BAULKHAM HILLS HIGH SCHOOL

YEAR 10

YEARLY EXAMINATION

2011

MATHEMATICS

Time allowed : 60 minutes

(As it is 80 min will be the ideal time for the paper)

Instructions:

- Attempt ALL questions
- Write using black or blue pen only.
- You may use pencil for diagrams
- Write your answers on the paper provided
- Start a new page for each question

Question 1 – Start a new page **marks**

a) Simplify $\frac{2^{3x+4}}{2^{2+x}}$. 1

b) If $\log_{10} x = 2$, find x. 1

c) Give the following in scientific notation: $(6\times10^{-3})^3$ 2

d) Find integers a and b such that $(5+\sqrt{3})^2 = a+b\sqrt{3}$ 2

e) If $\log_a 2 = 0.27$ and $\log_a 3 = 0.428$, find $\log_a 4.5$. 2

f) Solve $2\cos\alpha - 1 = 0$ for $0^0 \le \alpha \le 360^0$ 2

g) Factorise fully: $4x^3 - 32$ 2

Question 2 - Start a new page

a) Draw a number plane and plot P(-2,1), Q(1,6) and R(6,3).
(i) Show that the mid-point, M, of PR is (2, 2). 1
(ii) Find the gradient of PR. 1
(iii) If M is also the mid-point of QS, find the coordinates of S. 2
(iv) Show that QM is perpendicular to PR. 1

b) If I buy a car for $54000 now and keep it for three years, what will its value be at the end of this period if it has depreciated at 20% p.a.? 2

c) Solve for t: $7 - 4t < 12$ 2

d) If $f(x) = 13 - 2x$, find the value of
(i) $f(-5)$ 1
(ii) x if $f(x) = 100$. 2

Question 3 – Start a new page

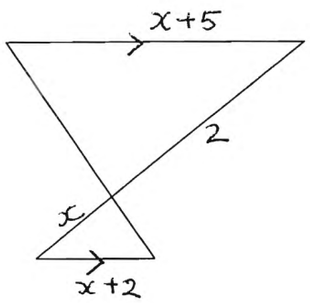
a) If $F(x)= 3x + 5$ and $y = F^{-1}(x)$ is its inverse.

(i) Find the inverse function. 1

(ii) Evaluate $F^{-1}(2) \times F(2)$ 1

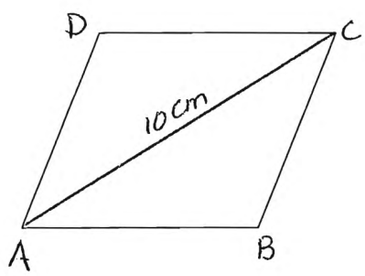
(iii) Sketch the graph of the inverse function 2

b) Find the value of x in the following: (No reasons required) 2



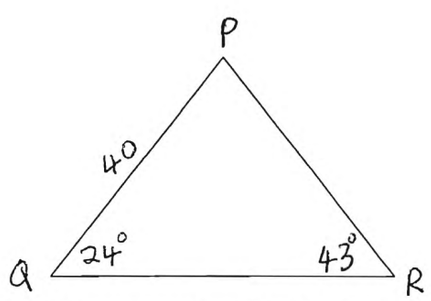
c) $\sin\theta = -\frac{3}{4}$ and $180^\circ < \theta < 270^\circ$. Find the exact value of $\tan\theta$. 2

d)



AC=10cm. Area of the rhombus is $80cm^2$. Find the length of AB to two significant figures. 2

e)



Find the length of RQ correct to ONE decimal place. 3

Question 4 – Start a new page

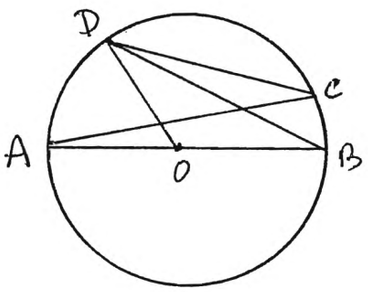
a) If $P(x) = (x-2)(x+1)^2$, sketch the graph of $y = P(x)$ on a number plane. 2

b) Sketch the following graphs showing all important features: 4

(i) $y = -2^x$

(ii) $x^2 + (y-2)^2 = 16$

c)



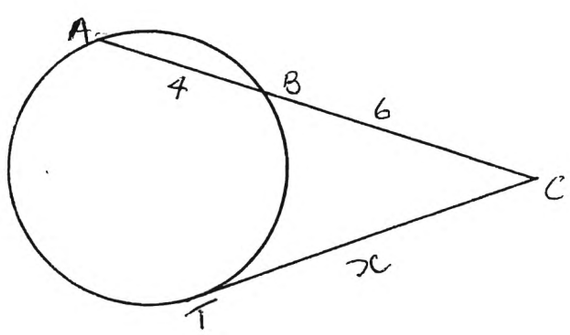
AB is the diameter of a circle with centre O. AC and BD are any two chords.
Copy the diagram into your answer sheet. Show that $\angle BDO = \angle ACD$. 3

d) If $(x-2)(x+p) = x^2 + qx + 10$, find the values of p and q. 2

e) Two similar statues were to be made at Jeya's studio. The smaller one was 1.6m tall, had a volume of $0.384m^3$ and needed 400mL of paint to complete the required two coats. If the height of the larger statue is 2.0m how much paint will be required to give the larger statue two coats? 3

Question 5 – Start a new page.

a) Find x. 1



Question 5 (continued)

b) An airplane takes off from an airport and travels in a direction 140°T at 600km/h for 1.5hours and then in a direction 280°T for 2 hours at 800km/h.

(i) Draw a neat diagram to represent the above information. 2

(ii) Find the distance, correct to nearest km, and bearing of the airplane from the airport after this time. 2

c) Find the coordinates of the vertex of $y = x^2 + 4x + 2$ 2

d) A rectangular swimming pool, with a flat floor, is 25m by 15m and is filled from pipes which deliver 500L of water per minute. How long will it take for the water to rise 10cm? 2

e) If $P(x) = x^3 - x^2 - 10x - 8$ and $Q(x) = x + 2$

(i) Find the remainder when P(x) is divided by Q(x). 1

(ii) Express P(x) as a product of its factors. 2

Question 6 – Start a new page

a) Thirty randomly chosen patients were surveyed about the length of time they spent waiting for a hospital service.

Stem	leaf
0	5 8 8 9
1	2 2 7 8 9
2	0 1 4 6 8
3	2 4 4 6 7 7 8 9 9
4	0 0 1 4 5 5 □

(i) if the range of scores is 44, find the value of □. 1

(ii) find the median waiting time. 1

(iii) Draw a box-and-whisker plot of this data. 3

b) Evaluate $\log_5 25 - \log_5 \sqrt{5}$. 2

c) Solve for x : $4^{2x+3} = 8^{x-5}$ 2

d) On a number plane shade in the region given by the two conditions: $x \leq -1$ and $y > x^2 - 9$ 2

Question 7 – Start a new page

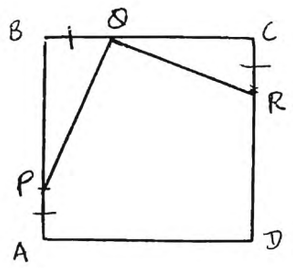
a) Three green and two yellow cards are in a basket. Two are drawn at random one after the other without replacement. What is the probability that

(i) both cards are yellow 1

(ii) both are the same colour. 2

b) Solve for x : $\log_{10} x - \dfrac{3}{\log_{10} x} = 2$ 3

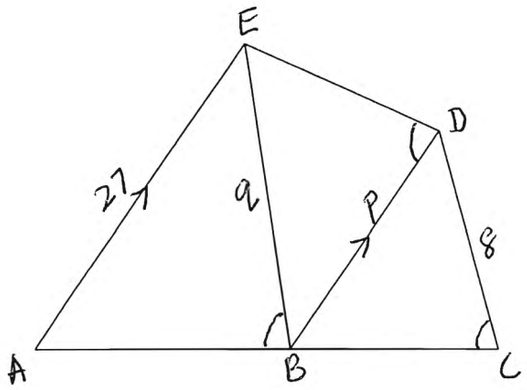
c) ABCD is a square.



Prove that (i) $\Delta PBQ \equiv \Delta QCR$ 3

(ii) $\angle PQR = 90^0$ 2

d)



Show that pq = 216. 2

End of Examination

YEAR 10, YEARLY EXAM 2011.

1) (a) $2^{3x+4-(2+x)} = 2^{2x+2}$ ✓

(b) $x = 10^2$; $x = 100$ ✓

(c) $6^3 \times 10^{-9} = 2.16 \times 10^{-7}$ ✓

(d) $(5+\sqrt{3})^2 = 25 + 10\sqrt{3} + 3$

$= 28 + 10\sqrt{3}$

$\therefore a = 28$ ✓; $b = 10$ ✓

(e) $\log_a 4.5 = \log_a \left(\frac{9}{2}\right)$

$= 2\log_a 3 - \log_a 2$ ✓

$= 0.586$ ✓

(f) $\cos\alpha = 1/2$

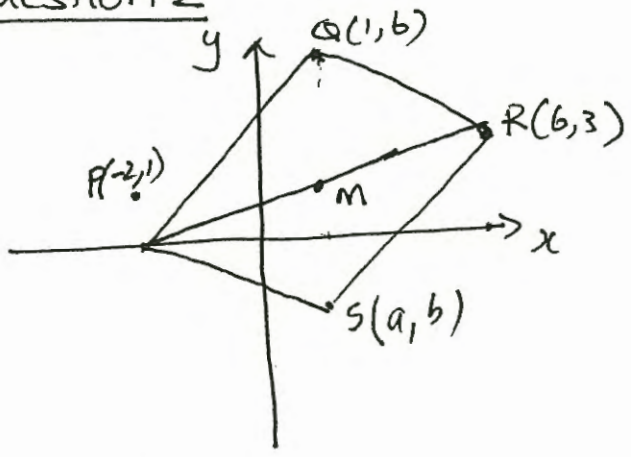
$\alpha = 60°, 300°$ ✓ ✓

(g) $4^{\checkmark}(x^3 - 8)$

$= 4 \cdot (x-2)(x^2+2x+4)$ ✓

(12)

Question 2



(i) $M\left(\frac{4}{2}, \frac{4}{2}\right) = (2,2)$ ✓

(ii) $m_{PR} = \frac{3-1}{6-(-2)} = \frac{2}{8} = \frac{1}{4}$ ✓

(iii) $\frac{a+1}{2} = 2 \Rightarrow a = 3$ ✓

$\frac{b+1}{2} = 2 \Rightarrow b = -2$ ✓

$S(3,-2)$

(iv) $m_{QS} = \frac{-2-6}{3-1} = \frac{-8}{2} = -4$

$m_{QS} \times m_{PR} = \frac{1}{4} \times -4 = -1$ ✓

$\therefore QS \perp PR$

(b) $V = 54000(1-0.2)^3$ ✓

$= \$27648$ ✓

(c) $-4t < 5$ ✓

$4t > -5$

$t > -5/4$ ✓

(d)(i) $f(-5) = 13 + 10 = 23$ ✓

(ii) $100 = 13 - 2x$ ✓

$2x = -87 \quad \therefore x = -87/2$ ✓

(12)

Question 3

(i) $y = 3x + 5$

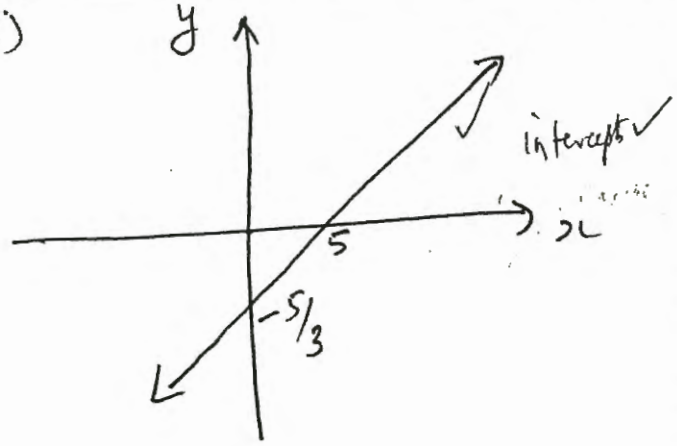
$x = 3y + 5 \Rightarrow y = \frac{x-5}{3}$

$\therefore f^{-1}(x) = \frac{x-5}{3}$ ✓

(ii) $f^{-1}(2) = -1 \quad f(2) = 11$.

$\therefore f^{-1}(2) \times f(2) = -11$ ✓

(iii)



$$f^{-1}(x) = \frac{1}{3}x - \frac{5}{3}$$

(b) $\frac{x+2}{x+5} = \frac{x}{2}$ ✓

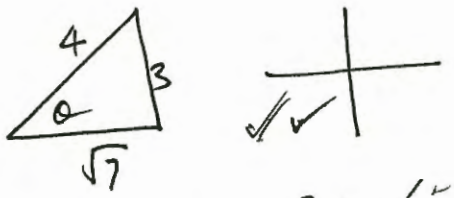
$2x+4 = x^2+5x$

$x^2+3x-4=0$

$(x+4)(x-1)=0$

$\therefore x = 1$ ✓ $\quad x=-4$ ✗

(c)



$\tan\theta = \frac{3}{\sqrt{7}}$ ✓

(d) $\frac{1}{2} \cdot 10 \times BD = 80$

$BD = 16$ ✓

$\sqrt{5^2+4^2} = AB = 6.4$ (2 sig).

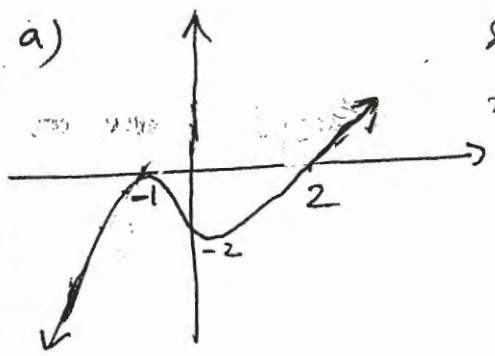
(e) $\frac{RQ}{\sin 113^\circ} = \frac{40}{\sin 43^\circ}$ ✓

$RQ = 54.0$ (1 decimal pl).

(13)

Question 4

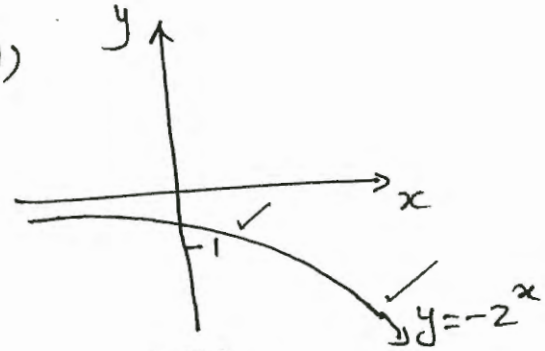
a)



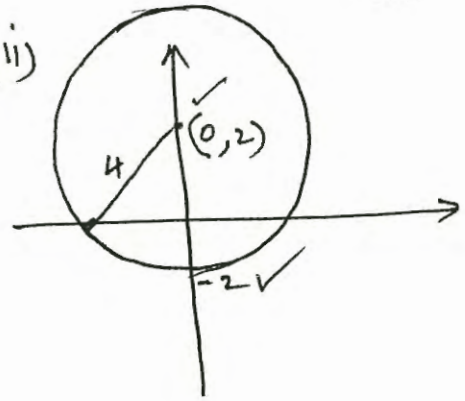
Slope -1

x, y-intercepts -1

b)(i)



(ii)



c) $\angle ODB = \angle OBD$ (Base $\angle$ of iso Δ; $DO = OB$ radii) ✓

$\angle OBD = \angle ACD$ ($\angle$ on the circumference by the same arc are $=$) ✓

$\therefore \angle BDO = \angle ACD$

d) $x^2+(p-2)x-2p = x^2+qx+10$

$-2p = 10 \implies p = -5$ ✓

$p-2 = q \implies q = -7$ ✓

(14)

e) $\frac{400}{V} = \frac{1.6^2}{2^2}$ ✓

$V = \frac{400 \times 2^2}{1.6^2} = 625\text{mL}$ ✓

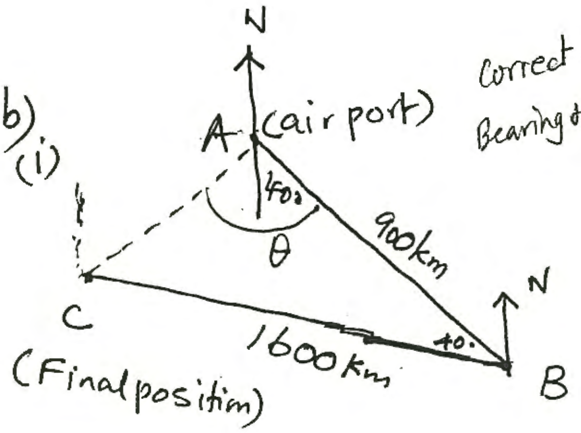
Question 5

a) $10 \times 6 = x^2$

$x^2 = 60$

$x = \sqrt{60} \simeq 7.75$ ✓

b) (i)



Correct labels ✓
Bearing & numerical data ✓

(ii) $AC^2 = 1600^2 + 900^2 - 2 \times 900 \times 1600 \times \cos 40^\circ$

$AC = 1078.79 \simeq 1079$ km ✓

$\frac{1079}{\sin 40} = \frac{1600}{\sin\theta}$

$\theta = 72^\circ 24'$

$\therefore$ Bearing $140 + 72^\circ 24'$

$= 212^\circ 24'$ T ✓

c) $x = -\frac{4}{2} = -2$ ✓

$\therefore y = 4 - 8 + 2 = -2$

Vertex $(-2, -2)$ ✓

d) $V = 25 \times 15 \times 0.1 = 37.5\,m^3$ ✓

0.5KL per minute.

$\therefore T = \frac{37.5}{0.5}$ min $= 75$ min

or 1 hour 15 min. ✓

e) (i) $R = P(-2)$

$= -8 - 4 + 20 - 8 = 0.$ ✓

(ii) $\therefore$ $(x+2)$ is a factor

$$\begin{array}{r} x^2 - 3x - 4 \\ x+2 \overline{) x^3 - x^2 - 10x - 8} \\ x^3 + 2x^2 \\ \hline -3x^2 - 10x \\ -3x^2 - 6x \\ \hline -4x - 8 \\ -4x - 8 \\ \hline \end{array}$$ ✓

(12)

$P(x) = (x^2 - 3x - 4)(x + 2)$

$= (x - 4)(x + 1)(x + 2)$ ✓

Question 6

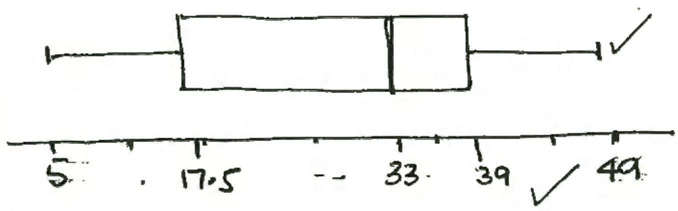
a) (i) $\square = 9$ ✓

(ii) $\frac{32 + 34}{2} = 33$ min ✓

(iii) $Q_1 = \frac{17 + 18}{2} = 17.5$

$Q_2 = 33$ ✓

$Q_3 = \frac{39 + 39}{2} = 39.00$



✓ ✓

b) $2\log_5 5 - \frac{1}{2}\log_5 5$ ✓

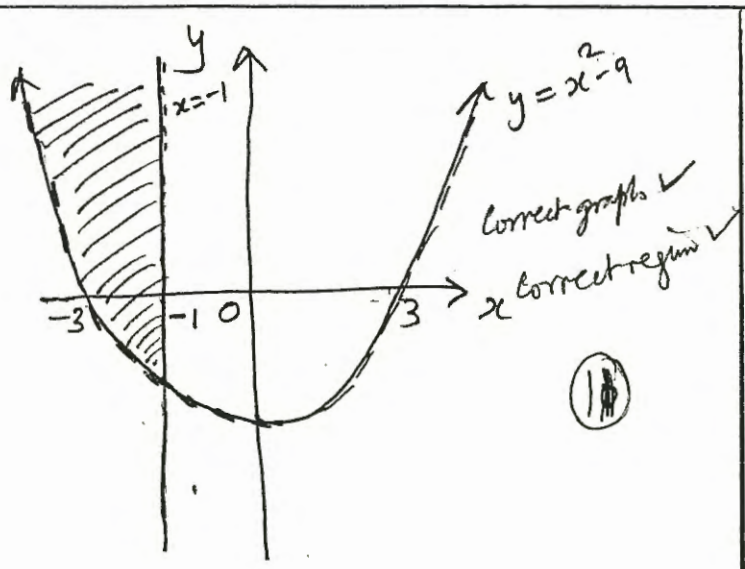
$= 1\frac{1}{2}$ ✓

c) $(2^2)^{2x+3} = (2^3)^{x+5}$ ✓

$4x + 6 = 3x + 15$

$x = 9$ ✓

1)



(11)

Question 7

i) $P(YY) = \frac{2}{5} \times \frac{1}{4} = \frac{1}{10}$ ✓

ii) $P(\text{same colour}) = P(YY) + P(GG)$ ✓

$= \frac{1}{10} + \frac{6}{25} = \frac{17}{50}$. ✓

b) let $\log_{10} x = m$

$m^2 - 3 = 2m$ ✓

$m^2 - 2m - 3 = 0 \Rightarrow m = 3$ or -1

$\log_{10} x = 3 \Rightarrow x = 1000$ ✓

$\log_{10} x = -1 \Rightarrow x = \frac{1}{10}$. ✓

c) (i) In ΔPBQ and ΔQCR

$BQ = CR$ (given)

$\angle PBQ = \angle QCR = 90°$ ✓ [∠ of a square]

$AB - BQ = BC - BQ$ ✓ [All sides of square are =]

$BP = QC$.

$\therefore \Delta PBQ \equiv \Delta QCR$ [S.A.S] ✓

(ii) Let $\angle BPQ = x$.

$\therefore \angle PQB = 90 - x$ [∠ sum of Δ]

$\angle CQR = x$ ✓ [matching ∠ of ≡ Δ]

$90 - x + \angle PQR + x = 180°$ [∠ sum on st. line]

$\therefore \angle PQR = 90°$ ✓

(d) $\Delta AEB \parallel\parallel \Delta EBD$

$\frac{AE}{EB} = \frac{EB}{BD} \Rightarrow \frac{27}{q} = \frac{q}{p}$ ✓

$\Delta EBD \parallel\parallel \Delta BDC$

$\frac{EB}{BD} = \frac{BD}{DC} \Rightarrow \frac{q}{p} = \frac{p}{8}$

$\therefore \frac{27}{q} = \frac{p}{8}$ ✓ $\Rightarrow pq = 216$.

(13)