BAULKHAM HILLS HIGH SCHOOL

YEAR 10 YEARLY MATHEMATICS November 2012

Time allowed: 70 minutes

Student's Name:__________________

Teacher's Name:__________________

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- Diagrams are not to scale unless specified.
- NO liquid paper/tape is to be used in the exam
- Write your teacher's name and your name on the booklet provided.

<u>*Topics Tested:*</u> *Polynomials & Curve Sketching, Probability, Trigonometry, Volume And Surface Area, Series and its Applications, Coordinate Geometry, Further Reasoning In Number, Algebraic Techniques, Further Geometry, Graphs, Statistics, Similarity And Congruency, Circle Geometry, Function And Logarithms, Radians.*

MULTIPLE CHOICE

Answer the multiple choice on the answer booklet provided.

1 If $4^{x-1} = 32$ then the value of x is

(A) 10 (B) 3.5 (C) 3 (D) 6

2 $2\log_a 3 - \log_a 2 = ?$

(A) $\log_a 7$ (B) $\log_a 4.5$ (C) $2\log_a 1.5$ (D) cannot be simplied.

3 A code of 2 letters and 3 numbers is to be made for a security panel, using A, B, C and $1, 2, 3, 4, 5, 6$
How many codes are possible if repeats are allowed?

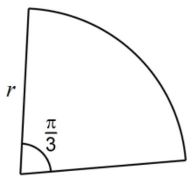
(A) 6 (B) 60 (C) 1944 (D) 531441

4 The probability that an archer at the Olympics will not hit a target in a single shot is 1 in 8. He fires two arrows. Find the probability that both arrows miss the target.

(A) $\frac{49}{64}$ (B) $\frac{1}{64}$ (C) $\frac{7}{64}$ (D) none of these

5 Given $f(x) = \frac{3}{x} - 4$, then $f^{-1}(4) = ?$

(A) $-\frac{13}{4}$ (B) $\frac{13}{4}$ (C) $\frac{3}{8}$ (D) $-\frac{3}{8}$

6 

The sector below has an area of $10\pi\ units^2$

What is the value of r?

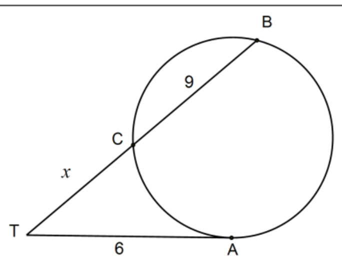
(A) $\sqrt{6}\,\pi$ (B) $\sqrt{\frac{\pi}{3}}$ (C) $\sqrt{60}$ (D) $\sqrt{\frac{1}{3}}$

7 Solve $|2x - 1| = 3x$

(A) $x = -1$ (B) $x = -1\ or\ x = \frac{1}{5}$ (C) $x = \frac{1}{5}$ (D) $x = 1$

8 Marks in a test have a mean of 78 and a standard deviation of 12. What z-score corresponds to a mark of 60?

(A) -2 (B) -1.5 (C) -1 (D) 1

9 

Line TA is a tangent to the circle at A and TB is a secant meeting the circle at B and C.

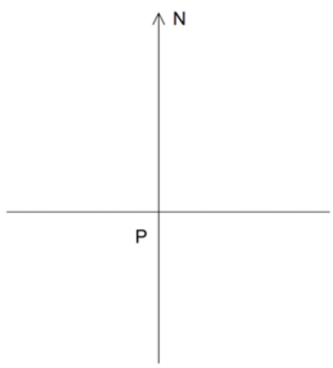
Given that $TA = 6, CB = 9, TC = x$, what is the value of x?

(A) -12 (B) 2 (C) 3 (D) 4

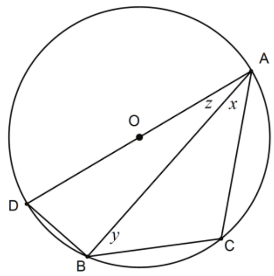
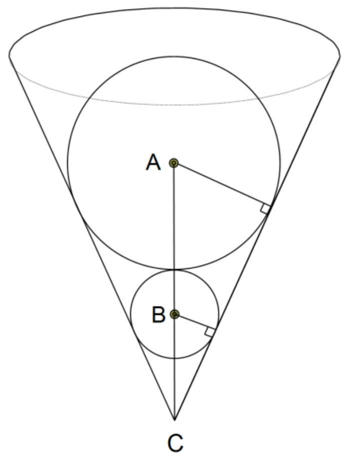
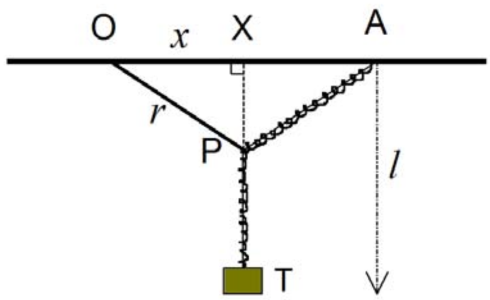
10 If $\frac{1}{x} = \frac{1}{y} + \frac{1}{z}$ then z equals

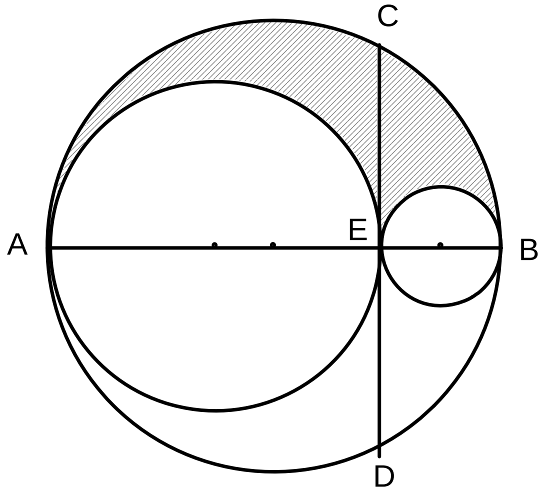
(A) $\frac{xy}{x-y}$ (B) $\frac{x-y}{xy}$ (C) $x - y$ (D) $\frac{xy}{y-x}$

		Marks
Question 11 (9 marks)		
a)	Factorise $3x^3 - 24$	**2**
b)	Line n has the equation $3x + y - 3 = 0$	
	i) Show that the gradient of the line n is -3	**1**
	ii) Line p is perpendicular to the line n and passes through the point $A(2,2)$. Show that the equation of the line p is $x - 3y + 4 = 0$	**2**
	iii) What acute angle (to the nearest degree) does the line p make with the x-axis?	**1**
	iv) Point B is the x-intercept of line p, find the coordinates of B	**1**
	v) Point C has co-ordinates $(2,6)$. Find the area of ΔABC	**2**
Question 12 (10 marks)		
a)	Find the exact value of $3\tan 210° + 3\sin 300°$	**3**
b)	i) Using the axis of symmetry, or otherwise, show that the vertex of the parabola $x^2 - 10x + 15 = 2y$ is at $(5, -5)$	**2**
	ii) State the domain and the range for this parabola.	**2**
c)	Simplify $(1 + \tan^2\theta)(1 - \sin^2\theta)$	**3**
Question 13 (9 marks)		
a)	The sum of n terms of a sequence of numbers is given by $S_n = 102n - 2n^2$ Find an expression for T_n, the nth term of the sequence	**2**
b)	Find the inverse function $f^{-1}(x)$ if $f(x) = \frac{1-2x}{x}, x \neq 0$	**2**
c)	Two separate canoes start off from a jetty, P, on a large lake. The first paddles on a bearing of $040°T$ for 12 nautical miles (nm) to a buoy Q. At the same time the second canoe paddles a distance of $8nm$ on a bearing of $100°T$ to another buoy R	



	i) Copy the sketch and add the relevant information	**1**
	ii) Calculate the distance (in nm) between the two canoes at the two buoys. (correct to 1dp)	**2**
	iii) Calculate the area of ΔPQR to 1 decimal place.	**2**

Question 14 (10 marks)		**Marks**
a)	Shade the region $y < \sqrt{9 - x^2}$	**2**
b)	 The diagram shows points B and C on a semi circle with centre O and diameter AD. Given that $\angle BAC = x$, $\angle ABC = y$ and $\angle OAB = z$, find the value of $x + y + z$, giving reasons.	**3**
c)	If $\sqrt{7} + \sqrt{28} + \sqrt{63} + \cdots T_n = 300\sqrt{7}$ How many terms are there in this series.	**3**
d)	Evaluate to 2 dp. $\dfrac{3{\cdot}1^{x} + 3{\cdot}1^{x+2}}{3{\cdot}1^{x-1}}$	**2**
Question 15 (9 marks)		
a)	Solve for θ $2\tan\theta = 3$ for $0 \le \theta \le 360°$	**2**
b)	What is the perpendicular distance of the point $(2, -1)$ from the line $y = 3x + 1$	**2**
c)	Solve $2\log_5 3 = \log_5 x - \log_5 6$	**2**
d)	i) Show that $(x + 2)$ is a factor of $P(x) = 6x^3 + 7x^2 - 9x + 2$ ii) And hence by division, or otherwise, express $P(x)$ in factored form.	**1** **2**
Question 16 (10 marks)		
a)	 In a cone of radius 22cm, two spheres are inserted with the larger sphere having radius $10cm$ and level with the top of the cone. The distance from the centre of this sphere to the apex of the cone, AC is $15cm$. The smaller sphere has radius rcm. i) Show that the distance of BC is $5 - r$ ii) Show that the radius of the smaller sphere is 2cm iii) If the remaining space in the cone is to be filled with sand, how many grams of sand are required. (Given $1cm^3 = 1g$)	 **1** **3** **3**
b)	A weight is attached to a string and hung by a metal rod as shown. The distance OA is 1 metre and the string from A to T is $2m$.  i) By using the triangles OXP and AXP find an expression for AP ii) Find an expression for l in terms of x and r	 **2** **1**

Question 7 **(12 marks)**		**Marks**
a)		
b)		
c)	i)	
d)	(harder question for circle geom. Yearly 20111) – thanks to Viera  AB, AE and EB are diameters of a circle. DC is a tangent to both circles. CD = 10 Find the shaded area.	

- END OF PAPER -

Yr Ten Yearly – 2012 Solutions

L.C.

Multiple Choice Answers

1)	A	~~B~~	C	D	6)	A	B	~~C~~	D
2)	A	~~B~~	C	D	7)	A	B	~~C~~	D
3)	A	B	~~C~~	D	8)	A	~~B~~	C	D
4)	A	~~B~~	C	D	9)	A	B	~~C~~	D
5)	A	B	~~C~~	D	10)	A	B	C	~~D~~

Question 11, 9 marks

i) $3(x^3-8) = 3(x-2)(x^2+2x+4)$ ① ①

ii) i) $y=-3x+3$ $\because y=mx+b$

$\therefore m=-3$ ① some method.

ii) $m_2 = -\frac{1}{m_1}$

$= \frac{1}{3}$ ①

$y-y_1 = m(x-x_1)$

$y-2 = \frac{1}{3}(x-2)$

$x-3y+4=0$ — some form that leads to this ①

iii) $\tan\theta = m$

$\tan\theta = \frac{1}{3}$

$\theta = 18\cdot43...$

$\doteqdot \underline{18°}$ ①

iv) B: $y=0$ $x=-4$ ① $(-4,0)$ – preferred – but $x=-4$, $y=0$ ok

v) many methods! let $D=(2,0)$

$A_{\text{Large}\triangle DCB} = \frac{1}{2}\times 6\times 6$

$= 18u^2$

$A_{\text{small}\triangle BAD} = \frac{1}{2}\times 6\times 2$

$= 6u^2$

① Method – "working towards"

$A_{\triangle ABC} = 18-6$

$= \underline{12u^2}$ ①

Question 12, 10 marks.

a) $3\tan 210° + 3\sin 300°$

$= 3\tan 30° - 3\sin 60°$

$= 3\times\frac{1}{\sqrt{3}}$ ① $- 3\times\frac{\sqrt{3}}{2}$ ①

$= \frac{3\sqrt{3}}{3} - \frac{3\sqrt{3}}{2}$

$= -\frac{\sqrt{3}}{2}$ ① If done on Calc must be complet

b) $y=\frac{1}{2}x^2-5x+\frac{15}{2}$

i) $x = \frac{-b}{2a}$

$= \frac{5}{2\times\frac{1}{2}}$ ①

$= 5$

$\therefore y = \frac{1}{2}\times 25 - 25 + \frac{15}{2}$ ①

$= -5$

$\therefore$ Vertex is $(5,-5)$

ii) D: all real x ①

R: $y \geq -5$ ①

c) $(1+\tan^2\theta)(1-\sin^2\theta)$

$= \sec^2\theta$ ① $\times \cos^2\theta$ ①

$= \frac{1}{\cos^2\theta}\times\cos^2\theta$

$= 1$ ①

Question 13. 9 marks

a) $S_n = 102n - 2n^2$

$S_{n-1} = 102(n-1) - 2(n-1)^2$ ①

$= 102n - 102 - 2n^2 + 4n - 2$

$= 106n - 104 - 2n^2$

$d = -4$

$T_n = S_n - S_{n-1}$

$= 102n - 2n^2 - 106n + 104 + 2n^2$

$= \underline{104 - 4n}$ ①

b) $f(x) = \dfrac{1-2x}{x} = y \quad (x \neq 0)$

$\therefore f^{-1}(x): \; x = \dfrac{1-2y}{y}$ ①

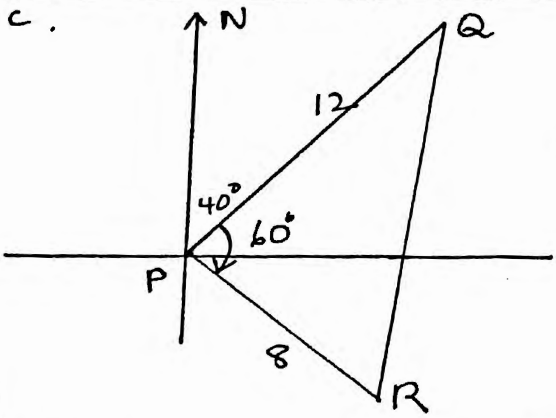
$xy = 1 - 2y$

$2y + xy = 1$

$y(x+2) = 1$

$y = \dfrac{1}{x+2}$ ①

c.



①

ii) $a^2 = b^2 + c^2 - 2bc\cos A$

$QR^2 = 12^2 + 8^2 - 2 \times 12 \times 8 \times \cos 60°$ ①

$= 112$

$QR = 10.583\ldots$ ①

$= 10.6\,\text{nm}$

iii) $A_\triangle = \frac{1}{2}ab\sin C$

$A = \frac{1}{2} \times 12 \times 8 \times \sin 60°$ ①

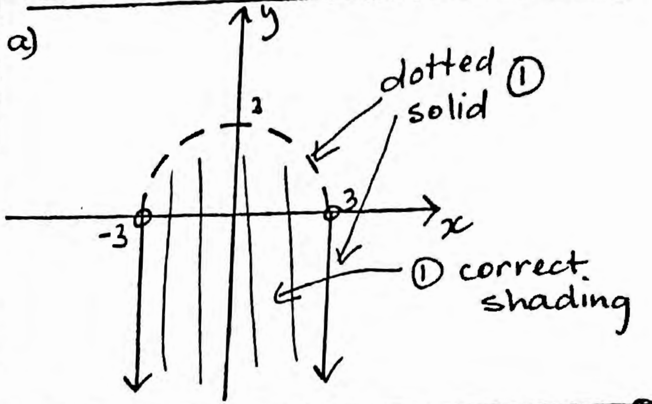
$= 41.569\ldots$

$= 41.6\,\text{nm}^2$ ①

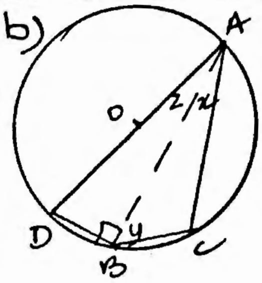
must be 1dp here.

Question 14. 10 marks

a)



b)



$A\hat{B}D = 90°$ (∠ in a semi circle) ①

DACB is a cyclic quadrilateral

$x + y + z + 90 = 180°$ ①

(opposite ∠s in a cyclic quadrilateral are supplementary)

$\therefore x + z + y = 90°$ ①

c) $\sqrt{7} + 2\sqrt{7} + 3\sqrt{7} + \ldots = 300\sqrt{7}$

AP: $a = \sqrt{7} \quad d = \sqrt{7}$ ①

$S_n = \frac{n}{2}(2a + (n-1)d)$

$300\sqrt{7} = \frac{n}{2}(2\sqrt{7} + (n-1)\sqrt{7})$

$600\sqrt{7} = \sqrt{7}\,n(2 + n - 1)$

$n^2 + n - 600 = 0$ ①

$n = \dfrac{-1 \pm \sqrt{1 + 2400}}{2}$

$= -25, 24$

but $n > 0 \quad \therefore n = 24$ ①

$\therefore$ 24 terms in series

would like this mark to reflect understan. of $n>0$?

d) $\dfrac{(3.1)^x(1 + (3.1)^2)}{(3.1)^x \times (3.1)^{-1}}$ ① evidence of factorizing

$= (3.1)(1 + (3.1)^2)$

$= 32.891$ ① not strict on 2dp.

$= 32.89$

Question 15 ~~10~~ 9 marks.

a) $2\tan\theta = 3 \qquad 0 \le \theta \le 360$

$\tan\theta = \frac{3}{2}$

$\theta = 56°19' \; (1), \; 236°19' \; (1)$

b) $3x - y + 1 = 0, \quad (2, -1)$

$\text{perp } d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$

$= \left|\frac{3\times 2 - 1\times -1 + 1}{\sqrt{9+1}}\right| \; (1)$

$= \frac{8}{\sqrt{10}}$ or $\frac{4\sqrt{10}}{5}$ units (1)

c) $\log_5 9 = \log_5 \frac{x}{6}$ (1) either LHS or RHS correct.

$\frac{x}{6} = 9$

$x = 54$ (1)

d) $P(x) = 6x^3 + 7x^2 - 9x + 2$

i) $(x+2): P(-2) = 6\times(-2)^3 + 7\times(-2)^2 - 9\times -2 + 2$ (1)

$= 0 \quad \therefore (x+2)$ is a factor.

ii)

$$\begin{array}{r} 6x^2 - 5x + 1 \\ x+2 \overline{)\, 6x^3 + 7x^2 - 9x + 2} \\ 6x^3 + 12x^2 \\ \hline -5x^2 - 9x \\ -5x^2 - 10x \\ \hline x + 2 \\ x + 2 \\ \hline 0 \end{array}$$

(1) method working towards.

$\therefore 6x^2 - 5x + 1 = (3x-1)(2x-1)$

$P(x) = (x+2)(3x-1)(2x-1)$ (1)

Should add to 9 paper said 10.

Question 16 10 marks.

a) i) $AC = 10 + 2r + x = 15$

$= 10 + r + BC = 15$ (1)

$\therefore r + BC = 5$

$BC = 5 - r$

ii) $\frac{10}{r} = \frac{AC}{BC}$ (1) – using similar Δs (1) (give a reason of some sort for ratio)

flip $\frac{r}{10} = \frac{5-r}{15}$

$15r = 50 - 10r$

$25r = 50$ (1)

$r = 2$ as required.

iii) V of cone $= \frac{1}{3}\pi r^2 h \quad (h = 15 + 10)$

$= \frac{1}{3}\pi \times 15^2 \times 25$

$= 5890.486\ldots \text{ cm}^3$ (1)

V of $S_1 = \frac{4}{3}\pi r^3 \qquad V S_2 = \frac{4}{3}\pi \times 2^3$

$= \frac{4}{3}\pi 10^3 \qquad = 33.510$

$= 4188.79$

$V_{S_1+S_2} = 4222.3\ldots$ (1)

Sand $= V_{cone} - V_{S_1+S_2}$

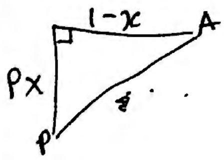
$= 1668.186 \text{ cm}^3$

$= 1668.19 \text{ g}$. ok (1)

b. $\Delta OXP \Rightarrow PX^2 = r^2 - x^2$ (1)

$PX = \sqrt{r^2 - x^2}$

$\Delta AXP \Rightarrow$



$AP^2 = (1-x)^2 + PX^2$

$= (1-x)^2 + r^2 - x^2$

$AP = \sqrt{(1-x)^2 + r^2 - x^2}$

$= \sqrt{1 - 2x^2 + r^2}$ (1) either ok

ii) $\ell = 2 - \sqrt{1 - 2x^2 + r^2} + \sqrt{r^2 - x^2}$ (1)