BAULKHAM HILLS HIGH SCHOOL

November 2016

Mathematics

YEAR 10 Yearly Exam

General Instructions

- Working time – 70 minutes
- Write using non-erasable black or blue pen
- Board-approved calculators may be used
- Show all necessary working in Section II
- Marks may be deducted for careless or badly arranged work

Total marks – 68
Exam consists of 7 pages.

This paper consists of TWO sections.

Section 1 – Page 2-4 (10 marks)
Questions 1-10

- Attempt Question 1-10
- Allow about **12** minutes for this section

Section II – Pages 5-7 (58 marks)

- Attempt questions 11-15
- Allow about **58** minutes for this section

Section I – Multiple choice questions (10 marks)
Use the multiple choice Answer Sheet for Question 1 – 10.

1. The solutions of $4m^2 = m$ are:

(A) $m = 0, \frac{1}{4}$ (B) $m = 0, -\frac{1}{4}$

(C) $m = \frac{1}{2}, -\frac{1}{2}$ (D) $m = 2, -2$

2. The first three terms of an arithmetic progression are 26, 23, 20. The sum of the first n terms of the series is:

(A) $S_n = \frac{n}{2}(-3n + 55)$ (B) $S_n = 29 - 3n$

(C) $S_n = \frac{n}{2}(-3n + 29)$ (D) $S_n = 26 - 3n$

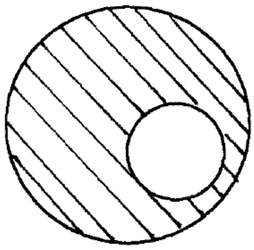
3. If $f(x) = 2x^2 - 3x + 4$ the value of $f(1) - f(-1) =$

(A) 2 (B) -2

(C) -6 (D) 6

4. In the diagram the radius of the larger circle is $2\frac{1}{2}$ times the radius of the smaller circle. The ratio of the unshaded area to the shaded area is:

Not to scale.



(A) 2 : 5 (B) 4 : 25

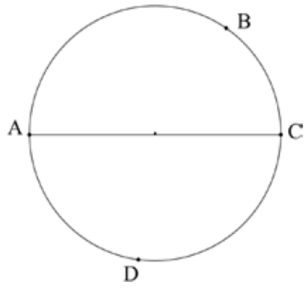
(C) 5 : 2 (D) 4 : 21

5. If $(x + 2)$ is a factor of the polynomial $2x^3 + kx^2 + 5x - 2$, the value of k is:

(A) 7 (B) -7

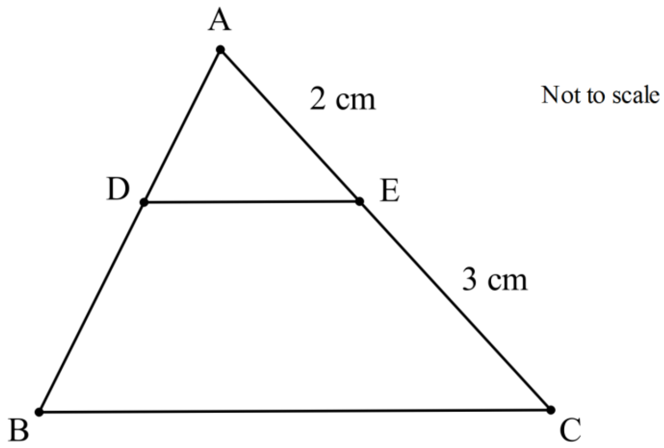
(C) -12 (D) 12

6. A, B, C and D are points on a circle as shown.
The circle has centre O and AC is a diameter of the circle.
If $\angle ABD = 75^0$ and $\angle BDC = 25^0$, then $\angle BCA$ is equal to:



(A) 15°
(B) 25°
(C) 65°
(D) 75°

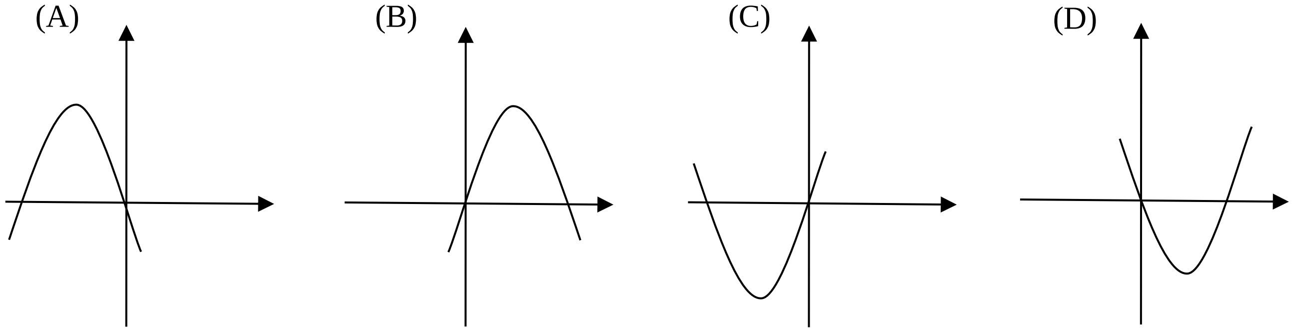
7. In the diagram below, ΔADE is similar to ΔABC.



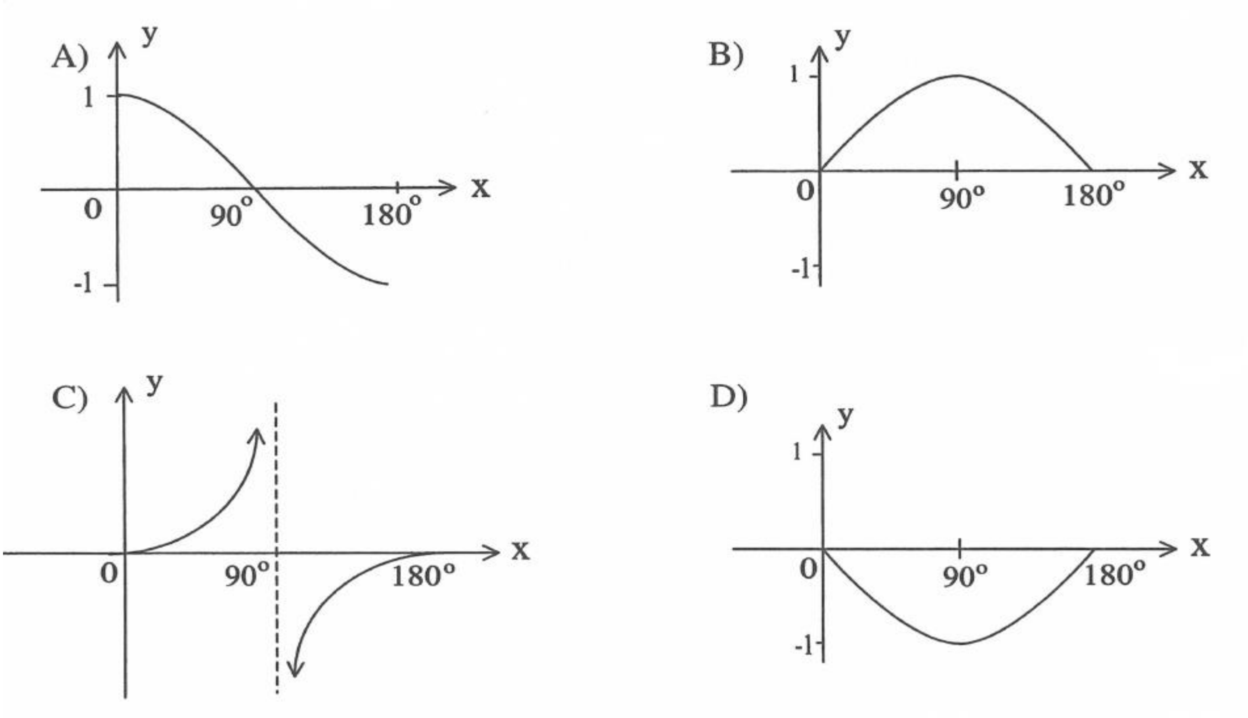
ΔADE has an area of 16 m². The area of ΔABC is

(A) $36m^2$
(B) $40m^2$
(B) $52m^2$
(D) $100m^2$

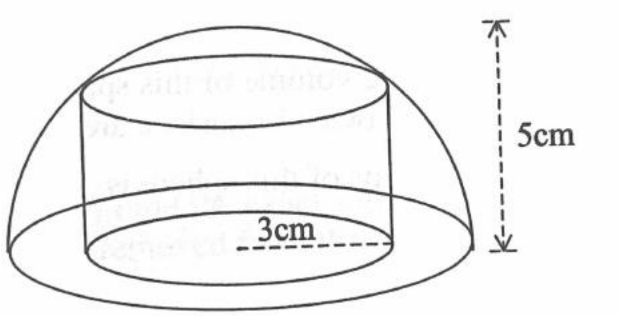
8. Which of the following graphs has an equation of the form
$y = ax^2 + bx$, where $a < 0$ and $b > 0$?



9. Which of the following curves represents $y = \sin x$ where $0 \leq x \leq 180^0$



10. The diagram shows a cylinder inscribed in a hemisphere



The exact volume of the cylinder is:

(A) 45π cm^3

(B) 36π cm^3

(C) 30π cm^3

(D) 24π cm^3

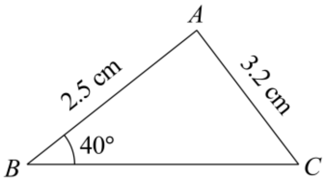
End of Section I

Section II – Extended response questions (53 marks)

Question 11 (12 marks) - Start a new page

a) Solve the equation $4\sin^2 A = 3$ for $0° \leq A \leq 360°$ **2**

b) In ΔABC, $AB = 2.5$ cm, $AC = 3.2$ cm and $\angle ABC = 40°$. **2**



Find the value of $\angle ACB$.

c) Find the equation of the line that passes through the point (2,-1) and is parallel to the line $2x + y = 4$ **2**

d) For the following parabola $y = 2x^2 - 2x + 1$

(i) Find the equation of the axis of symmetry. **1**

(ii) Find the minimum value of y. **1**

(iii) Sketch the parabola showing all the important features. **2**

e) Tap A can fill a tank full of water in 30 minutes; tap B can fill the same tank full of water in 15 minutes. **2**

How long will it take to fill the same tank using both taps simultaneously?

Question 12 (12 marks) - Start a new page

a) Factorise fully: $a^2c - b^2c - abc^2 + ab$. **2**

b) i) Show that the line through BC has equation $x + 7y - 26 = 0$ where $B = (5,3)$ and $C = (-2,4)$. **2**

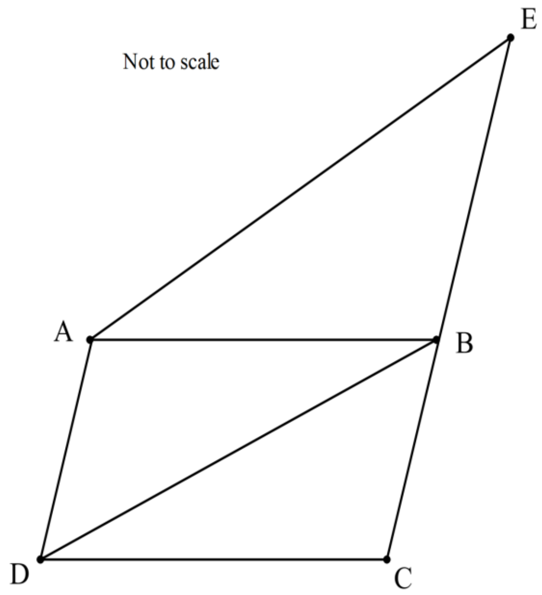
ii) Find the perpendicular distance from the point $A(2,1)$ to line BC. **2**

c) Solve for x: $5^x \times 25^{2x+1} = 125^x$ **2**

d) If $\tan A = -\dfrac{12}{5}$ and $\sin A > 0$, find the exact value of $\cos A$. **2**

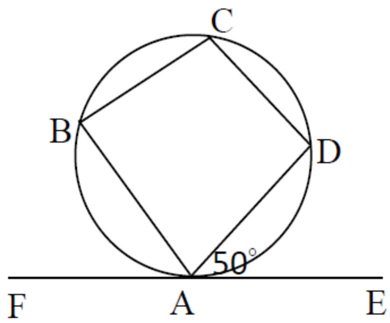
e) Write the domain and range of $y = \sqrt{25 - x^2}$ **2**

Question 13 (12 marks) - Start a new page

a)	Solve $3x^2 + 4x - 3 = 0$. (Leave your answer(s) in exact surd form).	**2**
b)	ABCD is a rhombus. CB is produced to E such that CB = BE. Copy the diagram into your answer booklet.  (i) Prove that $\Delta ABE \equiv \Delta DCB$ (ii) Hence explain why AE is parallel to DB (iii) State giving reasons, what type of quadrilateral is $AEBD$	 **3** **1** **2**
c)	Shade the region satisfying the inequality $(x-1)^2 + (y-1)^2 \leq 1$	**2**
d)	Find the value(s) of x such that the three following successive terms: $(x-1), (x+3), (5x+3)$ form a geometric sequence.	**2**

Question 14 (10 marks) - Start a new page

a)	Find the points of intersection of the graphs $y = 4 - 2x$ and $y = x^2 + 4x - 3$.	**3**
b)	Sketch the graph $f(x) = 3^x$, and the graph $y = -f(x)$ on the same number plane	**2**

c)	$ABCD$ is a cyclic quadrilateral and FAE is a tangent at A. $\angle DAE = 50°$ and $BD//FE$. Copy the diagram into your booklet.  (i) Calculate $\angle BAF$, giving reasons. (ii) Calculate $\angle BCD$, giving reasons	**2** **2**
d)	Mr Zhao has three children. The product of the children's ages is 200 and the two youngest are twins. What is the age of the oldest child? (if the ages of the children are integers and none of the children are over 40)	**1**

Question 15 (12 marks) - Start a new page

a)	(i) Graph $y = x(x+1)(x+3)^2$ (ii) Hence solve $x(x+1)(x+3)^2 \le 0$	**2** **1**
b)	Find the values of k for which $x^2 - 2kx + 6k = 0$ has real roots.	**3**
c)	Ivan lives in Parramatta and is starting a new job in the city. He needs to catch a train to get to work. His new boss says he cannot be late on the first two days of his new job or he will lose it. The probability that his train will arrive on time is 0.96. (i) What is the probability that Ivan's train is late on the first day? (ii) What is the probability of the train being late on the first two days? (iii) What is the probability of Ivan keeping his job? (iv) What is the probability that Ivan arrives late on exactly one of the first three days of his new job? (do not round off your answer).	**1** **1** **1** **1**
d)	(i) Show that $\dfrac{x}{n(n+1)} = \dfrac{x}{n} - \dfrac{x}{n+1}$ (ii) Hence or otherwise simplify the following $\dfrac{x}{2} + \dfrac{x}{6} + \dfrac{x}{12} + \dfrac{x}{20} + ... + \dfrac{x}{9900}$	**1** **1**

- END OF PAPER -

Y10 Yearly 2016

Multiple choice questions

1. A 2. A 3. C 4. D 5. A 6. C 7. D
8. B 9. B 10. B

Q11 a)

$$4\sin^2 A = 3$$
$$\sin^2 A = \frac{3}{4}$$
$$\sin A = \pm\frac{\sqrt{3}}{2}$$

$\sin A = \frac{\sqrt{3}}{2}$ $\quad$ $A = 60°, 120°$ $\quad$ 1

$\sin A = -\frac{\sqrt{3}}{2}$ $\quad$ $A = 240°, 300°$ $\quad$ 1

b)
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad 1$$

$$\frac{2.5}{\sin C} = \frac{3.2}{\sin 40°} \quad 1$$

$$\sin C = \frac{2.5 \sin 40°}{3.2}$$
$$= 0.5021778$$

$$\angle C = \angle ACB = \sin^{-1}(0.5021778) \quad 1$$
$$= 30.14° \text{ or } 30°9'$$

c)
$$2x + y = 4$$
$$y = -2x + 4$$

Parallel $m_1 = m_2$

$m_1 = -2$

$m = -2$ $\quad$ point $(2, -1)$

Equation of the line

1 $\quad y - y_1 = m(x - x_1)$
$$y - -1 = -2(x - 2)$$
$$y + 1 = -2x + 4$$

Equation of the line

$$\boxed{y = -2x + 3}$$

1

d) $y = 2x^2 - 2x + 1$

i) equation of axis of symmetry

$$x = -\frac{b}{2a}$$

$$= -\frac{-2}{2\times 2}$$

$$= \frac{1}{2} \quad 1$$

$y = f(\frac{1}{2}) = \frac{1}{2}$

Vertex $(\frac{1}{2}, \frac{1}{2})$

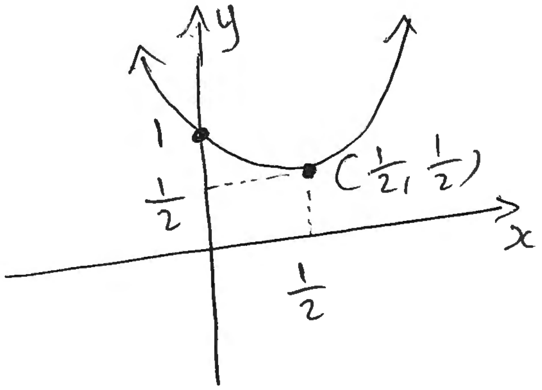
ii) Minimum value of y

$$f(\tfrac{1}{2}) = 2\times(\tfrac{1}{2})^2 - 2\times\tfrac{1}{2} + 1$$

$$= \frac{1}{2} - 1 + 1$$

$$= \frac{1}{2}.$$

iii)



~~Sp~~ shape 1

1 Vertex, y intercept

e) Let V = Volume of the tank (Full volume)

In one minute, Tap A can fill $\frac{V}{30}$

, Tap B can fill $\frac{V}{15}$.

Together, in one minute both taps can fill

$$1 \quad \frac{V}{30} + \frac{V}{15} = \frac{V + 2V}{30} = \frac{3V}{30} = \frac{V}{10}.$$

$\therefore$ To get full tank V, it take both taps 10 minutes.

1

Q12 (3)

a) $a^2c - b^2c - abc^2 + ab$

$= a^2c - abc^2 - b^2c + ab$

$= ac(a-bc) + b(a-bc)$ (1)

$= (ac+b)(a-bc)$ (1)

b) i) $B(5,3)$ $C(-2,4)$

$$m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4-3}{-2-5}$$

$$= -\frac{1}{7}$$ (1)

equation of the line

$y - y_1 = m(x - x_1)$

$y - 3 = -\frac{1}{7}(x-5)$

$7(y-3) = -x+5$

$7y - 21 = -x+5$

$x + 7y - 26 = 0$ (1)

ii) perpendicular distance from $A(2,1)$ to line BC.

$$d_{per} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2+B^2}}$$ (1)

$$= \frac{|1\times 2 + 7\times 1 - 26|}{\sqrt{1^2+7^2}} = \frac{17}{\sqrt{50}}$$ (1)

4

c) $5^x \times 25^{2x+1} = 125^x$

$5^x \times 5^{2(2x+1)} = 5^{3x}$ 1

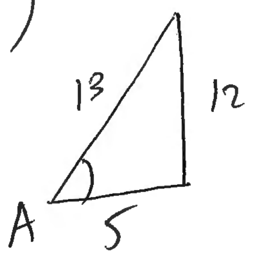
$5^{x+4x+2} = 5^{3x}$

$5x+2 = 3x$

$2x = -2$

$x = -1$ 1

d)



$\tan A < 0$

$\sin A > 0$

$\therefore \cos A < 0$

$\cos A = -\frac{5}{13}$

1 1

e) $f(x) = \sqrt{25-x^2}$

$25 - x^2 \geq 0$

Domain $-5 \leq x \leq 5$

Range $0 \leq y \leq 5$

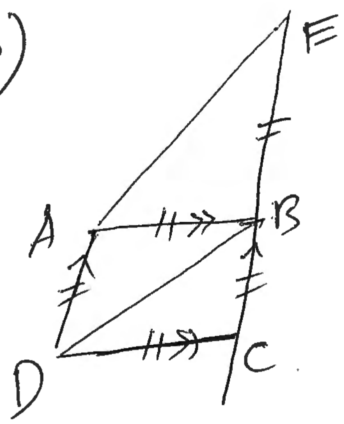
Q13

a) $3x^2+4x-3=0$

$\Delta = b^2-4ac = 4^2-4\times3\times-3$
$= 16+36$
$= 52$

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$= \frac{-4\pm\sqrt{52}}{2\times3}$$
$$= \frac{-4\pm2\sqrt{13}}{6}$$
$$= \frac{-2\pm\sqrt{13}}{3}$$

b)



i) In ΔABE & ΔDCB
$CB = BE$ (given)
$AB = DC$ (opposite sides of a rhombus are $=$)
$\angle DAB = \angle ABE$ (alternate $\angle$'s, $AD \parallel CE$)
$\angle DAB = \angle DCB$ (opposite $\angle$'s of a rhombus are equal)

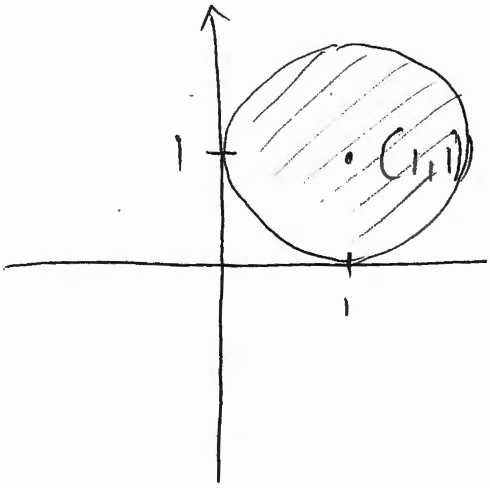
$\therefore \Delta ABE \equiv \Delta DCB$ (SAS)

ii) $\therefore \angle AEB = \angle DBC$ (matching $\angle$'s of congruent Δ's)

$\therefore AE \parallel DB$ (Corresponding $\angle$'s equal)

iii) $AD \parallel BE$ ($AB \parallel BC$ opposite sides of rhombus $\parallel$)
$AE \parallel DB$ (proven)
$\therefore AEBD$ is a parallelogram (2 pairs of opposite sides $\parallel$)

c)



centre (1,1)
x & y intercepts
(1,0) ; (0,1) 1

6

Region 1

d)

$$\frac{x+3}{x-1} = \frac{5x+3}{x+3} \quad 1$$

$$(x+3)(x+3) = (x-1)(5x+3)$$
$$x^2+6x+9 = 5x^2-2x-3$$
$$5x^2-x^2-2x-6x-3-9=0$$
$$4x^2-8x-12=0$$
$$4(x^2-2x-3)=0$$
$$x^2-2x-3=0$$
$$(x-3)(x+1)=0$$
$$x=3 \quad \text{or} \quad x=-1 \quad 1$$

Q14

a) $\begin{cases} y = 4 - 2x & (1) \\ y = x^2 + 4x - 3 & (2) \end{cases}$

$x^2 + 4x - 3 = 4 - 2x$

$x^2 + 4x + 2x - 3 - 4 = 0$

$x^2 + 6x - 7 = 0$

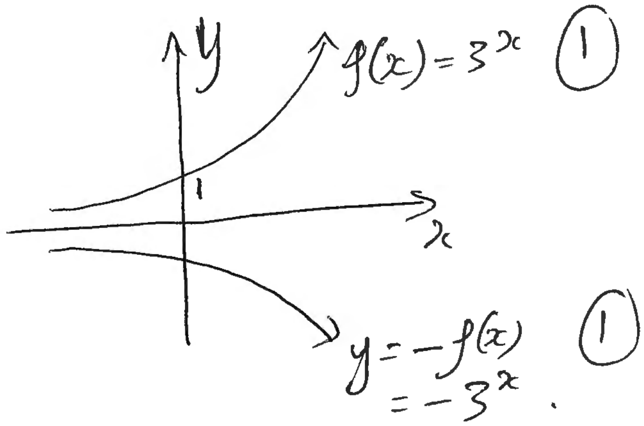
$(x-1)(x+7) = 0$

$x = 1$, $x = -7$

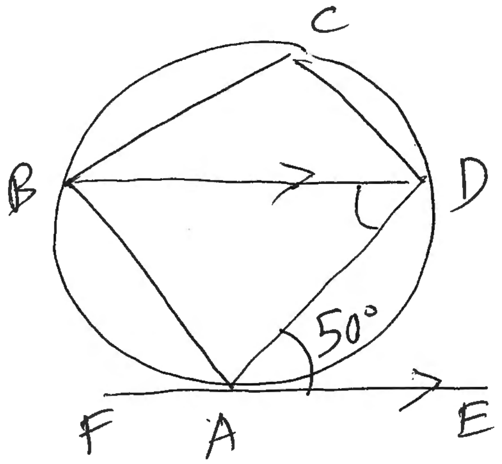
$y = 4 - 2 \times 1 = 2$; $y = 4 - 2 \times -7 = 18$

Points of intersection $(x = 1, y = 2)$; $(x = -7, y = 18)$

b)



c)



i) $\angle BDA = \angle DAE = 50^\circ$ (Alternate $\angle$'s, $BD \parallel FE$) 1

$\angle BAF = \angle BDA = 50^\circ$ (alternate segment theorem) 1

ii) $\angle BAD = 180^\circ - \angle BFA - \angle DAE$ (angle sum of $\angle$'s on straight line)

$= 180^\circ - 50^\circ - 50^\circ$

$= 80^\circ$ 1

$\angle BCD = 180^\circ - \angle BAD$

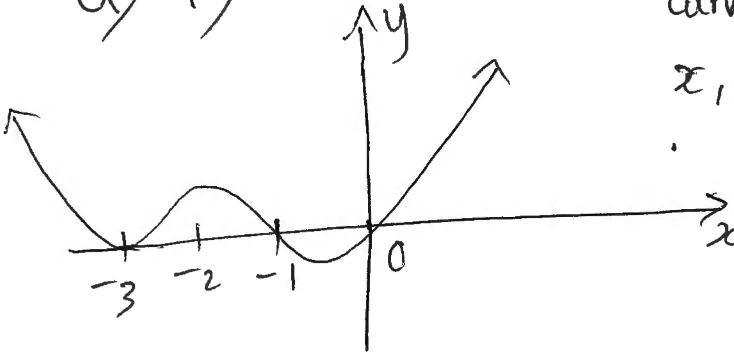
$= 100^\circ$ (Opposite $\angle$'s of a cyclic quadrilateral are supplementary)

1

d) Oldest child's age = 8 yrs old ($8 \times 4 \times 4 = 200$)

Q15

a) i)



Curve 1
x, y intercepts 1

ii) $-1 \le x \le 0$ from graph.

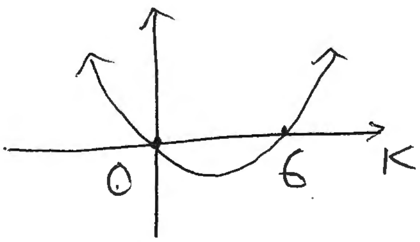
b) $x^2 - 2kx + 6k = 0$

To have real roots $\Delta = b^2 - 4ac \geq 0$ 1

$\Delta = (-2k)^2 - 4 \times 1 \times 6k \geq 0$

$4k^2 - 24k \geq 0$ 1

$4k(k-6) \geq 0$



$K \leq 0$ or $k \geq 6$

1

c) One day $P(\text{on time}) = 0.96$

i) $\therefore P(\text{late}) = 1 - 0.96$
$= 0.04$.

ii) ~~P(late on the first two days)~~

~~$= P(\text{late}) \times P(\text{late})$~~

~~$= 0.04 \times 0.04$~~

~~$= 0.0016$~~

10

iii) $P(\text{keep his job})$

$= \cancel{P(\text{not late on the first two days})}$

$= \cancel{1 - P(\text{late on the first two days})}$

$= \cancel{1 - 0.0016}$

$= \cancel{0.9984}$.

iv) $P(\text{late on exactly one of the first three days})$

$= 3 \times 0.04 \times 0.96^2$

$3 \begin{cases} \text{On time, late, late} \\ \text{Late, on time, late} \\ \text{Late, late, on time} \end{cases}$

Three arrangements — one day late — Two days on time

$= 0.110592$.

d) i) $\dfrac{x}{n} - \dfrac{x}{n+1} = \dfrac{(n+1)x - n \times x}{n(n+1)}$

$= \dfrac{nx + x - nx}{n(n+1)}$

$= \dfrac{x}{n(n+1)}$

ii) $\dfrac{x}{2} + \dfrac{x}{6} + \dfrac{x}{12} + \cdots + \dfrac{x}{9900}$

$= \dfrac{x}{1\times 2} + \dfrac{x}{2\times 3} + \dfrac{x}{3\times 4} + \cdots + \dfrac{x}{99\times 100}$

$= \dfrac{x}{1} - \cancel{\dfrac{x}{2}} + \cancel{\dfrac{x}{2}} - \cancel{\dfrac{x}{3}} + \cancel{\dfrac{x}{3}} - \cancel{\dfrac{x}{4}} + \cdots \cancel{\dfrac{x}{99}} - \dfrac{x}{100}$

$= x - \dfrac{x}{100} = \dfrac{99x}{100}$

ii) * If late on both days:

Scenario 1: P(late on 1st day) × P(late on 2nd day)

$= 0.04 \times 0.04$

$= 0.0016$

* If (late 1st day, on time 2nd day)

Scenario 2: + (On time 1st day, late 2nd day)

+ (late 1st day, late 2nd day)

$= 0.04 \times 0.96 + 0.96 \times 0.04$

$+ 0.04 \times 0.04$

$= 0.0784.$

iii) If students assume scenario 1.

Answer for part (iii)

$1 - 0.0016 = 0.9984.$

If students assume scenario 2:

Answer: $1 - 0.0784$

$= 0.9216.$