Sydney Boys High School

MOORE PARK, SURRY HILLS

YEAR 10 ADVANCED MATHEMATICS

Half Yearly Examination 2018

General Instructions:

- All questions may be attempted.
- Write using black pen.
- Marks may be deducted for careless or badly arranged work.
- All working and answers are to be written in this test booklet.
- If you wish to rewrite an answer, draw a line through your faulty answer and rewrite your answer on the back pages of this booklet. Show the number and part of the answer being rewritten
- Leave your answers in the simplest exact form, unless otherwise stated.
- Board approved calculators may be used.
- Clearly indicate your class by placing an **X** next to your class.

Time Allowed: 90 minutes

Reading Time: 5 minutes

Examiner: *B.Kilmore*

Name: _____

Class	Teacher	
10**A**	Mr Wang	
10**B**	Ms Ward	
10**C**	Ms Evans	
10**P**	Mr Fuller	
10**L**	Ms Millar	
10**U**	Ms Chan	
10**S**	Mr Choy	

Section	Marks
A	/ 15
B	/ 13
C	/ 13
D	/ 15
E	/ 14
F	/ 14
G	/ 15
Total	/ 99

Section A (15 Marks)

a) Express 0.375 as a percentage. **(1m)**

b) Find 48% of 1 km 400m. **(1m)**

c) Convert 160km/h to m/s **(2m)**

d) Find x if $x:6=5:16$ **(1m)**

e) Calculate the simple interest on \$600 for 9 months at 4% p.a. **(1m)**

f) Round 4.06352 to 4 significant figures. **(1m)**

g) If $a=-3$ find the value of $-a^2-a$ **(1m)**

h) True or False? All squares are parallelograms. **(1m)**

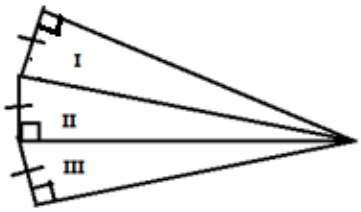
i) Solve the following equations:

(i) $(6+m)(7-m)=0$ **(1m)**

(ii) $2p^2-18=0$ **(2m)**

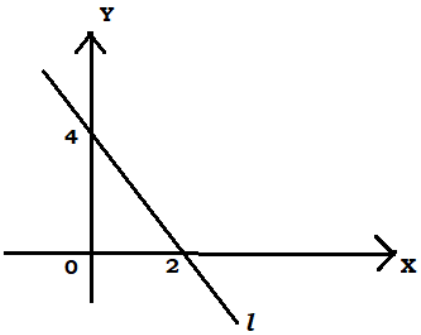
(iii) $8x^2=12x$ **(1m)**

j) Which triangles are congruent? **(1m)**



A. I and II only
B. I and III only
C. II and III only
D. I, II and III

k) Find the equation of the line *l*. **(1m)**



Section B (13 Marks)

a) Solve this quadratic equation by factorising

$6a^2 - a - 1 = 0$ **(2m)**

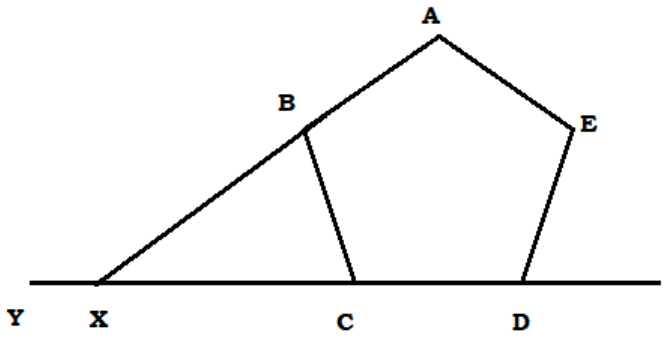
b) What is the percentage increase on an investment of $6000 which compounds annually for 5 years at 9% pa? **(2m)**

c) A laptop purchased for $3200 depreciates at the rate of 16% p.a.

(i) What is the value of the laptop after 4 years? **(1m)**

(ii) How long until the laptop is worth less than $500? **(1m)**

d) The diagram shows a regular pentagon ABCDE. The sides AB and DC are produced to meet at X. The point Y lies on DCX produced.



Find the size of $\angle YXB$ giving reasons. **(3m)**

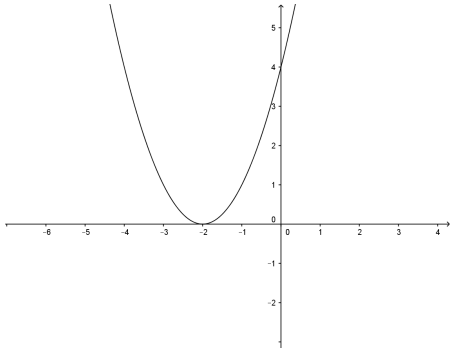
e) Find the distance between two points A(-3,7) and B(-8,-3) correct to 3 decimal places. **(1m)**

f) Match each equation with one of the graphs below. **(3m)**

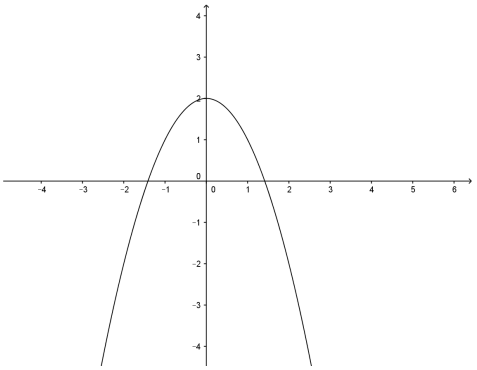
(i) $y=(x+2)^2$ _____ (ii) $y=-(x+2)^2$ _____ (iii) $y=x^2+2$ _____

(iv) $y=-x^2+2$ _____ (v) $y=(x-2)^2$ _____ (vi) $y=-(x-2)^2$ _____

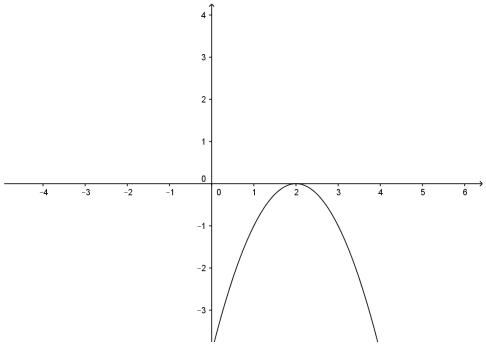
A



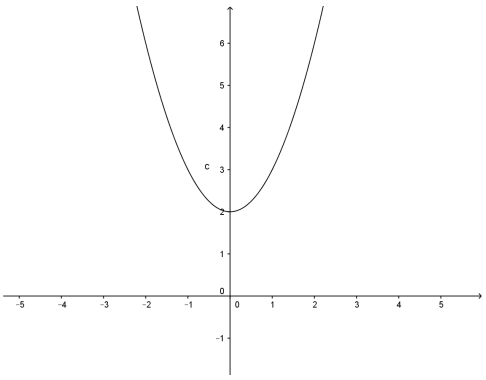
B



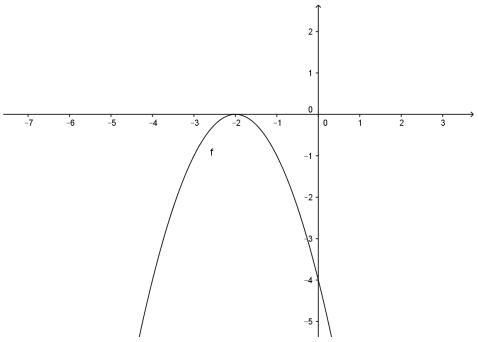
C



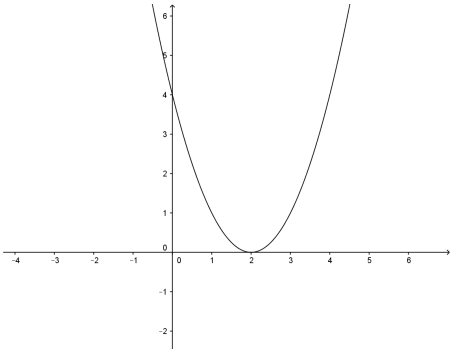
D



E



F



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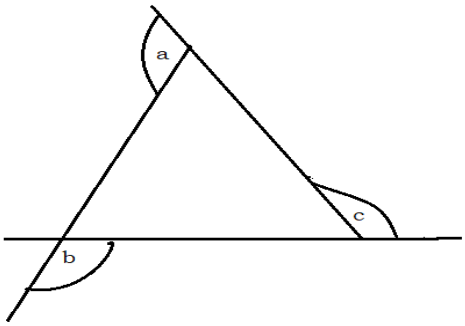
Section C (13 Marks)

a) State the value of $a+b+c$ in this diagram? **(1m)**



b) Solve by completing the square leaving your answer in exact surd form.

$$x^2-12x+10=0$$ **(2m)**

c) Solve by using the quadratic formula, leaving your answer in exact form.

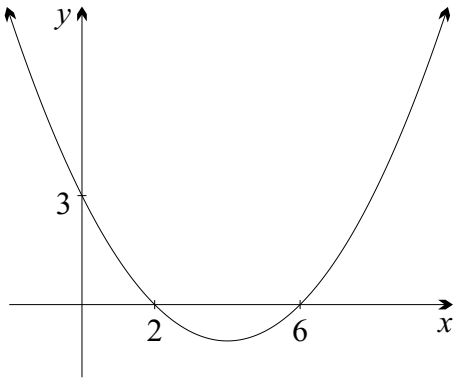
$$2x^2-9x+8=0$$ **(2m)**

d) Consider the quadratic x^2+3x+5.

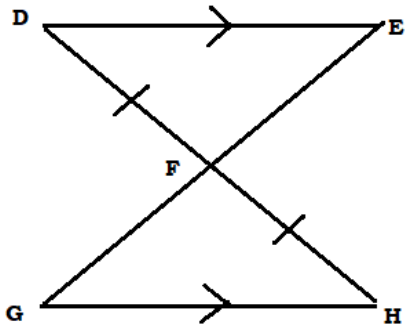
(i) What is the value of $b^2 - 4ac$ for this quadratic? **(1m)**

(ii) Explain what this tells you about the graph of $y=x^2+3x+5$ **(1m)**

e) The graph below shows a parabola with x-intercepts at $(2,0)$ and $(6,0)$. **(2m)**
Its y-intercept Is at $(0,3)$. Determine the equation of the parabola.
You may leave your answer in factorised form.



f) In the diagram below $DE \parallel GH$ and $DF = HF$. **(1m)**
State the congruence test that proves that $\Delta DEF \equiv \Delta HGF$.



g) A is (-2.1) and B is $(4,5)$

(i) Find the midpoint of AB. **(1m)**

(ii) Find the gradient of AB. **(1m)**

(iii) Find the equation of the line which is the perpendicular bisector of AB. **(1m)**

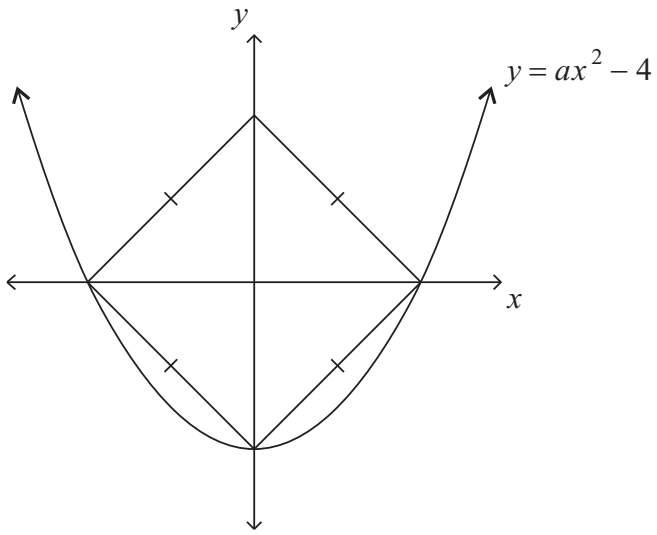
Section D (15 Marks)

a) How much must I invest now to save $50000 for a trip in 3 years time if interest is 9% pa compounding monthly? **(2m)**

b) By using a substitution, or otherwise, solve the following equation correct to 2 decimal places:

$$(x^2-20)^2-7(x^2-20)=-10$$ **(3m)**

c) A square centred on the origin has its vertices on the x and y-axes. The graph of the parabola $y=ax^2-4$, where $a>0$, passes through three of the square's vertices. Find the value of a. **(3m)**

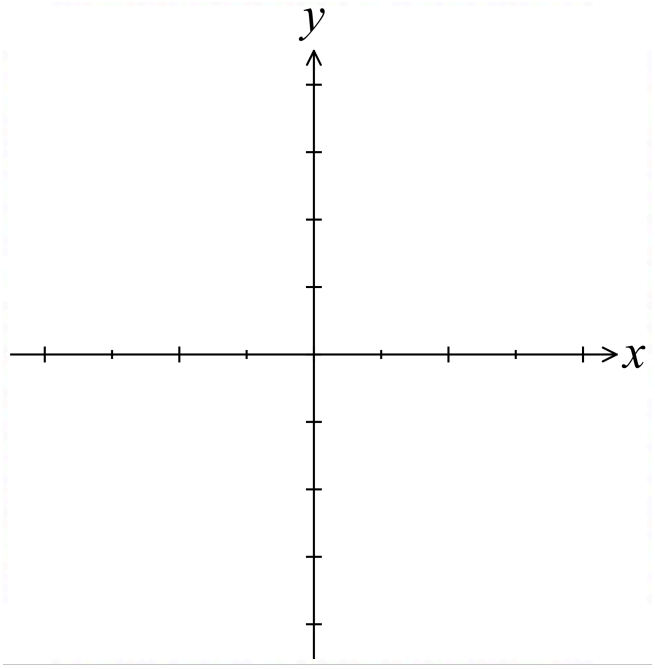


d) Consider the parabola $y = -(x+1)^2 - 2$

(i) What is its vertex? **(1m)**

(ii) What is the y-intercept? **(1m)**

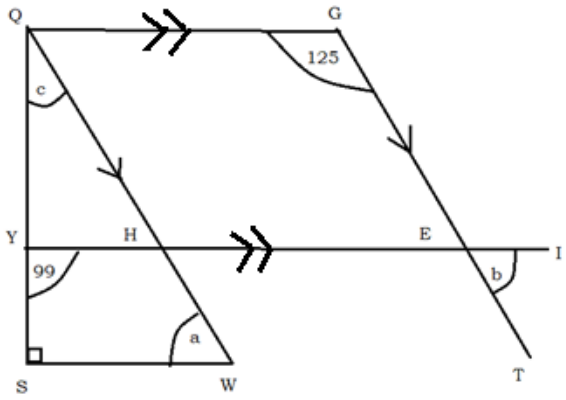
(iii) Sketch the parabola showing these features. **(2m)**



e) Prove, giving all reasons, that the diagonals of a parallelogram bisect each other. **(3m)**

Section E (14 Marks)

a) Find the value of the pronumerals a, b and c. You do NOT need to give reasons. **(3m)**



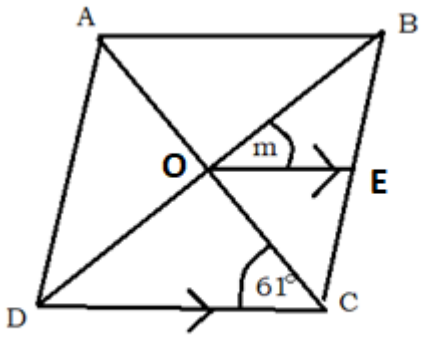
b) Solve, leaving your answer in exact form:

$$\frac{1}{3-5x}+\frac{4}{3+5x}=-1$$ **(3m)**

c) Keith invested $3000 in shares at the start of the year. The shares rose in value by 5% per month for the first 6 months and then fell in value by 5% per month for the next 6 months. Find the value of the shares at the end of the year. **(2m)**

d) A woman invested $40000 at 9% pa compounding monthly for 2 years. At the end of that time she reinvested the capital plus interest at an annual rate that compounded quarterly for 4 years. If the investment has accrued to $56000 at the end of the 6 years, what was the second annual interest rate? **(3m)**

e) ABCD is a rhombus. Find the value of m, giving reasons. **(3m)**

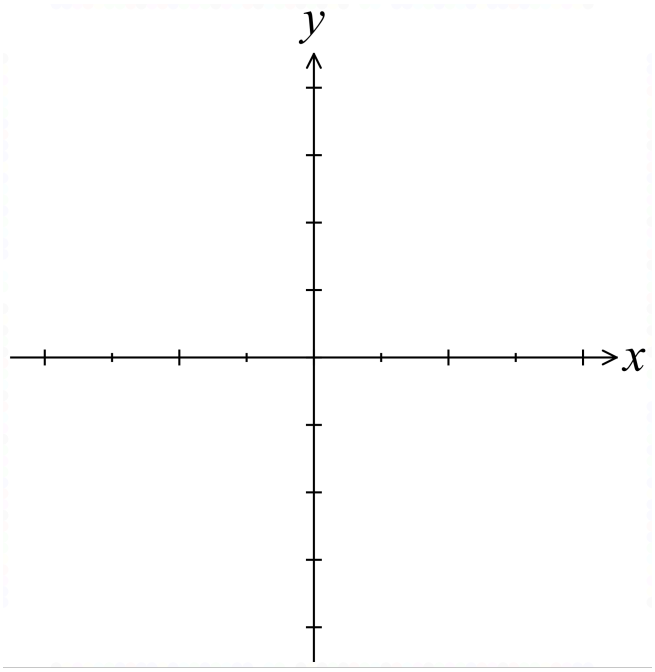


Section F (14 Marks)

a) Solve by completing the square: $3x^2 - 4x - 1 = 0$ **(3m)**

b) Solve the equation $3m - 11\sqrt{m} = 4$ **(3m)**

c) Sketch the graph of $y = -3x^2 - 5x + 2$ showing the x and y intercepts and the vertex. **(3m)**

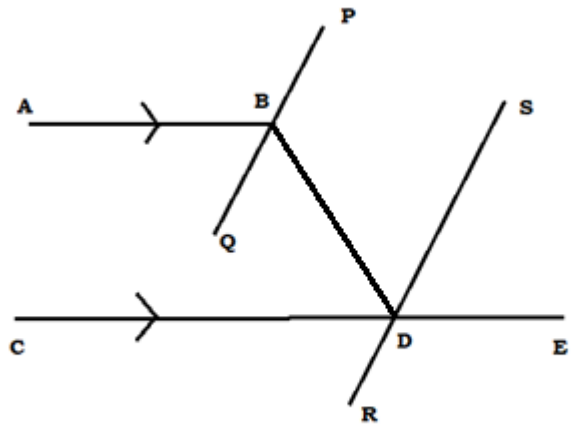


d) David has just bought a boat and has a mortgage of $120000 at 6%p.a. compounding monthly. He has to make payments of $750 per month. How much does he owe after the third payment? **(3m)**

e) Find the values of m for which $4x^2-(6+m)x+1=0$ has only one real root. **(2m)**

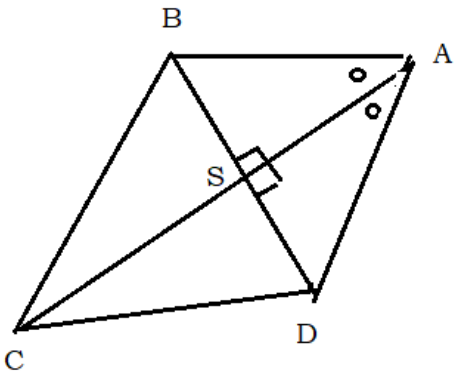
Section G (15 Marks)

a) In the diagram you are given that $AB \parallel CE$. Also PQ bisects $\angle ABD$ and RS bisects $\angle BDE$. Prove that $PQ \parallel RS$, giving reasons. **(3m)**



b) An amount of money compounds for 3 years at 12 % pa compounding 6 monthly and then for 4 years more at 11.75% pa compounding monthly. If the final value is $28 034, find the original amount invested. **(3m)**

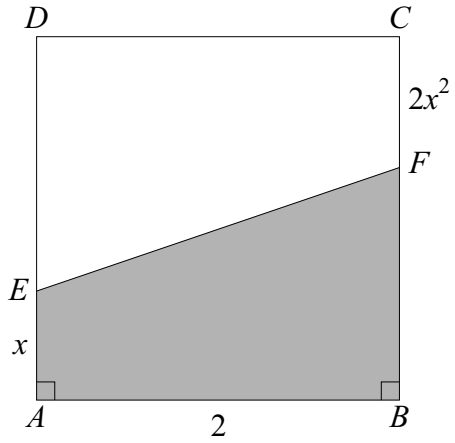
c) ABCD is a quadrilateral in which the diagonals BD and AC intersect at right angles. Also $\angle BAS = \angle DAS$.



(I) By using congruent triangles, prove that $AD = BA$ giving all reasons. **(2m)**

(II) Hence prove that $CD = BC$. (**2m)**

d) The diagram below shows a square *ABCD* with side length 2 cm. The point *E* is on side *AD* with *AE* = x. The point *F* is on side *BC* with *CF* = $2x^2$.
Let y be the area of the shaded region *ABFE.*



(i) What are the permissible values of x? **(1m)**

(ii) Show that $y = 2 + x - 2x^2$. **(2m)**

(iii) Find the maximum area of *ABFE.* **(1m)**

(iv) Find the minimum area of *ABFE.* **(1m)**

End of Test

Extra Working Page

Extra Working Page

SYDNEY
BOYS
HIGH
SCHOOL

2018

YEAR 10 HALF YEARLY
STAGE 5.3 MATHEMATICS

Sample Solutions

Section A (15 Marks)

a) Express 0.375 as a percentage. **(1m)**

37.5%

b) Find 48% of 1 km 400m. **(1m)**

$0.48 \times 1400 = 672\text{ m}$

c) Convert 160km/h to m/s. (Exact answer) **(1m)**

$160\text{ km/h} = 160\,000\text{ m/h}$

$= \frac{8000}{3}\text{ m/min}$

$= \frac{400}{9}\text{ m/s.}$

* It's a good idea to gradually work towards m/s.

d) Find x if $x:6 = 5:16$ **(1m)**

$\frac{x}{6} = \frac{5}{16}$

$x = \frac{15}{8}$

e) Calculate the simple interest on \$600 for 9 months at 4% p.a. **(1m)**

$I = Prt$

$= 600 \times 0.04 \times \frac{9}{12}$

$= \$18$

f) Round 4.06352 to 4 significant figures. **(1m)**

4.064

g) If $a = -3$ find the value of $-a^2 - a$ **(1m)**

$-(-3)^2 - (-3)$

$= -6$

* It's a good idea to rewrite putting (-3) in brackets wherever an a is.

h) True or False? All squares are parallelograms. **(1m)**

TRUE.

i) Solve the following equations:

(i) $(6+m)(7-m)=0$ **(1m)**

$m = -6, 7$

(ii) $2p^2 - 18 = 0$ **(2m)**

$2p^2 = 18$

$p^2 = 9$

$p = \pm 3$

(iii) $8x^2 = 12x$

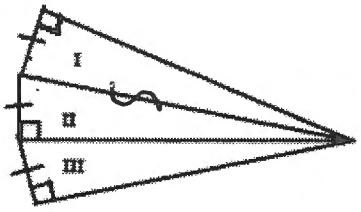
$8x^2 - 12x = 0$

$4x(2x - 3) = 0$

$x = 0, \frac{3}{2}$

A common mistake was to write only one solution. **(2m)**

j) Which triangles are congruent? **(1m)**



A. I and II only (RHS)

B. I and III only

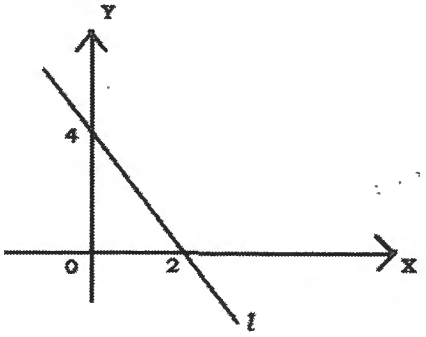
C. II and III only

D. I, II and III

* Many students got this wrong.

Note: corresponding sides must be equal not just any two

k) Find the equation of the line *l*. **(1m)**



$m = \frac{4-0}{0-2}$

$= -2$

$y = mx + b$

$y = -2x + 4$

or

$2x + y - 4 = 0$

$x-6 \quad -3+2$
$+-1$

Section B (13 Marks)

a) Solve this quadratic equation by factorising **(2m)**

$6a^2 - a - 1 = 0$

$6a^2 - 3a + 2a - 1 = 0$

$3a(2a-1) + 1(2a-1) = 0$

$(3a+1)(2a-1) = 0$ ✓

$a = -1/3, \quad a = 1/2$ ✓ (1dp)

Some students used Quad formula.

* full marks only for correct solns and by factorising

b) What is the percentage increase on an investment of $6000 which compounds annually for 5 years at 9% pa? **(2m)**

$A = 6000(1+0.09)^5$

$I = 9231.74 - 6000$

$\%$ increase $= \frac{3231.74}{6000} \times 100$

$= 53.862\%$ (3dp)

With working
(1/2) 9231.74
(1/2) 3231.74

53.9% ✓

c) A laptop purchased for $3200 depreciates at the rate of 16% p.a.

(i) What is the value of the laptop after 4 years? **(1m)**

$3200(1-0.16)^4$

$= \$1593.19$ ✓

6

(ii) How long until the laptop is worth less than $500? **(1m)**

$3200(1-0.16)^n < 500$

$0.84^n < 0.15625$

Guess & check is ok

$n < \frac{\ln 0.15625}{\ln 0.84}$

$n > 10.65$

$n = 11$ yrs ✓

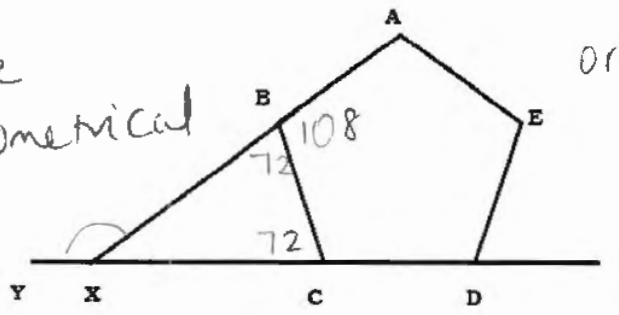
In 11yrs, $470.13

d) The diagram shows a regular pentagon ABCDE. The sides AB and DC are produced to meet at X. The point Y lies on DCX produced.

int ∠ $\frac{3 \times 180}{5}$

or $360 \div 5$ ext ∠

Students need to familiar themselve wi the SBHS Geometrical Abbreviations & Reasoning!



Find the size of $\angle YXB$ giving reasons. **(3m)**

① Reason 1 → int ∠ of regular pentagon & st line OR ext ∠ of regular pentagon

① 72° → st line

① $\angle BXY = 144^\circ$ (ext ∠ of Δ) or ∠ sum Δ and st line

e) Find the distance between two points A(-3,7) and B(-8,-3) correct to 3 decimal places. **(1m)**

$x_1\ y_1 \quad x_2\ y_2$

$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

$$= \sqrt{(-3-7)^2 + (-8--3)^2}$$

$$= \sqrt{100 + 25}$$

$$= \sqrt{125}$$

$$= 11.180 \text{ units}^5 \quad 3\text{dp}$$

4 ✓

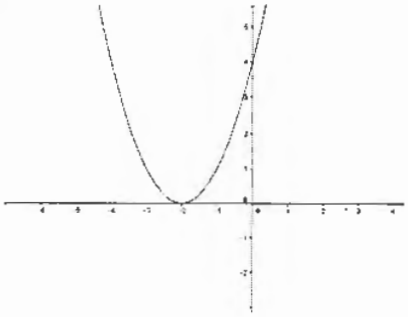
f) Match each equation with one of the graphs below. **(3m)**

(i) $y=(x+2)^2$ A (ii) $y=-(x+2)^2$ E (iii) $y=x^2+2$ D

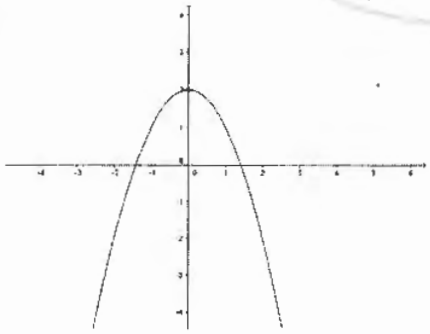
(iv) $y=-x^2+2$ B (v) $y=(x-2)^2$ F (vi) $y=-(x-2)^2$ C

$\frac{1}{2}$ each

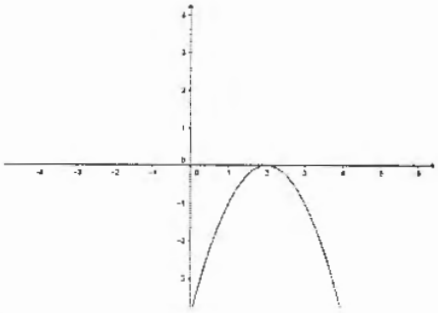
A



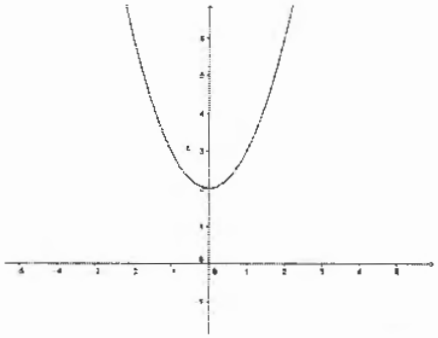
B



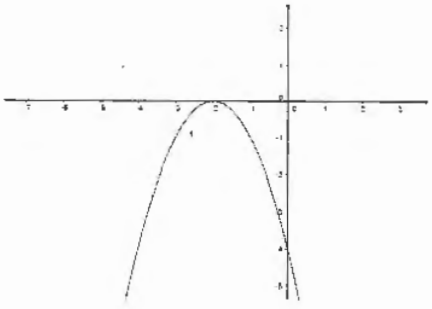
C



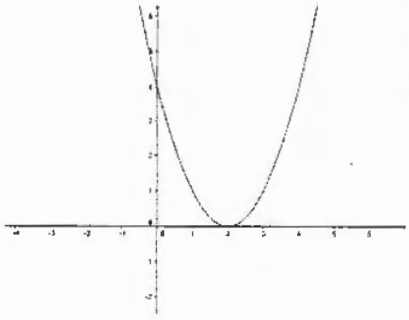
D



E



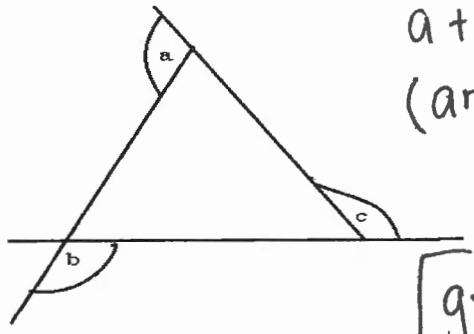
F



3

Section C (13 Marks)

a) State the value of $a+b+c$ in this diagram? **(1m)**



$a+b+c=360°$
(angle sum of exterior is 360°)

[generally well done]

b) Solve by completing the square leaving your answer in exact surd form.

$x^2-12x+10=0$ **(2m)**

$x^2-12x+\left(\frac{-12}{2}\right)^2=-10+\left(\frac{-12}{2}\right)^2$

$(x-6)^2=26$

$x-6=\pm\sqrt{26}$

$x=6\pm\sqrt{26}$

[generally well done]

c) Solve by using the quadratic formula, leaving your answer in exact form.

$2x^2-9x+8=0$ **(2m)**

$x=\frac{-(-9)\pm\sqrt{(-9)^2-4(2)(8)}}{2(2)}$

$=\frac{9\pm\sqrt{17}}{4}$

[some students gave decimals as their exact answers]

d) Consider the quadratic x^2+3x+5.

(i) What is the value of b^2-4ac for this quadratic? **(1m)**

$\Delta=3^2-4(1)(5)$
$=9-20$
$=-11$

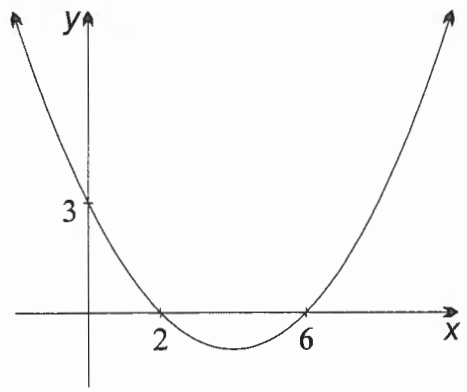
[well done]

(ii) Explain what this tells you about the graph of $y=x^2+3x+5$ **(1m)**

-the graph has no x-intercepts.

[very poorly answered]

e) The graph below shows a parabola with x-intercepts at $(2,0)$ and $(6,0)$. **(2m)**
Its y-intercept Is at $(0,3)$. Determine the equation of the parabola.
You may leave your answer in factorised form.



$y = a(x-2)(x-6)$

at $(0,3)$: $3 = a(0-2)(0-6)$

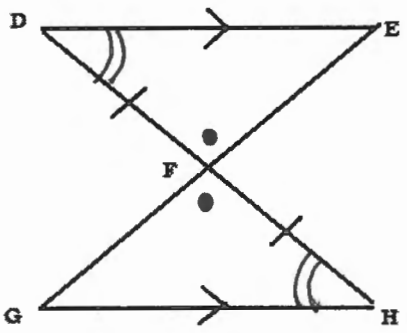
$3 = 12a$

$\therefore a = \frac{1}{4}$

$\therefore y = \frac{1}{4}(x-2)(x-6)$

[very poorly done]

f) In the diagram below $DE \parallel GH$ and $DF = HF$. **(1m)**
State the congruence test that proves that $\Delta DEF \equiv \Delta HGF$.



AAS

[many students said similarity tests]

g) A is $(-2,1)$ and B is $(4,5)$

(i) Find the midpoint of AB. **(1m)**

$mp_{AB} = \left(\frac{-2+4}{2}, \frac{1+5}{2}\right)$

$= (1,3)$

(ii) Find the gradient of AB. **(1m)**

$m = \left(\frac{5-1}{4-(-2)}\right) = \frac{4}{6} = \frac{2}{3}$

(iii) Find the equation of the line which is the perpendicular bisector of AB in general form. **(1m)**

$m_{\perp} = -\frac{3}{2}$, $(1,3)$

$\therefore y - 3 = -\frac{3}{2}(x-1)$

$2y - 6 = -3x + 3$

$\therefore 3x + 2y - 9 = 0$

[(i) and (ii) very well done
(iii) very poorly done]

Section D (15 Marks)

a) How much must I invest now to save \$50000 for a trip in 3 years time if interest is 9% pa compounding monthly? **(2m)**

Let P be the amount invested

$P(1{\cdot}0075)^{36} = 50000$

$\therefore P = 38207{\cdot}45$

b) By using a substitution, or otherwise, solve the following equation correct to 2 decimal places:

$(x^2-20)^2 - 7(x^2-20) = -10$ **(3m)**

Let $u = x^2 - 20$

$\therefore u^2 - 7u + 10 = 0$

$(u-5)(u-2) = 0$

i.e $x^2 - 25 = 0 \Rightarrow x = \pm 5$

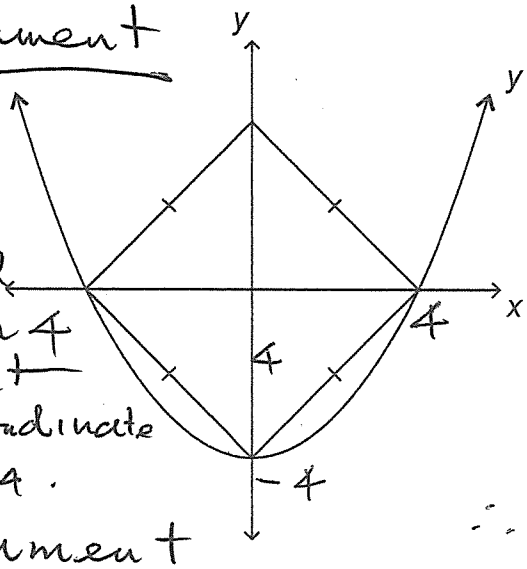
$x^2 - 22 = 0 \Rightarrow x = \pm\sqrt{22}$

c) A square centred on the origin has its vertices on the x and y-axes. The graph of the parabola $y = ax^2 - 4$, where $a > 0$, passes through three of the square's vertices. Find the value of a. **(3m)**

(c) Comment

Not well done at all; Most could not even 4 worked out the y-coordinate of $y = -4$.

Comment



$y = a\left(x^2 - \frac{4}{a}\right)$

$x^2 = \frac{4}{a}$.

When $x = 0$, $y = -4$

$\therefore 16 = \frac{4}{a} \Rightarrow a = \frac{1}{4}$.

(a) Quite a few students compound their interest in terms of yearly instead of monthly. A few used the formula for depreciation instead of appreciation.

(b) Most know how to reduce this quartic equation to quadratic but unable to form the quadratic equation. Quite a few [illegible] there are 2 solutions in a quadratic.

d) Consider the parabola $y = -(x+1)^2 - 2$

(i) What is its vertex? $(-1, -2)$ (Well done!) (1m)

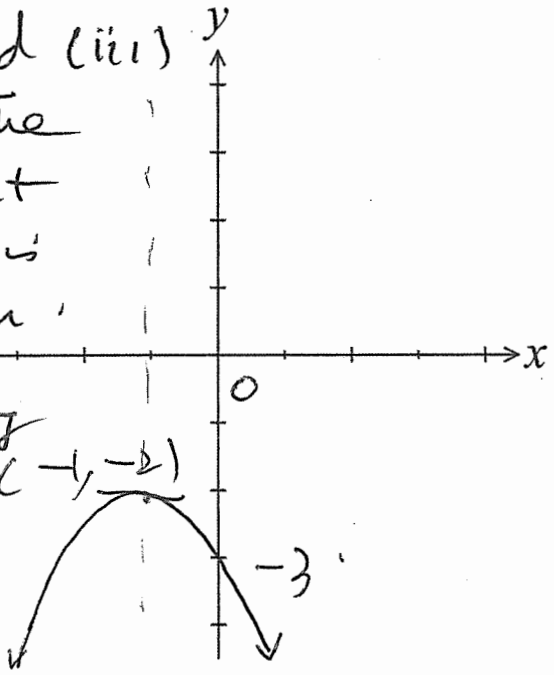
(ii) What is the y-intercept? When $x = 0$, $y = -3$ (Well done!) (1m)

$(0, -3)$

(iii) Sketch the parabola showing these features. (2m)

Comment — d (iii)

Most knew the basic work but the diagram is poorly drawn. Which almost of the diagram contradicts a cusp!

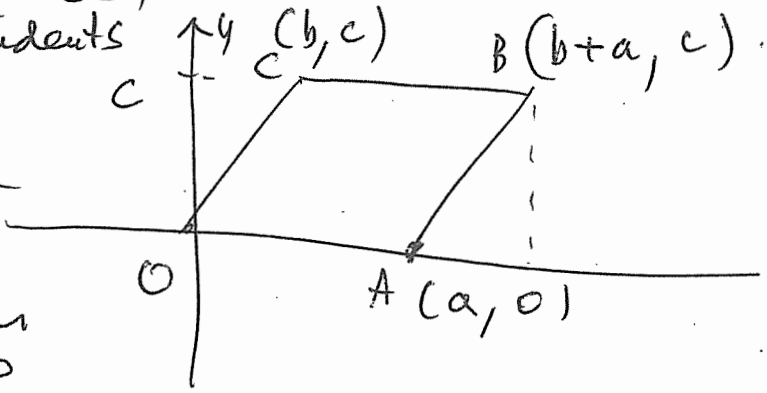


e) Prove, giving all reasons, that the diagonals of a parallelogram bisect each other. Do not assume any properties other than the property that opposite sides are parallel. (3m)

Comment (e)

Most students thinks proving the triangles are similar in order to get the marks.

Reasons were poorly set out:



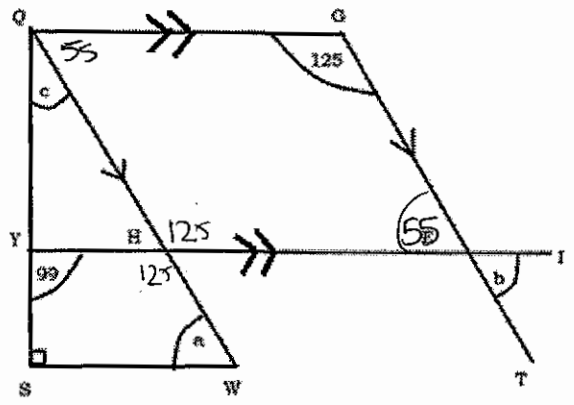
Mid pt of AC $\left(\frac{a+b}{2}, \frac{c}{2}\right)$.

Mid pt of OB $\left(\frac{a+b}{2}, \frac{c}{2}\right)$.

diagonals bisect each other.

Section E (14 Marks)

a) Find the value of the pronumerals a, b and c. You do NOT need to give reasons. **(3m)**



$a = 46°$

$b = 55°$

$c = 44°$

Generally well done. most common error was all 3 (or atleast 2) angles being 55°

b) Solve, leaving your answer in exact form:

$$\frac{1}{3-5x}+\frac{4}{3+5x}=-1$$ **(3m)**

$$\frac{1(3+5x)+4(3-5x)}{(3-5x)(3+5x)} = -1$$

$$3+5x+12-20x = -1(9-25x^2)$$

$$15-15x = -9+25x^2$$

$$25x^2+15x-24=0$$

$$x = \frac{-15\pm\sqrt{15^2-4\times25\times-24}}{2\times25}$$

$$= \frac{-15\pm5\sqrt{105}}{50}$$

$$= \frac{-3\pm\sqrt{105}}{10}$$

many students didn't:
- execute the difference of 2 squares correctly getting $9-25x$.
- multiply $(9-25x^2)$ by -1 correctly
- simplify the final fraction/surd correctly.

c) Keith invested \$3000 in shares at the start of the year. The shares rose in value by 5% per month for the first 6 months and then fell in value by 5% per month for the next 6 months. Find the value of the shares at the end of the year. **(2m)**

$A = 3000(1 + 0.05)^6$

$= 4020.29$

$A_2 = 4020.29(1 - 0.05)^6$

$= \$2955.28$

Common mistakes:
- not recognising the interest rate was already in months

d) A woman invested \$40000 at 9% pa compounding monthly for 2 years. At the end of that time she reinvested the capital plus interest at an annual rate that compounded quarterly for 4 years. If the investment has accrued to \$56000 at the end of the 6 years, what was the second annual interest rate? **(3m)**

$A_1 = 40000(1 + 9/12\%)^{24}$

$A_1 = 47856.54$

$56000 = A_1(1 + r/4)^{16}$

$\frac{56000}{A_1} = (1 + r/4\%)^{16}$

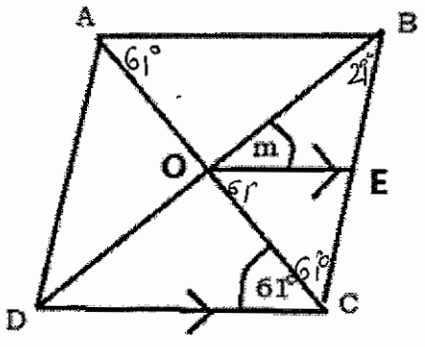
$1 + r/4\% = \sqrt[16]{\frac{56000}{A_1}}$

$r/4\% = \sqrt[16]{\frac{56000}{A_1}} - 1$

$r = 3.95\%$ p.a.

Poorly done:
common errors:
- not converting time frames & interest rates to correct time periods

e) ABCD is a rhombus. Find the value of m, giving reasons. **(3m)**



$\therefore \angle CBD = 29°$

$\therefore \angle m = (180 - (61 + 61 + 29))$ ($\angle$ sum $\triangle$)

$\therefore m = 29°$

Well done by most. Common error not providing correct reasoning.

There are many ways to do this question - this is just 1 long way ☺

$\angle CAB = 61°$ (alt $\angle$'s on 11 lines)

$\angle ACB = 61°$ (base $\angle$'s isos $\triangle$ =)

$\angle COE = 61°$ (alt $\angle$'s on 11 lines =)

$\angle COB = \angle CBO$ (base $\angle$s isos $\triangle$ =)

$\angle COB + \angle CBO = 180 - 122$ ($\angle$ sum $\triangle$)

$= 58°$

Section F (14 Marks)

a) Solve by completing the square: $3x^2 - 4x - 1 = 0$ **(3m)**

$x^2 - \frac{4}{3}x - \frac{1}{3} = 0$

$x^2 - \frac{4}{3}x + \left(-\frac{2}{3}\right)^2 = \frac{1}{3} + \left(-\frac{2}{3}\right)^2$

$\left(x - \frac{2}{3}\right)^2 = \frac{7}{9}$

$x = +\frac{2}{3} \pm \sqrt{\frac{7}{9}}$

$= \frac{2 \pm \sqrt{9}}{3}$

FAIRLY POORLY DONE

b) Solve the equation $3m - 11\sqrt{m} = 4$ **(3m)**

$x = \sqrt{m}$

$3x^2 - 11x - 4 = 0$

$3x^2 - 12x + x - 4 = 0$

$3x(x-4) + (x-4) = 0$

$(3x+1)(x-4) = 0$

$3\sqrt{m} = -1$ or $\sqrt{m} = 4$

NO SOLUTION $\sqrt{m} > 0$ $m = 16$

or

$43m - 4 = 11\sqrt{m}$

$(3m-4)^2 = 121m$

$9m^2 - 24m + 16 = 121m$

$9m^2 - 145m + 16 = 0$

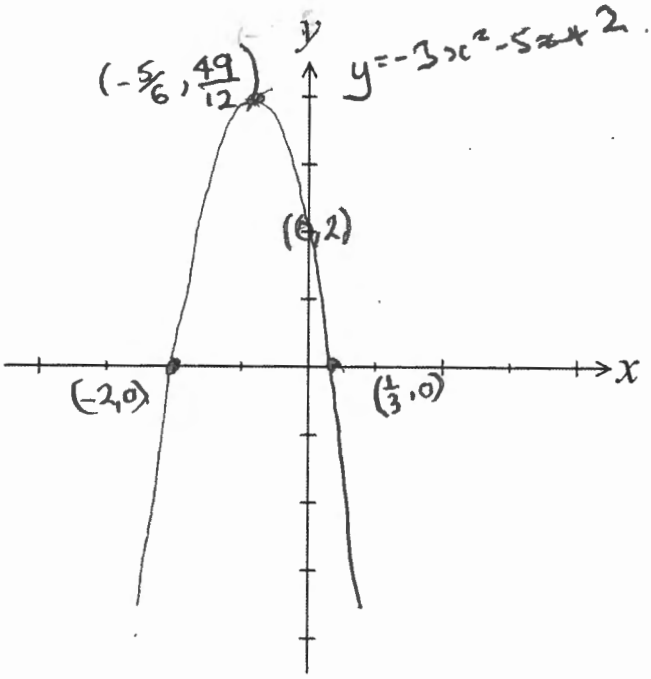
$m = \frac{145 \pm \sqrt{145^2 - 4(9)(16)}}{2(9)}$

$m = \frac{1}{9}$ or $m = 16$

2marks max.

VERY POOLY DONE

c) Sketch the graph of $y = -3x^2 - 5x + 2$ showing the coordinates of the x and y intercepts and the vertex. **(3m)**



Vertex $x = \frac{-b}{2a}$

$= \frac{-(-5)}{2(-3)} = \frac{-5}{6}$

$y = -3(-5/6)^2 - 5(5/6) + 2$

$= \frac{49}{12}$

Vertex $\left(-5/6, \frac{49}{12}\right)$

x intercept $y = 0 = -3x^2 - 5x + 2$

$0 = (1-3x)(x+2)$

$x = \frac{1}{3}, -2$

$\left(\frac{1}{3}, 0\right)$ $(-2, 0)$

y intercept $x = 0$

$y = 2$

$(0, 2)$

MANY STUDENTS DID NOT WRITE CO-ORDINATES AS REQUIRED
FAIRLY WELL DONE.

d) David has just bought a boat and has a mortgage of $120000 at 6%p.a. compounding monthly. He has to make payments of $750 per month. How much does he owe after the third payment? **(3m)**

$((20,000 \times 1.005 - 750) \times 1.005 - 750) \times 1.005 - 750$

$= \$119547.75$

Full marks awarded

e) Find the values of m for which $4x^2 - (6+m)x + 1 = 0$ has only one real root. **(2m)**

For only 1 root $\Delta = b^2 - 4ac = 0$

$(-(6+m))^2 - 4 \times 4 \times 1$

$(-6)^2 - (-12m) + (m)^2 - 16 = 0$

$m^2 + 12m + 20 = 0$

$(m+10)(m+2) = 0$

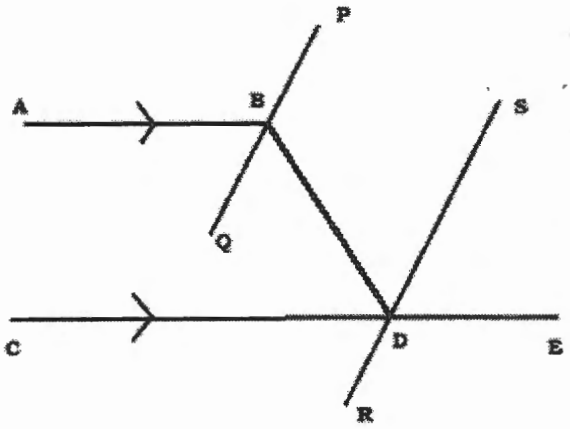
$m = -2$ or $m = -10$.

FAIRLY POORLY DONE

Many did not know the conclusion for one real root.

Section G (15 Marks)

a) In the diagram you are given that $AB \parallel CE$. Also PQ bisects $\angle ABD$ and RS bisects $\angle BDE$. Prove that $PQ \parallel RS$, giving reasons. **(3m)**



$\angle BDE = \angle ABD$ (Alternate angles, $AB \parallel CD$)

$\therefore \frac{1}{2}\angle BDE = \frac{1}{2}\angle ABD$

$\therefore \angle BDS = \frac{1}{2}\angle BDE$ (RS bisects $\angle BDE$)

$\angle QBD = \frac{1}{2}\angle ABD$ (PQ bisects $\angle ABD$)

$\therefore \angle BDS = \angle QBD$

$\therefore PQ \parallel RS$ (Alternate $\angle$'s are equal)

* Students should write down briefly on the test paper that PQ bisects $\angle ABD$ and RS bisects $\angle BDE$ if they want to state $\angle ABQ = \angle QBD$ and $\angle CDQ = \angle BDS$.

* Full marks given for correct reasoning and structure.

b) An amount of money compounds for 3 years at 12 % pa compounding 6 monthly and then for 4 years more at 11.75% pa compounding monthly. If the final value is \$28 034, find the original amount invested. **(3m)**

let P be the original amount invested.

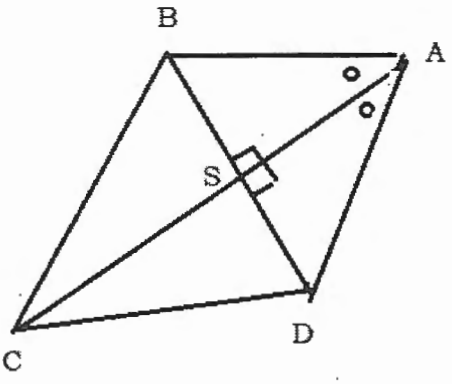
$$P\left(1+\frac{12\%}{2}\right)^{3\times 2}\left(1+\frac{11.75\%}{12}\right)^{4\times 12} = \$28034 \quad (1)$$

$$P = \frac{\$28034}{(1.06)^6\left(1+\frac{47}{48}\%\right)^{48}} \quad (1)$$

$$= \$12380.1049$$

$$\approx \$12380.11 \quad (1)$$ (To achieve \$28034 but will accept \$12380.10)

c) ABCD is a quadrilateral in which the diagonals BD and AC intersect at right angles. Also $\angle BAS = \angle DAS$.



(I) By using congruent triangles, prove that $AD = BA$ giving all reasons. **(2m)**

In $\triangle ABS$ and $\triangle ADS$

$\angle BAS = \angle DAS$ (given)

$\angle ASB = \angle ASD$ (given)

AS is common

$\therefore \triangle ABS \equiv \triangle ADS$ (AAS) ①

$\therefore AD = BA$ (corresponding sides of congruent Δ's) ①

* Students need to write the introduction for a congruent test proof.

(II) Hence prove that $CD = BC$. **(2m)**

In $\triangle ABC$ and $\triangle ADC$

$AD = BA$ (proved in (i))

AC is common

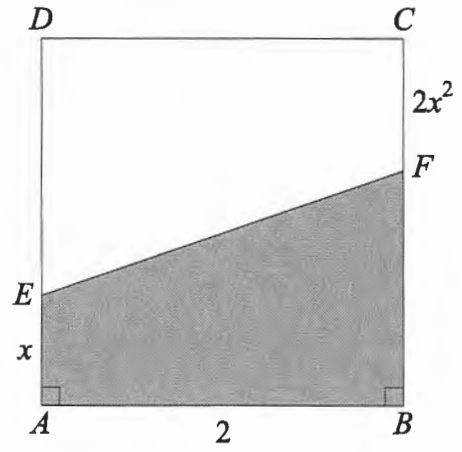
$\angle BAC = \angle DAC$ (given) ①

$\therefore \triangle ABC \equiv \triangle ADC$ (SAS)

$\therefore BC = CD$ (corresponding sides of congruent Δ's) ①

* Students whom used triangles $\triangle ABC$ and $\triangle ADC$ were more successful than $\triangle ABS$ and $\triangle ADS$.

d) The diagram below shows a square *ABCD* with side length 2 cm. The point *E* is on side *AD* with $AE = x$. The point *F* is on side *BC* with $CF = 2x^2$.
Let y be the area of the shaded region *ABFE*.



(i) What are the range of values that x can take? **(1m)**

$0 < 2x^2 < 2$

$0 < x^2 < 1$

$0 < x < 1$ ← 1 mark.

(ii) Show that $y = 2 + x - 2x^2$. **(2m)**

Area$_y$ = Area of trapezium

$\therefore y = \frac{h}{2}(a+b)$

$= \frac{2}{2}(x + 2 - 2x^2)$

$\therefore y = 2 + x - 2x^2$

* To gain full marks in the show question, students must explain the formulae they are using.

(iii) Find the maximum area of *ABFE*. **(1m)**

Since $y = -2x^2 + x + 2$, will produce a concave down parabola.

∴ Maximum Area can be found at vertex.

Axis of symmetry $= \frac{-b}{2a} = \frac{1}{4}$

$\therefore y = -2\left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right) + 2$

$= 2.125 \text{ units}^2$

(1 mark.

(iv) Find the minimum area of *ABFE*. **(1m)**

Test endpoints of (i)

At $x = 0$

$y = 2 + 0 - 2(0) = 2$

At $x = 1$

$y = 2 + 1 - 2(1) = 1$

End of Test

∴ Minimum Area = 1 unit^2

(1 mark