YEAR 10 YEARLY 2007

SECTION B

QUESTION 31 (20 Marks) START A NEW PAGE **Marks**

(a) Calculate the value of $\dfrac{(2.3+2.8)^2}{\sqrt{2.3\times 7.4}}$ to 2 decimal places. **2**

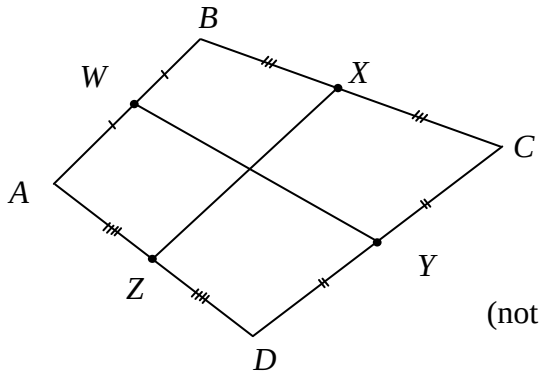
(b) What is the exact value of $\cos 150^\circ$? **2**

(c) Write as a single simplified fraction: $1 - \dfrac{\left(1-\dfrac{1}{a}\right)}{\left(a-\dfrac{1}{a}\right)}$ **3**

(d) Simplify: $\dfrac{1}{1+\tan^2 x}+\sin^2 x$ **3**

(e) Solve for x: $\sqrt{6x+1}=x$ **3**
Give your answer in simplified surd form.

(f) *ABCD* is a quadrilateral with midpoints *W, X, Y, Z* as shown in the diagram below.



(not to scale)

Copy the diagram and prove that the lines joining the midpoints of the opposite sides bisect each other, giving reasons. **4**

(g) Twenty tickets are sold in a raffle. There are 3 prizes.
First prize is \$30, second prize is \$20 and third prize is \$10.
John has bought 2 tickets.

What is the probability that John wins:

(i) The \$10 prize only? **1**

(ii) Exactly \$30 in prizes? **2**

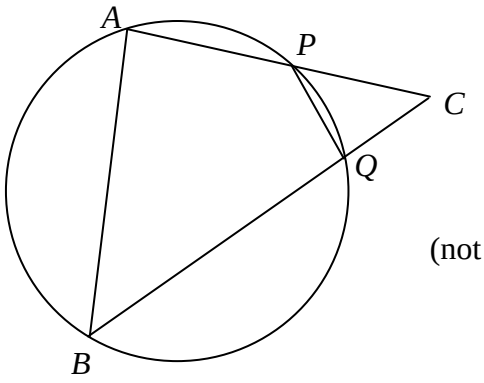
QUESTION 32 (20 Marks) START A NEW PAGE **Marks**

(a) *ABCD* is a rhombus with diagonals *AC* = 12 cm and *BD* = 9 cm. **3**
Construct *ABCD* accurately using a ruler and compasses only.
Show all construction lines.

(b) Find the exact value of $\sin^2 75^\circ$, given that $\sin 15^\circ = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$. **3**

Write your answer as a single simplified fraction.

(c) *ABC* is a triangle. The circle through *A* and *B* cuts *AC* at *P* and *BC* at *Q* so that $\angle BQP = 2 \times \angle CPQ$.

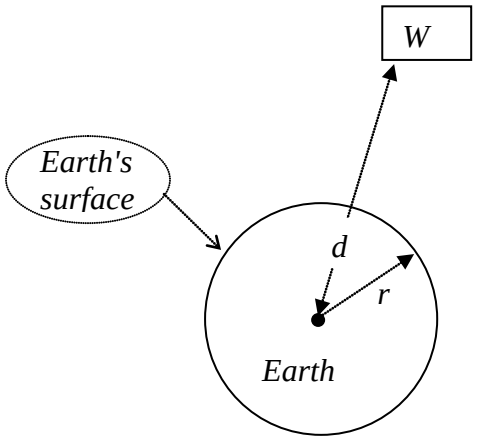


(not to scale)

Copy the diagram and prove that *AB=AC* . **3**

(d) The weight of an object varies inversely as the square of its distance from the centre of the Earth.

(i) Write an equation relating the weight (*W*) of an object to its distance (*d*) from the centre of the Earth. **1**



(ii) A body which weighs 72.0 units on the ground weighs 67.7 units at a height of 200 km above the Earth's surface.
Calculate the radius (*r*) of the earth in km to 3 significant figures. **4**

(e) Sketch the graph of $y = 1 - 2\sin x$ for $0° \le x \le 360°$. **4**
Show all intercepts with the x and y axes.

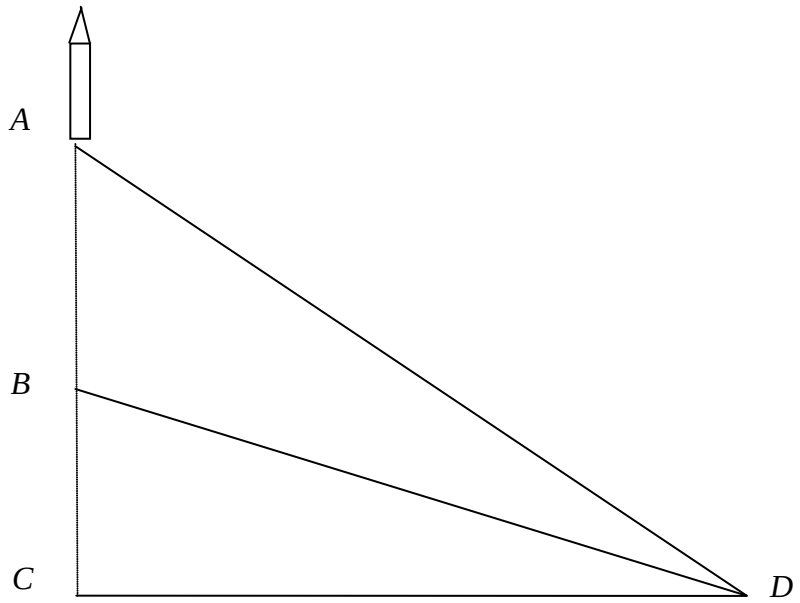
(f) In a particular town 60% of the population are women.
4% of the men and 1% of the women are taller than 180 cm.

(i) What percentage of the town's population is taller than 180 cm? **1**

(ii) If a person is chosen at random and is taller than 180 cm, what is the probability that the person is a woman? **1**

QUESTION 33 (20 Marks) START A NEW PAGE

(a) A rocket is fired vertically upwards from point C.
A monitoring station measures the distance BD to be 3000 m. Two seconds later the distance AD is measured to be 3200 m and $\angle ADB = 4°$.

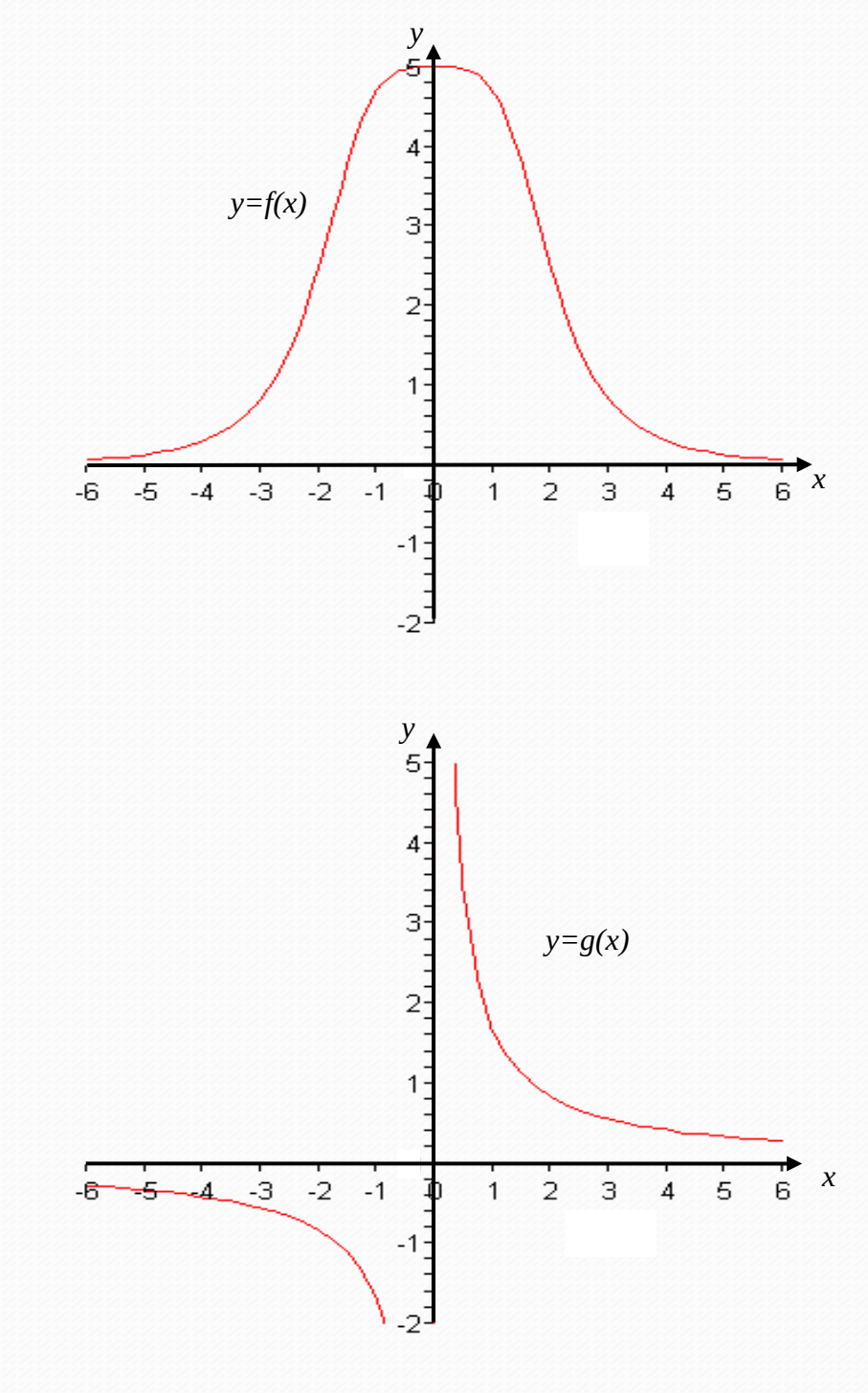


(i) Find the distance that the rocket moves from B to A (to the nearest m). **3**

(ii) Calculate the speed of the rocket in km per hour. **2**

Question 33 continued **Marks**

(b) The following diagrams show the graphs of $y = f(x)$ and $y = g(x)$.



(i) Estimate the value(s) of:

(α) $f(1) - g(1)$. **1**

(β) $f(-2) - g(-2)$. **1**

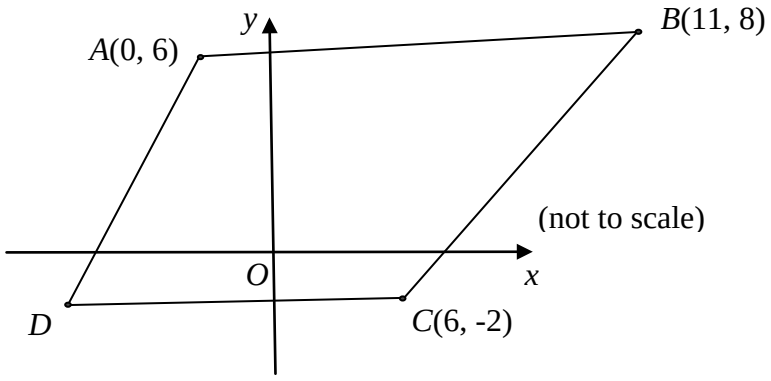
(γ) x where $f(x) = g(x)$. **2**

(ii) Sketch the graph of $y = f(x) - g(x)$. **3**

Label the x-intercepts and significant points.

Question 33 continued **Marks**

(c) The diagram below shows a quadrilateral *ABCD*.



The equations of sides *AD* and *CD* are $13x - 9y + 54 = 0$ and $x - 3y - 12 = 0$ respectively.

(i) Use the equation $13x - 9y + 54 + k(x - 3y - 12) = 0$ to find the equation of the diagonal *BD* in general form. **3**

(ii) Show that *BD* passes through the midpoint of *AC*. **2**

(iii) Show that *AC* is perpendicular to *BD*. **2**

(iv) Hence explain with reasons why *ABCD* is a kite. **1**

QUESTION 34 (20 Marks) START A NEW PAGE

(a) Find all real solutions for the equation $x^4 - x^2 - 12 = 0$ **4**

(b) Sketch the graph of $y = \dfrac{x^2 + x - 2}{x - 3}$. **6**

Clearly label the intercepts with the coordinate axes and all asymptotes.

(c) Solve: $2\sin\theta = \tan\theta$ for $0° \le \theta \le 360°$. **4**

(d) (i) Calculate the shortest distance from the point (1, -5) to the line $2x - y + 3 = 0$ **2**

(ii) Find the equation of the circle with centre (1, -5) that has the line $2x - y + 3 = 0$ as a tangent. **2**

(e) If $\dfrac{a}{x+y} = \dfrac{b}{y-z} = \dfrac{c}{z+x}$ prove that $a = b + c$ **2**

END OF EXAMINATION

YEAR 10 YEARLY 2007 EXAMINATION

ANSWER SHEET

SECTION A: 30 QUESTIONS [1 MARK EACH]

NAME: ANSWERS

CLASS: _____

Mark the appropriate answer with a cross X

1	A	X	C	D
2	A	B	C	X
3	A	X	C	D
4	A	B	X	D
5	A	B	X	D
6	A	B	C	X
7	A	B	C	X
8	X	B	C	D
9	A	X	C	D
10	A	B	X	D
11	X	B	C	D
12	A	B	C	X
13	A	X	C	D
14	A	X	C	D
15	A	B	X	D
16	A	X	C	D
17	A	B	C	X
18	A	B	X	D
19	X	B	C	D
20	X	B	C	D
21	A	B	C	X
22	A	X	C	D
23	A	X	C	D
24	X	B	C	D
25	A	B	C	X
26	A	B	X	D
27	A	B	C	X
28	X	B	C	D
29	X	B	C	D
30	A	X	C	D

Question	Mark
Section A 1 - 30	/30
Section B 31	/20
32	/20
33	/20
34	/20
Total	/110

HAND IN SEPARATELY AT THE END OF EXAM

Year 10 Yearly Solutions 2007

Q31 (a) $\dfrac{(2.3+2.8)^2}{\sqrt{2.3+7.4}} = 6.304644$

$= 6.30$ (2dp) (2) ① correct answer ① correct d.p

(b) $\cos 150^\circ = -\frac{\sqrt{3}}{2}$ (2) ① for $\sqrt{3}/2$ ① for neg sign

(c) $1 - \dfrac{(1-\frac{1}{a})}{(a-\frac{1}{a})} = 1 - \dfrac{(\frac{a-1}{a})}{(\frac{a^2-1}{a})}$

① for fractions on common denominator

$= 1 - \dfrac{(a-1)}{a} \times \dfrac{a}{(a-1)(a+1)}$

① simplifying fractions

$= 1 - \frac{1}{a+1}$

$= \dfrac{a+1-1}{a+1} = \dfrac{a}{a+1}$ (3) ① correct answer.

(d) $\dfrac{1}{1+\tan^2 x} + \sin^2 x \quad \dfrac{1}{\sec^2 x} + \sin^2 x$

② getting to $\cos^2 x$ (various methods)

$= \cos^2 x + \sin^2 x$

$= 1$ (3) ① answer.

(e) $x = \sqrt{6x+1}$

$x^2 = 6x+1$

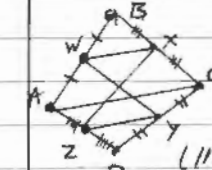
$x^2 - 6x - 1 = 0$ ① quad equation

$x = \dfrac{6 \pm \sqrt{36+4}}{2}$ ① solution

$x = \dfrac{6+\sqrt{40}}{2}$ as $x > 0$

$x = 3+\sqrt{10}$ (3) ① single correct ans

(f)



Construct WX, AC and ZY. In $\triangle ABC$

WX ∥ AC and $WX = \frac{1}{2}AC$ ①

(interval joining midpoints of two sides of a triangle is half the length and parallel to the third side) ①

Similarly in $\triangle ADC$ ZY ∥ AC, $ZY = \frac{1}{2}AC$

$\therefore$ WX ∥ ZY and WX = ZY

$\therefore$ WXZY is a parallelogram (2 sides equal and parallel) ①

$\therefore$ XZ bisects WY (diagonals of parallelogram bisect each other) (4) ①

31 (g) (i) $P(\$10) = P(\text{3rd Prize only})$

$= \frac{18}{20} \times \frac{17}{19} \times \frac{2}{18} = \frac{17}{190}$ (1) ① correct working

(ii) $P(\$30) = P(\text{1st prize}) + P(\text{2nd + 3rd Prize})$ (1)

$= \frac{2}{20} \times \frac{18}{19} \times \frac{17}{18} + \frac{18}{20} \times \frac{2}{19} \times \frac{1}{18}$

$= \frac{17}{190} + \frac{1}{190} = \frac{18}{190}$ (2) ① unsimplified fraction

Q32(a) construction (various methods) (3) ① construction lines ② Accuracy all sides = 7.5 cm.

(b) $\sin^2 75^\circ = 1 - \cos^2 75^\circ$

$= 1 - \sin^2 15^\circ$ ① identity

$= 1 - \left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^2$

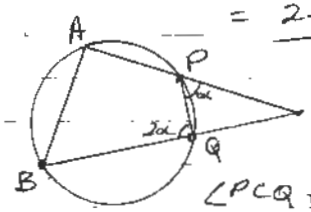
$= 1 - \left(\frac{3 - 2\sqrt{3} + 1}{8}\right)$ ① squaring $\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)$ correctly

$= \frac{8 - (4 - 2\sqrt{3})}{8}$

$= \frac{4 + 2\sqrt{3}}{8}$

$= \frac{2 + \sqrt{3}}{4}$ (3) correct simplified answer

(c)



Let $\angle CPQ = \alpha$

$\therefore \angle BQP = 2\alpha$ (given)

$\angle PCQ + \angle CPQ = \angle BQP$

(exterior angle of ΔPCQ equals sum of two opposite interior angles)

$\therefore \angle PCQ = \alpha$ ①

$\angle ABQ = \angle CPQ = \alpha$

(exterior angle of cyclic quad equals opposite interior angle) ①

$\therefore \angle ABC = \angle ACB = \alpha \quad \therefore AC = AB$

(equal sides opposite equal angles in ΔABC) (3) ①

32(d) (i) $W = \frac{k}{d^2}$ ①

(ii) $72 = \frac{k}{r^2} \qquad 67.7 = \frac{k}{(r+200)^2}$

$\frac{72}{67.7} = \frac{(r+200)^2}{(r)^2}$

$\sqrt{\frac{72}{67.7}} = \frac{r+200}{r}$ as $r > 0$

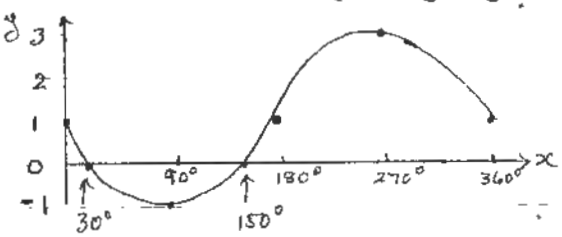
$r + 200 = r \times (1.03126885)$

$200 = 0.3126885r$

$r = 6396.188$

Radius is 6400 (3 sig figs) (4) ① 3 sig figs

32(e)



① y intercept ① amplitude ① x intercepts ① position + shape of graph (etc) (4)

(f) (i) 60% = women 1% of women > 180cm

$\therefore$ 0.6% women > 180 cm

40% = men 4% men > 180cm

$\therefore$ 1.6% men > 180cm

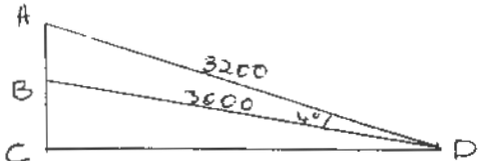
$\therefore$ 2.2% of population > 180cm ①

(ii) $P(\text{women if} > 180\text{cm}) = \frac{\%\ \text{women} > 180\text{cm}}{\%\ \text{popln} > 180\text{cm}}$

$= \frac{0.6}{2.2}$

$= \frac{3}{11}$ ①

Q33 (a) (i)



$AB^2 = (3000)^2 + (3200)^2 - 2 \times 3000 \times 3200 \times \cos 4^\circ$

$= 86770.235$

$AB = 294.5678784$

Dist AB = 295 (nearest m) (3)

(ii) speed = dist/time $= \frac{294.6}{1000} \times \frac{2}{3600}$

$= 530.22$

speed = 530 km/hr (2)

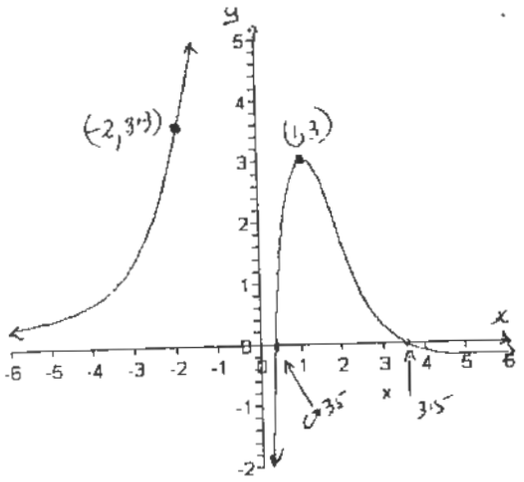
(b) (i) (α) $f(1) - g(1) \approx 4.7 - 1.7 \approx 3.0$ (1)

(β) $f(-2) - g(-2) \approx 2.5 - (-0.8) \approx 3.3$ (1)

(γ) $f(x) = g(x)$ when

$x \approx 3.5$ or $x \approx 0.35$ (2)

(ii)



No deduction for dp's or sig figs in ans.

① use of cos rule

① calc.

① answer.

① speed = D/T

① in km/hr.

accept $2.5 < y < 3.5$

$2.8 < y < 3.8$

$3 < x < 4$

$0 < x < 1$

① Left Branch

① Right Branch for $0 < x < 1$

① Right Branch $x > 1$

no loss of mark if graph not shown to cross x axis at $x = 3.5$ intercept (carried forward error)

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33 (c) (i) $3x - 9y + 54 + k(x - 3y - 12) = 0$

sub (11, 8) $143 - 72 + 54 + k(11 - 24 - 12) = 0$ ①

$125 + k(-25) = 0$

$k = 5$ ①

$13(x) - 9(y) + 54 + 5(x - 3y - 12) = 0$

$18x - 24y - 6 = 0$

$3x - 4y - 1 = 0$ (3) ① simplified eqⁿ

(ii) Mid Point AC $= \left(\frac{0+6}{2}, \frac{6-2}{2}\right)$

$= (3, 2)$ ①

Sub (3, 2) LHS $= 3(3) - 4(2) - 1$

$= 9 - 8 - 1 = 0$

$= RHS$ (2) ①

(iii) $m_{BD} = \frac{-3}{-4} = \frac{3}{4}$

$m_{AC} = \frac{6+2}{0-6} = \frac{-8}{6} = -\frac{4}{3}$ ①

$m_{BD} \times m_{AC} = \frac{3}{4} \times \frac{-4}{3} = -1$ ①

$\therefore$ BD is perp to AC (2)

(iv) BD bisects AC at right angles

$\therefore$ kite as one diagonal bisects the other at right angles. (1) ① reason.

Q34 (a) $x^4 - x^2 - 12 = 0$

$(x^2 - 4)(x^2 + 3) = 0$ ① factorization

$x^2 = 4$ or $x^2 = -3$

$x = \pm 2$ as there is no real soln to $x^2 = -3$ (4) ① reference to $x^2 + 3 \neq 0$ for real x. ② solutions

(b) $y = \frac{x^2 + x - 2}{x - 3}$ VA $x = 3$

$= \frac{(x+2)(x-1)}{(x-3)}$ y int $y = 2/3$

x int $x = -2$

$x = 1$

34(b) Oblique Asymptote : $y = x+4$

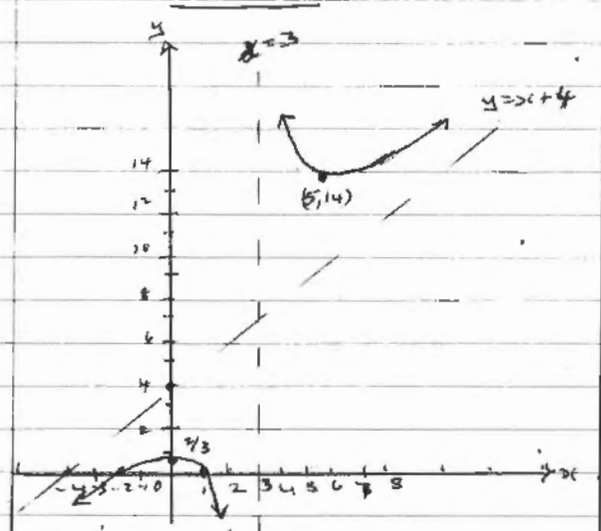
$$x-3 \overline{)\, x^2+x-2}$$

$x+4$

x^2-3x

$4x-2$

$4x-12$



① vert ast
① oblique ast
② x intercepts
① y intercepts
① shape/axes/scale and position of top branch.
⑥

(c) $2\sin\theta = \tan\theta$

$2\sin\theta = \frac{\sin\theta}{\cos\theta}$

$2\sin\theta\cos\theta = \sin\theta \quad (\cos\theta \neq 0)$ ①

$\sin\theta(2\cos\theta - 1) = 0$

$\sin\theta = 0$ or $\cos\theta = \frac{1}{2}$ ①

$\theta = 0°, 180°, 360°, 60°, 300°$ ④ ② solutions

(d) (i) $d = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}} = \frac{2(1)-1(-5)+3}{\sqrt{2^2+1^2}}$ ① correct formula + substitution

$= \frac{10}{\sqrt{5}} = 2\sqrt{5}$. ② ① distance

(ii) circle

$(x-1)^2+(y+5)^2 = 20$ ② ① centre ① (radius)^2

(e) $\frac{a}{x+y} = \frac{b}{y-z} = \frac{c}{z+x}$ ①

$b = \frac{a(y-z)}{x+y} \qquad c = \frac{a(z+x)}{x+y}$.

$b+c = \frac{a(y-z)}{x+y} + \frac{a(z+x)}{x+y}$

$= \frac{a(y-z+z+x)}{(x+y)} \quad = \frac{a(y+x)}{(x+y)}$

$\therefore a = b+c$ ② ① for complete proof.