# EPPING BOYS' HIGH SCHOOL 

## YEAR 10 STAGE 3 MATHEMATICS 2006 Yearly Examination

Date: 200610275
Name: $\qquad$
Time allowed: 70 minutes
Class 10M $\qquad$

All questions may be attempted.
Figures are not necessarily drawn to scale.
Except for multiple choice questions, working must be shown.
1 mark is awarded for each correct answer to multiple choice questions.

| Section | I | II | III | IV | Total | \% |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Topic | Algebra | Trig | Vol \& SA | Stats |  |  |
| Full marks | $\mathbf{2 0}$ | $\mathbf{2 0}$ | $\mathbf{1 5}$ | 15 | 70 | $\mathbf{1 0 0}$ |
| Score |  |  |  |  |  |  |
|  |  |  |  |  |  |  |

## SECTION I Algebra

## Questions

Answers

1. If $4 x+3=0$, then $x=$
A. $\frac{3}{4}$
B. $-\frac{3}{4}$
C. $\frac{4}{3}$
D. $-\frac{4}{3}$
2. If $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$, then $v=$
A. $\frac{u f}{u-f}$
B. $\frac{u f}{f-u}$
C. $f-u$
D. $u-f$
3. If $n$ is a non-zero integer, which statement below is always true?
A. $2 n \geq n-7$
B. $\frac{n}{100}<n$
C. $n^{2} \geq n$
D $2^{n}$ is an integer.
4. $a^{2}-b^{2}=$
A. $(a-b)^{2}$
B. $(b-a)^{2}$
C. $(b-a)(b+a)$
D. $(a-b)(a+b)$

## Questions

Answers
05. If $\sqrt{x^{2}-25}$ is a real number, then $x$ cannot be
A. 36
B. -7
C. 4
D. -5
06. Solve for $x$ :
(a) $(x-7)^{2}=8$ (Leave your answers in surd form.)
(b) $(x-5)(x+2)=0$
(c) $x^{2}-7 x+12=0$
(d) $3 x^{2}-7 x-8=0$ (Leave your answer in surd form.)
(e) $x+1=\frac{6}{x}$
07. Solve the following pair of simultaneous equations:
$2 x-5 y=30$
$14 x+15 y=-20$
08. John is six times as old as his son. In 24 years time, John will be double his son's age. How old is John now?

## SECTION II Trigonometry

## Questions

9. In $\triangle A B C, \angle A=59^{\circ}, \angle B=30^{\circ}$, and $B C=12 \mathrm{~cm}$.

The length of $A C$ in cm, correct to 3 decimal places is
A. $3 \cdot 051$
B. $7 \cdot 000$
C. $10 \cdot 286$
D. 11.800
10. $\sin 78^{\circ} \neq$
A. $\sin 102^{\circ}$
B. $\sin 258^{\circ}$
C. $\cos 12^{\circ}$
D. $-\cos 168^{\circ}$
11. Which of the following is the correct cosine rule?
A. $a^{2}=b^{2}+c^{2}+2 b c \cos A$
B. $a^{2}=b^{2}+c^{2}-2 b c \cos A$
C. $\cos B=\frac{b^{2}+c^{2}-a^{2}}{2 b c}$
D. $\cos C=\frac{a^{2}+b^{2}-c^{2}}{2 b c}$
12. $\tan \theta=$
A. $\frac{a d j}{o p p}$
B. $\frac{a d j}{h y p}$
C. $\frac{o p p}{h y p}$
D. $\frac{o p p}{a d j}$
13. Which of the following set of data gives rise to an ambiguous case in $\triangle A B C$ ?
A. $c=4 \mathrm{~cm}, b=3 \mathrm{~cm}$, and $a=6 \mathrm{~cm}$.
B. $a=20 \mathrm{~cm}, b=13 \mathrm{~cm}$, and $\angle B=29^{\circ}$.
C. $a=12 \mathrm{~cm}, \angle A=40^{\circ}$, and $\angle B=60^{\circ}$.
D. $b=7 \mathrm{~cm}, c=5 \mathrm{~cm}$, and $\angle A=45^{\circ}$.
14. In $\triangle A B C, \angle C=18^{\circ}, a=4 \mathrm{~cm}$, and $c=(\sqrt{5}-1) \mathrm{cm}$.

Find the magnitude of $\angle A$ and the area of $\triangle A B C$ to 3 decimal places.
15. In $\triangle A B C, a=23 \mathrm{~cm}, b=11 \mathrm{~cm}$, and $c=\sqrt{903} \mathrm{~cm}$.

Find the magnitude of $\angle C$, and the area of $\triangle A B C$ to 3 decimal places.
16. Town $A$ is 12 km north-east of Town B ; Town $B$ is 15 km from Town C .

The bearing of Town $C$ from Town B is $120^{\circ}$. Find the distance between
Towns $A$ and $C$ to 3 decimal places. Find also the bearing of Town $C$ from Town $A$ to the nearest degree.

## SECTION III Volume and Surface area

## Questions

17. The formula for the volume of a sphere is
A. $4 \pi r^{2}$
B. $\frac{4}{3} \pi r^{2}$
C. $\frac{4}{3} \pi r^{3}$
D $2 \pi r^{3}$
18. The formula for the volume of a cone is
A. $2 \pi r$
B. $\frac{1}{3} \pi r h$
C. $\frac{1}{3} \pi r^{2} h$
D. $\pi r^{2} h$
19. The formula for the curved surface area of a cone is
A. $3 \pi r^{2}$
B. $\frac{1}{3} \pi r h$
C. $\pi r h$
D. $\pi r s$
20. Given that $A B=11 \mathrm{~cm}, B C=28 \mathrm{~cm}$, and the perpendicular distance between $A D$ and $B C$ is 9 cm , the area of parallelogram $A B C D$ in $\mathrm{cm}^{2}$ is

A. 308
B. 252
C. 154
D. 126
21. Given that $A B=17 \mathrm{~cm}, A C=16 \mathrm{~cm}$, the area of rhombus $A B C D$ in $\mathrm{cm}^{2}$ is

A. 136
B. 240
C. 272
D. 480
22. Find the volume of the solid below which consists of a hemisphere surmounted by a cone of equal radii of 3 cm . The height of the cone is 9 cm . Give your answer in terms of $\pi$.
23. Find the volume and total surface area of a rectangular pyramid with height 60 cm , and the dimensions of its base is $50 \mathrm{~cm} \times 22 \mathrm{~cm}$.

## SECTION IV Statistics

24. John was awarded 76 marks for both his English and History tests. The mean and standard deviation of the English marks were 52 and 12 respectively; and those of the History marks were 65 and 4 respectively. In which of the two subjects did John do better? Justify your answer with $z$-scores.
25. Complete the following frequency distribution table:

| Score | frequency | Cumulative frequency |
| :---: | :---: | :---: |
| 1 | 13 |  |
| 2 | 26 |  |
| 3 | 37 |  |
| 4 | 33 |  |
| 5 | 29 |  |
| 6 | 12 |  |

Find the range, mean, mode and median of the above set of scores.

| Range $=$ | Mode $=$ |
| :--- | :--- |
| Mean $=$ | Median $=$ |

26. 

| Class 10MA |  | Class 10MB |
| ---: | :---: | :--- |
| Leaf | Stem | Leaf |
|  | 1 | 8 |
| 2 | 2 | 345 |
|  | 3 | 0226 |
| 9887 | 4 | 1222578 |
| 9986 | 5 | 457899 |
| 99988766 | 6 | 233 |
| 8876543 | 7 | 0027 |
| 766 | 8 |  |
| 985 | 9 | 13 |

The above stem-and-leaf plot shows the marks of a mathematics test of classes 10MA and 10 MB . Find the range, median and mode for each of the two classes.

|  | Class 10MA | Class 10MB |
| :--- | :--- | :--- |
| Range |  |  |
| Median |  |  |
| Mode |  |  |

## End of the Paper

## EPPING BOYS' HIGH SCHOOL

## YEAR 10 STAGE 3 MATHEMATICS 2006 Yearly Examination

Date: 200610275
Name: Solution

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Class 10M $\qquad$

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| Section | I | II | III | IV | Total | \% |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Topic | Algebra | Trig | Vol \& SA | Stats |  |  |
| Full marks | 20 | 20 | 15 | 15 | 70 | 100 |
| Score |  |  |  |  |  |  |
|  |  |  |  |  |  |  |

## SECTION I Algebra

## Questions

Answers

1. If $4 x+3=0$, then $x=$
A. $\frac{3}{4}$
B. $-\frac{3}{4}$
C. $\frac{4}{3}$
D. $-\frac{4}{3}$
B
2. If $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$, then $v=$
A. $\frac{u f}{u-f}$
B. $\frac{u f}{f-u}$
C. $f-u$
D. $u-f$

A
03. If $n$ is a non-zero integer, which statement below is always true?
A. $2 n \geq n-7$
B. $\frac{n}{100}<n$
C. $n^{2} \geq n$
D $2^{n}$ is an integer.
C
04. $a^{2}-b^{2}=$
A. $(a-b)^{2}$
B. $(b-a)^{2}$
C. $(b-a)(b+a)$
D. $(a-b)(a+b)$
D

## Questions

Answers
05. If $\sqrt{x^{2}-25}$ is a real number, then $x$ cannot be
A. 36
B. -7
C. 4
D. -5
C
06. Solve for $x$ :
(a) $(x-7)^{2}=8$ (Leave your answers in surd form.)

$$
\begin{aligned}
& x-7= \pm 2 \sqrt{2} \\
& x=7 \pm 2 \sqrt{2}
\end{aligned}
$$

(b) $(x-5)(x+2)=0$

$$
x=5 \text { or }-2
$$

(c) $x^{2}-7 x+12=0$
$(x-3)(x-4)=0$
$x=3$ or 4
(d) $3 x^{2}-7 x-8=0$ (Leave your answer in surd form.)

$$
\begin{aligned}
x & =\frac{7 \pm \sqrt{49+96}}{6} \\
& =\frac{7 \pm \sqrt{145}}{6}
\end{aligned}
$$

(e) $x+1=\frac{6}{x}$

$$
\begin{aligned}
& x^{2}+x-6=0 \\
& (x+3)(x-2)=0 \\
& x=2 \text { or }-3
\end{aligned}
$$

7. Solve the following pair of simultaneous equations:
$2 x-5 y=30$
$14 x+15 y=-20$
$x=3 \frac{1}{2}$ and $y=-4 \frac{3}{5}$
8. John is six times as old as his son. In 24 years time, John will be double his son's age. How old is John now?

Let $x$ be the present age of John.
$x+24=2\left(\frac{x}{6}+24\right)$
$3 x+72=x+144$
$x=36$

John is 36 years old now.

## SECTION II Trigonometry

## Questions

9. In $\triangle A B C, \angle A=59^{\circ}, \angle B=30^{\circ}$, and $B C=12 \mathrm{~cm}$.

The length of $A C$ in cm , correct to 3 decimal places is B
A. $3 \cdot 051$
B. $7 \cdot 000$
C. $10 \cdot 286$
D. $11 \cdot 800$
10. $\sin 78^{\circ} \neq$
A. $\sin 102^{\circ}$
B. $\sin 258^{\circ}$
C. $\cos 12^{\circ}$
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11. Which of the following is the correct cosine rule?
A. $a^{2}=b^{2}+c^{2}+2 b c \cos A$
B. $a^{2}=b^{2}+c^{2}-2 b c \cos A$
C. $\cos B=\frac{b^{2}+c^{2}-a^{2}}{2 b c}$
D. $\cos C=\frac{a^{2}+b^{2}-c^{2}}{2 b c}$

B
12. $\tan \theta=$
A. $\frac{a d j}{o p p}$
B. $\frac{a d j}{h y p}$
C. $\frac{o p p}{h y p}$
D. $\frac{o p p}{a d j}$

D
13. Which of the following set of data gives rise to an ambiguous case in $\triangle A B C$ ?
E. $c=4 \mathrm{~cm}, b=3 \mathrm{~cm}$, and $a=6 \mathrm{~cm}$.
F. $a=20 \mathrm{~cm}, b=13 \mathrm{~cm}$, and $\angle B=29^{\circ}$.
G. $a=12 \mathrm{~cm}, \angle A=40^{\circ}$, and $\angle B=60^{\circ}$.
H. $b=7 \mathrm{~cm}, c=5 \mathrm{~cm}$, and $\angle A=45^{\circ}$.
14. In $\triangle A B C, \angle C=18^{\circ}, a=4 \mathrm{~cm}$, and $c=(\sqrt{5}-1) \mathrm{cm}$.

Find the magnitude of $\angle A$ and the area of $\triangle A B C$ to 3 decimal places.

$$
\begin{array}{rlrl}
\frac{\sin A}{4} & =\frac{\sin 18^{\circ}}{\sqrt{5}-1} & \text { area of } \triangle A B C & =\frac{1}{2}(\sqrt{5}-1)(\sqrt{10+2 \sqrt{5}}) \\
A= & =20^{\circ} & & \\
C A & =\sqrt{16-(\sqrt{5}-1)^{2}} & \\
& =\sqrt{10+2 \sqrt{5}} & &
\end{array}
$$

15. In $\triangle A B C, a=23 \mathrm{~cm}, b=11 \mathrm{~cm}$, and $c=\sqrt{903} \mathrm{~cm}$.

Find the magnitude of $\angle C$, and the area of $\triangle A B C$ to 3 decimal places.

$$
\begin{aligned}
\cos C & =\frac{23^{2}+11^{2}-903}{2 \times 23 \times 11} \\
\angle C & =120^{\circ}
\end{aligned}
$$

Area of $\triangle A B C=\frac{1}{2} \times 23 \times 11 \times \sin 120^{\circ}$

$$
\begin{aligned}
& =\frac{253 \sqrt{3}}{4} \mathrm{~cm}^{2} \\
& =109 \cdot 552 \mathrm{~cm}^{2}
\end{aligned}
$$

16. Town $A$ is 12 km north-east of Town B ; Town $B$ is 15 km from Town C .

The bearing of Town $C$ from Town B is $120^{\circ}$. Find the distance between Towns $A$ and $C$ to 3 decimal places. Find also the bearing of Town $C$ from Town $A$ to the nearest degree.

$$
\begin{aligned}
& \qquad \begin{aligned}
& A C^{2}=12^{2}+15^{2}-2 \times 12 \times 15 \times \cos 75^{\circ} \\
&=369-90(\sqrt{6}-\sqrt{2}) \\
& A C=3 \sqrt{41-10 \sqrt{6}+10 \sqrt{2}} \\
&=16 \cdot 608(\text { correct to } 3 \text { dec. pl. }) \\
& \frac{\sin \angle B A C}{15}=\frac{\sin 75^{\circ}}{A C} \\
& \angle B A C=60^{\circ} 44^{\prime} 21 \cdot 56^{\prime \prime}
\end{aligned} \\
& \text { The bearing of } C \text { from } A \text { is } S 16^{\circ} E \text { or } 164^{\circ}
\end{aligned}
$$

## SECTION III Volume and Surface area

## Questions

Answers
17. The formula for the volume of a sphere is
A. $4 \pi r^{2}$
B. $\frac{4}{3} \pi r^{2}$
C. $\frac{4}{3} \pi r^{3}$
D $2 \pi r^{3}$
C
18. The formula for the volume of a cone is
A. $2 \pi r$
B. $\frac{1}{3} \pi r h$
C. $\frac{1}{3} \pi r^{2} h$
D. $\pi r^{2} h$
C
19. The formula for the curved surface area of a cone is
A. $3 \pi r^{2}$
B. $\frac{1}{3} \pi r h$
C. $\pi r h$
D. $\pi r s$
D
20. Given that $A B=11 \mathrm{~cm}, B C=28 \mathrm{~cm}$, and the perpendicular distance between $A D$ and $B C$ is 9 cm , the area of parallelogram $A B C D$ in $\mathrm{cm}^{2}$ is

A. 308
B. 252
C. 154
D. 126
21. Given that $A B=17 \mathrm{~cm}, A C=16 \mathrm{~cm}$, the area of rhombus $A B C D$ in $\mathrm{cm}^{2}$ is


B
A. 136
B. 240
C. 272
D. 480
22. Find the volume of the solid below which consists of a hemisphere surmounted by a cone of equal radii of 3 cm . The height of the cone is 9 cm . Give your answer in terms of $\pi$.

$$
\begin{aligned}
V & =\frac{1}{3} \pi \times 3^{2} \times 9+\frac{2}{3} \pi \times 3^{3} \\
& =45 \pi \mathrm{~cm}^{3}
\end{aligned}
$$

23. Find the volume and total surface area of a rectangular pyramid with height 60 cm , and the dimensions of its base is $50 \mathrm{~cm} \times 22 \mathrm{~cm}$.

$$
\begin{aligned}
V & =\frac{1}{3} \times 60 \times 50 \times 22 \\
& =22,000 \mathrm{~cm}^{3}
\end{aligned}
$$

S.A. $=50 \times 22+61 \times 50+65 \times 22$
$=5580 \mathrm{~cm}^{2}$

## SECTION IV Statistics

24. John was awarded 76 marks for both his English and History tests. The mean and standard deviation of the English marks were 52 and 12 respectively; and those of the History marks were 65 and 4 respectively. In which of the two subjects did John do better? Justify your answer with $z$-scores.

$$
\begin{aligned}
z_{E} & =\frac{76-52}{12} & z_{H} & =\frac{76-65}{4} \\
& =2 & & =2.75
\end{aligned}
$$

$z_{H}>z_{E}$
Therefore, John did better in History.
25. Complete the following frequency distribution table:

| Score | frequency | Cumulative frequency |
| :---: | :---: | :---: |
| 1 | 13 | $\mathbf{1 3}$ |
| 2 | 26 | $\mathbf{3 9}$ |
| 3 | 37 | $\mathbf{7 6}$ |
| 4 | 33 | $\mathbf{1 0 9}$ |
| 5 | 29 | $\mathbf{1 3 8}$ |
| 6 | 12 | $\mathbf{1 5 0}$ |

Find the range, mean, mode and median of the above set of scores.

$$
\text { Range }=\mathbf{5} \quad \text { Mode }=\mathbf{3}
$$

$$
\text { Mean }=3 \cdot 5 \quad \text { Median }=\mathbf{3}
$$

26. 

| Class 10MA |  | Class 10MB |
| ---: | :---: | :--- |
| Leaf | Stem | Leaf |
|  | 1 | 8 |
| 2 | 2 | 345 |
|  | 3 | 0226 |
| 9887 | 4 | 1222578 |
| 9986 | 5 | 457899 |
| 99988766 | 6 | 233 |
| 8876543 | 7 | 0027 |
| 766 | 8 |  |
| 985 | 9 | 13 |

The above stem-and-leaf plot shows the marks of a mathematics test of classes 10MA and 10MB. Find the range, median and mode for each of the two classes. [6]

|  | Class 10MA | Class 10MB |
| :--- | :---: | :---: |
| Range | $\mathbf{7 7}$ | $\mathbf{7 5}$ |
| Median | $\mathbf{6 9}$ | $\mathbf{5 1}$ |
| Mode | $\mathbf{6 9}$ | $\mathbf{4 2}$ |

## End of the Paper

