SECTION A (30 MARKS) QUESTIONS 1 – 30

1. Evaluate $3.1 - 0.1 \times 10$

(A) 0.21 (B) 2.1 (C) 21 (D) 30

2. When $2 + \frac{3}{\sqrt{5}}$ is simplified to a single fraction, then the numerator becomes

(A) $2 + 3\sqrt{5}$ (B) $2\sqrt{5} + 3$ (C) 6 (D) $5\sqrt{5}$

3. $2x - (x - 3)$ can be simplified to

(A) $x - 3$ (B) $x + 3$ (C) $2x + 3$ (D) $2x - 3$

4. Decreasing 250 by 30% gives

(A) 75 (B) 175 (C) 249.70 (D) 325

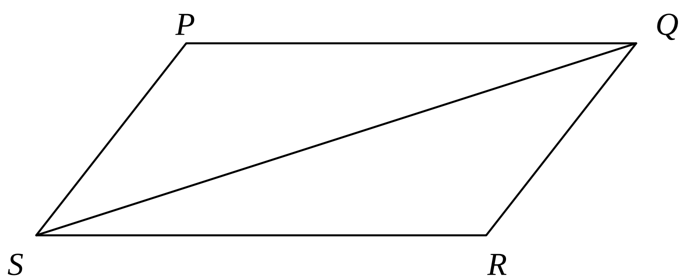
5. If 5 men take 3 hours to paint a brick fence of A m^2, how many hours will it take 3 men to complete the same task?

(A) 1.5 hours (B) 1.2 hours (C) 1.8 hours (D) 5 hours

6. If $\frac{m}{m+2n} = -3$ then the value of $\frac{m}{n}$ is

(A) $-\frac{3}{2}$ (B) $\frac{3}{2}$ (C) $\frac{2}{3}$ (D) $-\frac{2}{3}$

7. In the diagram below, $PQRS$ is a parallelogram and $\Delta PQS \equiv \Delta QRS$.



Which of the following are true?

(A) $\angle PSQ = \angle SQR$ (B) $\angle PSQ = \angle PQS$

(C) $SQ = PQ$ (D) $\angle PQS = \angle SQR$

8. For the quadratic equation $2y - 1 = 3x^2 - 5x$, the axis of symmetry has equation

(A) $x = -\frac{5}{6}$ (B) $x = \frac{5}{3}$ (C) $x = \frac{5}{6}$ (D) $x = \frac{5}{12}$

9. The mean of the set of scores from 0 to 20 inclusive is

(A) 1 (B) 10 (C) $\dfrac{200}{21}$ (D) 10.5

10. If $\sqrt{x+1} = 3$, then $(x + 1)^2$ equals

(A) 3 (B) 9 (C) 27 (D) 81

11. Given $\sin\theta = -\dfrac{1}{\sqrt{2}}$, where $0 \le \theta \le 180^0$, then the value of θ is

(A) 30^0 (B) 45^0 (C) 60^0 (D) 135^0

12. The equation of the straight line with a y – intercept of 4 and parallel to the straight line $3x - 2y = 8$ is

(A) $3x - 2y = -8$ (B) $3x + 2y = -8$

(C) $3x + 2y = 8$ (D) $2x + 3y = 12$

13. Which of the following gives the gradient of the line joining the point $A(-3, -2)$ to point $B(5, -7)$?

(A) $-\dfrac{8}{5}$ (B) $\dfrac{8}{5}$ (C) $-\dfrac{5}{8}$ (D) $\dfrac{5}{8}$

14. The sum of the interior angles, S, of a polygon, where n is the number of sides, is given by the equation

(A) $S = 180n^0$ (B) $S = 180(n - 2)^0$

(C) $S = 90(2n + 4)^0$ (D) $S = \dfrac{90(2n-4)^0}{n}$

15. Sam holds 40 cards of which 10 are red, 10 are blue, 10 are green and 10 are yellow. Find the probability that Sam picks two cards of the same colour, if 2 cards are drawn at random.

(A) $\dfrac{1}{20}$ (B) $\dfrac{1}{4}$ (C) $\dfrac{9}{156}$ (D) $\dfrac{9}{39}$

16. The interior angle of a regular dodecagon is

(A) 144^0 (B) 150^0 (C) 180^0 (D) 1800^0

17. For $3x - x^2 < 0$, the solutions of x are

(A) $0 < x < 3$ (B) $x < 0$ or $x > 3$

(C) $0 < x \le 3$ (D) $x < 0$ or $x \ge 3$

18. Given $P(x) = 8 + 4x - 2x^2 - x^3$, what values of x will $P(x) = 0$?

(A) −2 only (B) 2 only (C) −2 and 2 (D) −2, 0, 2

19. A circle with centre O, is drawn below. Which of the following properties gives the correct relationship between x and y?

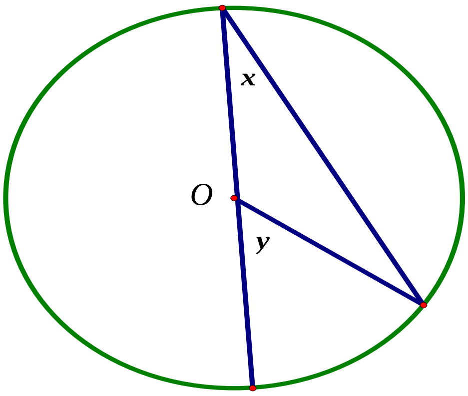


Diagram not to scale

(A) $2x = y$ [The angle at the circumference is twice the angle at the centre standing on the same arc.]
(B) $2x = y$ [The angle at the centre is twice the angle at the circumference standing on the same arc.]
(C) $x = y$ [The angle at the circumference is equal to the angle at the centre standing on the same arc.]
(D) $x = 2y$ [The exterior angle of a triangle is equal to the sum of the opposite two interior angles]

20. The acute angle between the line $2x + 3y - 4 = 0$ and the positive x – axis is closest to

(A) -33^0 (B) -34^0 (C) 33^0 (D) 34^0

21. If $4^{x+y} = 8$, then $x + y$ equals

(A) $\frac{3}{2}$ (B) 2 (C) 3 (D) 4

22. The area of a triangle ABC, where $AB = 5$ cm, $BC = 4$ cm and $\angle ABC = 30^0$ is

(A) 5 cm^2 (B) 10 cm^2 (C) $10\sqrt{3}$ cm^2 (D) $\frac{10}{\sqrt{3}}$ cm^2

23. The inter-quartile range of a set of scores is calculated as

(A) The upper quartile (Q_3) − the median (Q_2).
(B) The lower quartile (Q_1) − the median (Q_2).
(C) The upper quartile (Q_3) − the lower quartile (Q_1).
(D) The highest score − the lowest score.

24. The function $y = \frac{x+1}{2x}$ has which of the following characteristics?

(A) Horizontal asymptote $y = ½$, vertical asymptote $x = 0$.
(B) Horizontal asymptote $y = 1$, vertical asymptote $x = 0$.
(C) Horizontal asymptote $x = ½$, vertical asymptote $y = 0$.
(D) Horizontal asymptote $x = 1$, vertical asymptote $y = 0$

25. The 2005 tax table is given below.

Taxable income	***Tax on this income***
\$1 − \$6000	Nil
\$6001 − \$21600	17cents for each \$1 over \$6000
\$21601 − \$58000	\$2652 + 30 cents for each \$1 over \$21600
\$58001 − \$70000	\$13572 + 42 cents for each \$1 over \$58000
\$70001 and over	\$18612 + 47cents for each \$1 over \$70000

How much tax is payable on a taxable income of \$25 682?

(A) \$2652 (B) \$3876.60 (C) \$12347.40 (D) \$13 572

26. The quadratic formula is given by $x = \dfrac{-b \pm \sqrt{\Delta}}{2a}$, where $\Delta = b^2 - 4ac$. What type of roots occur when $\Delta = 0$.

(A) Equal and rational roots (B) Unequal and rational roots
(C) Equal and irrational roots (D) Unequal and irrational roots

27. The diagram shows a polynomial $P(x)$ of degree 4, with roots as indicated. Which of the following gives a possible equation for $P(x)$?

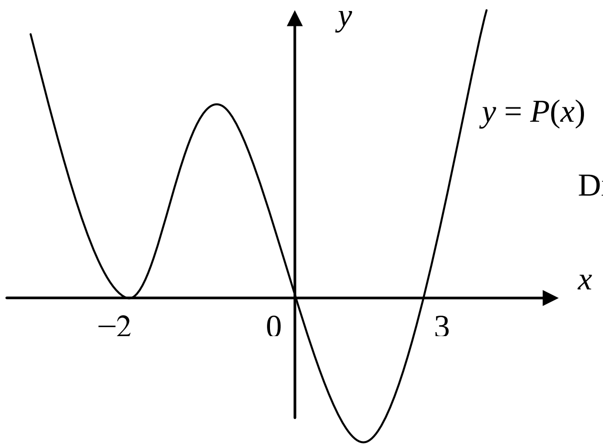


Diagram not to scale

(A) $P(x) = x^2(x-2)(x+3)$ (B) $P(x) = x^2(x+2)(x-3)$

(C) $P(x) = x(x-2)^2(x+3)$ (D) $P(x) = x(x+2)^2(x-3)$

28. Given that $\cos 2A = \cos^2 A - \sin^2 A$, find the value of $\cos 4A$, given that $\cos A = 0.6$.

(A) −0.8432 (B) −0.995 (C) 0.8432 (D) 0.995

29. An equivalent form of $\alpha^2 + \beta^2 + \gamma^2$ is given by

(A) $(\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\alpha\gamma+\beta\gamma)$ (B) $(\alpha+\beta+\gamma)^2 + 2(\alpha\beta+\alpha\gamma+\beta\gamma)$
(C) $(\alpha+\beta+\gamma)^2 - 2(\alpha\beta-\alpha\gamma-\beta\gamma)$ (D) $(\alpha+\beta+\gamma)^2 + 2(\alpha\beta-\alpha\gamma-\beta\gamma)$

30. The length of the side of the smallest square which will enclose three non-overlapping discs, each of radius 1 unit is

(A) 4 (B) $2+\sqrt{3}$ (C) $\dfrac{4+\sqrt{2}+\sqrt{6}}{2}$ (D) $3+\sqrt{2}$

SECTION B (Total 80 Marks)

Use your own writing paper. Clearly mark the Question Number on each page.

Question 31 (20 Marks) **MARKS**

(a) Factorise completely $2x^3 - 54$. 2

(b) Simplify $\dfrac{\tan x - 1}{\sec x}$. 2

(c) Find the domain and range for the function $f(x) = \dfrac{1}{\sqrt{4 - x^2}}$. 3

(d) The scores of two tests, each out of 20 marks, are as follows:

TEST A	9	11	12	12	13	13	14	15	15	16
TEST B	5	7	9	10	12	13	13	15	17	19

For a score of 15 in Test A, what is the equivalent score in Test B? 3
Answer to the nearest whole number & justify your answer.

(e) Solve $3\tan^2\theta - 1 = 0$ for $0^0 \le \theta \le 360^0$. 3

(f) (i) Express $4y = x^2 - 2x + 5$ in the form of $(x - h)^2 = 4(y - k)$. 1

(ii) Hence, sketch the curve $4y = x^2 - 2x + 5$. 2

(g) The speed of a worm varies as the square root of its length. If a worm which is 25 cm long can sprint at 40 cm/minute,

(i) How fast can a 50 cm worm go? (Leave your answer in exact form). 2

(ii) If a worm wants to go at 1 metre/minute, how long would it have to grow? (Answer correct to the nearest cm). 2

Question 32 (20 Marks) **<u>START A NEW PAGE</u>**

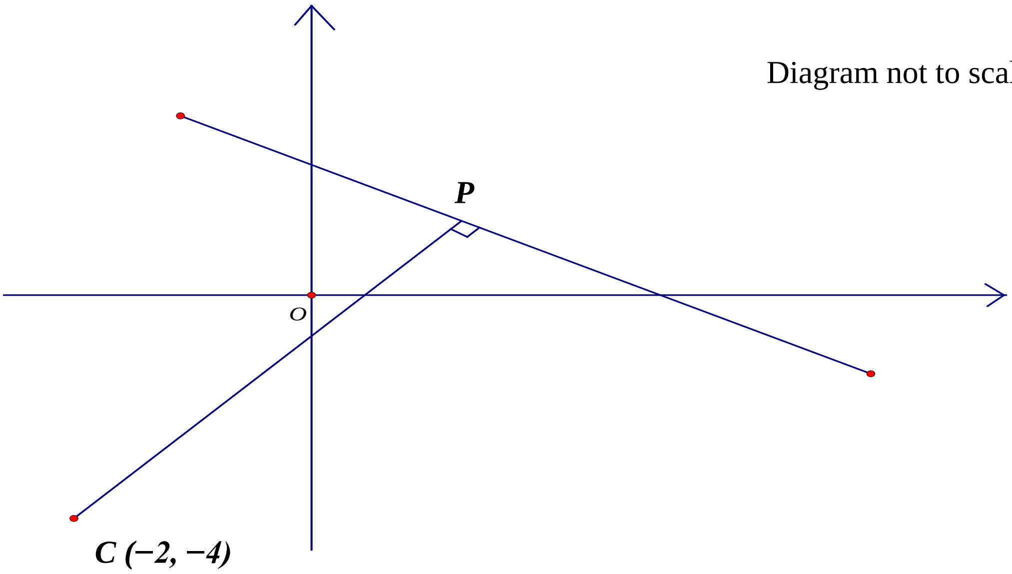
(a) (i) Solve for x: $16^x = \dfrac{1}{128}$ 1

(ii) Simplify $\dfrac{(3^{-2})^3 \times 12^{\frac{1}{4}}}{9\sqrt{2}}$ 3

Question 32 continued over the page

(b) In the diagram, AB is the interval joining the points $A(-1, 2)$ and $B(4, -1)$.
P is the foot of the perpendicular drawn from the point $C(-2, -4)$ to AB.

Diagram not to scale



Copy this diagram onto your answer sheet.

(i) Show that the distance from A to B is $\sqrt{34}$ units. 1

(ii) Show that the equation of the line AB is $3x + 5y - 7 = 0$. 2

(iii) Find the length of CP. 2

(iv) Find the coordinates of the point D such that $ABCD$ is a parallelogram. 1

(v) Find the area of the parallelogram $ABCD$. 1

(c) The equation of a circle is $x^2 + y^2 + 4x - 2y - 20 = 0$. From an external point $T(5, 2)$, a tangent is drawn to meet this circle at A. Find

(i) the length of the interval AT. 3

(ii) the length of the intercept on the y – axis cut by this circle. 2

(d) In the diagram $PMRQ$ is a quadrilateral where $PM \parallel QR$, $PM = MR$ and $PQ = PR$.

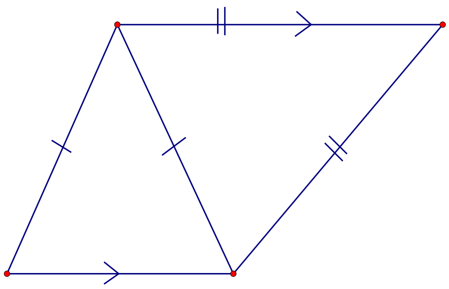


Diagram not to scale

(i) Prove that $\Delta PQR \; ||| \; \Delta MPR$. 2

(ii) If $PQ = 3$ cm, $PM = 2$ cm, find the length of QR. 2

Question 33 (20 Marks) **START A NEW PAGE** **MARKS**

(a) (i) State the period of $y = 1 + \cos 2x$. 1

(ii) Sketch the graph of $y = 1 + \cos 2x$ for $0^0 \le x \le 360^0$. 3

(b) Two identical cubes each have its faces numbered from 0 to 5. The cubes are rolled and the score is determined as the *product* of the two digits on the uppermost aces.

(i) If the cubes are rolled once, what is the probability that the score is

(α) 0? 1

(β) at least 16? 2

(ii) If the cubes are rolled twice and the scores for each roll are added, what is the probability of getting a score of 41 or more? 2

(c) When the polynomial $P(x)$ is divided by $(x + 1)(x - 4)$ the quotient is $Q(x)$ and the remainder is $R(x)$.

(i) Explain why the most general form of $R(x)$ is given by $R(x) = ax + b$? 1

(ii) Given that $P(4) = -5$, show that $R(4) = -5$. 2

(iii) Further, when $P(x)$ is divided by $(x + 1)$ the remainder is 5. Find $R(x)$. 2

(d) A lighthouse is 10 km North- West of a ship travelling due West at 16 km/hour.

(i) How far is the ship from the lighthouse 45 minutes later? Answer correct to 2 decimal places. 2

(ii) What is the bearing of the lighthouse from the ship then? 4

Question 34 (20 Marks) **START A NEW PAGE** **MARKS**

(a) The point $P(x, y)$ divides the interval joining the points $A(-1, 3)$ and $B(2, 8)$ internally in the ratio $k : 1$.

(i) Find the coordinates of P in terms of k. 2

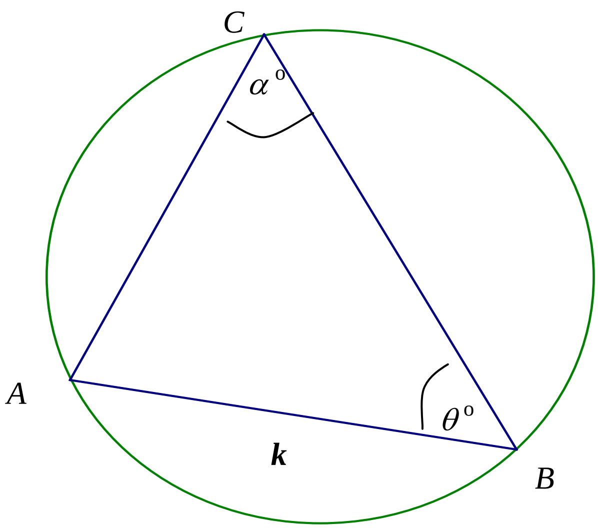
(ii) Find the ratio in which the line $5x + 2y - 10 = 0$ divides the interval AB. 3

(b) Sketch neatly showing all important features the curve $y = \dfrac{(x+1)^2}{x^2 - 4}$. 4

Question 34 continued over the page

(c) In the diagram below, points *A*, *B* and *C* lie on a circle. The length of the chord *AB* is a constant, *k*. 3

Diagram not to scale



Let $\angle ACB = \alpha^o$ and $\angle ABC = \theta^o$ respectively.

(i) Explain why α^o is a constant? 1

(ii) If *S* is the sum of the lengths of the chords *AC* and *BC*, show that *S* is given by $S = \dfrac{k}{\sin\alpha}\left(\sin\theta^o + \sin(\theta+\alpha)^o\right)$. 2

(iii) Find the expression for *S*, in simplified form when $\theta = \left(90 - \dfrac{\alpha}{2}\right)$. 2

(d) In the diagram below, *AC* is the diameter of the circle *AECFG* with centre *O* and *BD* is a tangent to the circle at *C*.

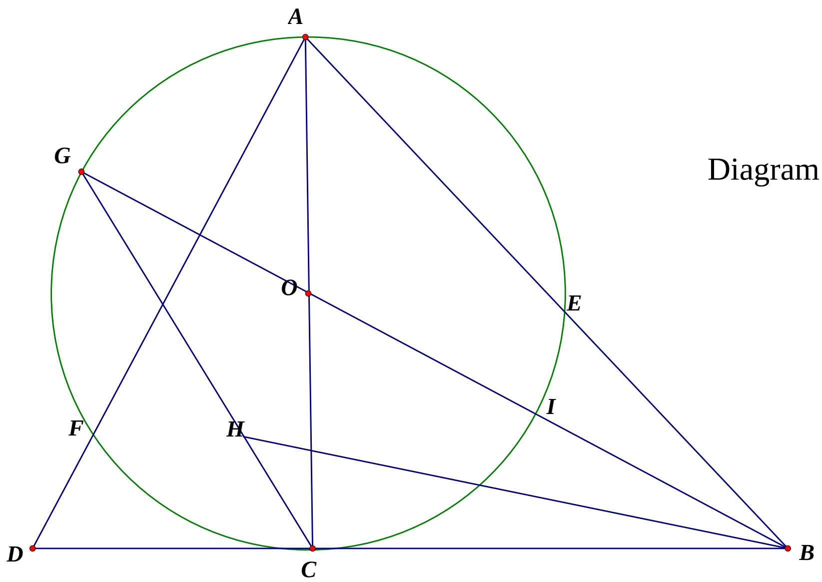


Diagram not to scale

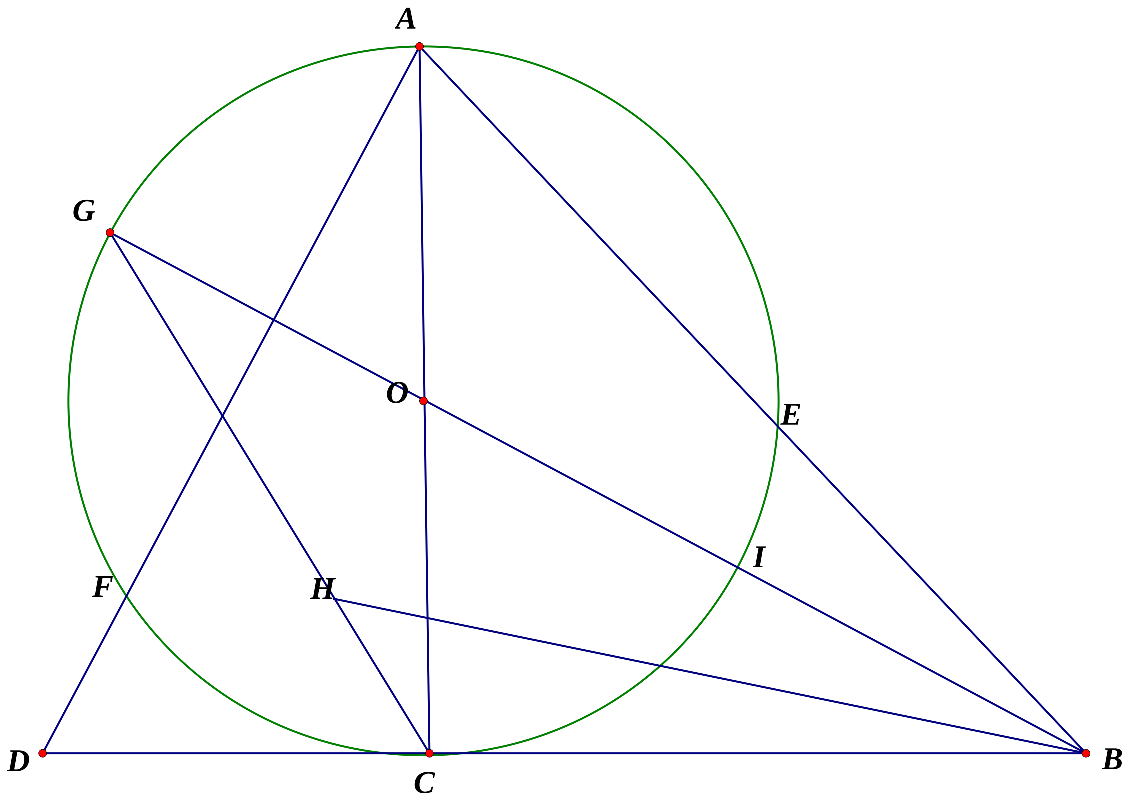
(i) State why $\angle FEC = \angle FCD$. 1

(ii) Hence, prove that *DFEB* is a cyclic quadrilateral. 3

(iii) If $\angle BHC = 45^0$, prove that $\angle HBG = \angle HBC$. 2

ANSWER Q34 (d) ON THIS PAGE, use the diagram as an aid

Name: ______________________ CLASS: _______________



SOLUTIONS YEAR 10 YEARLY PAPER

SECTION A – MULTIPLE CHOICE

(1)	B	(2)	B	(3)	B	(4)	B	(5)	D	(6)	A
(7)	A	(8)	C	(9)	B	(10)	D	(11)	D	(12)	A
(13)	C	(14)	B	(15)	D	(16)	B	(17)	B	(18)	C
(19)	B	(20)	D	(21)	A	(22)	A	(23)	C	(24)	A
(25)	B	(26)	A,C	(27)	D	(28)	A	(29)	A	(30)	C

Full worked SOLUTIONS for Multiple Choice.

1. $3.1 - 1 = 2.1$
   ➔ answer is B

2. $2 + \dfrac{3}{\sqrt{5}} = \dfrac{2\sqrt{5}+3}{\sqrt{5}}$
   $\therefore$ numerator is $2\sqrt{5}+3$
   ➔ answer is B

3. $2x - (x - 3) = x + 3$
   ➔ answer is B

4. $250 \times 0.7 = 175$
   ➔ answer is B

5. 5 men : 3 hours =
   1 men : 3×5 hours
   ∴ 3 men : 5 hours
   ➔ answer is D

6. $\dfrac{m}{m+2n} = -3$
   $\therefore m = -3m - 6n$
   $\therefore 4m = -6n$
   $\therefore \dfrac{m}{n} = -\dfrac{3}{2}$
   ➔ answer is A

7. Since $\Delta PQS \equiv \Delta QRS$
   Only A is true.

8. $2y - 1 = 3x^2 - 5x$
   $\therefore a = \dfrac{3}{2},\ b = -\dfrac{5}{2},\ c = ½$
   $\therefore x = -\dfrac{-\frac{5}{2}}{2\left(\frac{3}{2}\right)} = \dfrac{5}{6}$
   ➔ answer is C

9. $0 + 1 + 2 + \ldots + 20 = 210$
   $\therefore 210 \div 21 = 10$ ➔ answer is B

10. $\sqrt{x+1} = 3$
    $x + 1 = 9$ ➔ $\therefore (x + 1)^2 = 81$
    ∴answer is D

11. Question needs to read as $\cos\theta = -\dfrac{1}{\sqrt{2}}$
    In the given domain $0 \le \theta \le 180$, θ is in the 2nd quadrant. $\therefore\ \theta = 135^0$.
    ➔ answer is D

12. gradient is $\dfrac{3}{2}$, y intercept is 4
    $\therefore y = \dfrac{3x}{2} + 4$ or $3x - 2y = -8$
    ➔ answer is A

13. gradient AB = $\dfrac{-2--7}{-3-5} = \dfrac{-5}{8}$
    ➔ answer is C

14. $S = (n - 2)\times 180^0$ ➔ answer is B

15. Since there are 4 colours, and each has equal chance of being drawn,
    P(same colour) = $4 \times \dfrac{10}{40} \times \dfrac{9}{39} = \dfrac{9}{39}$
    ➔ answer is D

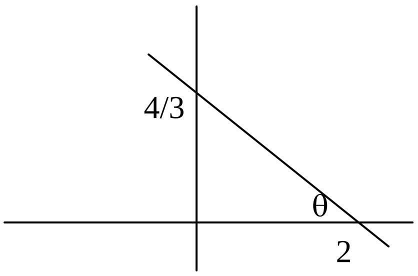
16. A dodecagon has 12 sides.
    ∴ sum of interior sides = 1800^0
    ∴ each interior angle = 1800 ÷ 12
    = 150^0
    ➔ answer is B

17. $3x - x^2 < 0$
    $x(3 - x) < 0\ \ \therefore x < 0$ or $x > 3$
    ➔ answer is B

18. $P(x) = 8 + 4x - 2x^2 - x^3$
$= 4(2 + x) - x^2(2 + x)$
$= (2 + x)(4 - x^2)$
$= (2 + x)^2(2 - x)$
$\therefore$ P(x) = 0 when x = 2 or x = -2
➔ answer is C

19. answer is B. See circle geometry theorems

20.



Since we want the acute angle we don't worry about the negative gradient.

$\therefore \tan\theta = \dfrac{\frac{4}{3}}{2} = \dfrac{2}{3}$

$\therefore \theta = 33.69\ldots$ which is closest to 34^0
➔ answer is D

21. $4^{x+y} = 8$
$2^{2(x+y)} = 2^3$
$\therefore 2(x + y) = 3$
$\therefore x + y = \dfrac{3}{2}$ ➔ answer is A

22. Area = $\dfrac{1}{2}(5)(4)\sin 30^0 = 5$ cm^2
➔ answer is A

23. IQR = $Q_3 - Q_1$
= upper quartile – lower quartile.
➔ answer is C

24. $y = \dfrac{x+1}{2x} = \dfrac{1}{2} + \dfrac{1}{2x}$

$\therefore$ as $x \to \infty, \dfrac{1}{2x} \to 0 \;\; \therefore y \to ½$

This is the horizontal asymptote
Vertical asymptote is when x = 0
➔ answer is A

25. Tax payable on \$21600 is \$2652.
Tax payable on remaining\$4082 = 0.3 × 4082 = \$1224.60
$\therefore$ total tax payable is \$3876.60
➔ answer is B

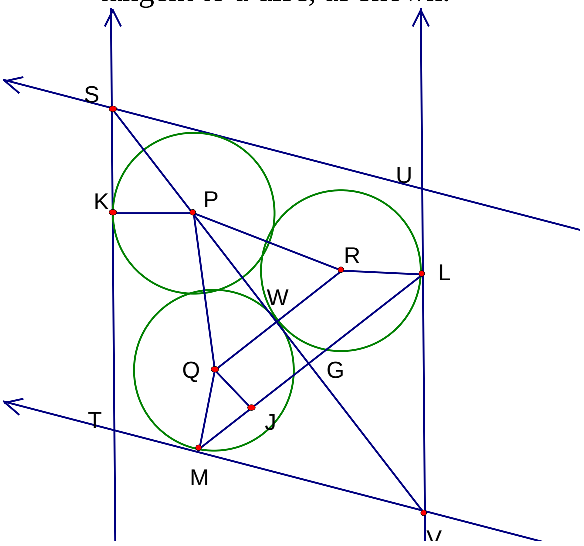
26. If $\Delta = 0$ then we have $x = -\dfrac{b}{2a}$.
Then we only have one solution. Or equal roots.
If a, b, c are rational then roots are rational ➔ answer is A.
[note: also accepted C, due to ambiguous case]

27. Double root at x = -2 ,
single roots at x = 0 and x = 3
➔ answer is D

28. $\cos 2A = \cos^2 A - \sin^2 A$
$= 2\cos^2 A - 1$
$= 2(0.6)^2 - 1 = -0.28$
Following the pattern,
$\cos 4A = \cos^2 2A - \sin^2 2A$
$= 2\cos^2 2A - 1$
$= 2(-0.28)^2 - 1 = -0.8432$
➔ answer is A

29. Now $(\alpha + \beta + \gamma)^2$
$= (\alpha+\beta)^2 + \gamma^2 + 2(\alpha + \beta)\gamma$
$= \alpha^2 + \beta^2 + 2\alpha\beta + \gamma^2 + 2\alpha\gamma + 2\beta\gamma$
$\therefore \alpha^2 + \beta^2 + \gamma^2$
$= (\alpha + \beta + \gamma)^2 - (2\alpha\beta + 2\alpha\gamma + 2\beta\gamma)$
➔ answer is A

30. For the smallest square to contain all 3 discs, a pair of parallel sides must be tangent to a disc, as shown.



The line joining the centres *P*, *Q*, *R* form an equilateral triangle.

ΔSKP is a right isosceles triangle KP = SK = 1 $\therefore$ SP = $\sqrt{2}$ units.

In Δ PQR, PW = $\sqrt{3}$ (altitude of equilateral triangle with sides 2 units)

In Δ QJM, QJ = $\frac{1}{\sqrt{2}}$ (pythag. Theorem with hypotenuse 1 unit)

Also MJ = $\frac{1}{\sqrt{2}}$ = WG. And JG = 1 unit.

In Δ MGV, GM = GV = 1 + $\frac{1}{\sqrt{2}}$ = 1 + $\frac{\sqrt{2}}{2}$(Right isos. Triangle)

$$\therefore \text{SV} = \text{SP} + \text{PW} + \text{WG} + \text{GV}$$

$$= \sqrt{3}+\frac{\sqrt{2}}{2}+\sqrt{2}+1+\frac{\sqrt{2}}{2}$$

$$= 1 + \sqrt{3}+2\sqrt{2}$$

Now by Pythagoras Theorem, $2\times \text{ST}^2 = \text{SV}^2$

$$\therefore \text{ST} = \frac{1}{\sqrt{2}}SV = \frac{\sqrt{2}}{2}(1+\sqrt{3}+2\sqrt{2}) = \frac{4+\sqrt{2}+\sqrt{3}}{2}$$ ➔ answer is C.

* Alternative ways are possible using $\cos 15^0$. Ref: 1994 AMC*

YEAR 10 YEARLY 2005 PART B

Q31

a) $2(x^3-27) = 2(x-3)(x^2+3x+9)$ #

b) $\dfrac{\frac{\sin x}{\cos x}-1}{\frac{1}{\cos x}} = \sin x - \cos x$

c) D: $-2 < x < 2$

R: $\frac{1}{2} \le y$

d) $\bar{x}_A = 13$ $SD_A = 2$ $15 = 13 + 2$ $(\bar{x}_A + 1SD_A)$

$\bar{x}_B = 12$ $SD_B = 4.15$ $x = 12 + 4.15 = 16.15 = 16$ (nearest number) #

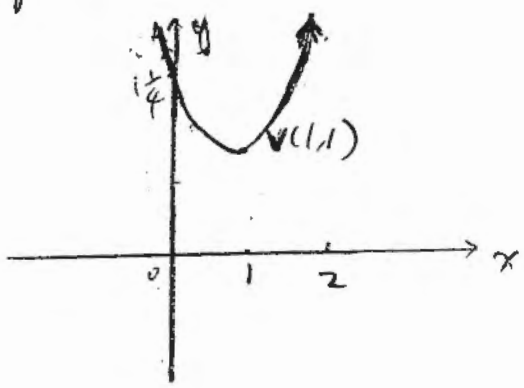
e) $\tan^2\theta = \frac{1}{3}$ $\therefore \tan\theta = \pm\frac{1}{\sqrt{3}}$

$\theta = 30°, 150°, 210°, 330°$

f) i) $4y = (x^2 - 2x + 1) + 4$

$4(y-1) = (x-1)^2$

ii)



Vertex

y-intercept

Shape

g) $S \propto \sqrt{l}$ $\therefore \frac{S}{\sqrt{l}} = k$

(i) $\frac{40}{\sqrt{25}} = \frac{S}{\sqrt{50}}$ $\therefore S = 40\sqrt{\frac{50}{25}} = 40\sqrt{2}$ cm/min

(ii) $\frac{100}{\sqrt{l}} = \frac{40}{\sqrt{25}}$ $l = \left(\frac{100 \times \sqrt{25}}{40}\right)^2 = 156.25$ cm $= 156$ cm (nearest cm) a (i) $2^{4x} = 2^{-7}$

$x = -7/4$

(ii) $\dfrac{3^{-1} \times (2^2 \times 3)^{\frac{1}{4}}}{3^2 \times 2^{\frac{1}{2}}} = \dfrac{3^{-6} \cdot 2^{\frac{1}{2}} \cdot 3^{\frac{1}{4}}}{3^2 \, 2^{\frac{1}{2}}} = 3^{-6+\frac{1}{4}-2} = 3^{-7\frac{3}{4}}$ #

b i) $AB = \sqrt{(4+1)^2 + (-1-2)^2} = \sqrt{25+9} = \sqrt{34}$ #

ii) $y - 2 = \left(\dfrac{2--1}{-1-4}\right)(x+1)$

$y - 2 = \dfrac{3}{-5}(x+1)$

$3x + 3 = -5y + 10$

$3x + 5y - 7 = 0$

iii) $CP = \dfrac{|3(-2) + 5(-4) - 7|}{\sqrt{3^2 + 5^2}} = \dfrac{+33}{\sqrt{34}}$ #

(iv) $D = (-7, -1)$

v) Area of ABCD $= +\dfrac{33}{\sqrt{34}} \times \sqrt{34} = 33$ units^2

c) $x^2 + 4x + y^2 - 2y = 20$

$(x^2 + 4x + 4) + (y^2 - 2y + 1) = 25$

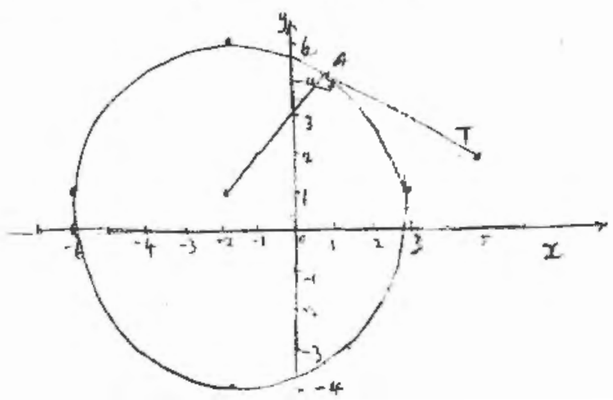
$(x+2)^2 + (y-1)^2 = 5^2$ centre $(-2, 1)$ radius $= 5$

$AT^2 = (5+2)^2 + (2-1)^2 - 5^2$

$= 49 + 1 - 25$

$= 25$

$\therefore AT = \sqrt{25} = 5$ #



P.

Q32 ct'd

c ii) At $x=0$, $2^2 + y^2 - 2y + 1 = 25$

$$y^2 - 2y - 20 = 0$$

$$y = \frac{2 \pm \sqrt{4 - 4(1)(-20)}}{2} = \frac{2 \pm \sqrt{84}}{2} = 1 \pm \sqrt{21}$$

$\therefore$ length of intercepts on y-axis = $\underline{\underline{2\sqrt{21}}}$

d i) To prove $\Delta PQR \,|||\, \Delta MPR$

Proof : PM ∥ QR (given)

$\angle MPR = \angle QRP$ (alternate angle equal, PM ∥ QR)

PM = MR (given)

$\angle MPR = \angle MRP$ (angles opposite equal sides are equal)

Similarly $\angle PQR = \angle PRQ$ as PR = RQ (given)

$\therefore \Delta PQR \,|||\, \Delta MPR$ (equiangular)

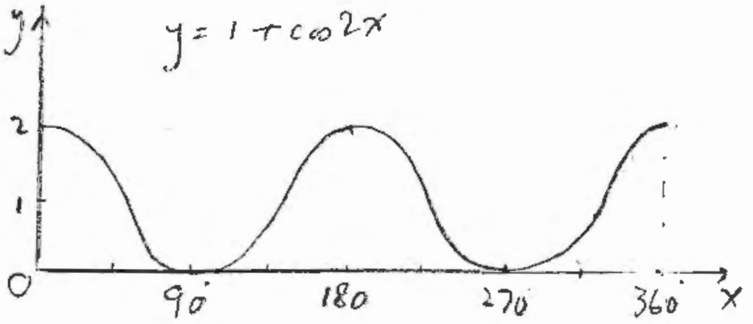
ii) $\frac{PQ}{PM} = \frac{QR}{PR}$ (corresponding sides of similar triangles, $\Delta PQR \,|||\, \Delta MPR$)

$$\frac{3}{2} = \frac{QR}{3}$$

$QR = \frac{3 \times 3}{2} = \underline{\underline{4.5 \text{ cm}}}$

Q 33

a (i) Period $= \frac{360°}{2} = \underline{\underline{180°}}$

ii) 

Amplitude

x intercept

shape

Qu 11a

y \ x	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	6	8	10
3	0	3	6	9	12	15
4	0	4	8	12	16	20
5	0	5	10	15	20	25

b(i) $\frac{11}{36}$

(ii) (4,4), (4,5), (5,4), (5,5)

$\frac{4}{36} = \frac{1}{9}$

(iii) (4,4)+(5,5), (4,5)+(5,5), (5,4)+(5,5), (5,5)+(4,4), (5,5)+(4,5), (5,5)+(5,4), (5,5)+(5,5)

$= \frac{7}{36^2} = \frac{7}{1296}$

c i) The remainder always has a lower degree than the divisor which is quadratic ie $R(x) = ax + b$

ii) $P(x) = (x+1)(x-4)Q(x) + R(x)$

$= (x+1)(x-4)\,Q(x) + ax + b$

$P(4) = 4a + b = -5$

$\therefore R(4) = 4a + b = -5$

iii) $P(-1) = a(-1) + b = 5$

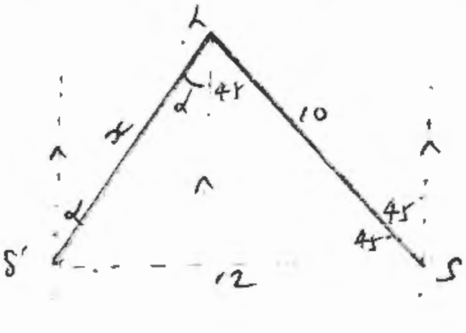
$-a + b = 5$

But $4a + b = -5$

$5a = -10 \quad \therefore a = -2$

$b = 5 + a = 3 \quad \therefore \underline{R(x) = -2x + 3}$

d i) $16 \text{ km} \times \frac{45}{60} = 12 \text{ km}$



$x^2 = 10^2 + 12^2 - 2(10)(12) \times \cos 45$

$x^2 = 74.29$

$x = \underline{8.62 \text{ km}}$

ii) $\frac{x}{\sin 45} = \frac{12}{\sin \theta} \quad \therefore \sin\theta = \frac{12 \sin 45}{8.62} = 0.9844$ (x = 8.62)

$\theta = 79.86 \quad \therefore \alpha = 79.86° - 45° = 34.86°$

$\therefore$ Lighthouse is $\underline{N\ 34°51'\ E}$ of the ship

Q34

a i) $P(x, y) = ?$ $x = \frac{2k-1}{k+1}$; $y = \frac{8k+3}{k+1}$

ii) Eq of line AB :

$$y - 8 = \left(\frac{8-3}{2--1}\right)(x-2)$$

$$y - 8 = \frac{5}{3}(x-2)$$

$$y = \frac{5x}{3} + \frac{14}{3}$$

Pt of intersection of the 2 lines: $5x + 2y - 10 = 0$

$$5x + 2\left(\frac{5x}{3} + \frac{14}{3}\right) - 10 = 0$$

$$\frac{25x}{3} - \frac{2}{3} = 0 \quad \therefore \; x = \frac{2}{25}$$

$$\frac{2k-1}{k+1} = \frac{2}{25} \quad \therefore \; 50k - 25 = 2k + 2$$

$$48k = 27$$

$$k = \frac{9}{16}$$

$\therefore$ Ratio is $9:16$ #

b) $y = \frac{(x+1)^2}{x^2-4}$

Asymptotes $x = \pm 2$

$y = 1$

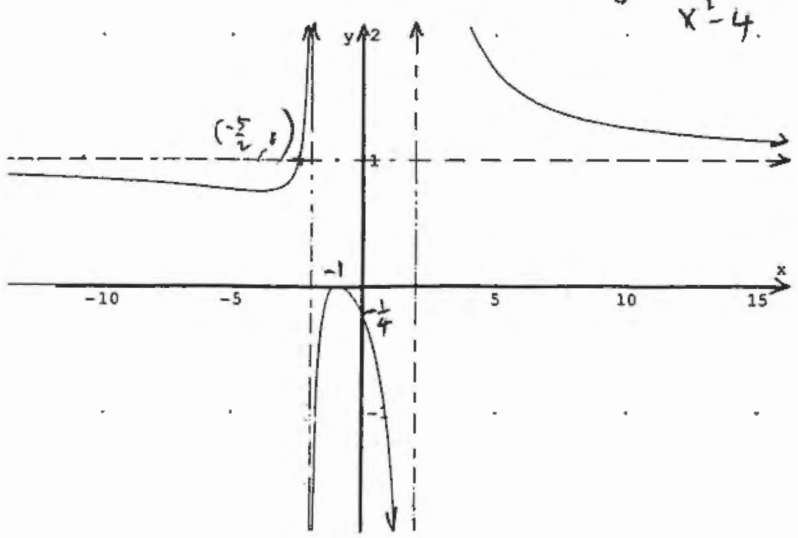
Pt of intersection with $y = 1$

$$\frac{(x+1)^2}{x^2-4} = 1$$

$$x^2 + 2x + 1 = x^2 - 4$$

$$x = -5/2$$

$$y = \frac{(x+1)^2}{x^2-4}.$$



upper 2 branches

lower part
(concave parabola with correct intercepts)
$(-1, 0)$, $(0, -\frac{1}{4})$

Q34

c i) The length of chord AB is a constant, k (given)
Angles subtended at the circumference on the same side of the circle by equal chord are equal ∴ α is a constant.

ii) $\frac{CA}{\sin\theta} = \frac{k}{\sin\alpha}$ ∴ $CA = \frac{k\sin\theta}{\sin\alpha}$

$\frac{CB}{\sin\angle CAB} = \frac{CB}{\sin[180° - (\theta+\alpha)]} = \frac{k}{\sin\alpha}$

But $\sin[180° - (\theta+\alpha)] = \sin(\theta+\alpha)$

∴ $\frac{CB}{\sin(\theta+\alpha)} = \frac{k}{\sin\alpha}$ ∴ $CB = \frac{k\sin(\theta+\alpha)}{\sin\alpha}$

$S = CA + CB = \frac{k}{\sin\alpha}\left[\sin\theta + \sin(\theta+\alpha)\right]$

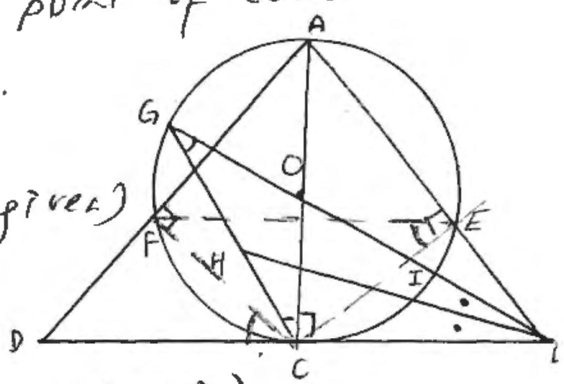
iii) when $\theta = 90 - \frac{\alpha}{2}$

$S = \frac{k}{\sin\alpha}\left[\sin\left(90-\frac{\alpha}{2}\right) + \sin\left(90-\frac{\alpha}{2}+\alpha\right)\right]$

$= \frac{k}{\sin\alpha}\left[\sin\left(90-\frac{\alpha}{2}\right) + \sin\left(180-\left(90-\frac{\alpha}{2}+\alpha\right)\right)\right]$

$= \frac{2k}{\sin\alpha}\sin\left(90-\frac{\alpha}{2}\right) = \frac{2k}{\sin\alpha}\cos\frac{\alpha}{2}$ #

d i) Angle between chord and tangent at point of contact equal angle in alternate segment.



ii) Join FE, FC, EC
AC is the diameter of circle AECFG (given)
$\angle AFC = 90°$ (angle in semi-circle)
similarly $\angle AEC = 90°$
$\angle FDC = 180° - 90° - \angle FCD$ (Angle sum of $\triangle FCD$)
$= 90° - \angle FCD$

$\angle AEF = 90° - \angle FEC$
But $\angle FCD = \angle FEC$ (proved in part i)
∴ $\angle FDC = \angle AEF$
∴ DFEB is a cyclic quadrilateral
(Exterior angle equals to interior opposite angle

Q34 iii) $\angle BHC = 45°$ (given)

~~$\angle GHB = 180° - \angle BHC = 180° - 45° = 135°$ (Angles sum on a straight line ...)~~

$\angle BGC + \angle HBG = 45°$ (exterior angle equals to sum of interior opposite angles in $\triangle HBG$)

$\angle ACB = 90°$ (line from centre is perpendicular to tangent at point of contact)

$\therefore \angle HBC + \angle ACG = 180° - 90° - 45° = 45°$ (angle sum of $\triangle HBC = 180°$)

$GO = CO$ (radii of same circle)

$\angle ACG = \angle BGC$ (angles opposite equal sides are equal)

$\therefore \angle HBC + \angle BGC = 45°$

$\angle B\not{G}C + \angle HBG = \angle HBC + \angle B\not{G}C = 45°$

$\therefore \angle HBG = \angle HBC$