EPPING BOYS HIGH SCHOOL

YEAR 10 STAGE 5.3 MATHEMATICS

2012 YEARLY EXAMINATION

Student's Name: .....

(Please put a cross in the box, next to your teacher's name)

	Class Teacher	Class
	Mr Garvey	102M53-1
	Mr Lachmaiya	102M53-2
	Mrs Liyanage	102M53-3
	Ms Tang	102M53-4

Time Allowed: 65 mins

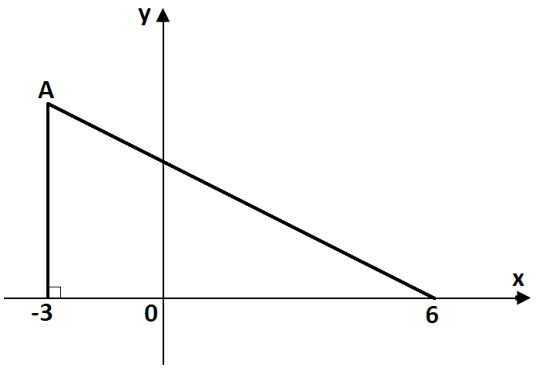
General Instructions

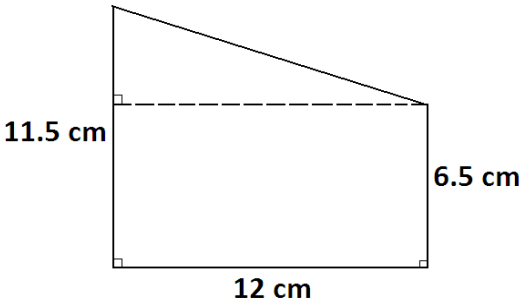
- Write your name and your teacher's name on each section
- Write using only **BLACK** or **BLUE** pen (pencils can only be used for diagrams)
- **ALL** necessary working should be shown in every question
- Marks may be deducted for careless and untidy work
- Only Board of Studies approved calculators may be used in section B onwards
- Attempt all sections

Section	A	B	C	D	E	Total	%
	Non-calculator	Similarity	Further Trigonometry	Further Algebra	Circle Geometry		
Out of	10	7	17	17	17	68	100
Mark							

PART A – NON-CALCULATOR

Time allowed: 10 mins NO calculators are to be used Write your answers in the space provided	Mark	Student's Name	
		Teacher	

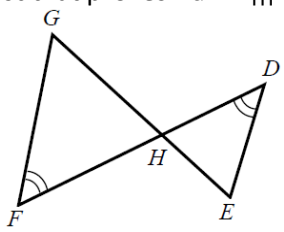
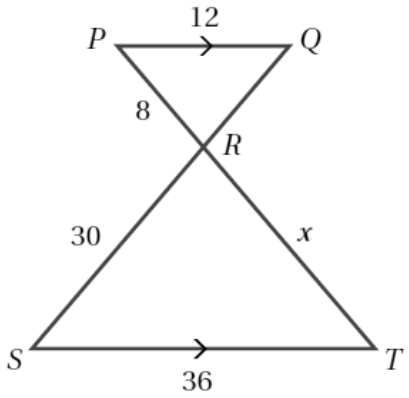
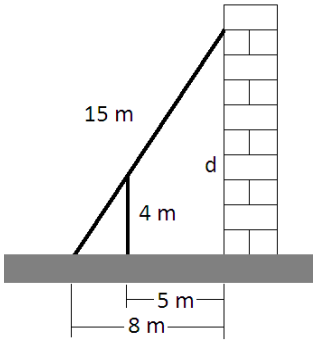
	Question	Answer	Marks
1.	Find the value of $$\frac{48 \div (-2)}{6 - 4 \times 3}$$		1
2.	It has been calculated that the probability of a male birth is 0.48. Over a period of time, there were 2700 babies born in Australia. How many babies were female?		1
3.	The area of the triangle drawn on the number plane is $27\ units^2$.  Find the coordinates of the point A.		1
4.	Laura works as a telephone operator selling home security systems. She is paid $\$440$ per week plus 4.5% of her sales of any systems. Last week her sales totalled $\$1400$. Find her total pay for the week?		1
5.	The surface area of a cube is $54\ cm^2$. Find the volume in cm^3		1

6.	The average of 6 scores is 41. If another score is included, the average increases by 3.5. What is the new score?		1
7.	Brian is hiring the local hall for a party for his 18th. He requires 120 chairs for the party and has been told that the ratio of tables to chairs at the hall is $2:9$. If there are 10 tables in the hall, how many extra chairs will he need to hire?		1
8.	Yesterday, a train left Westlakes at 5.47 pm and arrived at Edgeworth at 7.12 pm. If the train had left Westlakes on schedule but had arrived at its destination fifteen minutes late, how long should the journey normally take, in minutes?		1
9.	What is the perimeter of the following shape?  		1
10.	Simplify the following expression $\frac{3}{x^2-4}-\frac{5}{x^2+x-6}$		1

- End of Part A -

PART B – SIMILARITY

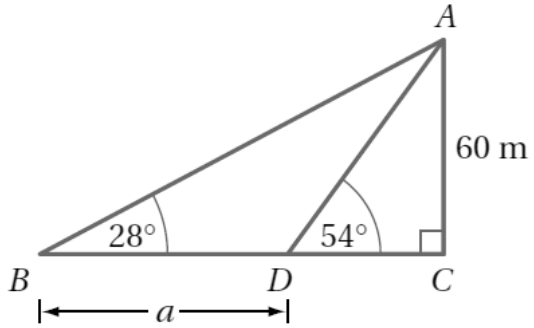
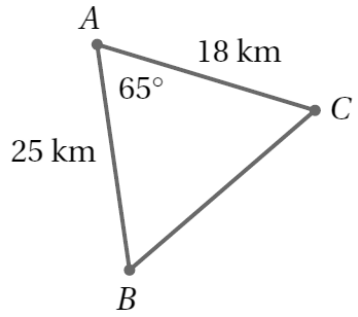
Calculators are to be used Write your answers in the space provided	Mark	Student's Name	
		Teacher	

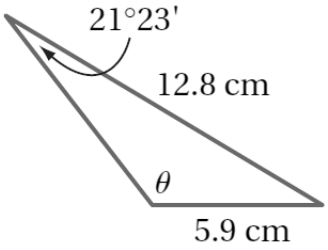
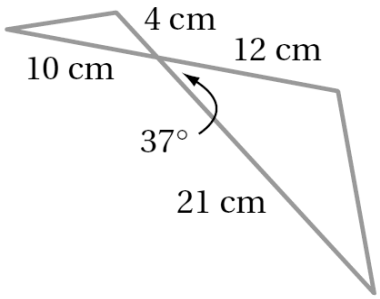
	Question	Answer	Marks
1.	State the test that proves $\Delta GHF \;|||\; \Delta DHE$  		1
2.	For the following diagram, given $PQ//ST$  **a.** Prove that $\Delta PQR \;|||\; \Delta TSR$ **b.** Find the value of x, giving reason	**a.**	2
		b.	2
3.	The wall of a building is being supported by a piece of timber 15 m long which is positioned 8 m from the wall.  Find the distance that the piece of timber reaches up the wall if a support 4m long is propped under the timber, 5 m from the wall.		2

– End of Part B –

PART C – FURTHER TRIGONOMETRY

Calculators are to be used Write your answers in the space provided	Mark	Student's Name	
		Teacher	

	Question	Answer	Marks
1.	Find the value of a correct to 1 decimal place 		2
2.	Three towns, A, B and C, are connected by straight roads. The distance from town A to town B is $25\ km$ and the distance from town A to town C is $18\ km$. If the angle between the two roads AB and AC is $65°$, what is the distance BC to the nearest kilometre? 		2
3.	Find the value of x $\sin 23 = \cos(x + 30)$		2
4.	Prove that $\dfrac{\sin\theta \times \sin(90° - \theta)}{\cos(90° - \theta)} = \cos\theta$		2

5.	If $\tan\theta < 0$ and $\cos\theta < 0$. State whether the angle θ is acute or obtuse where $0° < \theta < 180°$		1
6.	Find the exact value of $\tan 150°$		1
7.	Given θ is obtuse, find the value of θ correct to the nearest minute  		2
8.	Find the total area of both triangles correct to the nearest square centimetre  		2
9.	An aircraft leaves Darwin and flies on a bearing of $123°$ for $200\ km$. The aircraft then changes direction and flies on a bearing of $213°$, until it is due south of Darwin. How far south of Darwin is the aircraft, correct to the nearest kilometre? HINT: Drawing a diagram is required		3

– End of Part C –

PART D – FURTHER ALGEBRA

Calculators are to be used Write your answers in the space provided	Mark	Student's Name	
		Teacher	

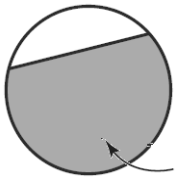
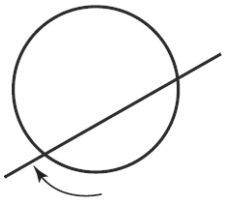
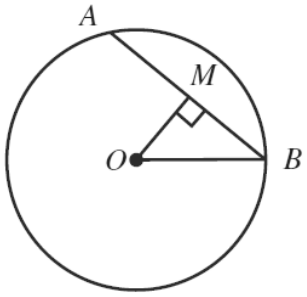
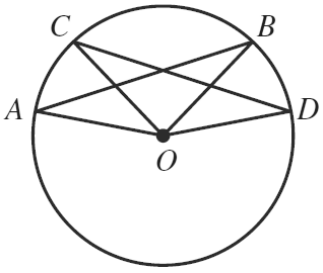
	Question	Answer	Marks
1.	Solve the following equations simultaneously $y = x^2 + 6x + 11$ and $y = 4 - 2x$		3
2.	Solve the following equations simultaneously $y = x^2 - 7x + 10$ and $y = -4x + 6$		2
3.	Rewrite the expression $x^6 - 4x^3 + 5$ in terms of a if $a = x^3$		1

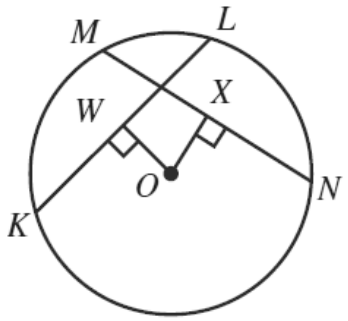
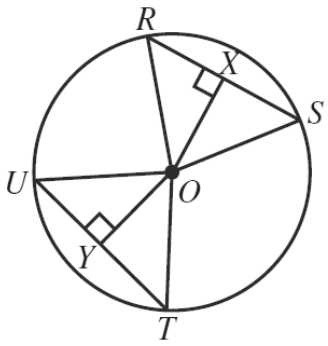
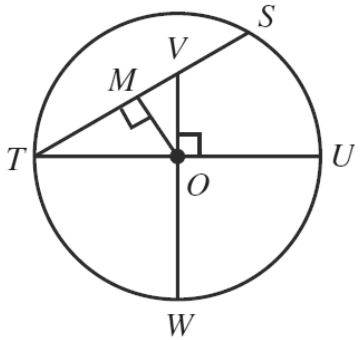
4.	Make y the subject of the following $$T = \frac{3(y+k)}{4c}$$		2
5.	Given $$A = \sqrt{\frac{p+q}{p-q}}$$ **a.** Make p the subject of the formula **b.** Considering the original equation, explain why $p \neq q$	**a.**	2
		b.	1
6.	Solve the following equation $$4y^4 - 37y^2 + 9 = 0$$		3
7.	Use the substitution $a = 1 - 2k$ to solve the following equation $$3(1-2k)^2 - 5(1-2k) = 22$$		3

- End of Part D -

PART E – CIRCLE GEOMETRY

Calculators are to be used Write your answers in the space provided	Mark	Student's Name	
		Teacher	

	Question	Answer	Marks
1.	Name the parts of the circle, shown in the diagrams below **a.**  **b.**  **c.** 	**a.**	1
		b.	1
		c.	1
2.	Given $AB = 24\ cm$ and $OM = 5\ cm$ Find OB 		2
3.	**Given $\boldsymbol{AB = CD}$** $\angle AOC = 32°$ and $\angle COD = 124°$ Find $\angle COB$, giving reasons 		2

4.	In the diagram below, Given $OW = OX = 9\ cm$ and $KL = 25\ cm$. Find MN. giving reasons  		2
5.	Given $\angle SOR = \angle TOU$ and $YT = 6.5\ cm$ Find RS, giving reasons  		3
6.	In the diagram shown, O is the centre of the circle, $TU = 120\ cm$, $VW = 105\ cm$ and $OM = 36\ cm$.  **a.** Find the value of SM, giving reasons **b.** Hence find SV.	**a.**	3
		b.	2

– THE END OF THE EXAMINATION ☺ –

EPPING BOYS HIGH SCHOOL

YEAR 10 STAGE 5.3 MATHEMATICS

2012 YEARLY EXAMINATION SOLUTIONS

Student's Name: .....

(Please put a cross in the box, next to your teacher's name)

	Class Teacher	Class
	Mr Garvey	102M53-1
	Mr Lachmaiya	102M53-2
	Mrs Liyanage	102M53-3
	Ms Tang	102M53-4

Time Allowed: 65 mins

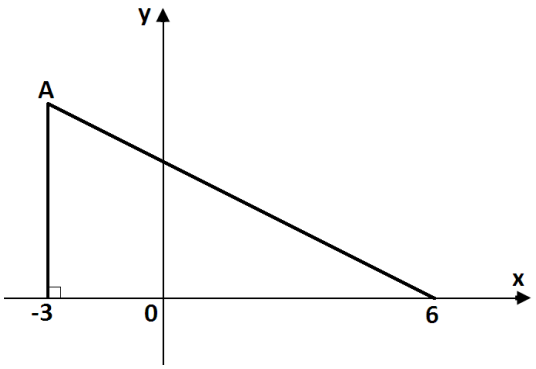
General Instructions

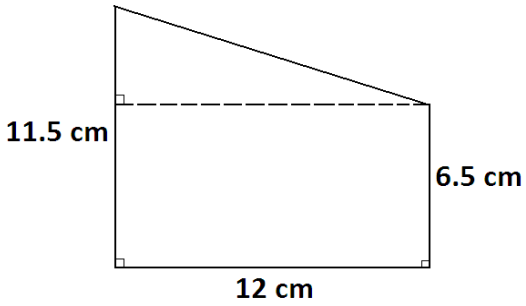
- Write your name and your teacher's name on each section
- Write using only **BLACK** or **BLUE** pen (pencils can only be used for diagrams)
- **ALL** necessary working should be shown in every question
- Marks may be deducted for careless and untidy work
- Only Board of Studies approved calculators may be used in section B onwards
- Attempt all sections

Section	A	B	C	D	E	Total	%
	Non-calculator	Similarity	Further Trigonometry	Further Algebra	Circle Geometry		
Out of	10	7	17	17	17	68	100
Mark							

PART A – NON-CALCULATOR

Time allowed: 10 mins **NO calculators are to be used** **Write your answers in the space provided**	**Mark**	**Student's Name**	
		Teacher	

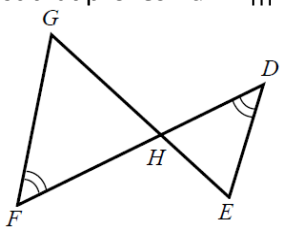
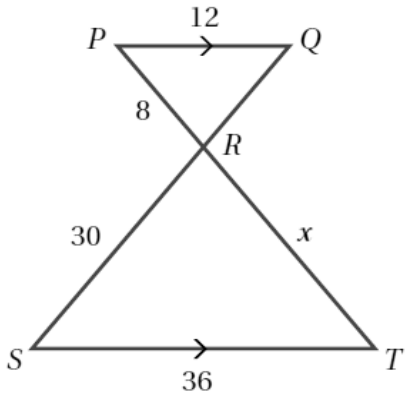
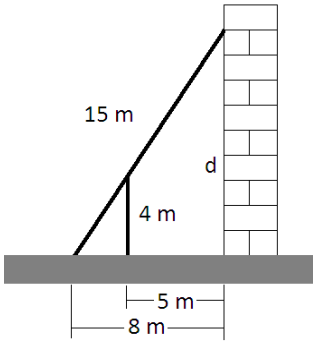
	Question	Answer	Marks
1.	Find the value of $$\frac{48 \div (-2)}{6 - 4 \times 3}$$	$= \frac{-24}{-6} = 4$	1
2.	It has been calculated that the probability of a male birth is 0.48. Over a period of time, there were 2700 babies born in Australia. How many babies were female?	Number of females $= 0.52 \times 2700$ $= 1404$	1
3.	The area of the triangle drawn on the number plane is $27\ units^2$.  Find the coordinates of the point A.	Let $h =$ height of triangle $27 = \frac{1}{2} \times 9 \times h$ $h = 54 \div 9$ $h = 6$ $\therefore A = (-3, 6)$	1
4.	Laura works as a telephone operator selling home security systems. She is paid $\$440$ per week plus 4.5% of her sales of any systems. Last week her sales totalled $\$1400$. Find her total pay for the week?	$1\% = \$14$ $4.5\% = \$63$ Total pay $= 440 + 63$ $= \$503$	1
5.	The surface area of a cube is $54\ cm^2$. Find the volume in cm^3	$SA = 6 \times s^2$ $54 = 6 \times s^2$ $s^2 = 9 \rightarrow s = 3$ $\therefore V = 3 \times 3 \times 3 = 27 cm^3$	1

6.	The average of 6 scores is 41. If another score is included, the average increases by 3.5. What is the new score?	New score $= 44.5 \times 7 - 41 \times 6$ $= 311.5 - 246$ $= 65.5$	1
7.	Brian is hiring the local hall for a party for his 18th. He requires 120 chairs for the party and has been told that the ratio of tables to chairs at the hall is $2:9$. If there are 10 tables in the hall, how many extra chairs will he need to hire?	$2\ parts\ =\ 10\ tables$ $1\ part\ =\ 5$ $\therefore$ Chairs $= 9 \times 5 = 45$ Number of extra chairs $= 120 - 45$ $= 75$	1
8.	Yesterday, a train left Westlakes at 5.47 pm and arrived at Edgeworth at 7.12 pm. If the train had left Westlakes on schedule but had arrived at its destination fifteen minutes late, how long should the journey normally take, in minutes?	$5.47pm\ to\ 6.47pm = 1\ hour$ $6.47pm\ to\ 7.12pm = 25\ mins$ Journey took $1\ hour\ 10\ mins$	1
9.	What is the perimeter of the following shape?  	Let hypotenuse$= x$ $x = \sqrt{12^2 + 5^2} = 13\ cm$ Perimeter $= 13 + 12 + 11.5 + 6.5$ $= 43\ cm$	1
10.	Simplify the following expression $\frac{3}{x^2-4} - \frac{5}{x^2+x-6}$	$\frac{3}{x^2-4} - \frac{5}{x^2+x-6}$ $= \frac{3}{(x-2)(x+2)} - \frac{5}{(x-2)(x+3)}$ $= \frac{3(x+3) - 5(x+2)}{(x-2)(x+2)(x+3)}$ $= \frac{-2x-1}{(x-2)(x+2)(x+3)}$	1

- End of Part A -

PART B – SIMILARITY

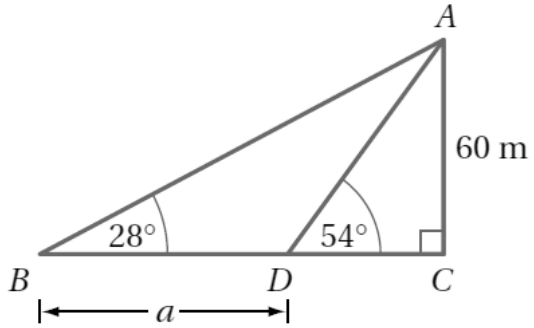
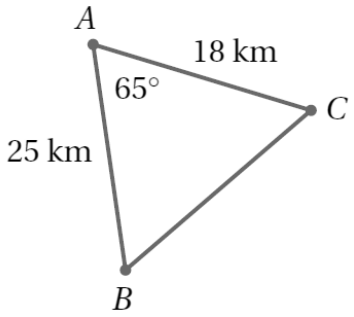
Calculators are to be used Write your answers in the space provided	Mark	Student's Name	
		Teacher	

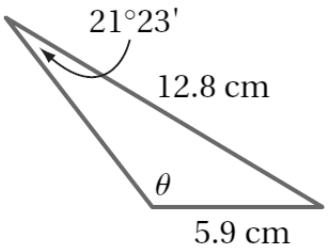
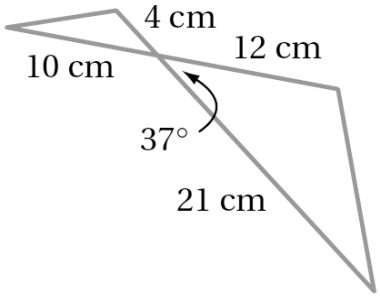
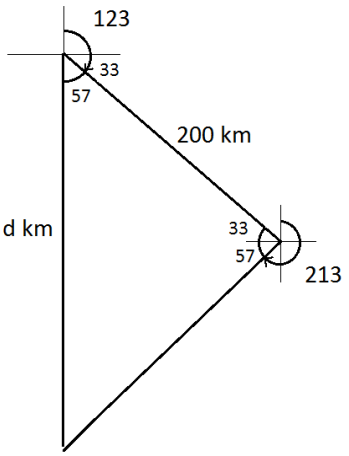
	Question	Answer	Marks
1.	State the test that proves $\Delta GHF \;|||\; \Delta DHE$  	Equiangular OR AAA	1
2.	For the following diagram, given $PQ//ST$  **a.** Prove that $\Delta PQR \;|||\; \Delta TSR$ **b.** *F*ind the value of x, giving reason	**a.** In ΔPQR and ΔTSR $\angle PQR = \angle RST$ (alternate angles, $PQ//ST$) $\angle PRQ = \angle SRT$ (vertically opposite angles are equal) $\therefore \Delta PQR \;|||\; \Delta TSR$ (equiangular)	2
		b. $\frac{RT}{ST} = \frac{RP}{PQ}$ (corresponding sides on similar triangles are in proportion) $\frac{x}{36} = \frac{8}{12}$ $x = \frac{8 \times 36}{12}$ $x = 24$	2
3.	The wall of a building is being supported by a piece of timber 15 m long which is positioned 8 m from the wall.  Find the distance that the piece of timber reaches up the wall if a support 4m long is propped under the timber, 5 m from the wall.	$\frac{d}{8} = \frac{4}{3}$ $d = \frac{4 \times 8}{3}$ $d = \frac{32}{3} = 10\frac{2}{3}\ m$ OR $d^2 = 15^2 - 8^2$ $d = \sqrt{161} = 12.69$ (to two decimal places)	2

– End of Part B –

PART C – FURTHER TRIGONOMETRY

Calculators are to be used Write your answers in the space provided	Mark	Student's Name	
		Teacher	

	Question	Answer	Marks
1.	Find the value of a correct to 1 decimal place 	$\tan 54 = \frac{60}{DC}$ $DC = \frac{60}{\tan 54}$ $\tan 28 = \frac{60}{BC}$ $BC = \frac{60}{\tan 28}$ $a = BC - DC$ $a = \frac{60}{\tan 28} - \frac{60}{\tan 54}$ $a = 69.3\ m$	2
2.	Three towns, A, B and C, are connected by straight roads. The distance from town A to town B is $25\ km$ and the distance from town A to town C is $18\ km$. If the angle between the two roads AB and AC is 65°, what is the distance BC to the nearest kilometre? 	$BC^2 = 18^2 + 25^2 - 2 \times 18 \times 25 \times \cos 65$ $BC^2 = 568.6434644$ $BC = 24\ km$	2
3.	Find the value of x $\sin 23 = \cos(x + 30)$	$\sin 23 = \cos(90 - 23)$ $\sin 23 = \cos 67$ $\therefore x + 30 = 67$ $\therefore x = 37$	2
4.	Prove that $\frac{\sin\theta \times \sin(90° - \theta)}{\cos(90° - \theta)} = \cos\theta$	$LHS = \frac{\sin\theta \times \sin(90° - \theta)}{\cos(90° - \theta)}$ $= \frac{\sin\theta \times \cos\theta}{\sin\theta}$ $= \cos\theta$ $LHS = RHS$	2

5.	If $\tan\theta < 0$ and $\cos\theta < 0$. State whether the angle θ is acute or obtuse where $0° < \theta < 180°$	angle θ is obtuse	1
6.	Find the exact value of $\tan 150°$	$\tan 150° = -\tan 30°$ $= -\frac{1}{\sqrt{3}}$	1
7.	Given θ is obtuse, find the value of θ correct to the nearest minute  	$\frac{\sin\theta}{12.8} = \frac{\sin 21°23'}{5.9}$ $\sin\theta = \frac{12.8 \times \sin 21°23'}{5.9}$ $\theta = \sin^{-1}\left(\frac{12.8 \times \sin 21°23'}{5.9}\right)$ $\theta = 52°17'$	2
8.	Find the total area of both triangles correct to the nearest square centimetre  	$A = \frac{1}{2} \times 4 \times 10 \times \sin 37 + \frac{1}{2} \times 12 \times 21 \times \sin 37$ $A = 20\sin 37 + 126\sin 37$ $A = 88\ cm^2$	2
9.	An aircraft leaves Darwin and flies on a bearing of 123° for 200 km. The aircraft then changes direction and flies on a bearing of 213°, until it is due south of Darwin. How far south of Darwin is the aircraft, correct to the nearest kilometre? HINT: Drawing a diagram is required	 $\cos 57 = \frac{200}{d}$ $d = \frac{200}{\cos 57}$ $d = 367\ km$	3

– End of Part C –

PART D – FURTHER ALGEBRA

Calculators are to be used Write your answers in the space provided	Mark	Student's Name	
		Teacher	

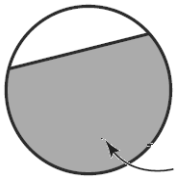
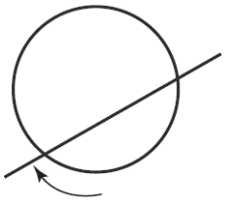
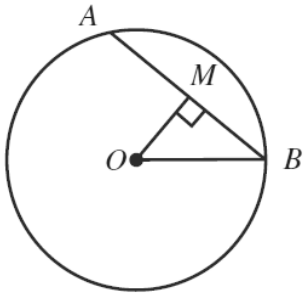
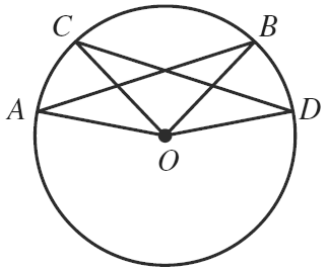
	Question	Answer	Marks
1.	Solve the following equations simultaneously $y = x^2 + 6x + 11$ and $y = 4 - 2x$	$y = x^2 + 6x + 11 \ldots (1)$ $y = 4 - 2x \ldots (2)$ Sub (1) into (2) $x^2 + 6x + 11 = 4 - 2x$ $x^2 + 8x + 7 = 0$ $(x + 1)(x + 7) = 0$ $x = -1, -7 \ldots (3)$ Sub (3) into (2) $y = 4 - 2(-1) = 6$ $y = 4 - 2(-7) = 18$ ∴ When $x = -1, y = 6$ and $x = -7, y = 18$	3
2.	Solve the following equations simultaneously $y = x^2 - 7x + 10$ and $y = -4x + 6$	$y = x^2 - 7x + 10 \ldots (1)$ $y = -4x + 6 \ldots (2)$ Sub (1) into (2) $x^2 - 7x + 10 = -4x + 6$ $x^2 - 3x + 4 = 0$ $x = \dfrac{3 \pm \sqrt{9 - 4(1)(4)}}{2(1)}$ $x = \dfrac{3 \pm \sqrt{-7}}{2}$ $x =$ no solutions	2
3.	Rewrite the expression $x^6 - 4x^3 + 5$ in terms of a if $a = x^3$	$x^6 - 4x^3 + 5$ $= (x^3)^2 - 4x^3 + 5$ $= a^2 - 4a + 5$	1

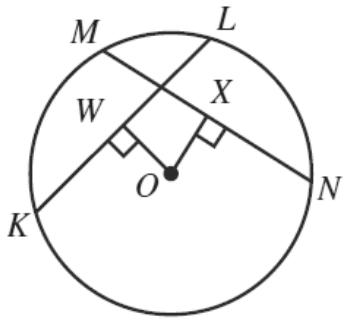
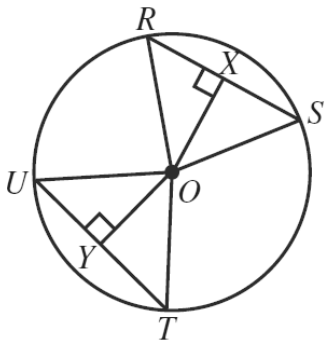
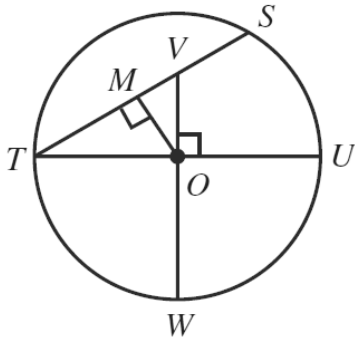
4.	Make y the subject of the following $T = \frac{3(y+k)}{4c}$	$T = \frac{3(y+k)}{4c}$ $4cT = 3(y+k)$ $y+k = \frac{4cT}{3}$ $y = \frac{4cT}{3} - k$	2
5.	Given $A = \sqrt{\frac{p+q}{p-q}}$ **a.** Make p the subject of the formula **b.** Considering the original equation, explain why $p \neq q$	**a.** $A = \sqrt{\frac{p+q}{p-q}}$ $A^2 = \frac{p+q}{p-q}$ $A^2(p-q) = p+q$ $A^2p - A^2q = p+q$ $A^2p - p = A^2q + q$ $p(A^2-1) = A^2q + q$ $p = \frac{A^2q+q}{A^2-1}$	2
		b. $\sqrt{\frac{p+q}{p-q}} = \frac{\sqrt{p+q}}{\sqrt{p-q}}$ $p - q \neq 0$ (denominator can't be 0) $\therefore p \neq q$	1
6.	Solve the following equation $4y^4 - 37y^2 + 9 = 0$	$4y^4 - 37y^2 + 9 = 0$ Let $a = y^2$ $4a^2 - 37a + 9 = 0$ $(4a-1)(a-9) = 0$ $a = \frac{1}{4}, 9$ $\therefore y^2 = \frac{1}{4}, 9$ $\therefore y = \frac{1}{2}, -\frac{1}{2}, 3, -3$	3
7.	Use the substitution $a = 1-2k$ to solve the following equation $3(1-2k)^2 - 5(1-2k) = 22$	$3a^2 - 5a = 22$ $3a^2 - 5a - 22 = 0$ $(3a-11)(a+2) = 0$ $a = \frac{11}{3}, -2$ $\therefore 1-2k = \frac{11}{3},$ $\therefore 3-6k = 11 \rightarrow -6k = 8 \rightarrow k = -\frac{4}{3}$ $\therefore 1-2k = -2$ $\therefore -2k = -3 \rightarrow k = \frac{3}{2}$ $\therefore$ When $a = \frac{11}{3}, k = -\frac{4}{3}$ and $a = -2,\ k = \frac{3}{2}$	3

- End of Part D -

PART E – CIRCLE GEOMETRY

Calculators are to be used Write your answers in the space provided	Mark	Student's Name	
		Teacher	

	Question	Answer	Marks
1.	Name the parts of the circle, shown in the diagrams below **a.**  **b.**  **c.** 	**a.** Major segment	1
		b. Secant	1
		c. Tangent	1
2.	Given $AB = 24\ cm$ and $OM = 5\ cm$ Find OB 	$AM = BM = 12\ cm$ $OB^2 = 12^2 + 5^2$ $OB^2 = 169$ $OB = 13\ cm$	2
3.	**Given $AB = CD$** $\angle AOC = 32°$ and $\angle COD = 124°$ Find $\angle COB$, giving reasons 	$\angle COD = \angle AOB = 124°$ (equal chords subtend equal angles at the centre of the circle) $\angle COB = \angle AOB - \angle AOC$ $\angle COB = 124 - 32$ $\therefore \angle COB = 92$	2

4.	In the diagram below, Given $OW = OX = 9\ cm$ and $KL = 25\ cm$. Find MN. Giving reasons  	$MN = 25\ cm$ (Chords equidistant from the centre of the circle are equal in length)	2
5.	Given $\angle SOR = \angle TOU$ and $YT = 6.5\ cm$ Find RS, giving reasons  	$YT = UY = 6.5\ cm$ (A perpendicular line from the centre of a circle to a circle bisects the chord) $UT = UY + YT = 6.5 + 6.5 = 13cm$ $UT = RS$ (Given $\angle SOR = \angle TOU$, equal angles at the centre of the circle subtend equal chords) $\therefore RS = 13cm$ (Since $UT = 13cm$ and proven that $UT = RS$)	3
6.	In the diagram shown, O is the centre of the circle, $TU = 120\ cm$, $VW = 105\ cm$ and $OM = 36\ cm$.  **a.** Find the value of SM, giving reasons **b.** Hence find SV.	**a.** Given $TU = 120\ cm$ and $OT + UT = UT$ $\therefore TO = UO = OW = 60cm$ (equal radii) $MT^2 = 60^2 - 36^2$ $MT = \sqrt{2304} = 48cm$ $SM = MT$ (A perpendicular line from the centre of a circle to a circle bisects the chord) $\therefore SM = 48cm$	3
		b. $OV = VW - OW = 105 - 60 = 45cm$ $MV^2 = 45^2 - 36^2$ $MV = 27cm$ $SV = SM - MV$ $SV = 48 - 27 = 21cm$	2

– THE END OF THE EXAMINATION ☺ –