BAULKHAM HILLS HS PERSEVERE

Year 10 - 2018
Common Test 4

MATHEMATICS

General Instructions

- Working time – 65 minutes
- Write using black or blue pen
- NESA-approved calculators may be used.
- In Questions, 6-11, show relevant mathematical reasoning and/or calculations

Total marks – 56

Section I Pages 1 - 3
5 marks

Section II Pages 4 - 7
51 marks

Section I

5 marks

Attempt questions 1-5

Use the page for the multiple-choice answers, for questions 1-5, in the answer booklet.

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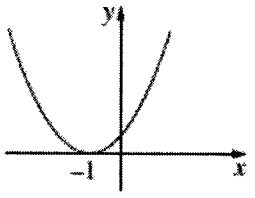
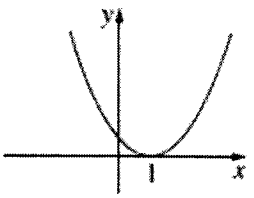
1. Which graph best represents $y = (x-1)^2$?

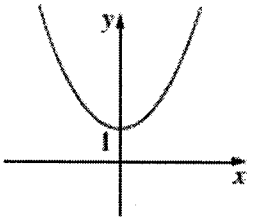
(A)



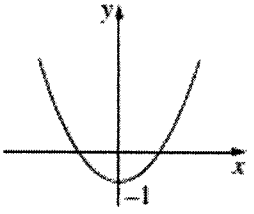
(B)



(C)



(D)



2. The probability that Bob's soccer team wins this weekend is $\frac{5}{7}$. The probability that Bob's rugby league team wins this weekend is $\frac{2}{3}$. What is the probability that neither team wins this weekend?

2/7 x 1/3

(A) $\frac{2}{21}$

(B) $\frac{10}{21}$

(C) $\frac{13}{21}$

(D) $\frac{19}{21}$

3. The coefficient of a^2 in the expansion of $(2a-1)^5$ is

(A) -80
(B) 40
(C) -20
(D) -40

4. Which expression is a term of the geometric series $3x - 6x^2 + 12x^3 - \cdots$??

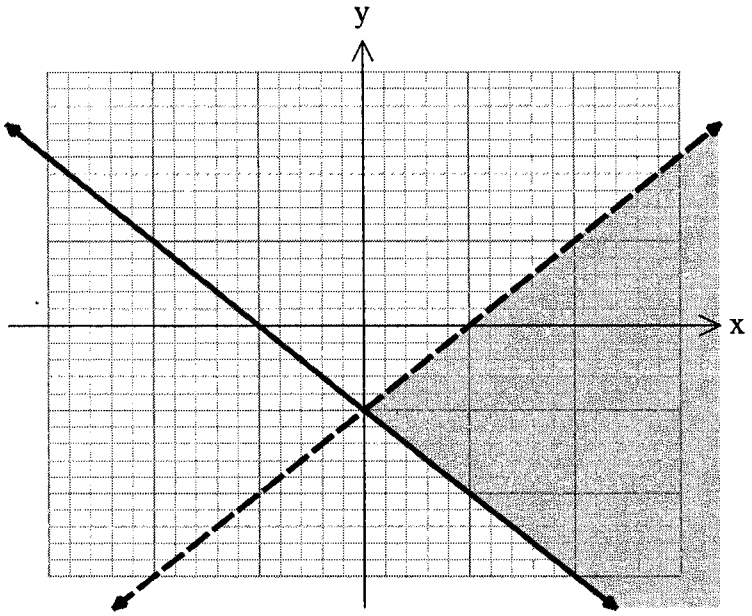
(A) $3072x^{10}$

(B) $-3072x^{10}$

(C) $3072x^{11}$

(D) $-3072x^{11}$

5.



The diagram shows the region enclosed by $y = x - 1$ *and* $y = -x - 1$. Which of the following pairs of inequalities describes the shaded region in the diagram?

(A) $y \leq -x - 1$ *and* $y \leq x - 1$

(B) $y \leq -x - 1$ *and* $y \geq x - 1$

(C) $y \geq -x - 1$ *and* $y < x - 1$

(D) $y \geq -x - 1$ *and* $y > x - 1$

END OF SECTION I

Section II

51 marks
Attempt questions 11-16.

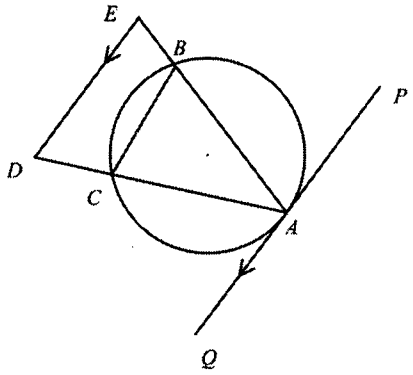
Answer each question in the **appropriate page of the writing booklet**. Extra pages, if needed, are available at the end of the booklet.
In Questions 6-11, your responses should include relevant mathematical reasoning and/or calculations.

QUESTION 6 (7 marks)

(a) When the polynomial $P(x) = x^3 - kx^2 - kx + 5$ is divided by $(x + 2)$ the remainder is 7. What is the value of k? **2**

(b) $P(x) = 2x^3 + x^2 - 5x - 1$ and $A(x) = (x^2 + 3)$. Express the result of $P(x) \div A(x)$ in the form of $P(x) = A(x) \times Q(x) + R(x)$. **2**

(c) In the diagram ΔABC is inscribed in a circle. PAQ is a tangent to the circle at A. E is a point on AB produced and D is a point on AC produced such that $ED \parallel PAQ$. Show that $BCDE$ is a cyclic quadrilateral. **3**



QUESTION 7 (9 marks)

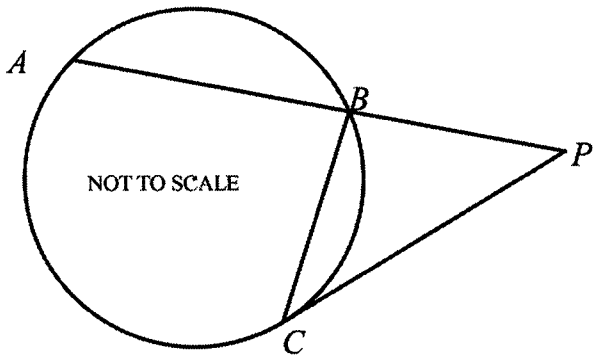
(a) Find the value(s) of k such that the roots of $x^2 + (3k + 1)x + 4k + 5 = 0$ are equal. **2**

(b) (i) Express $0.12\dot{3}$ as a numerical expression that includes a geometric series. **1**
(ii) Hence find a simple fraction that is equivalent to $0.12\dot{3}$ **2**

(c) A card is selected from twenty cards which are numbered using integers from 1 to 20. Numbers are not repeated.
$\boldsymbol{A} = \{cards\ with\ odd\ numbers\}$; $\boldsymbol{B} = \{cards\ with\ numbers\ that\ are\ multiples\ of\ 3\}$
Find (i) $n(A \cap B)$ **1**
(ii) $P(\bar{A})$ **1**
(iii) $P(A \cap B)$ **1**
(iv) $P(A|B)$ **1**

QUESTION 8 (7 marks)

(a) (i) In the diagram the points A, B and C lie on the circle and AB produced meets the tangent from C at the point P. Given that $PC = 12, AB = 7$ and $BP = x$, find the value of x. **2**



(ii) BC is the diameter of the circle passing through P, B and C. Find the length of BC. (No reasons required) **1**

(b) In an arithmetic progression, $T_2 = 7$ *and* $T_7 = 52$.

(i) Find the common difference and the first term. **2**

(ii) Find the value of the first term which is greater than 1000. **2**

QUESTION 9 (7 marks)

(a) Evaluate **2**

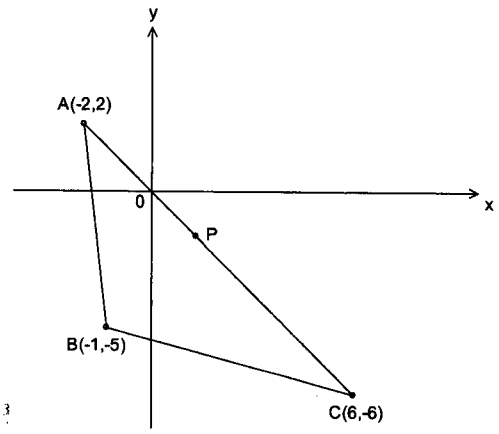
$$\sum_{n=4}^{7} \frac{1}{n-2}$$

(b) Sketch a neat graph of $y = (2 - x)(x + 1)^2$ showing all intercepts. **2**

(c) Solve $4\sin^3\theta - 3\sin\theta = 0$ for $0° \leq \theta \leq 360°$. **3**

QUESTION 10 (14 marks)

(a) The diagram shows ΔABC with vertices A (-2, 2), B (-1, -5) and C (6, -6).



NOT TO SCALE

(i) P is the midpoint of AC. Show that the coordinates of P are (2, -2). **1**

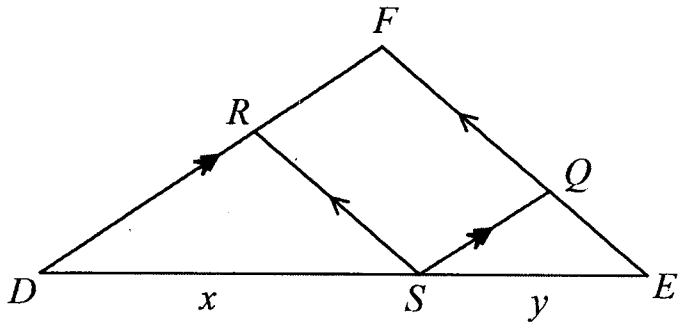
(ii) Show that the equation of BP is $y = x - 4$. **2**

(iii) Find the coordinates of D, if P is the midpoint of BD. **2**

(iv) Show that BD is perpendicular to AC. **2**

(v) Which shape best describes the geometric figure ABCD? Explain. **2**

(b)



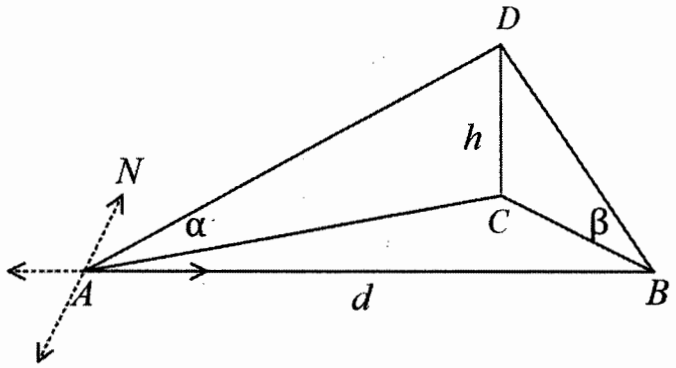
In ΔDEF, a point S is chosen on the side DE. The length of DS is x, and the length of ES is y. The line through S parallel to DF meets EF at Q. The line through S parallel to EF meets DF at R. The area of ΔDEF is M. The area of ΔDSR is M_1.

(i) Show that $\Delta DEF \;|||\; \Delta DSR$. **2**

(ii) Show that $\sqrt{\frac{M_1}{M}} = \frac{x}{x+y}$. **3**

QUESTION 11 (7 marks)

(a) A vertical pole CD is positioned so that the angles of elevations to the top of the pole, D, from the points A and B are α and β respectively. The ground is a level horizontal surface and the triangle ABC is right-angled at C. Point B is due east of point A and point C is on a bearing $060°T$ from point A.

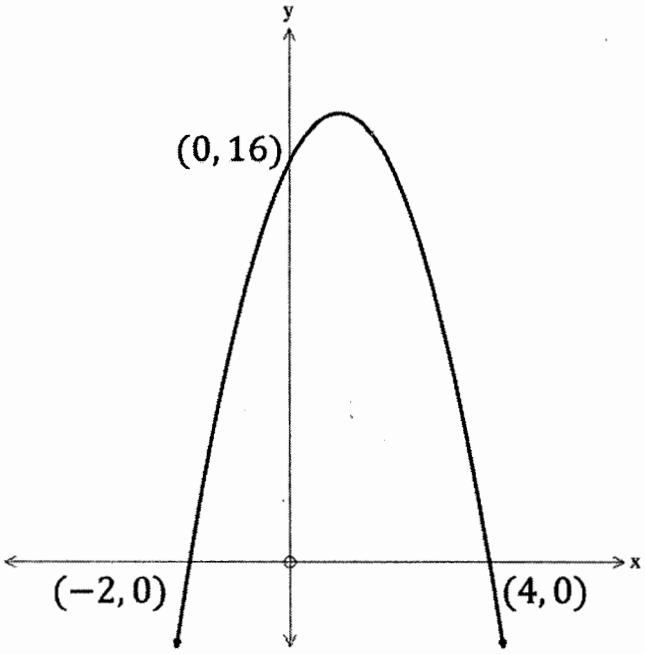


(i) Find the size of $\angle CAB$. **1**

(ii) Write an expression for the length AC in terms of d. **1**

(iii) Show that $\frac{tan\alpha}{tan\beta} = \frac{1}{\sqrt{3}}$ **2**

(b) If $y = a(x + b)^2 + c$ represents the parabola shown below, find the values of a, b and c. **3**



End of Test

Baulkham Hills High School
Year 10 Common Test 4 (Yearly Examination) 2018
Marking Guideline - Mathematics

Section I (5 marks)

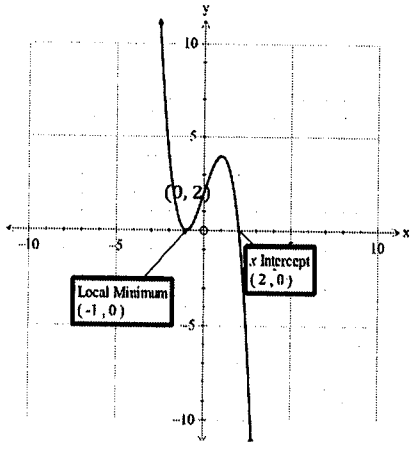
Award 1 mark to each correct answer.

Answers: 1. B 2. A 3. D 4. C 5. C

Section II (51 marks)

In all questions, award **FULL marks** for **correct answers** with **necessary working**.

Question	Suggested solution	Marking guideline
6(a)	**Answer: $k = -5$** $P(-2) = 7$ $(-2)^3 - k(-2)^2 - k(-2) + 5 = 7$ $k = -5$	2 – correct solution 1 - using the remainder theorem but incorrect answer
6(b)	**Answer**: $P(x) = A(x) \times Q(x) + R$ *where* $Q(x) = 2x + 1$ *and* $R = -11x - 4$.	2 - correct solution 1 – an attempt to find $Q(x)$ and R by long division
6(c)	***Proof***: $\angle BAP = \angle BCA$ (*∠ between a chord and a tangent at the point of contact is equal to the ∠ subtended by the chord in the alternate segment*) $\angle BAP = \angle DEA$ (*alternate ∠s on* $\parallel$ *lines,* $DE \parallel PAQ$) $\therefore\ \angle BCA = \angle DEA = \angle DEB$ $\therefore$ *In the quadrilateral BCDE, the exterior* $\angle BCA$ *is equal to the interior opposite* $\angle DEB$. $\therefore$ *BCDE is a cyclic quadrilateral.*	3 – correct solution 2 – proved $\angle BCA = \angle DEB$ 1 - recognised $\angle BAP = \angle BCA$ and/or $\angle BAP = \angle DEA$
7(a)	**Answer: $k = -1$ *and* $k = \frac{19}{9}$.** $\Delta = (3k + 1)^2 - 4 \times 1 \times (4k + 5)$ *For real and equal roots,* $\Delta = 0$. $9k^2 - 10k - 19 = 0$ $k = \dfrac{-(-10) \pm \sqrt{(-10)^2 - 4 \times 9 \times (-19)}}{2 \times 9}$	2 – correct solution 1 – recognised that *For real and equal roots,* $\Delta = 0$.
7(b)	(i) **Answer:** $0.12\dot{3} = 0.12 + 0.003 + 0.0003 + 0.00003 + \cdots$ (ii) **Answer:** $0.12\dot{3} = \frac{37}{300}$ $0.003 + 0.0003 + 0.00003 + \cdots = S_\infty$ *where* $S_\infty = \dfrac{0.003}{1 - \frac{1}{10}} = \dfrac{1}{300}$ $0.12\dot{3} = 0.12 + \dfrac{1}{300} = \dfrac{37}{300}$	1 2 – correct solution 1 – correct numerical expression for S_∞.

Question	Suggested solution	Marking guideline
7(c)	**Answer** (i) : $n(A \cap B) = 3$ (ii) : $P(\bar{A}) = \frac{1}{2}$ (iii) : $P(A \cap B) = \frac{3}{20}$ (iv) : $P(A\|B) = \frac{1}{2}$	1 1 1 1
8(a)	**Answer(i) :** $x = 9$ $(7 + x) \times x = 12^2$ $x^2 + 7x - 144 = 0 \quad \rightarrow as\ x > 0, x = 9$ **Answer (ii):** $BC = 15$ $\angle BPC = 90°;\ BC^2 = x^2 + 12^2$	2 – correct solution 1 - correct use of the intercept theorem 1
8(b)	**Answer (i):** $First\ term = -2; Common\ difference = 9$ **Answer (ii):** 1006 $(-2) + (n-1) \times 9 > 1000$ $n > 112\frac{1}{3} \quad \rightarrow n = 113$ $T_{113} = (-2) + (113 - 1) \times 9 = 1006$	2 – 1 each 2 – correct solution 1 – Calculates $n > 112\frac{1}{3} \quad \rightarrow n = 113$
9(a)	**Answer:** $\frac{77}{60}$ $\frac{1}{4-2} + \frac{1}{5-2} + \frac{1}{6-2} + \frac{1}{7-2}$ $= \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$	2 – correct value 1- correct expansion of the series
9(b)	**Answer:**  	2 – correct solution 1 – correct shape
9(c)	**Answer:** $\theta = 0°, 60°, 120°, 240°, 300°, 180^{\circ}, 360^{\circ}$ $sin\theta(4sin^2\theta - 3) = 0$ $As\ sin\theta = 0\ or\ sin\theta = \pm\frac{\sqrt{3}}{2}$	3 – correct solution 1 – correct factored form of the equation.

Baulkham Hills High School
Year 10 Common Test 4 (Yearly Examination) 2018
Marking Guideline - Mathematics

Question	Suggested solution	Marking guideline
10(a)	**Answer (i) : shows the use of the midpoint formula** **Answer (ii): $y = x - 4$** $m_{BP} = \frac{-5-(-2)}{(-1)-2} = 1$ $Equation\ of\ BP:\ y-(-5) = 1 \times \{x-(-1)\}$ **Answer (iii): $D(5, 1)$ (one for each of x and y)** **Answer (iv): $m_{BD} \times m_{AC} = -1$** **Answer (v): rhombus**	1 2 – correct solution 1 – found the gradient of BP 2 – correct solution 1- attempted to use the midpoint formula correctly 2 – correct solution 1- both gradients are correct 2 – correct solution 1 – correct shape but incorrect justification **OR** incorrect shape but consistent justification.
10(b)	**Answer (i) : showed $\Delta DEF \vert\vert\vert \Delta DSR$** $In\ \Delta DEF\ and\ \Delta DSR,$ $\angle DFE = \angle DRS\ (corresponding\ \angle s\ on\ \parallel\ lines; RS \parallel FE)$ $\angle D\ is\ common.$ $\therefore\ \Delta DEF \vert\vert\vert \Delta DSR\ (two\ \angle s\ of\ \Delta DFE\ = two\ \angle s\ of\ \Delta DSR)$ **Answer (ii) : $proof\ of\ \sqrt{\frac{M_1}{M}} = \frac{x}{x+y}$** $DE = x,\ DS = x + y$ $\frac{Area\ of\ \Delta DEF}{Area\ of\ \Delta DSR} = \frac{DE^2}{DS^2}$ $(Area\ of\ similar\ figures\ are\ in\ the\ ratio\ of\ squares\ of\ matching\ sides)$ $Hence, \sqrt{\frac{M_1}{M}} = \frac{x}{x+y}$	2 – correct solution 1 – recognised the required angles for the proof. 2 – correct solution 1- recognised the relationship between the area and lengths of similar figures. Students needed to prove beyond reasonable doubt they understood the relationship and did not use the answer to work backwords
10(a)	**Answer (i) : $\angle CAB = 30°$** **Answer (ii) : $AC = d\cos 30°$** **Answer (iii): $shows\ \frac{\tan\alpha}{\tan\beta} = \frac{1}{\sqrt{3}}$** $BC = d\sin 30°$ $In\ \Delta ADC, \tan\alpha = \frac{h}{AC};\ In\ \Delta BCD, \tan\beta = \frac{h}{BC}$ $\therefore \frac{\tan\alpha}{\tan\beta} = \frac{BC}{AC}$ $= \frac{d\sin 30°}{d\cos 30°}$ $= \tan 30° = \frac{1}{\sqrt{3}}$	1 1 2 – correct solution 1 - $\tan\alpha = \frac{h}{AC}\ and\ \tan\beta = \frac{h}{BC}$
10(b)	**Answer: $a = -2; b = -1\ and\ c = 18$** $y = a(x-4)(x+2)$ $y = a[x^2 - 2x - 8]$ $y = a[(x-1)^2 - 9]$ $passes\ through\ (0, 16)\ : 16 = a[(-1)^2 - 9] \rightarrow a = -2$	3 – correct solution 2 - any two correct values 1 – one correct value

Question	Suggested solution	Marking guideline
11 (a)	**Answer (i) :** $\angle CAB = 30°$ **Answer (ii) :** $AC = d\cos 30°$ **Answer (iii):** ***shows*** $\frac{\tan\alpha}{\tan\beta} = \frac{1}{\sqrt{3}}$ $BC = d\sin 30°$ $In\ \Delta ADC, \tan\alpha = \frac{h}{AC};\ In\ \Delta BCD, \tan\beta = \frac{h}{BC}$ $\therefore \frac{\tan\alpha}{\tan\beta} = \frac{BC}{AC}$ $= \frac{d\sin 30°}{d\cos 30°}$ $= \tan 30° = \frac{1}{\sqrt{3}}$	1 1 2 – correct solution 1 - $\tan\alpha = \frac{h}{AC}$ *and* $\tan\beta = \frac{h}{BC}$
11 (b)	**Answer:** $a = -2; b = -1$ ***and*** $c = 18$ $y = a(x-4)(x+2)$ $y = a[x^2 - 2x - 8]$ $y = a[(x-1)^2 - 9]$ *passes through* $(0,16)$: $16 = a[(-1)^2 - 9] \rightarrow a = -2$	3 – correct solution 2 - any two correct values 1 – one correct value