QUESTIONS 1 – 30

Answer the questions on the separate sheet provided.
Mark the appropriate box with a cross, **X.**

1. $25^3 \times 5^2 =$

A. 125^5 B. 125^6 C. 5^{12} D. 5^8

2. The equation $2x + y + 2 + k(x - y - 1) = 0$, where k is an constant, describes the set of lines through the point of intersection of the lines:

A. $y = x + 1$ and $y = 2x + 2$
B. $y = x - 1$ and $y = 2x + 2$
C. $y = x + 1$ and $y = -2x - 2$
D. $y = x - 1$ and $y = -2x - 2$

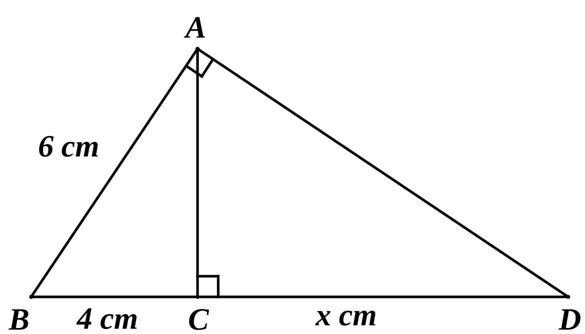
3. The equation of horizontal asymptote for the function $y = 1 + \dfrac{3}{x-2}$ is:

A. $x = 1$ B. $x = 2$ C. $y = 1$ D. $y = 4$

4. The intensity, I, of the light beam from a search lamp varies inversely as the square of the distance, D, away from the search lamp. If the intensity is 100 units when seen from 5 metres away, then the intensity at 8 metres away will be closest to:

A. 4 units B. 32 units C. 39 units D. 63 units

5.



In the above diagram, x =

A. 5 B. $2\sqrt{5}$ C. $3\sqrt{5}$ D. 9

6. An area of $5 \cdot 7\ \text{cm}^2$ is the same as:

A. $0 \cdot 0057\ \text{m}^2$ B. $0 \cdot 057\ \text{m}^2$ C. $57\ \text{mm}^2$ D. $570\ \text{mm}^2$

7. If $a = -2 + \sqrt{3}$ and $b = -2 - \sqrt{3}$, then $a^2 - b^2 =$

A. $8\sqrt{3}$ B. $-8\sqrt{3}$ C. 0 D. -14

8. If $x + y = 63$ and $\frac{x}{y} = 8$, then $x =$

A. 7 B. 56 C. 71 D. 119

9. If $8^{2x} = 16^{1-2x}$, then $x =$

A. $\frac{2}{7}$ B. $\frac{1}{3}$ C. $\frac{1}{4}$ D. $\frac{7}{12}$

10. *ABCD* is a quadrilateral with *AB* = *CD* and *AD* || *BC*. Quadrilateral *ABCD* must be a:

A. Trapezium but it might not be a parallelogram.
B. Parallelogram but it might not be a rhombus.
C. Rhombus but it might not be a square.
D. Square.

11. Which of the following is equivalent to $(25x)^{\frac{3}{2}}$?

A. $\frac{1}{5\sqrt{x^3}}$ B. $25\sqrt[3]{x^2}$ C. $\sqrt{125x^3}$ D. $125x\sqrt{x}$

12. The marks for a test of 10 students were recorded as:

1, 1, 1, 2, 3, 4, 4, 5, 7, 8

However, the student whose mark was recorded as 5, actually scored 6 in the test. When the correct mark is recorded, which of the following will increase?

A. Median B. Mode C. Range D. Interquartile Range

13. If $\sqrt{x+1} + 2 = 0$, then $x =$

A. -5 B. -3 C. 3 D. no real solution

14. If $2x^2 - 12x - 10 \equiv 2(x+b)^2 + c$, then $c =$

A. -28 B. -14 C. 8 D. 19

15. If $a = \frac{b+3}{2b-1}$, then $b =$

A. $\frac{a+3}{2a+1}$ B. $\frac{a-3}{2a+1}$ C. $\frac{a-3}{2a-1}$ D. $\frac{a+3}{2a-1}$

16. The graph of $3x+2y-1=0$ intersects the graph of $y=x^2$ at two points *A* and *B*. The *x*-coordinates of the points *A* and *B* are the solutions of the equation:

A. $2x^2-3x+1=0$ B. $2x^2+3x-1=0$ C. $3x^2-2x+1=0$ D. $3x^2+2x-1=0$

17. A farm produces *m* oranges which are packed into *n* cartons. Each carton contains *p* boxes. The average number of oranges which must be packed into each box is:

A. $\frac{m}{np}$ B. $\frac{mp}{n}$ C. $\frac{np}{m}$ D. $\frac{n}{pm}$

18. The number of points of intersection of the graphs $y=x^3+1$ and $y=\frac{4}{x-2}$ is:

A. 1 B. 2 C. 3 D. 4

19. A solution to the equation $\sin\theta=-\frac{\sqrt{3}}{2}$, is $\theta=$

A. 60° B. 150° C. 210° D. 300°

20. The range for the function $f(x)=1-2\sin x$, is:

A. $-1\le y\le 3$ B. $-2\le y\le 2$ C. $-1\le x\le 3$ D. $-3\le y\le 1$

21. Which of the following statements, about the diagonals of a rectangle, are True:

I. The diagonals bisect each other
II. The diagonals bisect the vertex angles
III. The diagonals intersect at right angles
IV. The diagonals are equal

A. I and II B. I and IV C. II and III D. I, II and IV

22. Which of the following is equivalent to $\frac{1}{1-\cos x}$?

A. $\frac{1+\cos x}{\sin^2 x}$ B. $\frac{\sin^2 x}{1+\cos x}$ C. $\frac{\cos x}{1-\cos^2 x}$ D. $\frac{1+\sin x}{1-\cos^2 x}$

23. The domain for the function $f(x)=\sqrt{25-(x-1)^2}$, is:

A. $0\le x\le 5$ B. $-4\le x\le 6$ C. $x\le -4$ or $x\ge 6$ D. $-6\le x\le 4$

24. In a survey, 45 students agreed that sport should be on Wednesdays. If the probability of a person agreeing was $\frac{9}{10}$, then the number of students in the survey was:

A. 45 B. 50 C. 95 D. 450

25. The following statistics were obtained from the Year 10 examinations.

Subject	Mean	Standard Deviation
Mathematics	55	12
English	66	8

The mark in Mathematics that is equivalent to 78 in English, is:

A. 67 B. 71 C. 73 D. 75

26. The exact value of $\sqrt{\sin 30^{o}} + \cos 45^{o}$ is:

A. $2\sqrt{2}$ B. $\frac{1+2\sqrt{2}}{\sqrt{2}}$ C. $\sqrt{2}$ D. $\frac{\sqrt{3}+1}{\sqrt{2}}$

27. The period for the function $y = \tan 2x$, is:

A. 90^{o} B. 180^{o} C. 360^{o} D. 720^{o}

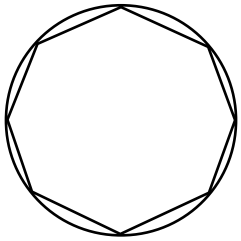
28. The probability that Ben will hit the target with his arrow in archery is 0·8. If he hits the target with his first arrow, then the probability that he will hit it with his next arrow is:

A. 0·16 B. 0·64 C. 0·8 D. 1.0

29. Given $ABCD$ is a parallelogram with vertices $A(-1, -4)$, $B(1, 2)$ and $C(3, 3)$.
The equation of BD is:

A. $x = 1$ B. $y = 2$ C. $y = x + 1$ D. $x - 2y + 3 = 0$

30. A circle is circumscribed about a regular octagon, as shown.



If the area of the octagon is 400 cm^2, then the area of the circle, in cm^2, is:

A. $100\pi\sqrt{2}$ B. $\dfrac{100\pi}{\sqrt{2}}$ C. $\dfrac{200\pi}{\sqrt{3}}$ D. $200\pi\sqrt{3}$

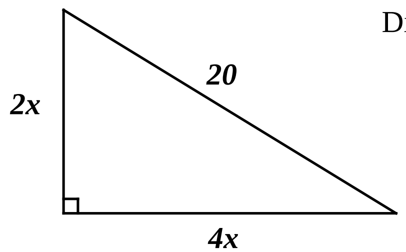
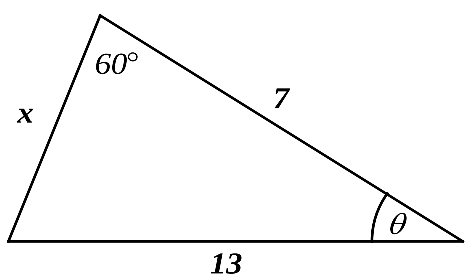
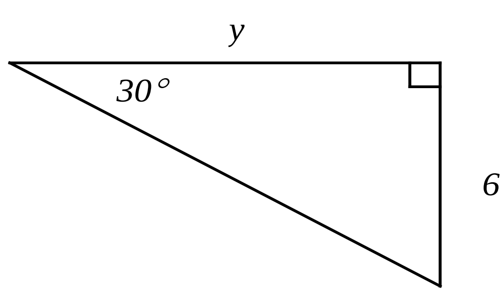
SECTION B (TOTAL 80 MARKS)

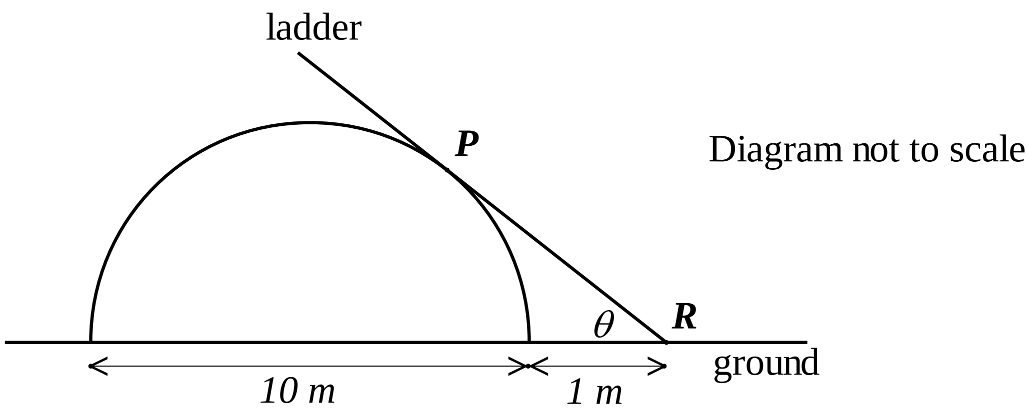
Questions 31 – 34

Use your own writing paper to answer each question.
Clearly write the question number at the top of each page.

Question 31 (20 marks)

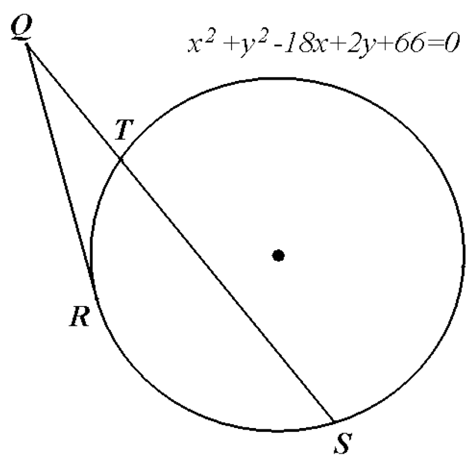
Marks

1.		Solve for x: $x(3x+5)=2(3x+5)$.	2
2.		Factorise: $x-27x^4$.	2
3.		Find the exact value of x :  Diagram not to scale	2
4.		State the domain **and** range for the function $y=\sqrt{x^2-4}$.	2
5.		 Diagram not to scale	
	a)	Find the exact value for x.	2
	b)	Find θ, giving your answer correct to the nearest degree.	2
6.		 Diagram not to scale Find the exact value for y.	2

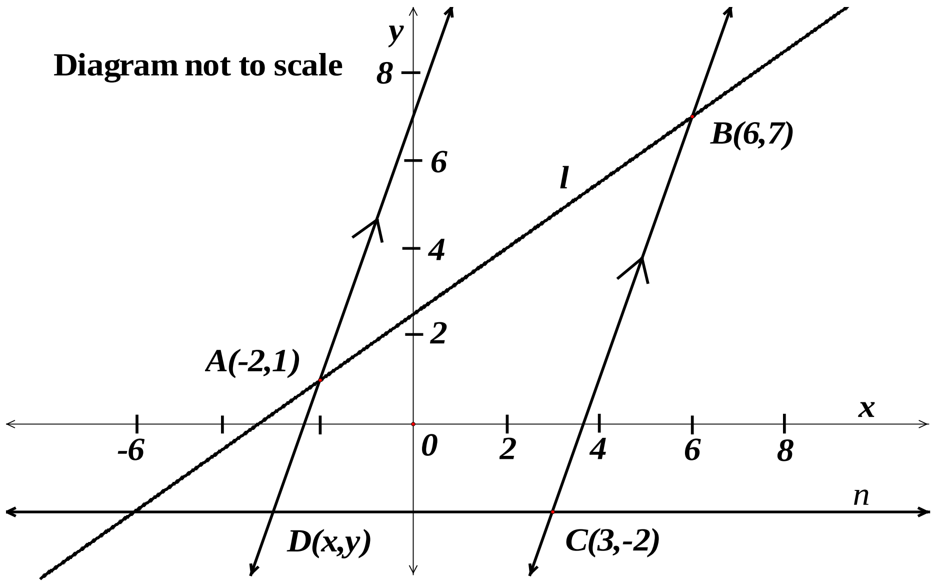
7.		Solve simultaneously: $\begin{aligned} 3x + 4y &= 1 \\ x^2 - 2y^2 &= 1 \end{aligned}$ where x and y are real numbers.	3
8.		A ladder leans against a 10 metre long building which has a semi-circular cross-section, as shown below. The foot of the ladder, R, is 1 metre from the wall of the building and the ladder touches the building at the point P.  	
	a)	Find θ, the angle that the ladder makes with the ground, correct to the nearest degree.	1
	b)	Find the height that the point P is above ground level, giving answer correct to the nearest tenth of a metre.	2

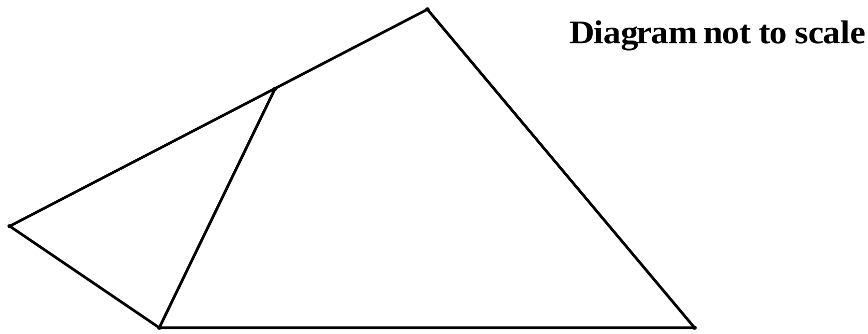
Question 32 (20 marks) START A NEW PAGE

Marks

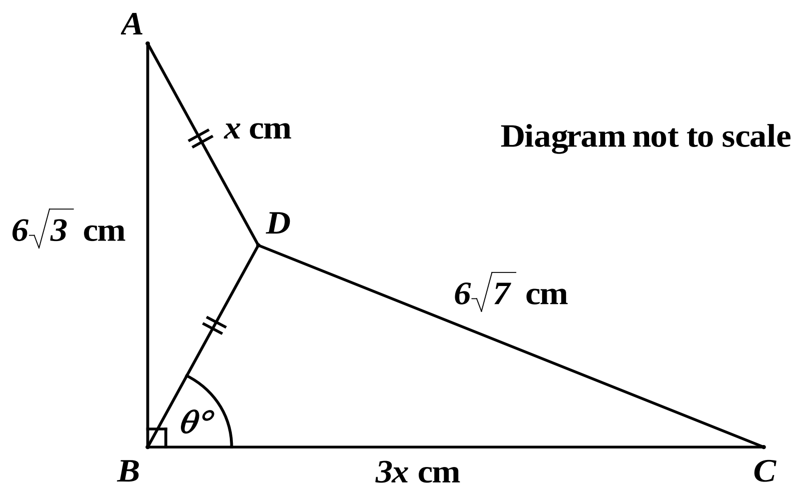
1.		A circle has equation $x^2+y^2-18x+2y+66=0$.	
	a)	By expressing the circle in the form $(x-h)^2+(y-k)^2=r^2$, where *h*, *k* and *r* are constants, state: (i) the co-ordinates of the centre, and (ii) the radius of the circle.	3
	b)	Show that the distance from the centre of the circle to the point $Q(1,5)$ is 10 units.	1
	c)	A tangent is drawn to the circle from *Q*, touching the circle at the point *R*, as indicated on the diagram below. Find the length of the interval *QR*, giving reasons.	2
		 Diagram not to scale	
	d)	Another interval is drawn to the circle from *Q*, intersecting the circle at *S* and *T* as shown above. If *QT* is 6 units, find the length of *ST*, giving reasons.	3
2.		Given $\cos\theta=-\frac{5}{12}$ and $180^o<\theta<270^o$, find the exact value of:	
	a)	sec θ.	1
	b)	tan θ.	2
3.		Solve for θ, $2\sin\theta+\sqrt{3}\cos\theta=0$ in the domain $0^o \le \theta \le 360^o$, giving your answer correct to the nearest minute.	3
4.		Two identical cubes each have faces numbered 0, 1, 2, 3, 4 and 5. The cubes are rolled once and a score is determined by the **product** of the two numbers on the uppermost faces.	
	a)	Draw a table that clearly shows all the possible outcomes.	1
	b)	What is the probability that the score is: (i) 0? (ii) at least 16? (iii) even, if it is known that at least one of the dice shows an odd number.	 1 1 2

Question 33 (20 marks) START A NEW PAGE

			Marks
1.		 The points $A(-2\,,1)$ and $B(6\,,7)$ lie on the line l whose equation is $3x-4y+10=0$. The line n is parallel to the x-axis. $C(3,-2)$ and $D(x,y)$ are two points on line n such that AD is parallel to BC.	
	a)	Find the distance AB.	1
	b)	Find the perpendicular distance from C to the line l.	2
	c)	Find the coordinates of the point D.	2
	d)	Find the area of ΔABC.	1
	e)	Find the area of quadrilateral $ABCD$.	2
2.	a)	Given $P(x)=x^2-2x+1$. Divide $P(x)$ by $x+1$ and express the result in the form $P(x)=(x+1)Q(x)+R(x)$, where $Q(x)$ is the quotient and $R(x)$ is the remainder.	2
	b)	Draw a neat sketch for $y=\dfrac{(x-1)^2}{x+1}$, clearly showing x and y intercepts and any asymptotes.	3
	c)	Hence, solve for x when $\dfrac{(x-1)^2}{x+1}\le 0$.	2

3.			
		In quadrilateral *ABCD* above, $\angle ABC = 40^o$, $\angle BCD = 100^o$ and $\angle CDA = 60^o$. *E* is a point on *BC* such that *AE* bisects $\angle BAD$.	
	a)	Copy the diagram onto your answer sheet and label the diagram with the above information.	1
	b)	Prove *AECD* is a cyclic quadrilateral, giving reasons.	2
	c)	If $\angle BCA = 44^o$, find the value of $\angle AED$, giving reasons.	2

Question 34 (20 marks) START A NEW PAGE

			Marks
1.		If $a = \tan\theta + \cot\theta$ and $b = \sin\theta + \cos\theta$, prove that $a\left(b^2 - 1\right) = 2$.	3
2.		A polynomial $P(x)$ is defined by $P(x) = ax\left(x^2 + b\right)$, where a, b are constants and $a \neq 0$. $P(x)$ has $(x+2)$ as a factor and when $P(x)$ is divided by $(x-3)$ the remainder is 21.	
	a)	Find the values for a and b.	4
	b)	Hence, sketch $y = P(x)$.	2
3.		 Given that $AD = DB = x$ cm, $BC = 3x$ cm, $AB = 6\sqrt{3}$ cm, $CD = 6\sqrt{7}$ cm and $\angle DBC = \theta^{\circ}$, where $\theta > 70$.	
	a)	By using the cosine rule in ΔDBC, show that: $\cos\theta = \dfrac{5x^2 - 126}{3x^2}$.	2
	b)	By considering ΔADB, show that: $\sin\theta = \dfrac{\sqrt{27}}{x}$.	2
	c)	Hence, using parts a) and b) and the identity $\sin^2\theta + \cos^2\theta = 1$, show that $16x^4 - 1\,017x^2 + 15\,876 = 0$.	2
	d)	Find all possible values of x.	3
	e)	Find the value of θ, to the nearest degree.	2

End Of Exam Paper

Section A (30 marks – 1 mark each)

1. D $25^3 \times 5^2 = 5^6 \times 5^2 = 5^8$.

2. D $2x+y+2+k(x-y-1)$ is a line through the intersection of $2x+y+2=0$ and $x-y-1=0$
$\therefore y = -2x-2$ and $y = x-1$

3. C $y = 1 + \frac{3}{x-2}$ as $x \to \pm\infty$ $y \to 1$
$\therefore$ horizontal asymptote $y=1$

4. C $I = \frac{k}{D^2}$ $100 = \frac{k}{5^2}$ $\therefore k = 2500$

$I = \frac{2500}{64} = 39 \cdot 0625$

5. A By similar triangles $\frac{4+x}{6} = \frac{6}{4}$ $\therefore 16+4x=36$
$4x = 20$ $x=5$

6. D $5 \cdot 7 \text{ cm}^2 = 5 \cdot 7 \times 10^2 \text{ mm}^2 = 570 \text{ mm}^2$

7. B $a = -2+\sqrt{3}$ $\therefore a^2 = 4-4\sqrt{3}+3$
$b = -2-\sqrt{3}$ $b^2 = 4+4\sqrt{3}+3$
$\therefore a^2-b^2 = -8\sqrt{3}$

8. B $x+y=63$ $\frac{x}{y} = 8$ $\therefore \frac{x}{63-x} = 8$ $\therefore x = 504-8x$
$\therefore 9x = 504$ $x = 56$

9. C $8^{2x} = 16^{1-2x}$ $\therefore 2^{6x} = 2^{4-8x}$
$6x = 4-8x$ $\therefore 14x = 4$ $\therefore x = \frac{2}{7}$

10. A

AB might not be parallel to CD

11. D $(25x)^{3/2} = (5\sqrt{x})^3 = 125x\sqrt{x}$

12. D Interquartile range changes from $5-1=4$ to $6-1=5$

13. D $\sqrt{x+1}+2=0$ $\therefore \sqrt{x+1} = -2$ but $\sqrt{x+1} > 0$
$\therefore$ no solution

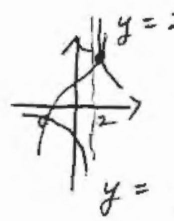
14. A $2x^2-12x-10 = 2(x+b)^2+c$
$2(x^2-6x-5) = 2(x-3)^2-28$ $\therefore c = -28$

15. D $a = \frac{b+3}{2b-1}$ $\therefore 2ab-a = b+3$
$b(2a-1) = a+3$ $\therefore b = \frac{a+3}{2a-1}$

16. B | $3x + 2y - 1 = 0 \quad y = x^2$.
$\therefore\ 3x + 2(x^2) - 1 = 0 \quad \therefore\ 2x^2 + 3x - 1 = 0$

17. C | m oranges $= n \times p \times x$ (x = no in box)
$\therefore\ x = \frac{m}{np}$

18. B | 2 points of intersection



19. D | $\sin\theta = -\frac{\sqrt{3}}{2} \qquad \sin 3\ 0^0 = -\sqrt{3}/2$

20. C | $f(x) = 1 - 2\sin x \qquad$ max value $y = 3$ $\quad$ min value $y = -1$
$\therefore$ range: $-1 \leq y \leq 3$

21. B

22. A | $\frac{1 + \cos x}{\sin^2 x} = \frac{1 + \cos x}{1 - \cos^2 x} = \frac{1 - \cos x}{(1 - \cos x)(1 + \cos x)} = \frac{1}{1 - \cos x}$

23. B | $f(x) = \sqrt{25 - (x-1)^2} \qquad -5 \leq (x-1) \leq 5$
$\therefore\ -4 \leq x \leq 6$

24. B | $\frac{9}{10} = \frac{45}{x} \qquad \therefore x = 50$

25. C | $\frac{78 - 66}{8} = 1.5 \qquad \therefore\ 1.5 \times 12 + 55 = 73$

26. C | $\sqrt{\sin 30} + \cos 45 = \sqrt{\frac{1}{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

27. A | $y = \tan 2x \qquad$ period $= \frac{180^\circ}{2} = 90^\circ$

28. C | 2nd arrow is independent of first arrow

29. A | Diagonals of parallelogram bisect each other. Mid Point of $AC = (1, -\frac{1}{2})$, $B(1, 2)$ $\therefore$
BD equation is $x = 1$

30. A | Area of octagon $= 8 \times \underline{r \times r} \times \sin 45^\circ$

$\therefore\ 400 = 4 \times \frac{1}{\sqrt{2}} r^2$

$\therefore\ r^2 = 100\sqrt{2}$.

$\therefore$ Area of circle $= \pi r^2$
$= \pi \times 100\sqrt{2}$

Section B

Question 31 (20 Marks)

1. $x(3x+5) = 2(3x+5)$

   $x = 2$ or $-\frac{5}{3}$ 1 each

2. $x - 27x^4$

   $= x(1-27x^3)$ 1

   $= x(1-3x)(1+3x+9x^2)$ 1

3. $(2x)^2 + (4x)^2 = 20^2$

   $4x^2 + 16x^2 = 400$ 1

   $20x^2 = 400$

   $x^2 = 20$

   $x = \sqrt{20}$ or $2\sqrt{5}$ 1

4. $x^2 - 4 \geq 0$

   $(x-2)(x+2) \geq 0$

   $x \leq -2$ or $x \geq 2$ (domain) 1

   $y \geq 0$ (Range) 1

5. a) $13^2 = x^2 + 7^2 - 14x\cos 60°$ 1

   $x^2 - 7x - 120 = 0$

   $(x-15)(x+8) = 0$

   $x = 15$ only $(x > 0)$ 1

   b) $\cos\theta = \frac{13^2 + 7^2 - 15^2}{2.13.7}$ 1

   $\theta = 92°$ (nearest degree) 1

6. $\tan 60° = \frac{y}{\sqrt{6}}$ 1

   $\sqrt{3} = \frac{y}{\sqrt{6}}$

   $\sqrt{18}$ or $3\sqrt{2} = y$ 1

7. $3x + 4y = 1$, $x^2 - 2y^2 = 1$ $\quad x = \frac{1-4y}{3}$

   $\left(\frac{1-4y}{3}\right)^2 - 2y^2 = 1$ 1

   $1 - 8y + 16y^2 - 18y^2 = 9$

   $2y^2 + 8y + 8 = 0$

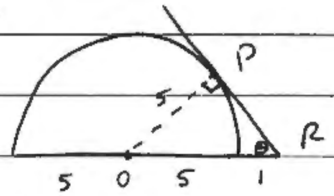
   $y^2 + 4y + 4 = 0$

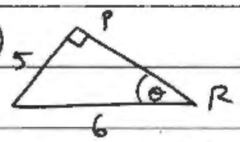
   $(y+2)^2 = 0$

   $y = -2$ 1

   $\therefore x = \frac{1-4(-2)}{3}$

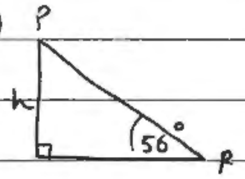
   $x = 3$ 1

8. 

   a) 

   $\sin\theta = \frac{5}{6}$

   $\theta = 56°$ (nearest degree) 1

   b) 

   $PR^2 + 5^2 = 6^2$

   $PR = \sqrt{11}$ 1

   $\therefore \sin 56° = \frac{h}{\sqrt{11}}$

   $h = \sqrt{11}.\sin 56°$

   $h \doteqdot 2.7496$ m

   $\therefore h = 2.7$ m (nearest tenth) 1

NB: alternative solutions are possible which may produce an answer for $h = 2.8$m.

Question 32 (20 Marks)

1. a) $x^2 + y^2 - 18x + 2y + 66 = 0$

$x^2 - 18x + 81 + y^2 + 2y + 1 = -66 + 81 + 1$

$(x-9)^2 + (y+1)^2 = 16$ 1

(i) centre $(9, -1)$ 1

(ii) radius is 4 1

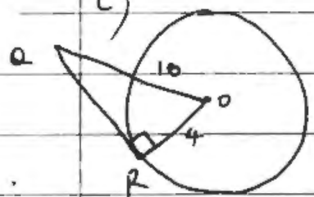
b) Centre $(9,-1)$ $Q(1,5)$

$\therefore$ distance $= \sqrt{(9-1)^2 + (-1-5)^2}$ 1

$= \sqrt{64 + 36}$

$= 10$

c)



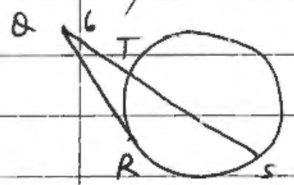
* By Pythagoras' th:

$QR^2 + 4^2 = 10^2$

$QR = \sqrt{84}$ or $2\sqrt{21}$ 1

* $\angle QRO = 90°$ (a line touches a circle at 90° from an external point). 1

d)



1

$QR^2 = SQ \cdot QT$ (The square of the intercept on tangent equals product of the intercept of the secant)

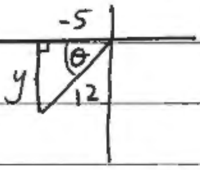
$84 = SQ \cdot 6$ 1

$SQ = 14$ 1

$\therefore ST = SQ - QT = 14 - 6 = \underline{8 \text{ units}}$

2. a) $\sec\theta = -\frac{12}{5}$ 1

b) $\tan\theta = \frac{\sqrt{119}}{5}$ 1



By Pythag:

$y^2 + 25 = 144$

$y^2 = 119$

1 $y = -\sqrt{119}$

3. $2\sin\theta + \sqrt{3}\cos\theta = 0$

$2\sin\theta = -\sqrt{3}\cos\theta$

$\tan\theta = -\frac{\sqrt{3}}{2}$, $0 \le \theta \le 360°$

θ lies in 2nd & 4th quad.

acute θ: $\tan\theta = \frac{\sqrt{3}}{2}$ 1

$\theta = 40°54'$ (nearest minute)

$\therefore \underline{\theta = 139°6', 319°6'}$

1 1

4. a)

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	6	8	10
3	0	3	6	9	12	15
4	0	4	8	12	16	20
5	0	5	10	15	20	25

1

b) (i) $\frac{11}{36}$ 1

(ii) $\frac{4}{36} = \frac{1}{9}$ 1

(iii) $\frac{12}{36-9} = \frac{12}{27}$ ① ① 2

Question 33 (20 Marks)

1 a) $AB = \sqrt{(6--2)^2 + (7-1)^2}$
$= \sqrt{64+36}$
$= \underline{10 \text{ units}}$ 1

b) $d = \dfrac{|3.3 - 2(-4) + 10|}{\sqrt{3^2+4^2}}$ 1
$= \dfrac{27}{5}$ or $5\frac{2}{5}$ 1

c) $D(-3, -2)$ 2

d) $\frac{1}{2} \times 10 \times \frac{27}{5}$
$= \underline{27\ u^2}$ 1

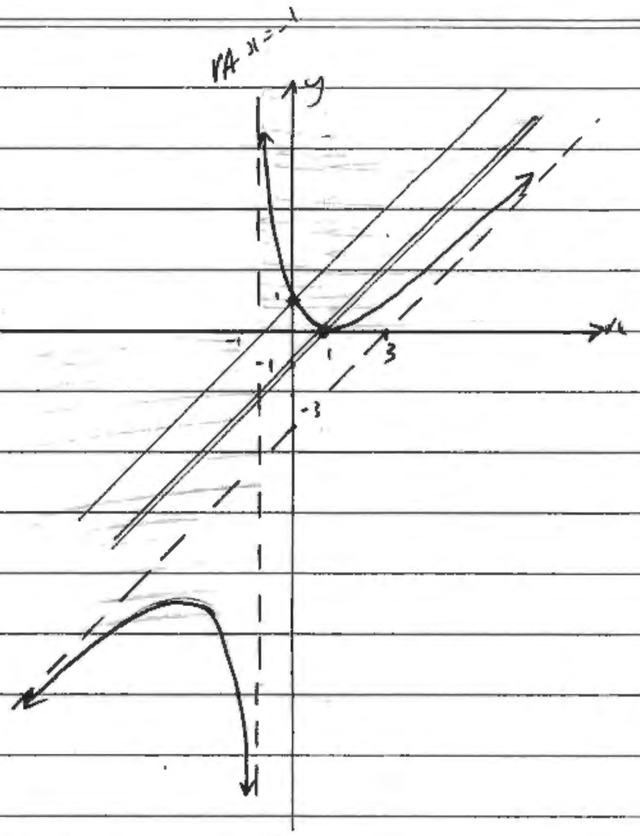
e) Area quad ABCD
$= A_{\Delta ABC} + A_{\Delta ACD}$
$= 27 + \frac{1}{2} \times 6 \times 3$
$= 27 + 9$ 1
$= \underline{36 u^2}$ 1

2 a)
$$\begin{array}{r} x - 3 \\ x+1 \overline{) x^2 - 2x + 1} \\ x^2 + x \\ \hline -3x + 1 \\ -3x - 3 \\ \hline 4 \end{array}$$
1

$\therefore x^2 - 2x + 1 = (x+1)(x-3) + 4$ 1

b) $y = \dfrac{(x-1)^2}{x+1}$

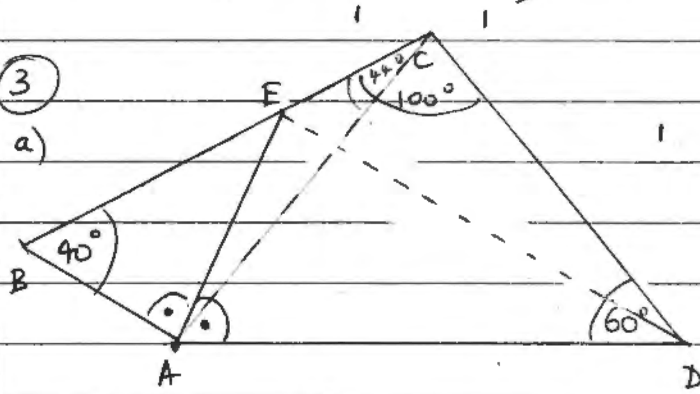
VA : $x = -1$ $\frac{1}{2}$
OA : $y = x - 3$ (See above) $\frac{1}{2}$
x int. $(1, 0)$ $\frac{1}{2}$
y int $(0, 1)$ $\frac{1}{2}$
Shape $\frac{1}{2}$
scale $\frac{1}{2}$
3



c) Solving $\dfrac{(x-1)^2}{x+1} \le 0$

Sol^n : $\{x : x < -1,\ x = 1\}$
1 1

3 a)



1

b) $\angle BAD = 160$ (angle sum of quadrilateral is $360°$) $\frac{1}{2}$

$\angle EAD = \frac{1}{2} \times 160$ (AE bisects $\angle BAD$) $\frac{1}{2}$
$= 80°$

$\therefore$ AECD is cyclic (opposite angles ECD & EAD are supplementary) 1

c) $\angle ACD = 56°$ (by subtraction) 1

$\angle AED = 56°$ (AECD are concyclic points; angles to the circumference of a circle standing on same arc are equal) 1

Question 34 (20 Marks)

1. $LHS = (\tan\theta + \cot\theta)\left((\sin\theta + \cos\theta)^2 - 1\right)$

$= (\tan\theta + \cot\theta)(\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1)$ 1

$= (\tan\theta + \cot\theta)(2\sin\theta\cos\theta)$

$= \left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)(2\sin\theta\cos\theta)$

$= 2\sin^2\theta + 2\cos^2\theta$ 1

$= 2(\sin^2\theta + \cos^2\theta)$ 1

$= 2 \times 1$

$= 2$

$= RHS$

2. $P(x) = ax(x^2 + b)$

a) Now $P(-2) = 0 \implies -2a(4+b) = 0$

$4a + ab = 0$ --- (1) 1

$P(3) = 21 \implies 3a(9+b) = 21$

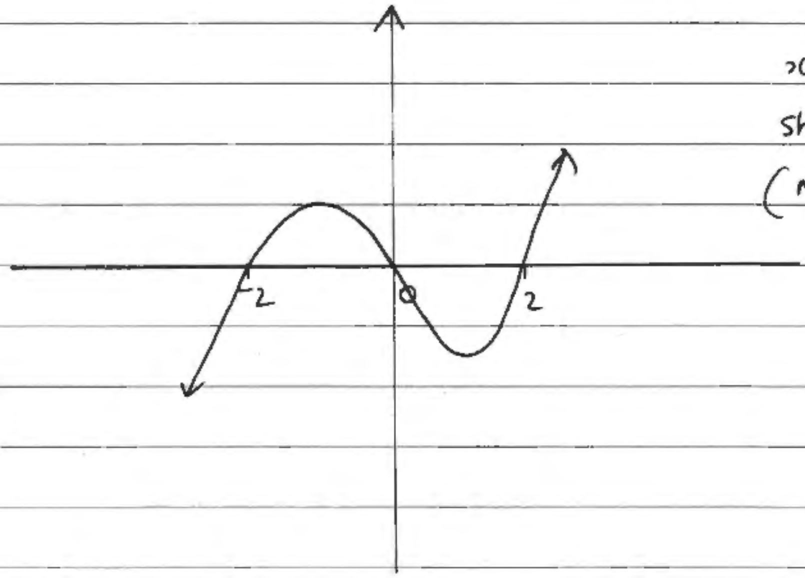
$9a + ab = 7$ --- (2) 1

Solving (1) & (2): $5a = 7$

$a = \frac{7}{5}$ 1

sub. $a = \frac{7}{5}$ into any above: $\underline{b = -4}$ 1

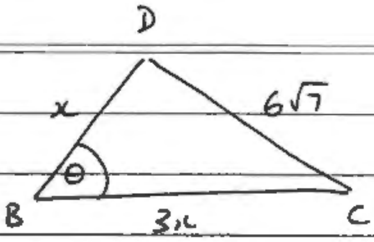
b) $y = \frac{7}{5}x(x^2 - 4) = \frac{7}{5}x(x-2)(x+2)$



x.int. 1

Shape. 1

(No other point required)

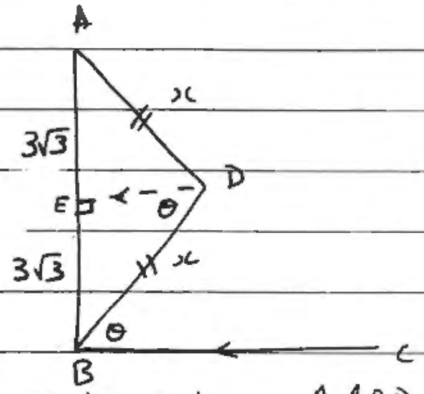


3 a) $\cos\theta = \dfrac{x^2+(3x)^2-(6\sqrt{7})^2}{2 \cdot x \cdot 3x}$ 1

$= \dfrac{x^2+9x^2-252}{6x^2}$

$= \dfrac{10x^2-252}{6x^2}$ 1

$\therefore \cos\theta = \dfrac{5x^2-126}{3x^2}$



b) ΔABD is isoscele so altitude bisects side AB, $EB = 3\sqrt{3}$ 1

$\angle EDB = \theta$ (alternate angles equal, $ED \parallel BC$) 1

$\sin\theta = \dfrac{3\sqrt{3}}{x} = \dfrac{\sqrt{27}}{x}$

NB: Alternative sol^{n}s possible including sine or cosine rule in ΔABD.

c) $\sin^2\theta + \cos^2\theta = 1$ (given)

$\left(\dfrac{5x^2-126}{3x^2}\right)^2 + \left(\dfrac{\sqrt{27}}{x}\right)^2 = 1$

$\dfrac{25x^4-1260x^2+15876}{9x^4} + \dfrac{27}{x^2} = 1$ 1

$25x^4-1260x^2+15876 + 243x^2 = 9x^4$ 1

$16x^4 - 1017x^2 + 15876 = 0$

d) $x^2 = \dfrac{1017 \pm \sqrt{(1017)^2 - 4 \cdot 16 \cdot 15876}}{32}$ 1

$= \dfrac{1017 \pm 135}{32}$

$\therefore x^2 = 36$ or $27\frac{9}{16}$ 1

$\therefore x = 6$ or 5.25 only $(x > 0)$. 1

e) Using $\sin\theta = \dfrac{\sqrt{27}}{x}$

If $x = 6$, $\sin\theta = \dfrac{\sqrt{27}}{6} \Rightarrow \theta = 60^\circ \Rightarrow$ not possible as $\theta > 70^\circ$ 1

If $x = 5.25$, $\sin\theta = \dfrac{\sqrt{27}}{5.25} \Rightarrow \theta \doteqdot 82^\circ \Rightarrow$ only answer possible 1