

# Sydney Girls High School



## MATHEMATICS

### Year 10

#### HALF-YEARLY EXAMINATION

2011

**Time Allowed: 60 minutes + 3 minutes reading time**

**Topics:** Quadratic Equations, Probability, Consumer Arithmetic, Number Plane Graphs, Surface Area & Volumes

**Total:** 60 marks

**Instructions:**

- There are FIVE (5) questions which are of equal value.
- Attempt all questions.
- Show all necessary working. Marks may be deducted for badly arranged work or incomplete working.
- Start each question on a new page.
- Write on one side of the paper only.
- Diagrams are NOT to scale.
- Board-approved calculators may be used.
- Write your name and your teacher's name clearly at the top of each question and clearly number each question.

**Student's Name:** \_\_\_\_\_ **Teacher's Name:** \_\_\_\_\_

### Question 1 (12 marks)

a. Factorise and hence solve the following quadratic equations:

i)  $2p^2 - 6p = 0$  2

ii)  $m^2 - 2m - 8 = 0$  3

b. Sketch the following, on separate number planes, showing all relevant features:

i)  $y = 3^{-x}$  2

ii)  $xy = -4$  2

c. A sample of 100 matchboxes was taken and the number of matches in each box counted and recorded. The results are given in the table:

No. of matches	No. of matchboxes
47	5
48	13
49	26
50	45
51	10
52	1

Find the probability that a box selected at random from this sample contains:

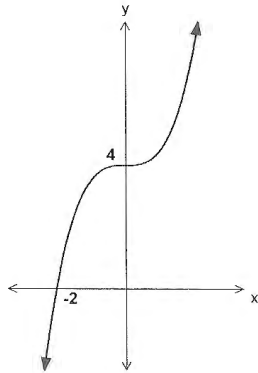
i) exactly 50 matches? 1

ii) more than 50 matches? 1

iii) 48 or fewer matches? 1

Question 2 (12 marks)

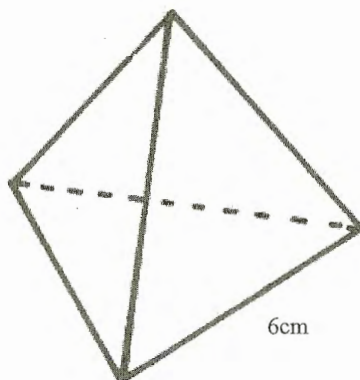
- a. Find the simple interest earned if one million dollars is invested for 5 months at 0.75%p.a. 2
- b. Use the quadratic formula to solve the equation  $3x^2 = 6x - 2$ . 3  
Leave your answer in surd form.
- c. Determine the cubic equation of the following graph: 2



- d. The parabola  $y = 2x^2 + kx - 7$  has an axis of symmetry with equation  $x = 3$ . 2  
Find the value of  $k$ .
- e. Find the compound interest earned on an investment of \$3000 at 3.8% p.a. over 2 3  
years if the interest is calculated annually.

### Question 3 (12 marks)

- a. A 6-sided die has the numbers 0, 1, 2, 3, 4 and 5 on its faces.  
In a game, two of these dice are rolled and the total of the faces is taken.
- i) List the possible outcomes of the game. 2
  - ii) What is the chance of obtaining a total of 7? 1
  - iii) What total has the greatest probability? 1
  - iv) What is the probability of getting a total of at least 3? 1
- b. A computer bought for \$2500 was sold 5 years later for \$750. 3  
Find its annual depreciation rate, correct to one decimal place.
- c. i) Show that the height of an equilateral triangle with side length 6cm is  $3\sqrt{3}$  cm. 2  
ii) Hence, find the exact surface area of this tetrahedron. 2



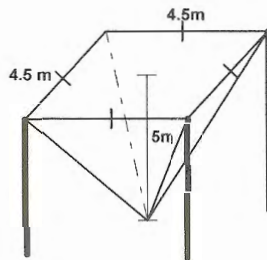
Question 4 (12 marks)

- a. Mary borrows \$25000 to buy a car. She repays the loan in fortnightly instalments of \$256 over 5 years. Calculate:
- i) the total amount repaid. 1
  - ii) the amount of interest charged on the loan. 1
  - iii) the flat rate of interest (p.a.). 2
- b. The members of a certain family have a tendency to inherit "ergophobia". When a child is born the chance of it being an ergophobic or non-ergophobic is equally likely.
- i) Draw a tree diagram to show the outcomes of 3 children born. 2
  - ii) What is the probability that the family will have two ergophobiacs and one non-ergophobiatic in any order? 1
  - iii) What is the probability of at least one ergophobiatic being born? 2
- c. A sphere with radius  $r$  fits tightly into a cylinder as shown.
- 
- Show that the ratio of the surface area of the sphere to the surface area of the cylinder is 2:3. 3

### Question 5 (12 marks)

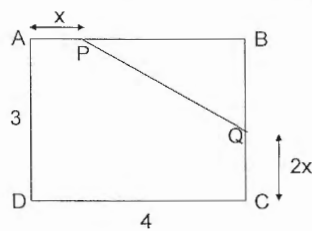
- a. For the parabola with equation  $y = -x^2 + 4x - 5$ :
- i) find any  $x$  or  $y$  intercepts. 2
  - ii) find the vertex. 2
  - iii) sketch its graph. 1

- b. A grain hopper is in the shape of an inverted square pyramid as shown.  
The perpendicular height of the pyramid is 5 metres.



- i) Find the volume of the square pyramid. 2
- ii) If there are 750kg of wheat per  $m^3$ , find the mass of grain in the hopper when it is filled to three-quarters of its capacity. 2  
Give your answer in tonnes, correct to two decimal places.

- c. ABCD is a rectangle,  $CD = 4$  and  $AD = 3$ . If  $AP = \frac{1}{2}QC = x$ :



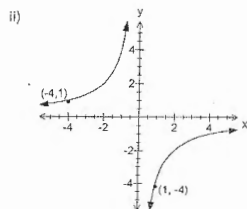
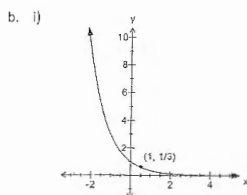
- i) find an expression for the area of pentagon APQCD. 1
- ii) hence, find its maximum area. 2

-----END OF TEST-----

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Question 1

- a. i)  $2p(p-3) = 0$   
 $p = 0$  or  $p = 3$
- ii)  $(m-4)(m+2) = 0$   
 $m = 4$  or  $m = -2$



- c. i)  $\frac{45}{100} = \frac{9}{20}$
- ii)  $\frac{11}{100}$
- iii)  $\frac{18}{100} = \frac{9}{50}$

Question 2

a.  $I = \$1000000 \times 0.75\% \times \frac{5}{12}$   
 $= \$3125$

b.  $3x^2 - 6x + 2 = 0$   
 $x = \frac{6 \pm \sqrt{6^2 - 4(3)(2)}}{2(3)}$   
 $= \frac{6 \pm \sqrt{12}}{6}$   
 $= \frac{3 \pm \sqrt{3}}{3}$

c.  $y = ax^3 + d$   
 $y = ax^3 + 4$   
 When  $y = 0, x = -2$ :  
 $0 = a(-2)^3 + 4$   
 $-4 = -8a$   
 $a = \frac{1}{2}$

$\therefore y = \frac{1}{2}x^3 + 4$

d.  $x = \frac{-b}{2a}$   
 $3 = \frac{-k}{4}$   
 $\therefore k = -12$

e.  $CI = \$3000 \times (1 + 3.8\%)^2 - \$3000$   
 $= \$3000(1.038)^2 - \$3000$   
 $= \$3232.332 - \$3000$   
 $= \$232.33$

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Question 3

a. i)

	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	6
2	2	3	4	5	6	7
3	3	4	5	6	7	8
4	4	5	6	7	8	9
5	5	6	7	8	9	10

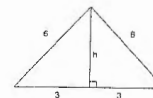
ii)  $P(7) = \frac{4}{36} = \frac{1}{9}$

iii) 5

iv)  $P(\geq 3) = \frac{30}{36} = \frac{5}{6}$

b.  $\$750 = \$2500(1-r)^2$   
 $0.3 = (1-r)^2$   
 $0.3^{\frac{1}{2}} = 1-r$   
 $r = 1 - 0.3^{\frac{1}{2}}$   
 $r = 0.2139969144$   
 $= 21.4\%$

c. i)



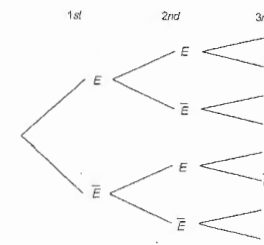
$6^2 = h^2 + 3^2$   
 $h^2 = 27$   
 $h = 3\sqrt{3}$  cm

ii)  $SA = 4 \times \frac{1}{2} \times 6 \times 3\sqrt{3}$   
 $= 36\sqrt{3}$  cm<sup>2</sup>

Question 4

- a. i) Total paid =  $\$256 \times 26 \times 5$   
 $= \$33280$
- ii) Interest =  $\$33280 - \$25000$   
 $= \$8280$
- iii) Flat interest rate p.a.  
 $= \frac{8280}{25000} \times 5 \times 100$   
 $\approx 6.6\%$  p.a.

b. i)



ii)  $P(\overline{E}\overline{E}\overline{E}) = \frac{3}{8}$

iii)  $P(\text{at least one } E) = 1 - P(\overline{E}\overline{E}\overline{E})$   
 $= 1 - \frac{3}{8}$   
 $= \frac{5}{8}$

c.

Surface Area of Sphere =  $4\pi r^2$

Surface Area of Cylinder  
 $= 2\pi r^2 + 2\pi rh$  ( $h = 2r$ )  
 $= 2\pi r^2 + 4\pi r^2$   
 $= 6\pi r^2$

$\therefore$  SA of sphere : SA of cylinder  
 $4\pi r^2 : 6\pi r^2$   
 $2 : 3$

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Question 5

a) i) y-intercept:  $x=0, y=-5 \therefore (0,-5)$

x-intercept:  $y=0,$

$$0 = -x^2 + 4x - 5$$

$$x = \frac{-4 \pm \sqrt{16 - 4(-1)(-5)}}{2(-1)}$$

$$= \frac{-4 \pm \sqrt{16 - 20}}{-2}$$

$$= \frac{-4 \pm \sqrt{-4}}{-2}$$

$\therefore$  No solution

$\therefore$  No x-intercept

ii) Axis of Symmetry:  $x = \frac{-b}{2a} = \frac{-4}{2(-1)} = 2$

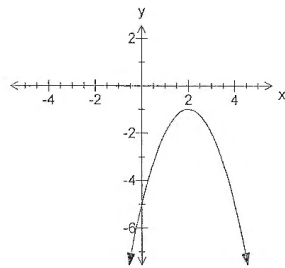
$$y = -(2)^2 + 4(2) - 5$$

$$= -4 + 8 - 5$$

$$= -1$$

$\therefore$  Vertex =  $(2, -1)$

iii)



b) i)

$$V = \frac{1}{3}Ah$$

$$= \frac{1}{3} \times (4.5)^2 \times 5$$

$$= 33.75\text{m}^3$$

ii)

$$\text{Mass} = 33.75 \times \frac{3}{4} \times 750\text{kg}$$

$$= 18984.375\text{kg}$$

$$= 18.98 \text{ tonnes}$$

c) i)

Area of Pentagon APQCD

= Area of rectangle - Area of triangle PQB

$$= 3 \times 4 - \frac{1}{2} \times (3-2x)(4-x)$$

$$= 12 - \frac{1}{2}[-12 - 3x - 8x + 2x^2]$$

$$= 12 - 6 + \frac{11}{2}x - x^2$$

$$= -x^2 + \frac{11}{2}x + 6$$

ii) Maximum area occurs when:

$$x = \frac{-b}{2a} = \frac{-\frac{11}{2}}{2(-1)} = \frac{11}{4}$$

$\therefore$  Maximum Area

$$= -\left(\frac{11}{4}\right)^2 + \left(\frac{11}{2}\right)\left(\frac{11}{4}\right) + 6$$

$$= 13.56 \text{ units}^2$$