place. **2**

Question 3 (16 Marks)

(a) Given *A* and *B* are the points (1, 2) and (−5, −4).

(i) Find the gradient of *AB*. **1**

(ii) Find the midpoint of *AB*. **1**

(iii) Find the equation of the line that is perpendicular to *AB* and passes through the midpoint of *AB*. **1**

(b) If the sides of a cube are increased by 15%, then what is the percentage increase of the volume? **2**

(c) Factorise completely:

(i) $x^2 - 13x + 42$ **1**

(ii) $a^2 - 4(b+c)^2$ **2**

(iii) $4x^3 + 28x^2 - 9x - 63$ **2**

(iv) $x^4 - 13x^2 + 36$ **2**

(d) Express as a single fraction in the simplest form $\dfrac{2x-3}{2} - \dfrac{x-1}{5}$. **2**

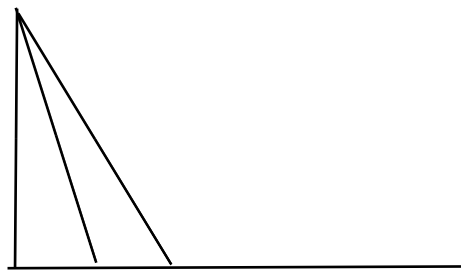
(e) Solve these equations simultaneously. **2**

$$5x + 2y = 16$$
$$x - y = -1$$

Question 4 (17 Marks)

(a) From the top of a cliff 300 metres above sea level. The angles of depression of two boats, out at sea are 28° and 22°.

(i) Draw a diagram representing the above information, and show that the distance from the base of the cliff to the closest boat is $300\tan 62^\circ$. **1**



(ii) Calculate the distance between the two boats, (correct to two decimal places). **2**

(b) Solve the following quadratic equations.

(i) $x^2 - 12x - 64 = 0$ **2**

(ii) $2x^2 - 15x - 11 = 0$ **2**

(iii) $(2x+3)^2 = 4$ **2**

(c) A hot chocolate drink is 6% pure chocolate, by volume. If 12 litres of pure milk are added to 50 litres of this drink, what is the percentage of chocolate in the new drink? **2**

(d) Solve for p if $\dfrac{p-1}{p+1} + \dfrac{1}{p^2-1} = \dfrac{p}{p-1}$. **3**

(e) A gift box is made from a rectangular piece of cardboard that is three times as long as it is wide. 5 cm squares are cut from each corner and the ends are then folded up to make the box. If the box's volume is 4340 cm^3, find the length and width of the cardboard. **3**

Question 5 (15 Marks)

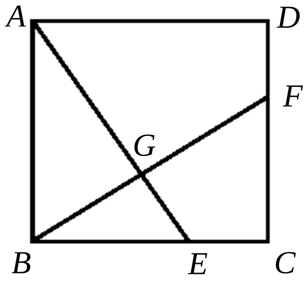
(a) If $x = 3 + \sqrt{8}$.

(i) Show that $x + \dfrac{1}{x} = 6$ **2**

(ii) Hence or otherwise evaluate $x^2 + \dfrac{1}{x^2}$. **2**

(b) Simplify and evaluate $\dfrac{81^{n+1} \times 9^n}{9^{3n+2}}$. **2**

(c) In the square, $ABCD$, E and F are points lying on the sides BC and CD respectively, such that $AE = BF$. Also, AE and BF intersect at G.

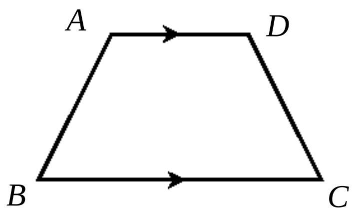


(i) Prove that ΔABE is congruent to ΔBCF. **2**

(ii) Show that ΔBGE is right angled. **2**

(iii) If $CF = 15$ cm and $EG = 9$ cm, find the length of BG. **2**

(d) In the trapezium $ABCD$ has three equal sides $AB = AD = DC$. The base BC is 2 cm less than the sum of the lengths of the other three sides. The distance between the parallel sides is 5 cm. **3**



Calculate the area of trapezoid $ABCD$.

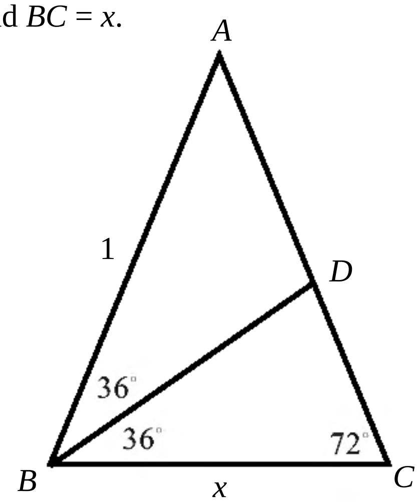
Question 6 (15 Marks)

(a) If $\left(2^4\right)\left(3^6\right)=9\left(6^x\right)$, what is the value of x? **2**

(b) By completing the square, find the values of a, b and c in $2x^2-12x+25=a\left(x-b\right)^2+c$. **3**

(c) Luke and Jayden run a 10-lap race around a housing block. For the first 5 laps Luke ran at an average speed of 2 km/h faster than Jayden. For the last 5 laps Luke ran at an average speed of 2 km/h slower than Jayden. Who won the race? **3**

(d) In the diagram ABC is an isosceles triangle where $\angle ABC = \angle BCA = 72^\circ$ and $AB = AC = 1$. Angle ABC is bisected by BD, and $BC = x$.



(i) Show that triangles ABC and BCD are similar. **2**

(ii) By using (i) find the exact value of x. **2**

(e) In a pack of construction paper, the numbers of blue and red sheets are originally in the ratio 2:7. Each day, Laura uses 1 blue sheet and 3 red sheets. One day, she uses 3 red sheets and the last blue sheet, leaving her with 15 red sheets. How many sheets of construction paper were in the pack originally? **3**

Question 7 (14 Marks)

(a) Ned adds the degree measures of the interior angles of a convex polygon and arrives at a sum of 2017. He then discovers that he forgot to include one angle. What is the size of the forgotten angle? **2**

(b) Three rugs have a combined area of 200 m^2. By overlapping the rugs to cover a floor area of 140 m^2, the area which is covered by exactly two layers of rug is 24 m^2. What area of floor is covered by three layers of rug? **3**

(c) Two identical perfect cubes (similar to dice) each having faces numbered 0, 1, 2, 3, 4, 5 are rolled. A score for the roll is determined as the product of the two numbers on the two uppermost faces. **3**

What is the probability that if the cubes are rolled twice and the scores for each roll are added, what is the probability of a combined score of at least 41?

(d) The *y*-intercepts of three parallel lines are 2, 3 and 4. The sum of the *x*-intercepts of the three lines is 36. What is the slope of these parallel lines? **3**

(e) Solve for x: $\dfrac{x-\sqrt{x+1}}{x+\sqrt{x+1}}=\dfrac{11}{5}$. **3**

END OF EXAMINATION

EXTRA WORKING PAGE 1:

EXTRA WORKING PAGE 2:

SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2017

Year 10 Half Yearly

Advanced Mathematics

Suggested Solutions

Questions	Marker
1	AYW
2	TE
3	AMG
4	EC
5	TE
6	JWC
7	BK

Question 1 (16 Marks)

(a) Calculate 68% of $\frac{14}{25}$, leaving your answer as a decimal. **1**

$0.68 \times 0.56 = 0.3808$

(b) Express your answer to $2017 \times (28)^4$ in scientific notation correct to 4 significant figures. **2**

$1239761152 \doteqdot 1.240 \times 10^9$

(c) Expand and simplify:

(i) $8x + 7 - 3x$ $= 5x + 7$ **1**

(ii) $3(4x-5)+12$ $= 12x - 15 + 12$ **1**

$= 12x - 3$

(iii) $(5-x)(5+x)$ $5^2 - x^2$ **1**

$= 25 - x^2$

(iv) $(3x-7)(2x+5)$ $6x^2 + 15x - 14x - 35$ **2**

$= 6x^2 + x - 35$

(d) Find the value of k if, $7\sqrt{3} = \sqrt{k}$. **1**

$49 \times 3 = k$

$k = 147$

(e) Simplify:

(i) $2\sqrt{3} - 5\sqrt{2} + 3\sqrt{3}$ $= 5\sqrt{3} - 5\sqrt{2}$ **1**

(ii) $3\sqrt{2} \times \sqrt{6}$ $= 3\sqrt{12} = 6\sqrt{3}$ **1**

11

(iii) $\dfrac{4\sqrt{20}}{2\sqrt{12}}$ $= \dfrac{\overset{2}{\cancel{8}}\sqrt{5}}{\cancel{4}\sqrt{3}} = \dfrac{2\sqrt{5}}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{2\sqrt{15}}{3}$ **1**

(Rationalize denominator)

(iv) $2\sqrt{2}\left(\sqrt{3}-5\right)$ $2\sqrt{6} - 10\sqrt{2}$ **1**

(f) At Rhonda High School, 36 students are on either the baseball team, the hockey teams, or both. If there are 25 students on the baseball team and 19 students on the hockey team, how many students play both sports? **2**

B= Baseball

H = Hockey

$N(B \cap H) = N(B) + N(H) - N(B \cup H)$

$= (25+19) - 36$

$= 8$

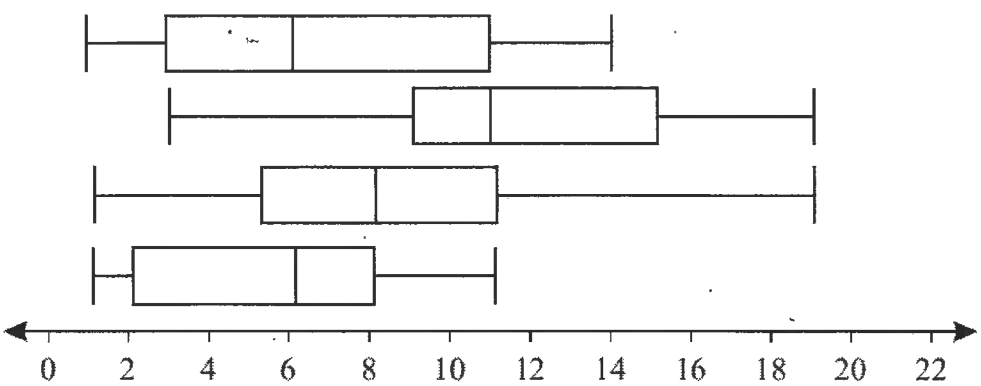
(g) Which of the data sets displayed in the following box-and-whisker plots has the largest interquartile range? **1**

(A)

(B)

(C)

(D)



5

Question 2 (16 Marks)

(a) Simplify $\sqrt{72}+\sqrt{18}$. **2**

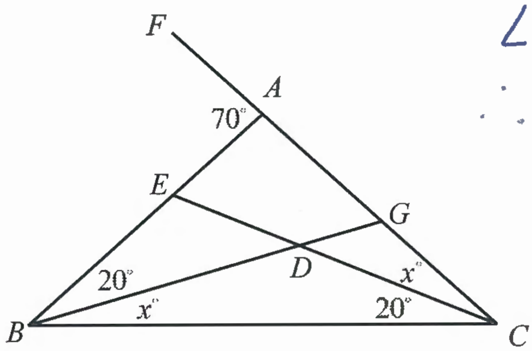
$= \sqrt{36}\times\sqrt{2} + \sqrt{9}\times\sqrt{2}$

$= 6\sqrt{2} + 3\sqrt{2}$

$= 9\sqrt{2}$

-1 not fully simplified.

(b) Find the value of x, giving reasons? **2**



$\angle EAG = 110°$ ($\angle$ sum st line).

$\therefore 110° + 20 + x + 20 + x = 180°$

($\angle$ sum $\triangle$)

$\therefore 2x = 30$

$\therefore \boxed{x = 15}$

no reasons -1

(c) If $\frac{3}{4}$ of the boys in a junior class use a backpack, and $\frac{2}{5}$ are not in full uniform, find the probability that a boy chosen by lot would be in uniform and using a backpack. **3**

$\frac{3}{5} \times \frac{3}{4} = \frac{9}{20}$

most common error not using correct fraction for in uniform

(d) The mean of three numbers, x, y and z is x. What is the mean of y and z? **2**

$\frac{x+y+z}{3} = x$

$x+y+z = 3x$

$\therefore y+z = 2x$

$\frac{y+z}{2} = \frac{2x}{2}$

$\boxed{\frac{y+z}{2} = x}$

no working -1.

(e) David is paid at these rates:

Weekday rate	$18.00 per hour
Saturday rate	Time-and-a-half
Sunday rate	Double time

His time sheet for last week is:

	Start	*Finish*	*Unpaid break*	
Friday	9.00 am	1.30 pm	30 minutes	4
Saturday	9.00 am	4.00 pm	1 hour	6
Sunday	8.00 am	2.00 pm	1 hour	5.

(i) Calculate David's gross pay for last week. **2**

$(18 \times 4) + (18 \times 1.5 \times 6) + (18 \times 2 \times 5)$

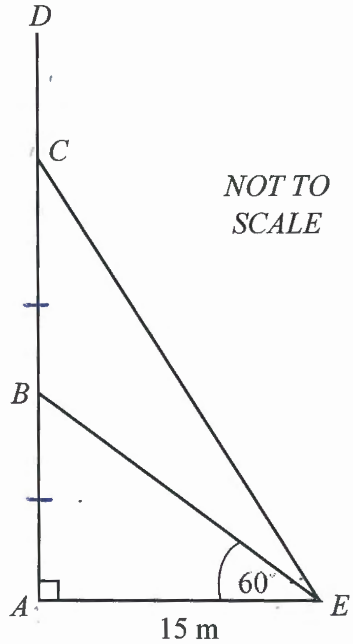
$= \$414$ –

(ii) David decides not to work on Saturdays. He wants to keep his weekly gross pay the same. How many extra hours at the weekday rate must he work? **1**

$6 \times 1.5 = 9$ extra week day hours

this question was poorly done.

(f) The diagram shows a radio mast AD with two of its supporting wires, BE and CE. The point B is the bisector of A and C.



(i) Calculate the height AB in metres, leaving your answer in exact form. **2**

$\tan 60 = \dfrac{AB}{15}$

$AB = 15\sqrt{3}$

–1 not in exact form or leaving as $15 \tan 60°$

(ii) Calculate the distance CE in metres, correct to one decimal place. **2**

$CE^2 = 15^2 + (2 \times 15\sqrt{3})^2$

$CE = \sqrt{2925}$

$CE = 54.1\text{m}$

Year 10 Half Yearly 2017

Question 3 (16 Marks)

(a) Given A and B are the points $(1,2)$ and $(-5,-4)$. **1**

(i) Find the gradient of AB.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-4-2}{-5-1} \\ &= \frac{-6}{-6} \\ &= 1 \end{aligned}$$

[Comment: Generally well answered, but several candidates cancelled the fraction in the second last line to get -1.]

(ii) Find the midpoint of AB. **1**

$x = \frac{1-5}{2} = -2$ $\quad y = \frac{2-4}{2} = -1$ $\quad$ So Mid-Point $(-2,-1)$

[Comment: Again well answered. The most common error was to subtract, rather than add, in the formula.]

(iii) Find the equation of the line that is perpendicular to AB and passes through the midpoint of AB. **1**

$$\begin{aligned} m_\perp &= -1; \quad MP(-2,-1) \\ y - y_1 &= m(x - x_1) \\ y + 1 &= -1(x+2) \\ &= -x - 2 \\ x + y + 3 &= 0 \end{aligned}$$

[Comment: Many failed to simplify the formula correctly, some used the wrong gradient, and most failed to give the answer in general form (but lost no marks for this).]

(b) If the sides of a cube are increased by 15%, then what is the percentage increase of the volume? **2**

$$\begin{aligned} V_2 &= (1.15)^3 \\ &= 1.520875 \end{aligned}$$

$\therefore$ Increase is 52.0875%

[Comment: Generally well answered. The most common error was to state the increase as 152%, or thereabouts.]

(c) Factorise completely: **1**

(i) $x^2 - 13x + 42$

$$= (x-7)(x-6)$$

[Comment: Almost everyone got this right. Some thought they were solving a quadratic equation.]

(ii) $a^2 - 4(b+c)^2$ **2**

$$= (a+2(b+c))(a-2(b+c))$$

[Comment: Many thought they could factorise this further, but they were wrong.]

(iii) $4x^3 + 28x^2 - 9x - 63$ **2**

$$= 4x^2(x+7) - 9(x+7)$$
$$= (4x^2 - 9)(x+7)$$
$$= (2x-3)(2x+3)(x+7)$$

[Comment: The most common error was to stop at the second last line.]

(iv) $x^4 - 13x^2 + 36$ **2**

$$= (x^2-4)(x^2-9)$$
$$= (x-2)(x+2)(x-3)(x+3)$$

[Comment: Generally well answered by those who attempted it (most), but as above some stopped short, and some tried to solve a quartic.]

(d) Express as a single fraction in the simplest form $\frac{2x-3}{2} - \frac{x-1}{5}$. **2**

$$\frac{2x-3}{2} - \frac{x-1}{5} = \frac{5(2x-3) - 2(x-1)}{10}$$
$$= \frac{10x - 15 - 2x + 2}{10}$$
$$= \frac{8x-13}{10}$$

[Comment: Then most common error (it was very common) was to write -2 at the end of the numerator in the second line. How can candidates persist in making this error after three and a half years of advanced maths?]

(e) Solve these equations simultaneously. **2**

$$5x + 2y = 16$$
$$x - y = -1$$

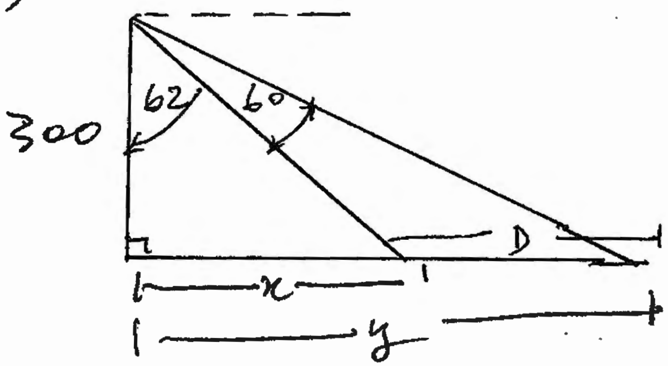
$$
\begin{aligned}
& & 5x + 2y &= 16 \quad \text{---(1)} \\
& & x - y &= -1 \quad \text{---(2)} \\
&\text{2x(2):} & 2x - 2y &= -2 \quad \text{---(3)} \\
&\text{(1)+(3):} & 7x &= 14 \\
& & x &= 2 \\
&\text{Sub in (2):} & 2 - y &= -1 \\
& & -y &= -3 \\
& & y &= 3
\end{aligned}
$$

Solution: $(2,3)$

[Comment: Generally well answered. Some made addition or subtraction errors.]

Question (4).

(i)



$$\frac{x}{300} = \tan 62 \text{ ———— } (1)$$

$$\Rightarrow x = 300 \tan 62^\circ$$

Comment: To get the 1 mk: diagram with angle 62° in the right place and the expression

(ii) $D = y - x$

$= 300(\tan 68^\circ - \tan 62^\circ)$

$= 178.31^\circ$.

Comment: To get 2 mks

i) Correct expression and $D = y - x$

ii) and correctly evaluate to 2d.p.

(b) $x^2 - 12x - 64 = 0$

(i) $(x-16)(x+4) = 0$

$\Rightarrow x = 16, -4$.

(ii) $2x^2 - 15x - 11 = 0$

Apply formula

$$x = \frac{15 \pm \sqrt{313}}{4} \quad \text{2 mks}$$

Comment (I.S.F) and rc

(iii) $(2x+3)^2 = 4$,

$2x + 3 = \pm 2$

$x = -\frac{5}{2}, -\frac{1}{2}$

Comment: 1 mark for each answer

(c)

$0.06 \times 50 = 3L$ (choc.).

$\therefore \frac{3}{50+12} \times \frac{100}{1} = 4.84\%$ (2.d.p).

(d)

$$\frac{(p-1)(p-1)+1}{p^2-1} = \frac{p}{p-1}$$

$(p \neq 1)$

$p^2 - 2p + 2 = p(p+1)$

$-2p + 2 = p$

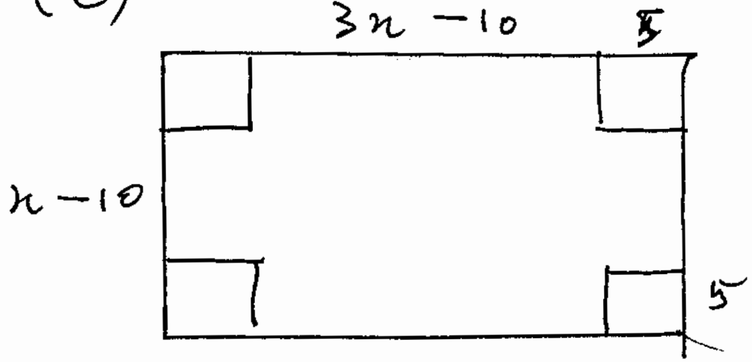
$3p = 2$

$\Rightarrow p = 2/3.$

Comment:

$p \neq 1$ must be acknowledge.

(e)



$V = (3x-10)(x-10).$

$3x^2 - 40x - 768 = 0$

$x = 24, \quad x = \frac{40 \pm \sqrt{10816}}{6}.$

length, width.

$x = 24 \quad \therefore W = 14$

$L = 62$

Comment:

Comment on negative root (inadmissibel

$x = 24$ .... but not L, W asked

Question 5 (15 Marks)

(a) If $x = 3 + \sqrt{8}$.

(i) Show that $x + \dfrac{1}{x} = 6$ **2**

LHS

$= 3 + \sqrt{8} + \dfrac{1}{3+\sqrt{8}}$

$= 3 + \sqrt{8} + \left(\dfrac{1}{3+\sqrt{8}} \times \dfrac{3-\sqrt{8}}{3-\sqrt{8}}\right)$

$= 3 + \sqrt{8} + \left(\dfrac{3-\sqrt{8}}{9-8}\right)$

$= 3 + \sqrt{8} + 3 - \sqrt{8}$

$= 6 \quad = \text{RHS}$

(ii) Hence or otherwise evaluate $x^2 + \dfrac{1}{x^2}$. **2**

$(3+\sqrt{8})^2 + \dfrac{1}{(3+\sqrt{8})^2}$

$= 9 + 6\sqrt{8} + 8 + \dfrac{1}{17+6\sqrt{8}}$

$= 17 + 6\sqrt{8} + \left(\dfrac{1}{17+6\sqrt{8}} \times \dfrac{17-6\sqrt{8}}{17-6\sqrt{8}}\right)$

$= 17 + 6\sqrt{8} + \left(\dfrac{17-6\sqrt{8}}{289-288}\right)$

$= 17 + 6\sqrt{8} + 17 - 6\sqrt{8}$

$= 34$

OR: $\left(x + \dfrac{1}{x}\right)^2 = x^2 + \dfrac{1}{x^2} + 2$

$\therefore x^2 + \dfrac{1}{x^2} = \left(x + \dfrac{1}{x}\right)^2 - 2$

$= 6^2 - 2$

$= 34$

OR: put $(3+\sqrt{8})^2 + \dfrac{1}{(3+\sqrt{8})^2}$ in your calculator

(b) Simplify and evaluate $\dfrac{81^{n+1} \times 9^n}{9^{3n+2}}$. **2**

$81^{n+1} = (9^2)^{n+1}$

$= 9^{2n+2}$

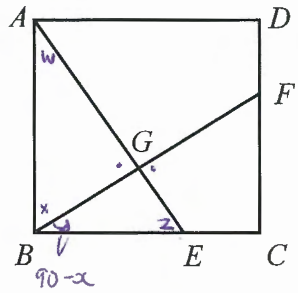
$\dfrac{9^{2n+2} \times 9^n}{9^{3n+2}}$

$= \dfrac{9^{3n+2}}{9^{3n+2}}$

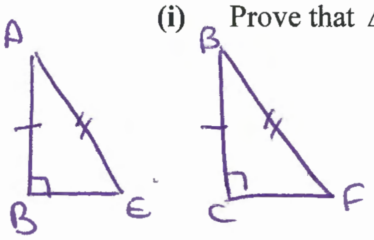
$= 1$

Poorly done

(c) In the square, $ABCD$, E and F are points lying on the sides BC and CD respectively, such that $AE = BF$. Also, AE and BF intersect at G.



(i) Prove that ΔABE is congruent to ΔBCF. **2**



$AB = BC$ (sides of square)

$AE = BF$ (given)

$\angle ABE = 90° = \angle BCF$ (prop of square)

$\therefore \Delta ABE \equiv \Delta BCF$ (RHS)

-1 for wrong $\equiv \Delta$ test.

(ii) Show that ΔBGE is right angled. **2**

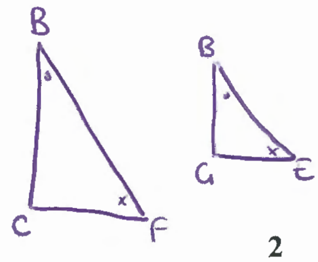
$\angle CBF = \angle GBE$ (common)

$\angle BFC = \angle BEG$ (corresp. $\angle$'s in $\equiv \Delta$'s)

$\therefore \angle BCF = \angle BGE$ ($\angle$ sum Δ)

& $\angle BCF = 90°$

$\therefore \Delta BGE$ is right angled



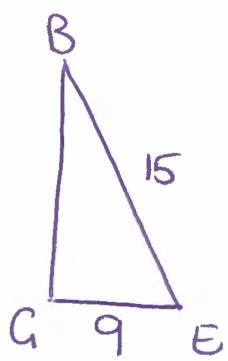
(iii) If $CF = 15$ cm and $EG = 9$ cm, find the length of BG. **2**

$CF = 15 \therefore BE = 15$ (corresp. sides in $\equiv \Delta$'s)

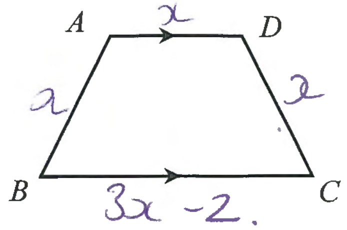
$BG = \sqrt{15^2 - 9^2}$

$= \sqrt{144}$

$= 12$ cm



(d) In the trapezium $ABCD$ has three equal sides $AB = AD = DC$. The base BC is 2 cm less than the sum of the lengths of the other three sides. The distance between the parallel sides is 5 cm. **3**



Calculate the area of trapezoid $ABCD$.

let AB, AD & DC $= x$

$\therefore$ $BC = 3x - 2$

$BE + FC = (3x-2) - x$

$= 2x - 2$

$BE = FC$

$\therefore$ $BE = \frac{2x-2}{2}$

$\boxed{BE = x - 1}$

In ΔABE

$x^2 = 5^2 + (x-1)^2$

$= 25 + x^2 - 2x + 1$

$\therefore$ $2x = 26$

$\boxed{x = 13}$

$\therefore$ $AD = 13$

$BC = 3 \times 13 - 2$

$= 37$

$\therefore$ Area ABCD

$= \frac{1}{2}(37 + 13) \times 5$

$= 125$ cm

1 mark for

$A = \frac{1}{2}(3x - 2 + x) \times 5$

$= 10x - 5$

this question was poorly done.

Question 6 (15 Marks)

(a) If $(2^4)(3^6)=9(6^x)$, what is the value of x? **2**

$$2^4 \times \frac{3^6}{3^2} = 6^x$$

$$2^4 \times 3^4 = 6^x$$

$$6^4 = 6^x$$

$$\therefore x = 4$$

① working

① answer.

(b) By completing the square, find the values of a, b and c in $2x^2-12x+25=a(x-b)^2+c$. **3**

$$2(x^2-6x) = -25$$

✗ No reason to try to solve for 'x'. Find the values of a, b, c.

$$2(x^2-6x+9) = -25+18$$

$$2(x-3)^2 = -7$$

$$\therefore 2x^2-12x+25 = 2(x-3)^2+7$$

$$\therefore a=2,\quad b=3,\quad c=7$$

① comp. squares

① factorised form

① solutions: a, b, c

No marks for a=2 simply by equating without proper completing squares

(c) Luke and Jayden run a 10-lap race around a housing block. For the first 5 laps Luke ran at an average speed of 2 km/h faster than Jayden. For the last 5 laps Luke ran at an average speed of 2 km/h slower than Jayden. Who won the race? **3**

let Jayden run at x km/h

* 1st 5 laps

Luke: $T = \frac{D}{x+2}$ Jayden: $\frac{D}{x} = T$

* Last 5 laps.

Luke: $T = \frac{D}{x-2}$ Jayden: $T = \frac{D}{x}$.

Jayden's time − Luke's time.

$$= \frac{2D}{x} - \left(\frac{D}{x+2} + \frac{D}{x-2}\right)$$

$$= \frac{2D(x^2-4) - D\left(x(x-2)+x(x+2)\right)}{x(x+2)(x-2)}$$

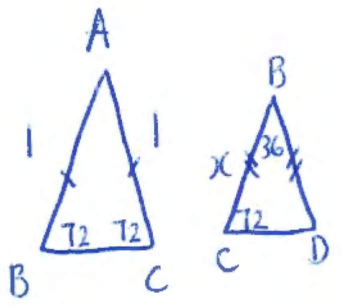
$$= \frac{D\left(\cancel{2x^2}-8-(\cancel{x^2}-\cancel{2x}+\cancel{x^2}+\cancel{2x})\right)}{x(x+2)(x-2)}$$

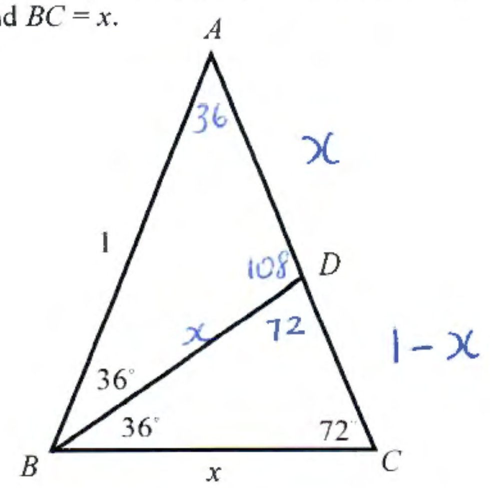
$$= \frac{-8D}{x(x+2)(x-2)} < 0 \quad \therefore \text{Jayden faster}$$

② logical working.

① Jayden

(d) In the diagram ABC is an isosceles triangle where $\angle ABC = \angle BCA = 72^\circ$ and $AB = AC = 1$. Angle ABC is bisected by BD, and $BC = x$.





(i) Show that triangles ABC and BCD are similar. **2**

In $\triangle ABC$, BCD

$\angle BAC = 36^\circ$ ($\angle$ sum of $\triangle$) ① reason

$\angle BDC = 72^\circ$ ($\angle$ sum of $\triangle$) ① equiangular w/ appropriate reasoning

$\therefore \triangle ABC \sim \triangle BCD$ equiangular

(ii) By using (i) find the exact value of x. **2**

$\angle BDC = 108^\circ$ (st $\angle$) and $\angle BAD = 36^\circ$ ($\angle$ sum of $\triangle$)

$\therefore \triangle BAD$ is an isos $\triangle$ $\therefore AD = x$ (sides oppo equal $\angle$s)

$\therefore \dfrac{1}{x} = \dfrac{x}{1-x}$ (matching sides in proportion in $\sim \triangle$s) ① ratio

$x^2 = 1 - x$ ie $x^2 + x - 1 = 0$ ① answer

$\therefore x = \dfrac{-1+\sqrt{5}}{2}$ $(x > 0)$ $-\frac{1}{2}$ for neg. soln.

(e) In a pack of construction paper, the numbers of blue and red sheets are originally in the ratio 2:7. Each day, Laura uses 1 blue sheet and 3 red sheets. One day, she uses 3 red sheets and the last blue sheet, leaving her with 15 red sheets. How many sheets of construction paper were in the pack originally? **3**

Each day 1 blue sheet for x number of days

$\therefore$ there are x blue sheets. ① eqn. or equivalent

$\dfrac{x}{15+3x} = \dfrac{2}{7}$ $\therefore 7x = 30 + 6x$ ① soln.

$x = 30$ ① final answer.

Total amount of papers originally is

$\dfrac{30}{2} \times 9 = 135$ sheets OK for ③ if they work through the ratios successfully

Question 7 (14 Marks)

(a) Ned adds the degree measures of the interior angles of a convex polygon and arrives at a sum of 2017. He then discovers that he forgot to include one angle. What is the size of the forgotten angle? **2**

Angle sum of n-sided polygon $= 180(n-2)$

Let $n=13$, $\angle sum = 180\times 11 = 1980 \Rightarrow$ too small

Let $n=14$, $\angle sum = 180 \times 12 = 2160$

Let $x =$ missed angle

$\angle Sum - x = 2017$

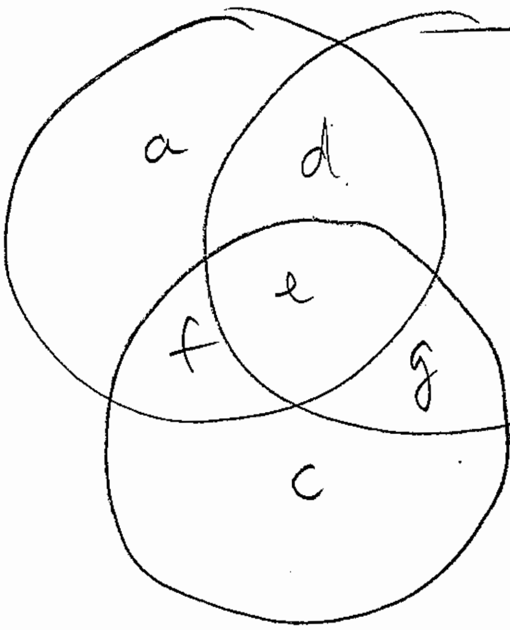
$2160 - x = 2017$

$\underline{x = 143^\circ}$ (convex $\Rightarrow$ all $\angle s < 180^\circ$)

Some students were able to get this question out. Many did not read the question correctly and assumed the polygon was regular.

(b) Three rugs have a combined area of 200 m^2. By overlapping the rugs to cover a floor area of 140 m^2, the area which is covered by exactly two layers of rug is 24 m^2. What area of floor is covered by three layers of rug? **3**

Use Venn Diagram



$A_1 + A_2 + A_3 = 200$

$\Rightarrow a+b+c+2(d+f+g)+3e = 200$ (1)

Also $d+f+g = 24$ (2)

Sub (2) in (1)

$\Rightarrow a+b+c+48+3e = 200$

$a+b+c+3e = 152$ (3)

Also $\underline{a+b+c} + \underline{d+f+g} + e = 140$ (4)

Sub (2) in (4) $\Rightarrow a+b+c+24+e = 140$

$a+b+c+e = 116$ (5)

(3) $-$ (5) $\Rightarrow 2e = 36$

$\underline{e = 18m^2}$ = area covered by 3 rugs.

This question was done very poorly. The shape of the rugs is irrevelant. A Venn diagram is the best approach.

(c) Two identical perfect cubes (similar to dice) each having faces numbered 0, 1, 2, 3, 4, 5 are rolled. A score for the roll is determined as the product of the two numbers on the two uppermost faces. **3**
What is the probability that if the cubes are rolled twice and the scores for each roll are added, what is the probability of a combined score of at least 41?

2 \ 1	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	6	8	10
3	0	3	6	9	12	15
4	0	4	8	12	16	20
5	0	5	10	15	20	25

$P(\text{score} \geq 41)$ roll twice

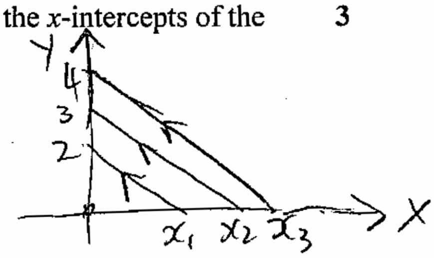
$= P(25 \text{ and } 16 \text{ or } 25 \text{ and } 20 \text{ or } 25 \text{ and } 25)$

$= \left(\frac{1}{36} \times \frac{1}{36} \times 2\text{ways}\right) + \left(\frac{1}{36} \times \frac{2}{36} \times 2\text{ways}\right) + \left(\frac{1}{36} \times \frac{1}{36} \times 1\text{way}\right)$

$= \frac{7}{1296}$

1 mark was awarded for the sample space. If students had correct denominator and the correct combinations of 16,20 and 25 then they scored 2 marks. The last mark was for correctly finding the number of ways each combination could occur.

(d) The y-intercepts of three parallel lines are 2, 3 and 4. The sum of the x-intercepts of the three lines is 36. What is the slope of these parallel lines? **3**



$y = mx + 2$ (1)

$y = mx + 3$ (2)

$y = mx + 4$ (3)

When $y = 0$, $x_1 = \frac{-2}{m_1}$, $x_2 = \frac{-3}{m}$, $x_3 = -\frac{4}{m}$

$\Rightarrow \frac{-2}{m} + \frac{-3}{m} + \frac{-4}{m} = 36$

$\times m \Rightarrow -2 + -3 + -4 = 36m$

$m = \frac{-9}{36} = -\frac{1}{4}$

Most students did not get this question out. Marks awarded as above.

(e) Solve for x: $\dfrac{x-\sqrt{x+1}}{x+\sqrt{x+1}} = \dfrac{11}{5}$. **3**

$$5x - 5\sqrt{x+1} = 11x + 11\sqrt{x+1}$$

$$-6x = 16\sqrt{x+1}$$

Squaring $\Rightarrow$ $36x^2 = 256(x+1)$

$$36x^2 - 256x - 256 = 0$$

$$9x^2 - 64x - 64 = 0$$

$$x = \frac{64 \pm \sqrt{64^2 + 4(9)(64)}}{18}$$

$$x = 8 \text{ or } \frac{-8}{9}$$

Again most students did not get this question out. Squaring or multiplying by the conjugate and then expanding was generally not helpful.

END OF EXAMINATION