BAULKHAM HILLS HIGH SCHOOL

YEAR 10 YEARLY EXAM
MATHEMATICS
November 2014

Time allowed: 70 minutes

Student's Name:_____

Teacher's Name:_____

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- You must show all necessary working.
- Diagrams are not to scale unless specified.
- NO liquid paper/tape is to be used in the exam
- Write your teacher's name and your name on the booklet provided.

Topics Tested: Polynomials & Curve Sketching, Probability, Trigonometry, Volume And Surface Area, Series and its Applications, Coordinate Geometry, Further Reasoning In Number, Algebraic Techniques, Further Geometry, Graphs, Statistics, Similarity And Congruency, Circle Geometry, Function And Logarithms.

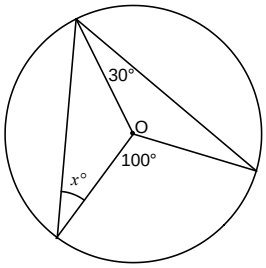
MULTIPLE CHOICE

Answer the multiple choice in the answer booklet provided.

1 Two cubes have their surface area in the ratio $1:9$. What is the ratio of their volumes?

(A) $1:3$ (B) $1:27$ (C) 1:729 (D) 1:81

2



O is the centre of the circle. What is the value of x?

(A) 20° (B) 30°

(C) 50° (D) 70°

3 The following marks were scored by 9 students in a test

0, 1, 2, 2, 2, 3, 3, 6, 8

Two students were absent and did the test on a later day. When their marks were included, the mean stayed the same but the mode was changed. What could their marks have been?

(A) 0 and 6

(B) 1 and 5

(C) 2 and 4

(D) 3 and 3

4 If $(x-2)(x-k) = x^2 + ax + 10$ then

(A) $k = 5, a = -7$

(B) $k = 5, a = 3$

(C) $k = -5, a = 3$

(D) $k = -5, a = -7$

5 Results for an aptitude test are given as z-scores. In this test Di gained a z-score of 3. The test has a mean of 55 and a standard deviation of 6.

What was Di's actual mark in this test?

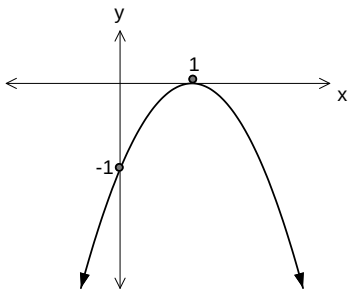
(A) 57

(B) 58

(C) 64

(D) 73

6



What is the equation of this graph?

(A) $y = x^2 - 1$ (B) $y = -x^2 - 1$

(C) $y = -x^2 + 2x - 1$ (D) $y = -x^2 - 2x - 2$

7 If $2^x = 10$ then 32^x equals

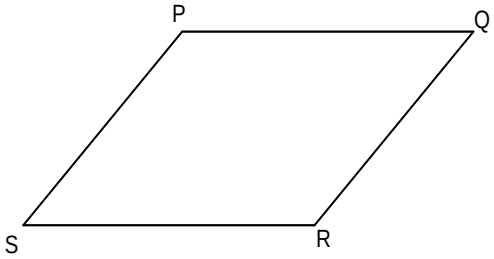
(A) 6 400

(B) 3 200

(C) 10 000

(D) 100 000

8



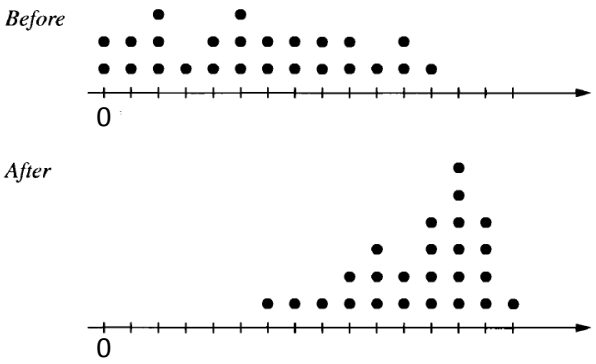
$PQRS$ is a parallelogram.
Which of the following statements would definitely make $PQRS$ a rectangle.

(A) PR bisects $\angle SPQ$
(B) PR and QS are perpendicular
(C) PS and SR are equal
(D) PR and QS are equal

9 If $f(x) = 2^x$, which of the following is equivalent to $f(-x)$?

(A) $f(x)$ (B) $\frac{1}{f(x)}$ (C) $\frac{2}{f(x)}$ (D) $-f(x)$

10 The dot plots below are drawn on the same scale. They show the class scores in tests taken before and after a unit of work was completed.



Which statement about the change in scores is correct:

(A) The mean increased and the standard deviation decreased.

(B) The mean increased and the standard deviation increased.

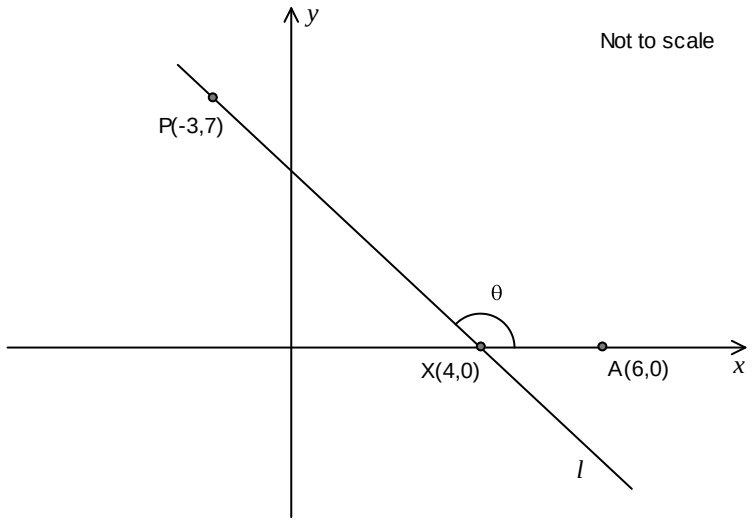
(C) The mean decreased and the standard deviation decreased.

(D) The mean decreased and the standard deviation increased.

Question 11 (10 marks)		**Marks**
a)	Solve $(2x-1)(x+5)=0$	**2**
b)	Simplify $\frac{2}{x-1}+\frac{4}{x^2+2x-3}$	**2**
c)	Find the limiting sum of the series $$\frac{2}{3}+\frac{4}{9}+\frac{8}{27}+\cdots$$	**2**
d)	Find the last digit of 7^{2011}. (Show all necessary working)	**2**
e)	Evaluate $\sum_{k=1}^{100} 4k+1$	**2**

Question 12 (10 marks)		
a)	Given $f(x)=x^2-5$	
	i) Find $f(3)$	**1**
	ii) Find the simplest expression for $\frac{f(x+h)-f(x)}{h}$	**2**
b)	Solve for $2\cos^2\theta=1$ for $0°\leq\theta\leq 360°$	**2**
c)	Solve $\lvert x-1\rvert=2x+1$	**3**
d)	Solve $x^2-7x+12\geq 0$	**2**

Question 13 (10 marks)

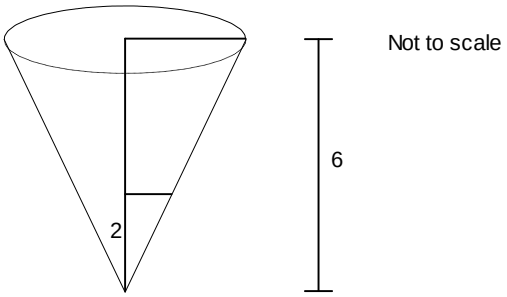


The line l cuts the x-axis at $X(4,0)$ and also passing through point $P(-3,7)$.
Point $A(6,0)$ lies on the x-axis.

a)	Find the gradient of the line l.	**1**
b)	Find the equation of the line l in general form.	**2**
c)	Find the angle θ ($\theta = \angle PXA$).	**1**
d)	Find the equation of the line passing through A, parallel to l.	**2**
e)	Find the perpendicular distance of point A from line l	**1**
f)	Find the distance PX.	**1**
g)	Find the area of ΔPXA	**2**

Question 14 (10 marks)

a) A medicine glass in the shape of a cone has a height of 6cm. 3mL of liquid medicine fill the cone to a height of 2cm. **3**



How many more millilitres of medicine are required to fill the cone to a height of 6cm?

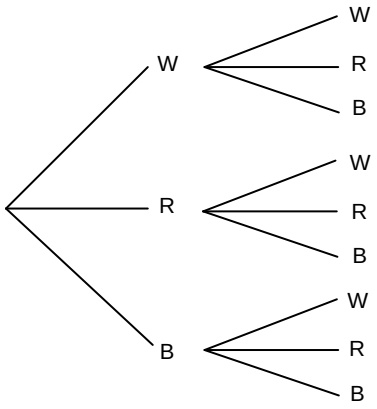
b) i) How many ways can six people sit around a circular table? **1**

ii) How many ways can six people sit in a straight line? **1**

c) Solve $2^{x-5} = 10^{x}$, correct to 2 decimal places. **2**

d) 5 white balls (W), 4 red balls (R) and 3 blue balls (B) are placed in a hat.
Two balls are selected at random without replacement.

i) Copy the probability tree diagram in your booklet and complete it. **1**



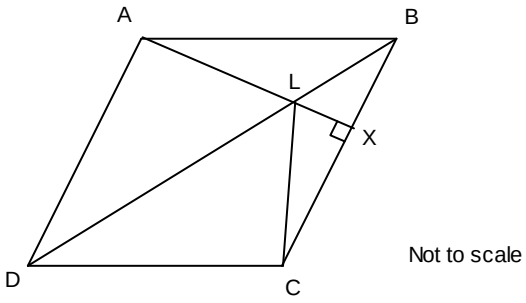
ii) Find the probability that both balls are red **1**

iii) Find the probability that at least one ball is white. **1**

Question 15 (10 marks)		**Marks**
a)	Solve $2\log_{10}(x-3) = \log_{10}(x+17)$.	**3**
b)	If $\sin\theta = \frac{2}{5}$ and θ is obtuse, find the exact value of $\tan\theta$.	**2**
c)	Given the polynomial $P(x) = x^3 + 2x^2 - 11x - 12$	
	i) Show that $(x-3)$ is a factor of $P(x)$.	**1**
	ii) Hence fully factorise $P(x)$.	**2**
	iii) Sketch the function $y = P(x)$ showing all x and y intercepts.	**2**

Question 16 (11 marks)

a)



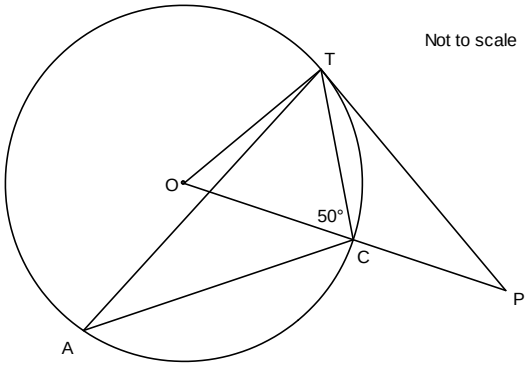
$ABCD$ is a rhombus, AX is perpendicular to BC and intersects BD at L.

i) Prove that the triangles ALD and CLD are congruent **3**

ii) Show that $\angle DAL$ is a right angle **2**

iii) Hence or otherwise, find the size of $\angle LCD$ giving reasons. **1**

b)



O is the centre of a circle. A tangent to the circle has been drawn at T and it has been produced to point P. OP intersects the circle at C. $\angle OCT = 50°$

Giving reasons,

i) Show that the size of $\angle PTC = 40°$. **3**

ii) Find the size of $\angle TAC$. **2**

Question 17 (4 marks)	**Marks**
Given the function $f(x): y = \frac{x}{x+1}$, where $x < -1$	
i) Find the inverse function $y = f^{-1}(x)$	**2**
ii) On the same set of axes, draw and clearly label the graphs of $y = f(x)$ and $y = f^{-1}(x)$, showing all asymptotes	**2**

~ End of exam ~

BAULKHAM HILLS HIGH SCHOOL

YEAR 10 YEARLY ASSESSMENT

2014

Name: _____

Teacher: _____

Section I – Multiple Choice

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

correct ↓

A ⊗ B ● C ○ D ○

Start Here →

1.	A ○	B ●	C ○	D ○	6.	A ○	B ○	C ●	D ○
2.	A ●	B ○	C ○	D ○	7.	A ○	B ○	C ○	D ●
3.	A ○	B ○	C ○	D ●	8.	A ○	B ○	C ○	D ●
4.	A ●	B ○	C ○	D ○	9.	A ○	B ●	C ○	D ⊗
5.	A ○	B ⊗	C ○	D ●	10.	A ●	B ○	C ○	D ○

FINAL MARK

BAULKHAM HILLS HIGH SCHOOL
YEAR 10 YEARLY EXAMINATION 2014

Name: _____

Teacher: _____

QUESTION	Basic	Problem
MC	*Q1-8* /8	*Q9-10* /2
Q11	*All* /10	
Q12	*All* /10	
Q13	*All* /10	
Q14		*All* /10
Q15		*All* /10
Q16		*All* /11
Q17		*All* /4
Sub-Total	/38	/37
Total	/75	

Question 11 **Name** _____

a) $(2x-1)(x+5)=0$

$x=\frac{1}{2}$ ① $\quad x=-5$ ①

b) $\frac{2}{x-1}+\frac{4}{x^2+2x-3}=\frac{2}{x-1}+\frac{4}{(x-1)(x+3)}$ ①

$=\frac{2(x+3)+4}{(x-1)(x+3)}=\frac{2x+10}{(x-1)(x+3)}$ ①

c) $\frac{2}{3}+\frac{4}{9}+\frac{8}{27}+\ldots=S_\infty=\frac{a}{1-r}$ ① $a=\frac{2}{3}$ $r=\frac{2}{3}$

$\therefore S_\infty=\frac{\frac{2}{3}}{1-\frac{2}{3}}=2$ ①

Bald answer 1/2

d) 7^{2011}

$7^1=7$

$7^2=49$

$7^3=343$

$7^4=2401$

$7^5=16807$ ①

$\therefore 2011\div 4=502$ $\therefore$ 2008th term ... last digit ①

2009-th 7

2010-th 9

2011-th term → 3

$\therefore$ last digit (3) ①

e) $\sum_{K=1}^{100} 4K+1=5+9+13+\ldots 401$ ①

AP $a=5$ $d=4$

$\therefore S_{100}=\frac{100}{2}(5+401)$ or $\frac{100}{2}(2\times 5+99\times 4)=20300$ ①

Question 12 **Name** _____

a) $f(x) = x^2 - 5$

i) $f(3) = 3^2 - 5 = 4$ (1)

ii) $\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - 5 - (x^2 - 5)}{h}$ (1)

$= \frac{x^2 + 2xh + h^2 - 5 - x^2 + 5}{h} = \frac{2xh + h^2}{h} = 2x + h$ (1)

b) $2\cos^2\theta = 1$ $\quad 0° \leq \theta \leq 360°$

$\cos^2\theta = 1/2$

$\cos\theta = \pm\frac{1}{\sqrt{2}}$ $\quad \therefore \theta = 45°, 135°, 225°, 315°$ (1) (1)

c) Solve $|x-1| = 2x+1$

$x - 1 = 2x + 1$ $\qquad x - 1 = -2x - 1$

$-2 = x$ (1) $\qquad 3x = 0$

$x = 0$ (1)

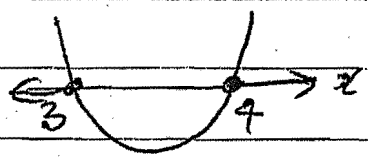
checking: $x = -2$ is not a solution $|-2-1| \neq -4+1$

$x = 0$ is O.K $\quad |0-1| = 2(0)+1$

$\therefore$ the only solution is $x = 0$ (1)

d) $x^2 - 7x + 12 \geq 0$

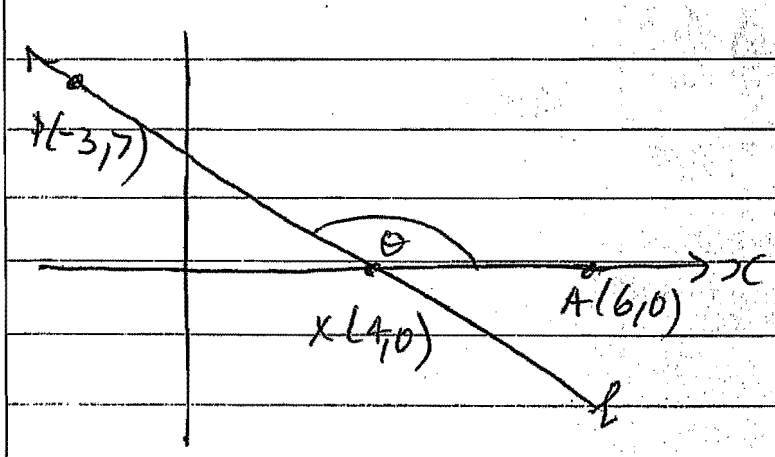
$(x-3)(x-4) \geq 0$ (1)



$\therefore x \leq 3 \; , \; x \geq 4$ (1)

You may ask for extra writing paper if you need more space to answer question 12

Question 13 **Name** _____



a) $m_l = \frac{7-0}{-3-4} = -1$ (1)

b) $l: \quad y - 0 = -1(x-4)$ (1)

$\therefore y = -x + 4$

$\therefore$ in general form $l: \quad x + y - 4 = 0$ (1)

c) $\theta = \angle PXA$

$\tan\theta = -1 = m_l \quad \therefore \quad \theta = 135°$ (1)

d) $m = -1$ & $A(6,0)$

$\therefore y - 0 = -1(x-6)$ (1)

$y = -x + 6$ or (1) $x + y - 6 = 0$

e) $A(6,0)$ from $l: x + y - 4 = 0$

$\therefore d = \frac{|6+0-4|}{\sqrt{1^2+1^2}} = \frac{2}{\sqrt{2}}$ OR $\sqrt{2}$ (1)

Question 13 **Name** _____

f) $PX = \sqrt{(-3-4)^2 + (7-0)^2} = \sqrt{98}$ (1)

g) $\text{Area}_{\Delta PXA} = \frac{1}{2} \times PX \cdot XA \cdot \sin\theta$

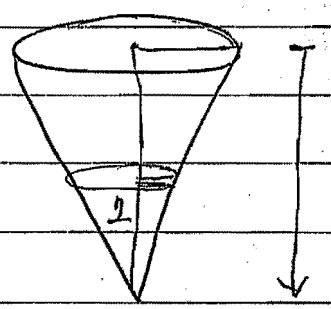
$= \frac{1}{2} \times \sqrt{98} \times 2 \times \sin 135^\circ$ (1)

$= \frac{1}{2} \times \sqrt{98} \times 2 \times \frac{1}{\sqrt{2}} = \sqrt{\frac{98}{2}} = 7 \text{ (units}^2\text{)}$ (1)

OR $\text{Area}_{\Delta PXA} = \frac{1}{2} \times 9 \times 7 - \frac{1}{2} \times 7 \times 7 = 7 \text{ (units}^2\text{)}$

or $\text{Area}_{\Delta PXA} = \frac{1}{2} b \times h = \frac{1}{2} \times \underbrace{\sqrt{2}}_{\text{from (e)}} \times \underbrace{7\sqrt{2}}_{\text{from (f)}} = 7 \text{ (units}^2\text{)}$

Question 14 **Name**_____

a) 

$V_1 = 3\,mL$

$V_2 = full$

$h : H = \frac{2}{6} = \frac{1}{3}$

$V_1 : V_2 = 1^3 : 3^3 = 1 : 27$ (1)

$3 : V_2 = 1 : 27$

$\frac{3}{V_2} = \frac{1}{27} \therefore V_2 = 81\,mL$ (1)

i. Volume required $= 81 - 3 = 78\,mL$ (1)

b) i) $(6-1)! = 5! = 120$ (1)

ii) $6! = 720$ (1)

c) $2^{x-5} = 10^x$

$\log_{10} 2^{x-5} = \log_{10} 10^x$

$(x-5)\log_{10} 2 = x$ (1)

$x \cdot \log_{10} 2 - x = 5\log_{10} 2$

$x(\log_{10} 2 - 1) = \log_{10} 2^5$ if in exact form 0.x

$x = \frac{\log_{10} 32}{\log_{10} 2 - 1}$ (1) or $\frac{5\log_{10} 2}{\log_{10} 2 - 1}$

$\therefore x = -2.15$ (2dp) or $\frac{5\ln 2}{\ln 2 - \ln 10}$

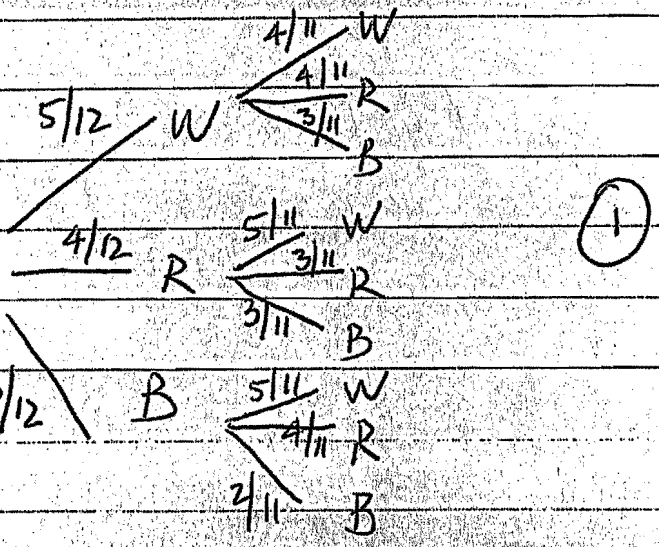
OR

$\ln 2^{x-5} = \ln 10^x$

$(x-5)\ln 2 = x \cdot \ln 10$

$x(\ln 2 - \ln 10) = 5\ln 2$

d) i)



(1)

ii) $P(RR) = \frac{4}{12} \times \frac{3}{11} = \frac{12}{12 \times 11}$ (1) $= \frac{1}{11}$

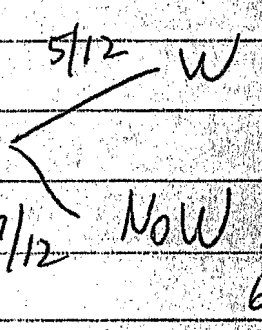
iii) $P(WW) + P(W\ Not\ W) + P(Not\ W\ W)$

$= \frac{5}{12} \times \frac{4}{11} + \frac{5}{12} \times \frac{7}{11} + \frac{7}{12} \times \frac{5}{11} = \frac{25}{22}$ (1) $\left[\frac{90}{132}\right]$

OR $P(\text{at least one is white}) = 1 - P(\text{no W, no W})$

$= 1 - \frac{7}{12} \times \frac{6}{11}$

$= \frac{15}{22}$



(OR) $\frac{5}{12} \times \frac{4}{11} + \frac{4}{12} \times \frac{5}{11} + \frac{3}{12} \times \frac{5}{11}$

Question 15 **Name**_____

a) $2\log_{10}(x-3) = \log_{10}(x+17)$

$\log_{10}(x-3)^2 = \log_{10}(x+17)$ (1)

$x^2 - 6x + 9 = x + 17$

$x^2 - 7x - 8 = 0$

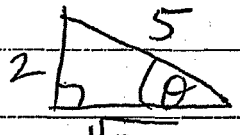
$(x-8)(x+1) = 0$

$x = 8 \quad x = -1$ (1)

but $x - 3 > 0 \therefore x > 3$

$\therefore$ the only (1) solution is $\boxed{x = 8}$

b) $\sin\theta = \frac{2}{5}$ θ - obtuse (II quadrant)



$\therefore \tan\theta = \ominus \frac{2}{\sqrt{21}}$

(1) (1)

c) $P(x) = x^3 + 2x^2 - 11x - 12$

i) $(x-3)$ is a factor of $P(x)$ if $P(3) = 0$

$\therefore P(3) = 3^3 + 2(3)^2 - 11(3) - 12 = 0$ (1)

$\therefore (x-3)$ is a factor

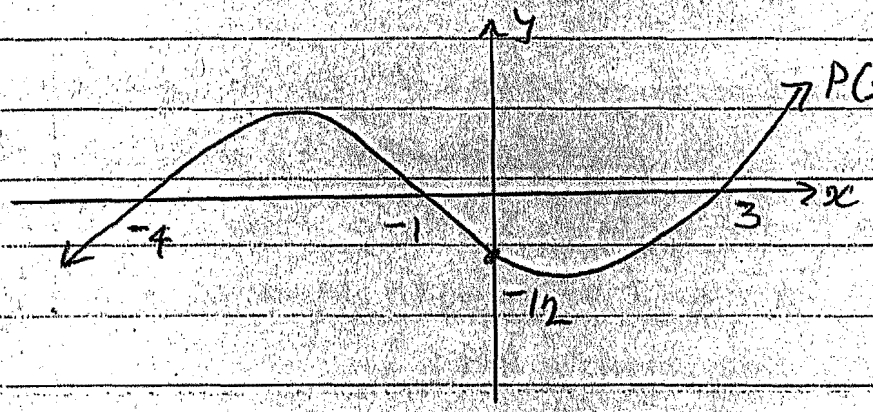
ii) $(x^3 + 2x^2 - 11x - 12) \div (x-3) = x^2 + 5x + 4$ (1)

$$\begin{array}{r} x^2 + 5x + 4 \\ x-3 \overline{)\, x^3 + 2x^2 - 11x - 12} \\ -(x^3 - 3x^2) \\ \hline 5x^2 - 11x - 12 \\ -(5x^2 - 15x) \\ \hline 4x - 12 \\ -(4x - 12) \\ \hline \text{r. } 0 \end{array}$$

$\therefore P(x) = (x-3)(x^2+5x+4) = (x-3)(x+4)(x+1)$ (1)

or other factors of -12 ..... o.k.

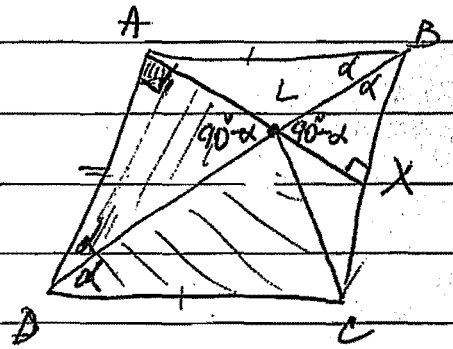
iii)



(1) shape
(1) intercepts

Question 16

a)



i) in $\triangle ALD$ & $\triangle CLD$

$AD = DC$ (all sides of rhombus =) (1)

side DL in common

$\angle ADL = \angle CDL$ (diagonal of rhombus bisects the $\angle$'s passing through) (1)

$\therefore \triangle ALD \equiv \triangle CLD$ (SAS) (1)

ii) let $\angle ADL = \angle CDL = \alpha$ opposite $\angle$'s in rhombus =

but $\angle ABL = \angle CBL = \alpha$ (1) x bisected by diagonal AB

$\therefore \angle BLX = 90° - \alpha$ ($\angle$ sum in $\triangle BXL$ where $\angle BXL = 90°$)

$\therefore \angle ALD = \angle BLX = 90° - \alpha$ (vertically opposite $\angle$'s =) (1)

$\therefore \angle DAL = 90°$ ($\angle$ sum in $\triangle DAL$)

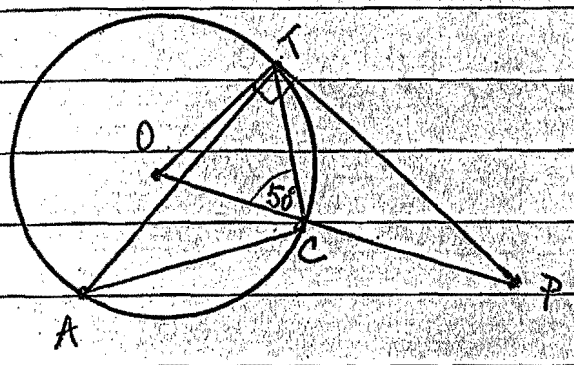
iii) $\angle LCD = 90°$ (matching $\angle$s in congruent $\triangle$ =) (1)

$= \angle DAL$

You may ask for extra writing paper if you need more space to answer question 13

Question 16 **Name:** _____

b)



i) $\angle OTC = 50°$ ($OT = OC$, $\triangle OCT$ is isosceles $\therefore$ base $\angle$s equal ①)

$\angle PTO = 90°$ (tangent ⊥ radius OT at pt. of contact)

$\angle PTC + 50° = 90° \therefore \angle PTC = 40°$ (① sum of adjacent $\angle$s $= 90°$ or complementary)

ii) $\angle TOC = 80°$ ($180° - \angle OTC \times 2 \therefore$ $\angle$ sum in $\triangle TOC$)

$\therefore \angle TAC = 40°$ (angle at circumference is half the angle at the centre standing on the same arc)

OR since $\angle CTP = 40° \therefore \angle TAC = 40°$ (① $\angle$ between tangent & chord equals the angle at the circumference in alternate segment)

Question 17

Given the function $f(x): y = \frac{x}{x+1}$ where $x \leq -1$.

i) Find the inverse function $f^{-1}(x)$

$f(x): y = \frac{x}{x+1}$

$f^{-1}(x): x = \frac{y}{y+1}$ ①

$xy + x = y$

$y(x-1) = -x$

$f^{-1}(x): y = \frac{x}{1-x}$ ① or $\frac{-x}{x-1}$ or

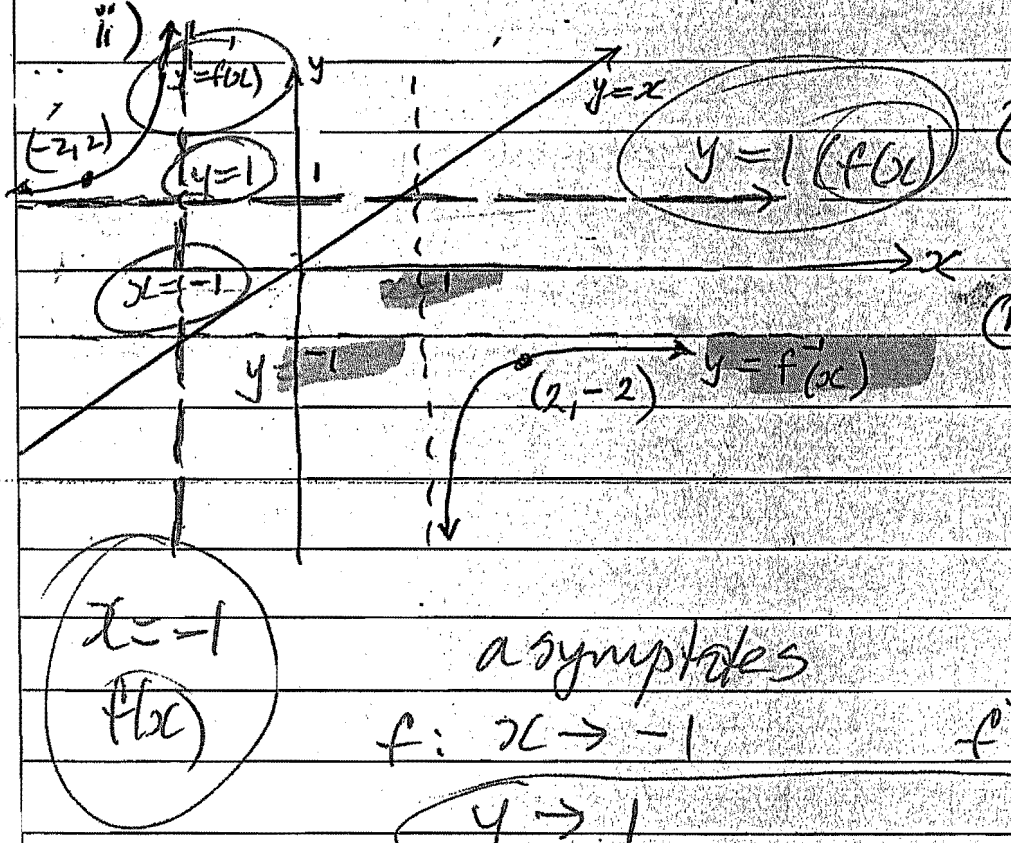
$x = \frac{y+1-1}{y+1} = 1 - \frac{1}{y+1}$

$x - 1 = -\frac{1}{y+1}$

$y = \frac{x}{1-x} = \frac{-x+1-1}{-(1-x)}$

$= -1 + \frac{1}{1-x}$

ii)



① making them symmetrical around $y = x$

① clearly stating asymptotes for both $f(x)$ and $f^{-1}(x)$

$x = -1$ $f(x)$

asymptotes

$f: x \to -1$ $\quad f^{-1}: y \to -1$

$y \to 1$ $\quad x \to 1$

$f: x \to -\infty \therefore y \to 0^+$

$f^{-1}: x \to 0^+ \therefore y \to -\infty$

Nada Dominik Vien
Pam Sam Gemini