

BAULKHAM HILLS HIGH SCHOOL



YEAR 10

MATHEMATICS

Yearly Examination, November 2013

Time allowed: 70 minutes

Student's Name: _____

Teacher's Name: _____

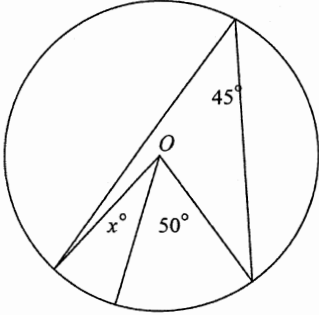
DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- Diagrams are not to scale unless specified.
- NO liquid paper/tape is to be used in the exam
- Write your teacher's name and your name on the booklet provided.

Topics Tested: Polynomials & Curve Sketching, Probability, Trigonometry, Volume And Surface Area, Series and its Applications, Coordinate Geometry, Further Reasoning In Number, Algebraic Techniques, Further Geometry, Graphs, Statistics, Similarity And Congruency, Circle Geometry, Function And Logarithms, consumer arithmetic.

MULTIPLE CHOICE (10 marks)

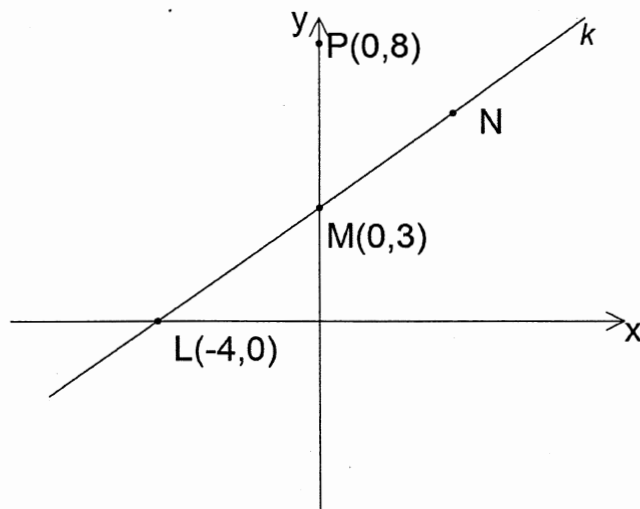
Answer the multiple choice on the answer booklet provided.

1	Jars of jam have a net weight of 115g each. They are packed in a cardboard box so that there are 8 layers with 84 jars in each layer. The net weight of jam in the box, in kilograms, is closest to (A) 50 (B) 80 (C) 8000 (D) 880
2	$2^{n+1} + 2^{n+1}$ equals (A) 2^{n+2} (B) 2^{2n+2} (C) 4^{2n+2} (D) 4^{2n+1}
3	A formula in physics states $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$. If $R_1 = 3$ and $R_2 = 6$ then R is equal to (A) $\frac{1}{2}$ (B) 2 (C) $\frac{1}{9}$ (D) 9
4	 <p>Find the value of x.</p> (A) 40 (B) 25 (C) 20 (D) 35
5	Six balls numbered 1 to 6 are in a hat and two are drawn at random. The probability that the two numbers drawn differ by 1 is (A) $\frac{1}{3}$ (B) $\frac{1}{6}$ (C) $\frac{1}{5}$ (D) $\frac{11}{30}$
6	A series is defined as $16 - 8 + 4 - 2 + 1 - \dots$ S_∞ of the series is (A) 4 (B) 16 (C) 32 (D) $\frac{32}{3}$
7	Mel invests \$1000 for 2 years at 12% per annum, with interest compounding every 6 months. How much, to the nearest cent, should Mel receive after 2 years? (A) \$1240.00 (B) \$1254.40 (C) \$1262.48 (D) \$1573.52
8	The last digit in the sum $3^{17} + 7^{13}$ is (A) 1 (B) 6 (C) 4 (D) 0
9	The area enclosed by the graphs of $x + y = 6$, $y = 4$, $x = 0$ and $y = 0$ is (A) 8 (B) 16 (C) 17 (D) 18

10	Given $S_n = 1 - 2 + 3 - 4 + 5 - 6 + \dots + (-1)^{n+1}n$ where n is a positive integer, then $S_{1992} + S_{1993}$ equals (A) -2 (B) -1 (C) 0 (D) 1
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Question 11 (10 marks)	Marks
a) Factorise $27 - 8x^3$	2
b) Find the exact value of $2 \sin 135^\circ + 2 \cos 240^\circ$	2
c) simplify $\frac{x}{x^2 - 4} + \frac{2}{x - 2}$	2
d) Solve $\cos \Theta - \frac{2}{5} = 0$ to the nearest minute where $0^\circ \leq \Theta \leq 360^\circ$	2
e) Find the value of $\log_{10}(2^{1000})$ correct to 3 decimal places.	2

Question 12 (10 marks)	
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The line k cuts the x -axis at $L(-4, 0)$ and the y -axis at $M(0, 3)$ as shown. N is a point on the line k , and P is the point $(0, 8)$.

Copy the diagram into your writing booklet.

a) Find the equation of the line k .	1
b) Find the angle α between k and the x axis.	1
c) Show that $\triangle LMP$ is isosceles.	2
d) Calculate the gradient of the line PL .	1
e) M is the midpoint of the interval LN . Find the coordinates of N .	1
f) Show that angle NPL is 90° .	2
g) Find the area of triangle LPN	2

Question 13 (13 marks)

- (a) A factory assembles torches. Each torch requires one battery and one bulb. It is known that 6% of all batteries and 4% of all bulbs are defective. Find the probability that, in a torch selected at random, both the battery and the bulb are NOT defective. Give your answer in exact form.

2

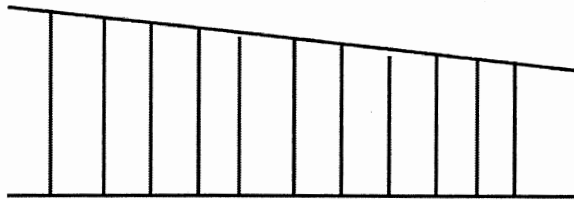
- (b) Consider the parabola with equation $y = x^2 - 7x - 8$

- i) Find the coordinates of the vertex.
ii) Sketch the parabola showing all intercepts and the vertex.

2

2

(c)



The rungs of a ladder decrease uniformly in length from 40cm at the bottom rung to 30 cm at the top rung. The total length of the rungs is 5.25m.

- i) Find the number of rungs.
ii) Find the difference in length between adjacent rungs.

2

2

- (d) Find all solutions of the equation: $(x^2 + 2x - 3)^2 + 2(x^2 + 2x - 3) - 3 = 0$

3

Question 14 (13 marks)**Marks**

a) Solve $6x^2 - 5x - 6 > 0$

2

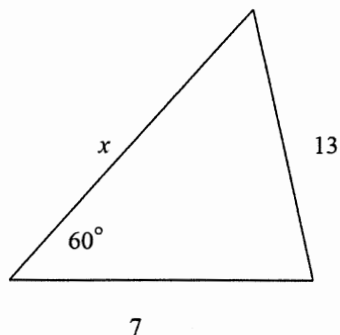
b) Find the integers a and b such that $(5 - \sqrt{2})^2 = a + b\sqrt{2}$

2

c) Let $\log_a 2 = x$ and $\log_a 3 = y$. find an expression for $\log_a 12$ in terms of x and y.

2

d)



The diagram shows a triangle with its sides and an angle of 60° as marked.

2

- (i) Show that $x^2 - 7x = 120$
(ii) Hence find the exact value of x.

1

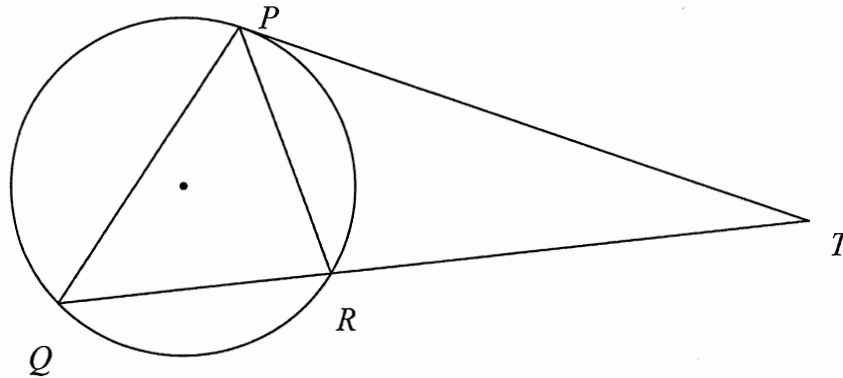
e) For a class of 20 students, the statistics of Maths and English marks are given below:

Subject	Mean	SD
English	60	10
Maths	70	14

- (i) If Tom scored 77 in Maths and 70 in English, calculate the Z score of Tom for each of his subjects. 2
- (ii) Which is the better mark? Explain. 2

Question 15 (9 marks)

(a)



PT is a tangent to the circle PRQ, and QR is a secant intersecting the circle in Q and R. The line QR intersects PT at T.

Copy the diagram into your answer booklet.

- i) Prove that triangles PRT and QPT are similar. 3
- ii) Hence prove that $PT^2 = QT \times RT$. 2
- (b) Find the quotient $Q(x)$, and the remainder $R(x)$, when the polynomial $P(x) = x^4 - x^2 + 1$ is divided by $x^2 + 1$. 2
- (c) Solve for m if $\log_5 m - \log_5 4 = 2$ 2

Question 16 (9 marks)

a) Consider the function

$$f(x) = 1 + \frac{3}{x-2} \text{ for } x > 2.$$

- (i) Give the equations of the vertical and horizontal asymptotes for $y = f(x)$. 2
- (ii) Find the inverse function of $f(x)$. 2
- (iii) State the domain of $f^{-1}(x)$. 1
- b) A and B are radar bases. B is 120km due east of A. Radar base A picks up a destroyer ship D, at $63^\circ T$ and B observes the ship at $333^\circ T$ at the same time.
- (i) Draw a neat diagram to show the above information and calculate angle ADB 2
- (ii) How far is the ship from A? 2

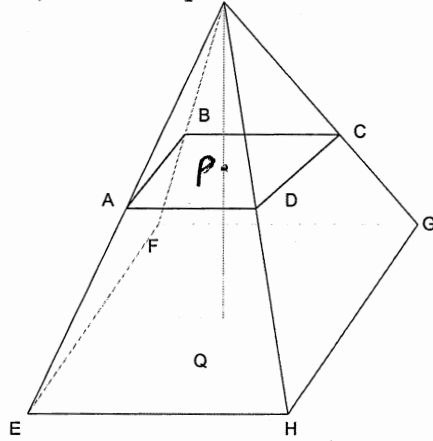
Question 17 (5 marks)

Marks

a)

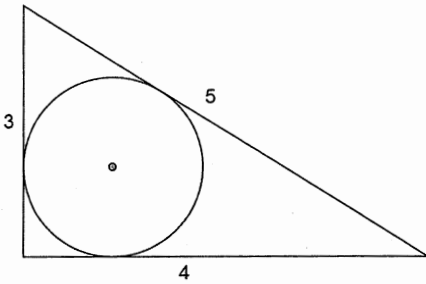
ABCD and EFGH are squares. $AD = 15$ cm, $EH = 20$ cm, $PQ = 16$ cm. Hence find the volume of the solid (frustum) below the plane ABCD.

3



b) A circle is inscribed inside a triangle with sides 3 cm, 4 cm, and 5 cm. What is the radius of the circle?

2



- END OF PAPER-



BAULKHAM HILLS HIGH SCHOOL

YEAR 10 YEARLY EXAM

2013

Name: _____ SOLUTIONS _____

Teacher: _____ N.J _____

Mathematics

Section I – Multiple Choice

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9
 A B C D

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A B C D

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

A B C D
 correct

- start here →
- | | |
|---|--|
| 1. A <input type="radio"/> B <input checked="" type="radio"/> C <input type="radio"/> D <input type="radio"/> | 6. A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input checked="" type="radio"/> |
| 2. A <input checked="" type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> | 7. A <input type="radio"/> B <input type="radio"/> C <input checked="" type="radio"/> D <input type="radio"/> |
| 3. A <input type="radio"/> B <input checked="" type="radio"/> C <input type="radio"/> D <input type="radio"/> | 8. A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input checked="" type="radio"/> |
| 4. A <input checked="" type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> | 9. A <input type="radio"/> B <input checked="" type="radio"/> C <input type="radio"/> D <input type="radio"/> |
| 5. A <input checked="" type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input type="radio"/> | 10. A <input type="radio"/> B <input type="radio"/> C <input type="radio"/> D <input checked="" type="radio"/> |

MCQ:

- 1) $\frac{115 \times 8 \times 84}{1000} \text{ kg}$ (B)
 2) $2 \cdot 2^{n+1} = 2^{n+2}$ (A)
 3) $\frac{1}{R} = \frac{1}{3} + \frac{1}{6} = \frac{9}{18}$
 $R = 2$ (B)
 4) $90 - 50 = 40^\circ =$ (A)

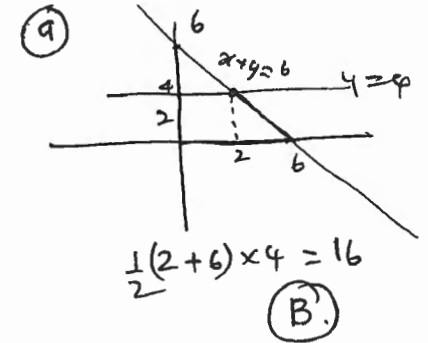
5) $\begin{matrix} 12 \\ 23 \\ 34 \\ +5 \\ \hline 56 \\ 65 \\ \hline 54 \\ 43 \\ 32 \\ 21 \end{matrix}$ (A) $\frac{10}{36}$

6) $a = 16$ $r = -\frac{1}{2}$ (D)
 $S_n = \frac{16}{1 + \frac{1}{2}} = \frac{16 \times 2}{3}$

7) $n = 2 \div \frac{1}{2} = 4$
 $r = 6\% = 0.06$ (C)
 $A = 1000 \times (1.06)^4$

8) $3^1 = 3$
 $3^2 = 9$
 $3^3 = 27$
 $3^4 = 81$
 $3^{17} = (3^4)^4 \times 3$
 last digit 3
 $7^1 = 7$
 $7^2 = 49$
 $7^3 = -3$
 $7^4 = -1$
 $(7^4)^3 \times 7 = \dots$ last digit 7
 $3 + 7 = 10$ (D)

9)



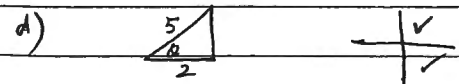
10) $S_{1992} = 1 - 2 + 3 - \dots - 1992$
 $= -1 \times 996 = -996$
 $S_{1993} = 1 - 2 + 2 - \dots - 1992 + 1993$
 $= -996 + 1993$
 $S_{1992} + S_{1993} = 1993 - 2(996)$
 $= 1$ (D)

Question 11

(a) $27 - 8x^3$
 $= 3^3 - (2x)^3 \checkmark$
 $= (3 - 2x)(9 + 6x + 4x^2) \checkmark$

(b) $2\sin 135^\circ + 2\cos 240^\circ$
 $= 2 \times \frac{1}{\sqrt{2}} + 2(-\frac{1}{2}) \checkmark$
 $= \frac{2}{\sqrt{2}} - 1$
 $= \sqrt{2} - 1 \checkmark$

(c) $\frac{x}{(x-2)(x+2)} + \frac{2}{x-2}$
 $= \frac{x + 2(x+2)}{(x-2)(x+2)} \checkmark$
 $= \frac{3x+4}{(x-2)(x+2)} \checkmark$

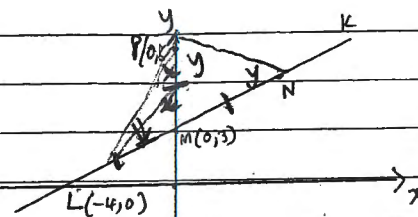


$\theta = \cos^{-1}(\frac{2}{5})$ $\theta = 66^\circ 25', 293^\circ 35'$

e) $\log_{10} 2^{1000} = 1000 \log_{10} 2$
 $= 1000 \times 0.3010 \checkmark$
 $= 301000$
 $= 301 \cdot \sqrt{030}$

You may ask for extra writing paper if you need more space to answer question 11

Question 12



a) $m_k = \frac{3-0}{0-(-4)} = \frac{3}{4}$
 $y-3 = \frac{3}{4}(x+0) \implies y = \frac{3}{4}x + 3$ or $3x - 4y + 12 = 0 \checkmark$

b) $\alpha = \tan^{-1}(\frac{3}{4}) = 36^\circ 52' 12'' \approx 37^\circ$

c) $d_{PM} = 8-3 = 5$; $d_{LM} = \sqrt{(0-(-4))^2 + (3-0)^2} = 5 \checkmark$
 In $\triangle LMP$, $PM = LM \dots \therefore \triangle$ is isosceles.

d) $m_{PL} = \frac{8-0}{0-(-4)} = 2 \checkmark$

e) $N(a,b)$ $\frac{a+(-4)}{2} = 0 \therefore a = 4$

$\frac{b+0}{2} = 3 \quad b = 6$

$\therefore N(4,6) \checkmark$ (both correct)

f) $m_{PN} = \frac{8-6}{0-4} = -\frac{1}{2} \checkmark$; $m_{PL} = \frac{8-0}{0-4} = 2$

note that $m_{PN} \times m_{PL} = -\frac{1}{2} \times 2 = -1 \checkmark$

$\therefore \hat{NPL}$ is 90°

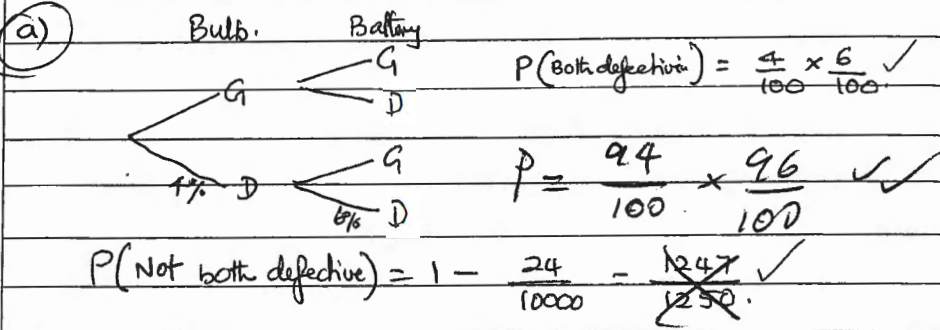
$$g) d_{LN} = \sqrt{(4-4)^2 + (6-0)^2} = \sqrt{100} = 10 \checkmark$$

$$h = \left| \frac{3 \times 0 - 4 \times 8 + 12}{5} \right| = 4.$$

$$\therefore \text{Area} = \frac{1}{2} \times 10 \times 4 = \underline{\underline{20 \text{ u}^2 \checkmark}}$$

You may ask for extra writing paper if you need more space to answer question 12

Question 13

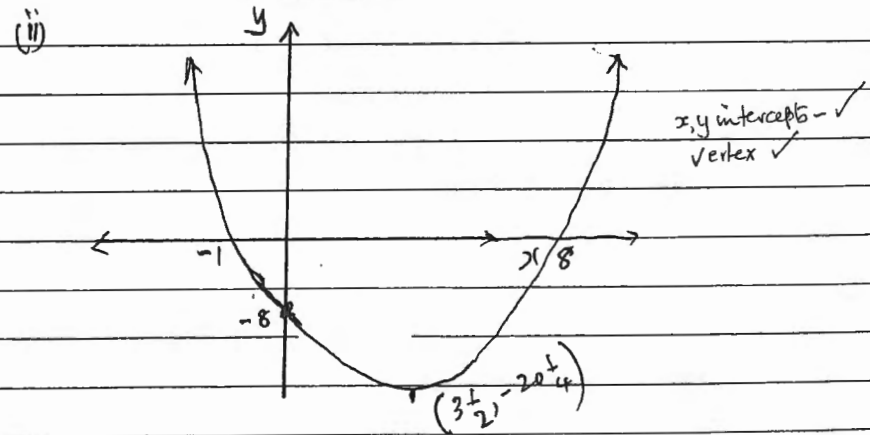


b) (i) $y = x^2 - 7x - 8$

$$a=1 \quad b=-7 \quad c=-8. \quad x = \frac{-b}{2a} = \frac{7}{2} \checkmark$$

$$\therefore y = \left(\frac{7}{2}\right)^2 - 7 \times \frac{7}{2} - 8 = -20\frac{1}{4} \checkmark$$

$$\therefore V \left(3\frac{1}{2}, -20\frac{1}{4}\right)$$



(c) (i) $a = 30$ $l = 40$. $S_n = 525 \checkmark$ (all three)

$$S_n = \frac{n}{2}(a+l)$$

$$525 = \frac{n}{2}(30+40)$$

$$35n = 525$$

$$n = \frac{525}{35} = 15 \checkmark$$

\therefore There are $\frac{35}{15}$ rungs on the ladder.

(ii) $T_{15} = 40 \checkmark$

$$40 = 30 + 14xd$$

$$d = \frac{10}{14} = \frac{5}{7} \checkmark$$

Difference in length is $\frac{5}{7}$ cm.

d) $(x^2+2x-3)^2 + 2(x^2+2x-3) - 3 = 0$

let $m = x^2+2x-3$.

$$m^2 + 2m - 3 = 0$$

$$(m+3)(m-1) = 0 \checkmark \implies m = -3; m = 1$$

$\therefore x^2+2x-3 = -3$

$\therefore x^2+2x = 0$

$$x(x+2) = 0 \implies x = 0; x = -2 \checkmark$$

$$x^2+2x-3 = 1$$

$$x^2+2x-4 = 0 \implies x = \frac{-2 \pm \sqrt{4+16}}{2}$$

$$= \frac{-2 \pm 2\sqrt{5}}{2}$$

$$x = -1 + \sqrt{5}; -1 - \sqrt{5} \checkmark$$

Question 14

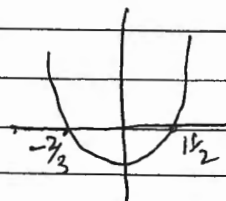
a) $6x^2 - 5x - 6 > 0$

$$6x^2 - 9x + 4x - 6 > 0$$

$$3x(2x-3) + 2(2x-3) > 0$$

$$(3x+2)(2x-3) > 0 \checkmark$$

$$x > \frac{3}{2} \text{ or } x < -\frac{2}{3} \checkmark$$



b) $(5 - \sqrt{2})^2$

$$= 25 - 10\sqrt{2} + 2$$

$$= 27 + (-10\sqrt{2})$$

$$\therefore a = 27; b = -10 \checkmark$$

c) $\log_a 12 = \log_a (4 \times 3)$

$$= \log_a 4 + \log_a 3 \checkmark$$

$$= 2 \log_a 2 + \log_a 3 \checkmark$$

$$= 2x + y \checkmark$$

d) (i) Use the cosine rule.

$$13^2 = x^2 + 7^2 - 2x \times 7 \cos 60^\circ \checkmark$$

$$169 = x^2 + 49 - 2x \times 7 \times \frac{1}{2} \checkmark$$

$$x^2 - 7x = 120.$$

(ii) $x^2 - 7x - 120 = 0$

$$x^2 - 15x + 8x - 120 = 0$$

$$(x-15)(x+8) = 0.$$

$$\therefore x = 15 \text{ or } -8$$

As x represents the length, $x = 15 \checkmark$

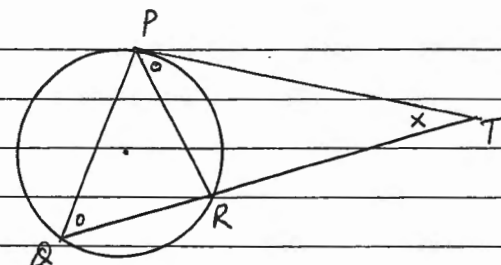
e) i) $Z = \frac{x - \bar{x}}{\sigma}$

$Z_{\text{English}} = \frac{70 - 60}{10} = +1 \quad \checkmark$

$Z_{\text{maths}} = \frac{77 - 70}{14} = \frac{7}{14} = +0.5 \quad \checkmark$

ii) English is better as its Z score is higher.

Question 15



a)

i) In $\triangle PRT$ and $\triangle QPT$

$\angle TPR = \angle TQR$ [alternate segment theorem] \checkmark

$\angle PTR$ is common for both \triangle s. \checkmark

$\therefore \triangle PRT \parallel \triangle QPT$. [Two pairs of matching \angle are \equiv]

ii) $\frac{PT}{RT} = \frac{QT}{PT}$ \checkmark [matching sides of $\parallel \triangle$ are in the same proportion] \checkmark

$PT^2 = QT \cdot RT$.

(b)
$$\begin{array}{r} x^2 + 1 \overline{) x^4 - x^2 + 1} \\ \underline{x^4 + x^2} \\ -2x^2 + 1 \\ \underline{-2x^2 - 2} \\ 3 \end{array}$$

\therefore Remainder = 3 \checkmark
 Quotient = $x^2 - 2$ \checkmark

c) $\log_5 m - \log_5 4 = 2$
 $\log_5 \left(\frac{m}{4}\right) = 2 \implies \frac{m}{4} = 5^2 \quad \checkmark$
 $\therefore m = 100 \quad \checkmark$

Question 16

a) (i) $x = 2\sqrt{y}$; $y = 1\sqrt{x}$

(ii) $x = 1 + \frac{3}{y-2}$ ✓

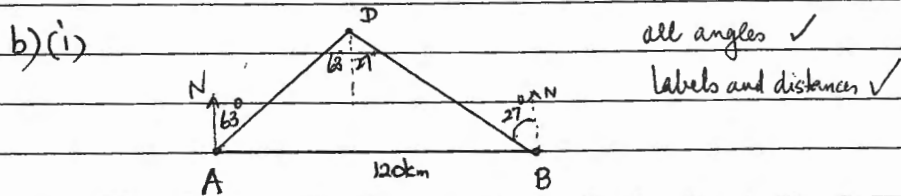
$$x-1 = \frac{3}{y-2}$$

$$y-2 = \frac{3}{x-1}$$

$$y = \frac{3}{x-1} + 2$$

$$\therefore f^{-1}(x) = 2 + \frac{3}{x-1}$$
 ✓

(iii) all real x such that $x > 1$. ✓



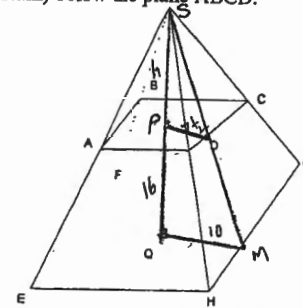
(ii) Note $\angle ADB = 90^\circ$. ✓
 $AD = 120 \cos 27^\circ$
 $\approx 106.9 \text{ km}$ ✓

$$\frac{xL}{164x} = \frac{3}{4}$$

$$4xL = 48 + 3xL$$

Question 17

a) ABCD and EFGH are squares. AD = 15 cm, EH = 20 cm, PQ = 16 cm. Hence find the volume of the solid (frustum) below the plane ABCD.



$$\frac{h}{15} = \frac{h+16}{10}$$
 ✓

$$\frac{2h}{15} = \frac{h+16}{10}$$

$$20h = 15h + 240$$

$$5h = 240$$

$$h = 48$$
 ✓

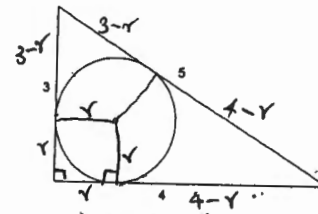
Volume of Pyramid with Base ABCD = $\frac{1}{3} \times 225 \times 48$

Volume of Pyramid with Base EFGH = $\frac{1}{3} \times 400 \times 64$

$$\therefore \text{Required volume} = \frac{1}{3} \times 64 \times 400 - \frac{1}{3} \times 225 \times 48$$

$$\approx \underline{\underline{4933.33 \text{ cm}^3}}$$
 ✓

b) A circle is inscribed inside a triangle with sides 3 cm, 4 cm, and 5 cm. What is the radius of the circle?



3, 4, 5 \rightarrow a pythagorean triad. ✓

$$3-r + 4-r = 5$$

$$7-5 = 2r$$

$$\underline{\underline{r = 1}}$$
 ✓

with some water