BAULKHAM HILLS HIGH SCHOOL

YEAR 10

MATHEMATICS
Yearly Examination, November 2017

Time allowed: 70 minutes

Student's Name__________________
Teacher's Name__________________

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- Diagrams are not to scale unless specified.
- **Do NOT use** liquid paper/correction tape in the exam.
- Write your teacher's name and your name on the answer booklet provided.
- You may use an approved calculator.

SECTION I - MULTIPLE CHOICE QUESTIONS (10 marks)

Answer the multiple choice questions by **shading** the correct option on the answer booklet provided.

1 $(\sqrt{5}-\sqrt{2})^2 =$

(A) 3 (B) 7 (C) $7-\sqrt{20}$ (D) $7-2\sqrt{10}$

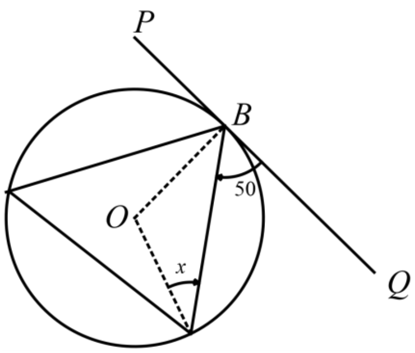
2 $2^{n+1} + 2^{n+1}$ equals

(A) 2^{n+2} (B) 2^{2n+2} (C) 4^{2n+2} (D) 4^{2n+1}

3 Solve for x: $2x^2-5x-1=0$.

(A) $x=\frac{5\pm\sqrt{17}}{4}$ (B) $x=\frac{-5\pm\sqrt{17}}{4}$ (C) $x=\frac{5\pm\sqrt{33}}{4}$ (D) $x=\frac{-5\pm\sqrt{33}}{4}$

4

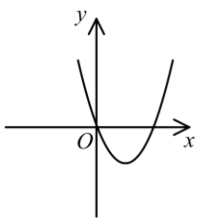


PQ is a tangent to the circle centre O and B is the point of contact. Find the value of x.

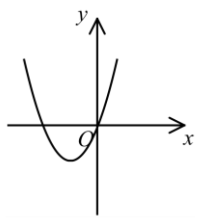
(A) 40° (B) 50° (C) 65° (D) 80°

5 Which of the following could be the graph of $y=x(4-x)$?

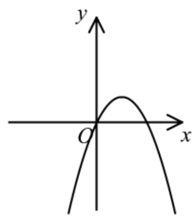
(A)



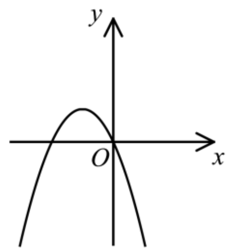
(B)



(C)



(D)



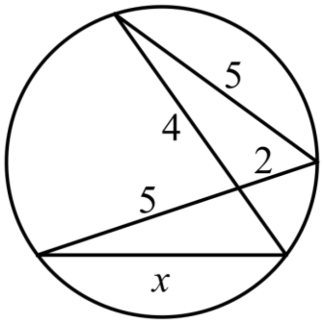
6 The last digit of $3^{17} + 7^{13}$ is

(A) 1 (B) 6 (C) 4 (D) 0

7 The solutions of $m^4 - m^2 - 6 = 0$ are

(A) $m = \sqrt{3}$, $m = \sqrt{2}$.
(B) $m = \pm\sqrt{3}$, $m = \pm\sqrt{2}$.
(C) $m = \pm\sqrt{3}$.
(D) $m = \pm\sqrt{2}$.

8



The value of x is

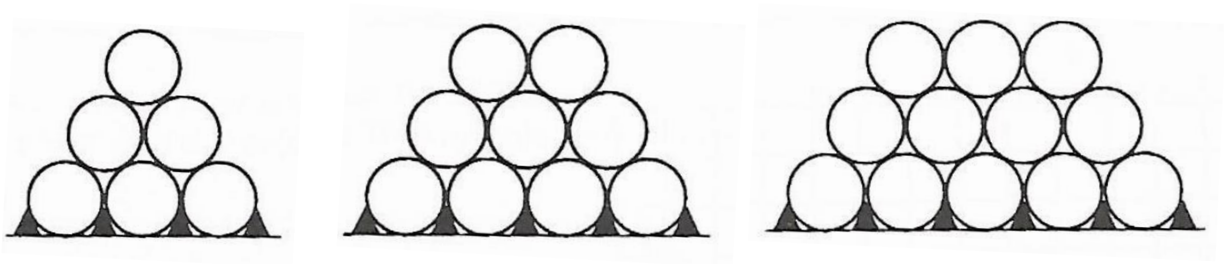
(A) 2.5

(B) 6.25

(C) 10

(D) 12.5

9

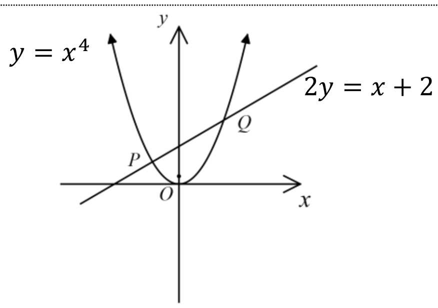


Rows of drums are stacked three high, as shown in the diagrams above. They are held in position by wedges. How many drums can be held in position by n wedges?

 - represents a wedge

(A) $n(n-1)(n-2)$ (B) $(n-1)(n-2)(n-3)$ (C) $3n-6$ (D) $3n-3$

10



P and Q are the points of intersection of the two graphs shown above.
The $x -$ values of P and Q are the solutions of :

(A) $x^4 - x - 2 = 0$ (B) $x^4 + x + 2 = 0$ (C) $2x^4 + x + 2 = 0$ (D) $2x^4 - x - 2 = 0$

SECTION II (57 marks)

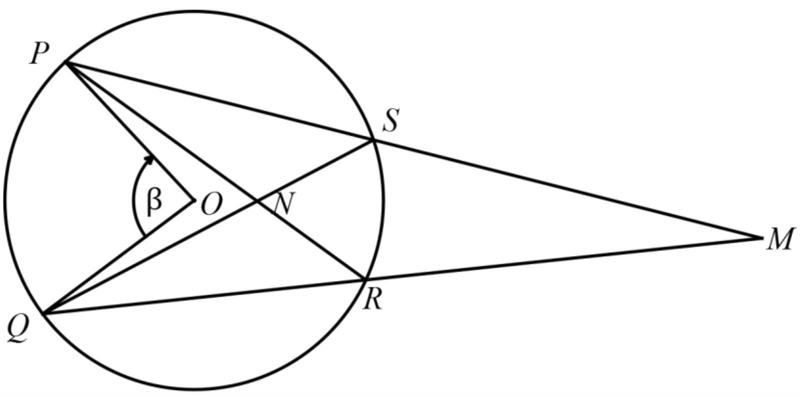
There are SEVEN questions in this section. Attempt ALL the questions. Show your **working and answers** on the appropriate page of your answer booklet.

Question 11 (9 marks**)**	Marks
a) Find the exact value of $\sin 150° \cos 45° + \sin 210°$.	2
b) Rewrite the expression $ax^2 - a - 2x^2 + 2$ as a product of linear factors.	3
c) If $\tan\theta = -\frac{9}{40}$ and $270° \leq \theta \leq 360°$, find the value of $\cos\theta$.	2
d) Solve $\sqrt{2}\sin x - 1 = 0$ where $0° \leq x \leq 360°$.	2

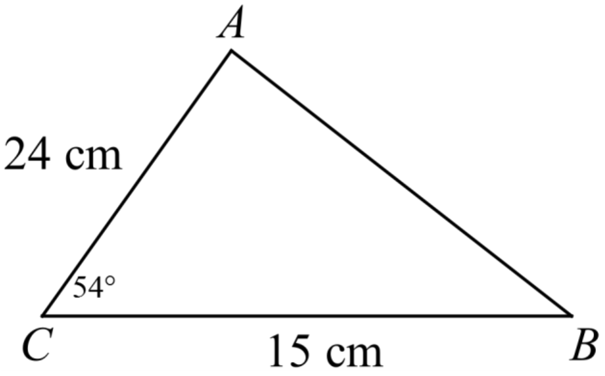
Question 12 (11 marks**)**	
(a) (i) On a number plane, mark the origin O and the points $A(2,1)$ and $B(3,-1)$.	1
(ii) Find the gradients m_1 of OA and m_2 of AB.	1
(iii) Show that OA is perpendicular to AB.	1
(iv) Find the equation of AB.	1
(v) Find the coordinates of the midpoint, D, of the interval OB.	1
(vi) Find the coordinates of the point C such that D is the midpoint of AC.	1
(vii) What shape best describes the geometric figure $OABC$? Justify.	2
(b) Shade the region, on a number plane, defined by $2x - 3y - 6 \geq 0\ AND\ y < \sqrt{4 - x^2}$.	3

Question 13 (7 marks**)**

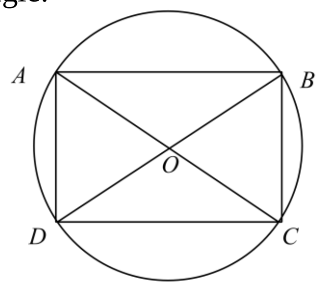
(a) In the diagram, chords PS produced and QR produced intersect at M. Lines PR and SQ intersect at the point N, and $\angle POQ = \beta$ where O is the centre of the circle.

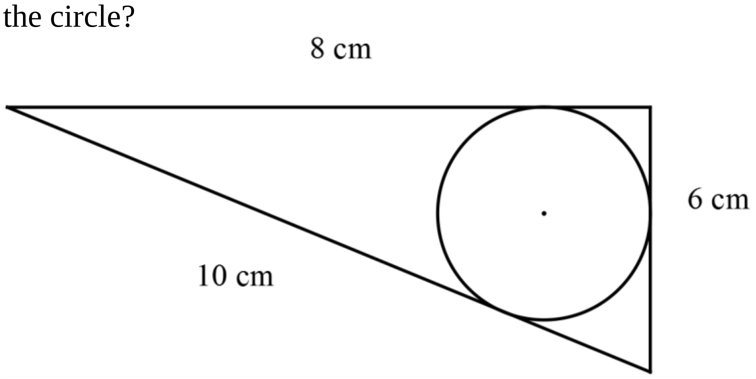


Prove that $\angle PRM = 180° - \frac{1}{2}\beta$ — 2

Question 13 (continued)	Marks
(b) When the polynomial $P(x)$ is divided by $(x^2 - 1)$ the remainder is $3x - 1$. What is the remainder when $P(x)$ is divided by $x - 1$?	2
(c) Find the perimeter of the triangle correct to two significant figures. 	3

Question 14 (10 marks)	
(a) Consider the function $f(x) = \frac{3}{x-2} + 3$ for $x > 2$.	
(i) Sketch the graph of $y = f(x)$.	2
(ii) Find the inverse function of $f(x)$.	2
(iii) State the domain of $f^{-1}(x)$.	1
(b) The numbers 2, 5, 6, 7, 8 and 9 are written on cards. A six-digit number is formed by picking one card at a time and placing it on a table in the order of the pick.	
(i) How many different six-digit numbers can be formed?	1
(ii) What is the probability that the six-digit number is greater than 900000 and is divisible by 5?	2
(c) The matching sides of two similar kites are in the ratio 11:16. Find the area of the smaller kite, correct to two decimal places, if the area of the larger kite is 1.44 m^2.	2

Question 15 (7 marks)	
(a) The quadrilateral $ABCD$ is inscribed inside a circle with centre O. AC and BD intersect at O. Prove that $ABCD$ is a rectangle. 	2

Question 15 (continued)	
(b) Determine the number of solutions of $2x^2 - 3x + 7 = 0$. Justify your answer. (c) Let $x = 0.2\dot{7}$. (i) Write x as an infinite series. (ii) Hence, express x as a simple fraction.	2 1 2
Question 16 (7 marks)	
(a) A telecommunications company sells 1800 mobile phones in the first month of operating. The owners plan to increase their sales by 200 mobile phones each month. How many mobile phones do they plan to sell in the last month of the third year of operation? (b) On 1st July 2005, Suba invested \$10000 in a bank account that paid interest at a fixed rate of 8% per annum, compounded annually. (i) How much would be in the account after the payment of interest on 1 July 2015 if no additional deposits were made? (ii) In fact, Suba added \$1000 to her account on 1 July each year, beginning on 1 July 2006. How much was in her account on 1 July 2015 after the payment of interest and her deposit?	2 2 3
Question 17 (6 marks)	Marks
(a) After a certain number of tests, Alison has scored a total of 180 marks. In the next two tests, Alison did no work and scored zero for each test and reduced her average by 3 marks. (i) Write an algebraic equation to represent the above situation. (ii) How many tests did Alison do altogether? (b) A circle is inscribed inside a triangle with sides 6 cm, 8 cm, and 10 cm. What is the radius of the circle? 	2 2 2

END OF PAPER

BAULKHAM HILLS

HS

PERSEVERE

BAULKHAM HILLS HIGH SCHOOL

YEAR 10 - MATHEMATICS

Section I – Multiple Choice

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ~~●~~ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

correct

A ~~●~~ B ~~●~~ C ○ D ○

Start Here →

1.	A ○	B ○	C ○	D ●	**6.**	A ○	B ○	C ○	D ●
2.	A ●	B ○	C ○	D ○	**7.**	A ○	B ○	C ●	D ○
3.	A ○	B ○	C ●	D ○	**8.**	A ○	B ●	C ○	D ○
4.	A ●	B ○	C ○	D ○	**9.**	A ○	B ○	C ●	D ○
5.	A ○	B ○	C ●	D ○	**10.**	A ○	B ○	C ○	D ●

Question 11

(a) $\sin 150^\circ \cos 45^\circ + \sin 210^\circ$

$= \sin 30^\circ . \cos 45 - \sin 30^\circ$

$= \frac{1}{2} . \frac{1}{\sqrt{2}} - \frac{1}{2}$

$= \frac{1}{2}\left(\frac{1-\sqrt{2}}{\sqrt{2}}\right)$

$= \frac{1-\sqrt{2}}{2\sqrt{2}}$

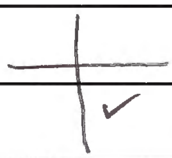
(b) $ax^2 - a - 2x^2 + 2$

$= a(x^2-1) - 2(x^2-1).$

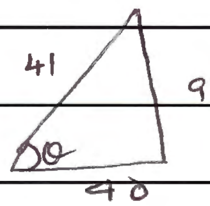
$= (a-2)(x^2-1)$

$= (a-2)(x-1)(x+1)$

(c) $\tan\theta = -\frac{9}{40}$



$\therefore \cos\theta = +(?)$



$\therefore \cos\theta = \frac{40}{41}$

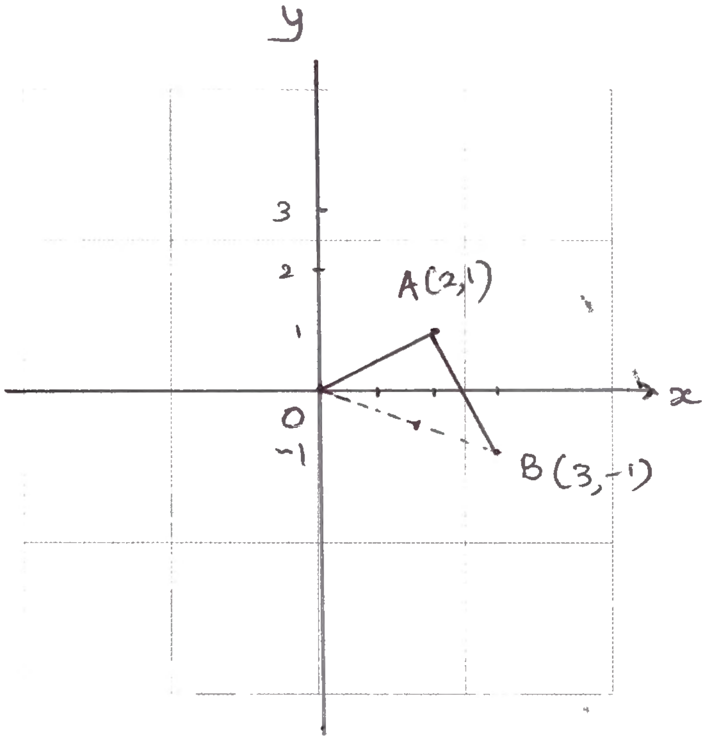
(d) $\sqrt{2}\sin x - 1 = 0$

$\sin x = \frac{1}{\sqrt{2}}$ reference angle $= 45^\circ$

$\therefore x = 45^\circ, 135^\circ$

Question 12

(a)(i)



(ii) gradient of OA $= m_1$; gradient of AB $= m_2$

$m_1 = \dfrac{1-0}{2-0}$ $\qquad m_2 = \dfrac{-1-1}{3-2} = -2$

$= \dfrac{1}{2}$

(iii) Note that $m_1 \times m_2 = -1$, $\because$ $m_1 \times m_2 = \frac{1}{2} \times -2 = -1$

$\therefore$ $OA \perp AB$.

(iv) $y - y_1 = m_2 (x - x_1)$

$y-(-1) = -2(x-3)$

$\therefore y+1 = -2x+6$

$2x + y - 5 = 0$

(v) O (0,0) ; B (3, −1) let D be (a, b)

$a = \dfrac{3+0}{2} \implies a = 1\frac{1}{2}$

$b = \dfrac{-1+0}{2} \implies b = -\frac{1}{2}$ $\quad \therefore D(1\frac{1}{2}, -\frac{1}{2})$.

(vi) Let C be (p, q); $A(2,1)$; $D(1\frac{1}{2}, -\frac{1}{2})$

$$\frac{2+p}{2} = \frac{3}{2} \Rightarrow p = 1$$

$$\frac{1+q}{2} = -\frac{1}{2} \Rightarrow q = -2$$

$\therefore C(1,-2)$.

(vii) From the above work, in the quadrilateral OABC

$OA \perp AB$ (Note these are adjacent sides).

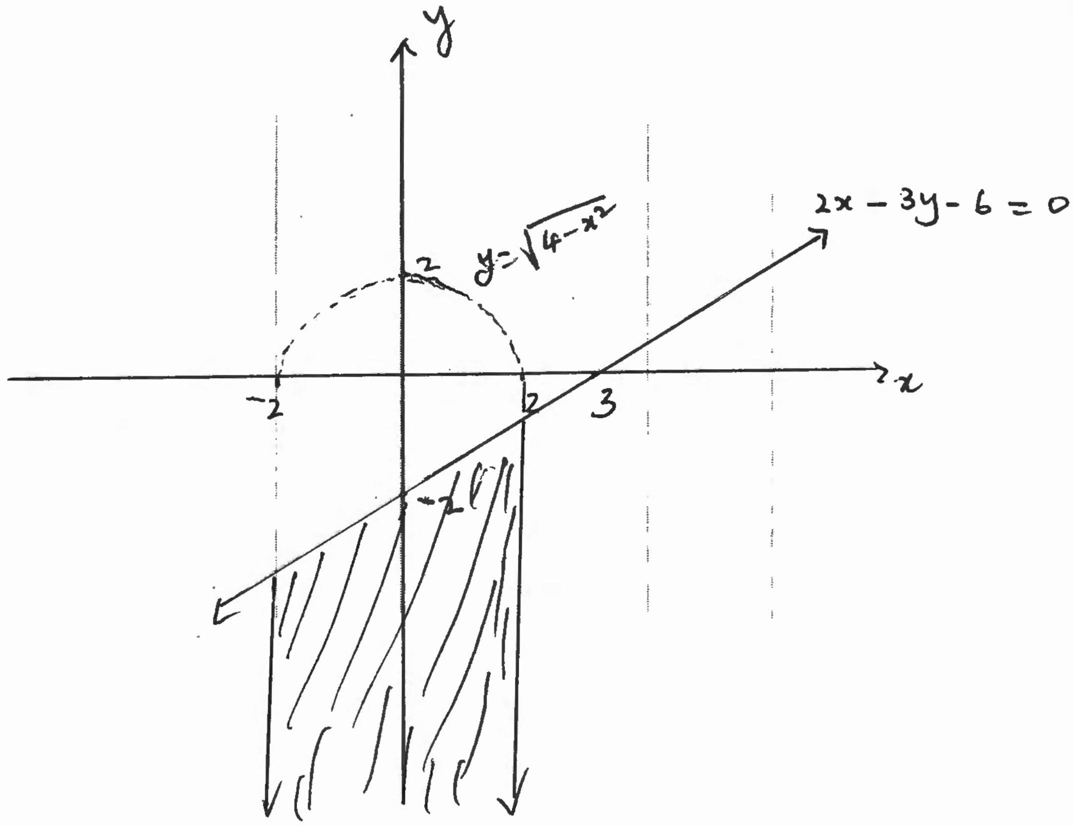
diagonals OB and AC bisect each other at D.

$\therefore$ OABC is a rectangle.

Also note $OA = AB = \sqrt{5}$ units $\Rightarrow$ adjacent sides are equal.

$\therefore$ OABC is a SQUARE.

12 (b)



Question 13

(c) Let AB be x.

$\therefore$ Perimeter $= 24 + 15 + x$.

$x^2 = 24^2 + 15^2 - 2 \times 24 \times 15 \times \cos 54°$

$= 377.7946\ldots$

$x = 19.436\ldots$

Perimeter is 58 cm (2 sig. figs).

(b) $P(x) = Q(x)(x^2-1) + 3x - 1$

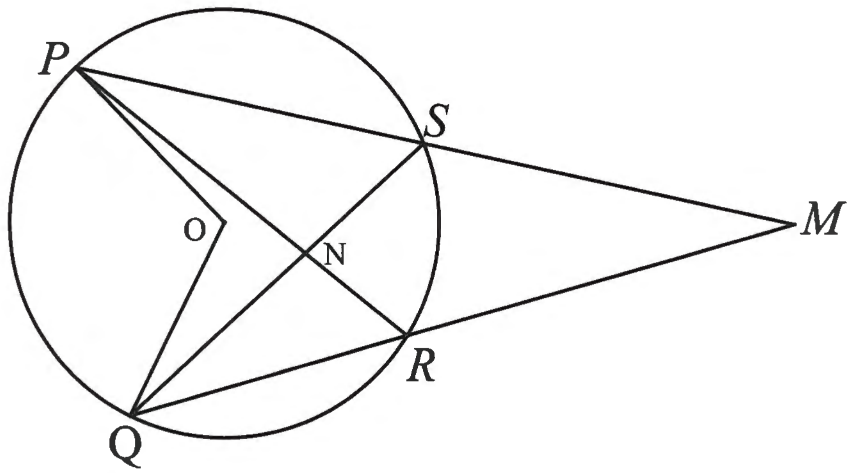
$= Q(x)(x-1)(x+1) + 3x - 1$

When $P(x)$ is divided by $(x-1)$, $R = P(1)$.

$P(1) = 0 + 3 - 1 = 2$

$\therefore$ Remainder is 2

13(a)



$\angle POQ = \beta$

$\therefore \angle PRQ = \beta/2$ [$\angle$ at the centre is twice the $\angle$ at the circumference standing on the same arc, PQ]

$\angle PRM + \angle PRQ = 180^\circ$ [$\angle$ sum on a straight line QRM]

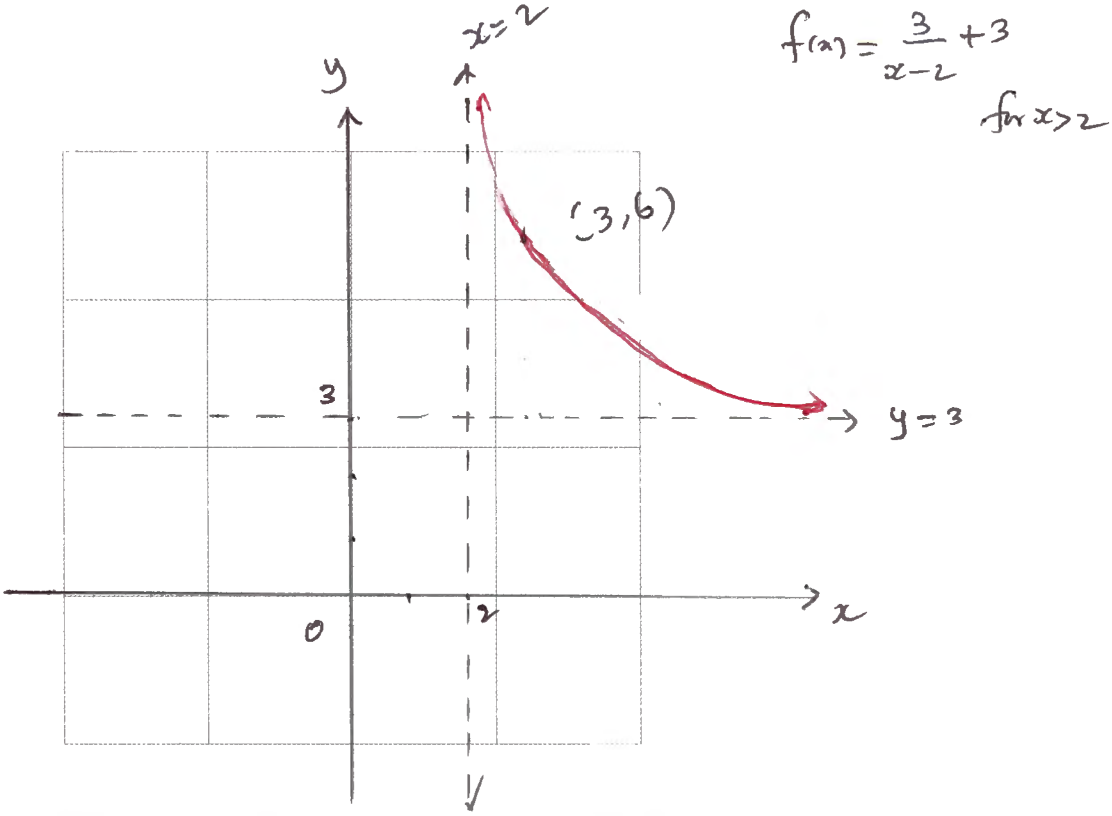
$\angle PRM + \beta/2 = 180^\circ$

$\therefore \angle PRM = 180 - \beta/2$.

You may ask for extra writing paper if you need more space to answer question 13.

Question 14

(a) (i)



(ii) $y = \frac{3}{x-2} + 3$

to find the inverse, interchange x and y.

$x = \frac{3}{y-2} + 3.$

$\frac{3}{y-2} = \cancel{x} - 3. \implies \frac{y-2}{3} = \frac{1}{x-3}.$

$\therefore\ y - 2 = \frac{3}{x-3} \implies y = \frac{3}{x-3} + 2$

$\therefore\ f^{-1}(x) = \frac{3}{x-3} + 2.$

(iii) Domain of $f^{-1}(x)$: all real values of x such that $x > 3$

(b)

(i) $\boxed{6}\ \boxed{5}\ \boxed{4}\ \boxed{3}\ \boxed{2}\ \boxed{1}$

$= 61.$

(ii) $\boxed{9}\ \boxed{?}\ \boxed{?}\ \boxed{?}\ \boxed{?}\ \boxed{5}$

$1 \times 4 \times 3 \times 2 \times 1 \times 1 = 24.$

$\therefore P(\text{number greater than 900000 and divisible by 5}) = \frac{24}{61}.$

$= \frac{\cancel{24}}{30}$

$= \frac{41}{61} = \frac{1}{30}$

(c) $A_L : A_s = l_L^2 : l_s^2.$

$\frac{1.44}{A_s} = \frac{16^2}{11^2}.$

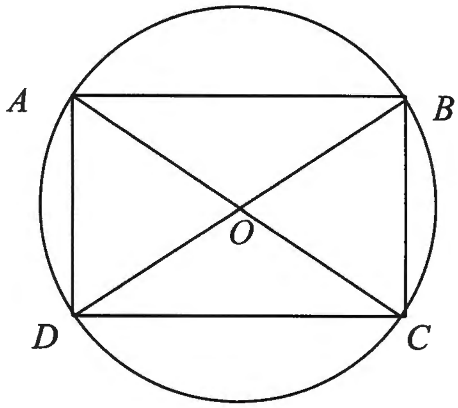
$\therefore A_s = \frac{1.44 \times 11^2}{16^2} = 0.68062\ldots$

Area of the smaller kite is $0.68\, m^2$ (2dp).

You may ask for extra writing paper if you need more space to answer question 14.

Question 15

(a)



(a) In the quadrilateral ABCD,

diagonals AC and BD bisect each other [O is the centre]

$\angle DAB = 90^\circ$ [$\angle$ in a semi circle is 90°]

$\therefore$ ABCD is a rectangle.

(b) $y = 2x^2 - 3x + 7$

$y = 0 \Rightarrow 2x^2 - 3x + 7 = 0$

$a = 2$; $b = -2$; $c = 7$

$\Delta = b^2 - 4ac$

$= 4 - 4(2)(7)$.

$= -52$.

ie $\Delta < 0$.

$\therefore$ $y = 0$ has no real solutions.

(c) (i) $x = 0.2\dot{7}$

$= 0.2 + 0.0\dot{7}$

$= 0.2 + 0.07 + 0.007 + 0.0007 + \cdots$

(ii) $0.07 + 0.007 + 0.0007 + \cdots$ is a GP with ~~the~~ first term 0.07 and common ratio, r, $\frac{1}{10}$. As $r < 1$

$S_\infty = \frac{a}{1-r}$

$= \frac{0.07}{1-0.1} = \frac{7}{90}$

$\therefore x = 0.2 + \frac{7}{90}$

$= \frac{2}{10} + \frac{7}{90}$

$= \frac{25}{90}$

$= \frac{5}{18}$

You may ask for extra writing paper if you need more space to answer question 15.

Question 16

(a) 1st month 2nd 3rd . . . .

Sales 1800 2000 2200 - - -

Sales in a month forms the terms of an AP.

$\therefore$ In 3 years.

T_{36} $T_n = a + (n-1)d$

$T_{36} = 1800 + (36-1) \times 200$

$= 8800.$

$\therefore$ 8800 mobile phones.

(b) Investment is \$10000 @ 8% p.a. from 1st July 2005 to 30th June 2015

$\therefore$ (i) $A_{10} = P(1+r)^n$ (10 years.)

$= 10000\left(1 + \frac{8}{100}\right)^{10}$

$= 21589.2499 \ldots$

$\therefore$ \$21589.25 will be in the account on the 1st Jul 2015

(ii)

Extra deposits \$1000 on 1st July 2006. 1st July 2007 1st Jul 2008

9 years 8 years 7 years

Maturity date 30th Jun 2015

Investment will equal to $1000(1.08)^9$ $1000(1.08)^8$ $1000(1.08)^7$

On the 1st July 2015,

Deposit for the month $\qquad$ Grown ~~mon~~thly deposits.

$$1000 + \left(1000\times1.08^1 + 1000\times1.08^2 + \cdots + 1000\times1.08^9\right)$$

This is a GP.

with the first term 1000 and Common radio 1.08 . and ~~1000~~ there are 10 terms.

$$\therefore S_{10} = \frac{1000\left(1.08^{10}-1\right)}{0.08} = 14486.56$$

$\therefore$ On the 1st July 2015,

Extra deposits worth = \$14486.56

\$10000 initial investmen worth = \$ 21589.25

Total = 36075.81.

$\therefore$ Account balance is \$36075.81

You may ask for extra writing paper if you need more space to answer question 16.

Question 17

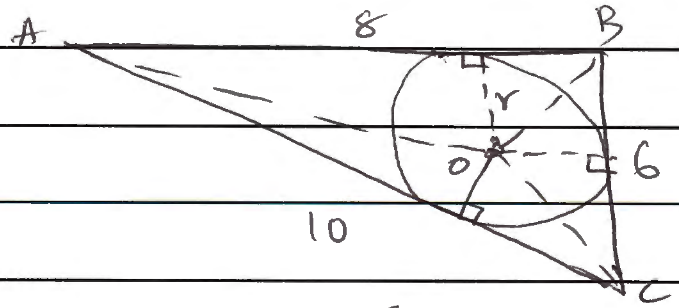
(a) (i) Let Alison sat for n tests in total.

$$\therefore \frac{180}{n-2} - \frac{180}{n} = 3.$$

(ii)
$$n(180) - 180(n-2) = 3n(n-2).$$
$$\cancel{180n} - \cancel{180n} + 360 = 3n^2 - 6n$$
$$n^2 - 2n - 120 = 0$$
$$(n-12)(n+10) = 0$$
$$\therefore n = 12 \ ; \ n = -10 \text{ (reject)}.$$

$\therefore$ Alison sat for 12 tests.

(b)



Note that $\{6, 8, 10\}$ is a Pythagorean triad.

$\therefore$ The Δ is a right $\angle$ed Δ at B.

$$|\Delta ABC| = |\Delta AOB| + |\Delta AOC| + |\Delta BOC|$$

$$\frac{1}{2} \cdot 8 \cdot 6 = \frac{1}{2} \cdot 8 \times r + \frac{1}{2} \cdot 10 \cdot r + \frac{1}{2} \cdot 6 \cdot r$$

$$48 = 24r \implies r = 2$$

$\therefore$ radius is 2cm