James Ruse 2008 Year 10 Final Examination

SECTION A 30 Marks (1 mark each)

1. The expression $(a-b)^2-(a^2-b^2)$ can be simplified to

A $-2b^2$ B $2b^2$ C $2b(b-a)$ D $2b(a-b)$

2. The value of $\sqrt{\dfrac{2.8+1.4}{6.2\times 0.02}}$ rounded to 2 decimal places is

A 5.82 B 1.67 C 0.12 D 3.75

3. The solution to the equation $(x^2+8x+16)(x^3-9x)=0$ is $x=$

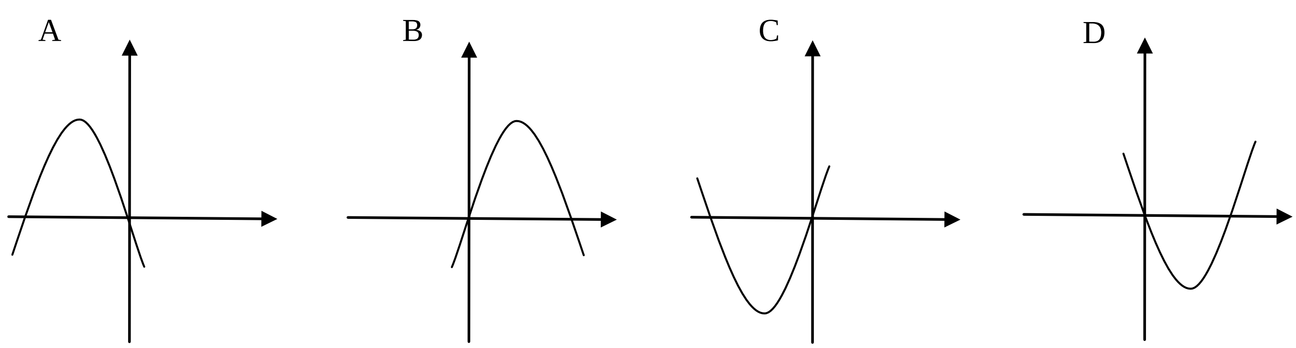
A -4, -3, 0 or 3 B -4, -3, 0, 3 or 4 C -4, -3, 1, 3 or 4 D -4, 0, 4 or 9

4. If $\cos x^{\circ} = 2\cos 60^{\circ}$, then a solution for x is

A 0° B 60° C 90° D 120°

5. Which of the following graphs has an equation of the form

$y = ax^2 + bx,$ where $a<0$ and $b>0$?



6. The statement *"Twelve more than half a number n is five less than the square of the number"* may be represented by

A $\dfrac{n}{2}+12=n^2-5$ B $\dfrac{n}{2}+12=(n-5)^2$

C $\dfrac{n+12}{2}=n^2-5$ D $\dfrac{n+12}{2}=(n-5)^2$

7. The set of unequal positive scores *a, b, c, d, e, f* is listed in order of size, with *a* being the smallest. If *a* is decreased by 10% and *f* is increased by 10% then the

A Mean and the median both increase.
B Mean and the median remain unchanged.
C Mean increases but the median is unchanged.
D Mean is unchanged but the median increases.

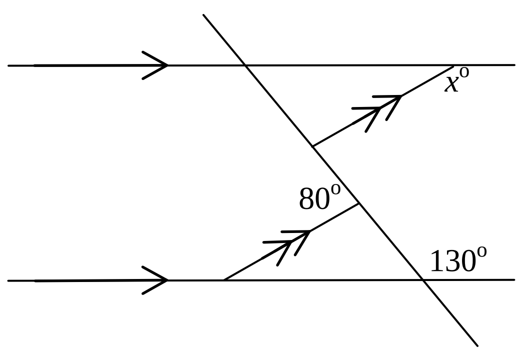
8. If $x^{\circ} + y^{\circ} = 90^{\circ}$ $\left(0 < x < 90,\ 0 < y < 90,\ x \neq y\right)$ then

A $\sin x^{\circ} + \sin y^{\circ} = 1$
B $\sin^2 x^{\circ} + \cos^2 y^{\circ} = 1$
C $\cos(x^{\circ} + y^{\circ}) = 1$
D $\sin x^{\circ} = \cos y^{\circ}$

9. The solution set for $2 - x \leq \frac{1}{2}\left(5 - 2x\right)$ is

A $\{x : x \geq 0,\ x \in R\}$
B $\left\{x : x \geq -\frac{1}{4},\ x \in R\right\}$
C $\{\ \}$
D $\{x \in R\}$

10. In the diagram below the value of *x* is



not to scale

A 100
B 130
C 140
D 150

11. The relative frequency of the letter **S** in the statement **MATHS IS FUN** is

A 2
B 0.2
C $\frac{2}{9}$
D 5 and 7

12. For the relation $S = \{(1,2), (2,3), (3,4), (4,5)\}$, the domain of S is

A {positive integers}
B {1, 2, 3, 4}
C {1, 2, 3, 4, 5}
D {2, 3, 4, 5}

13. If $M = \sqrt{\dfrac{N}{2L^2}}$ and $L > 0$ then $L =$

A $\sqrt{\dfrac{N}{2M}}$ B $\dfrac{1}{M}\sqrt{\dfrac{N}{2}}$ C $\dfrac{\sqrt{2N}}{M}$ D $\dfrac{2}{M}\sqrt{N}$

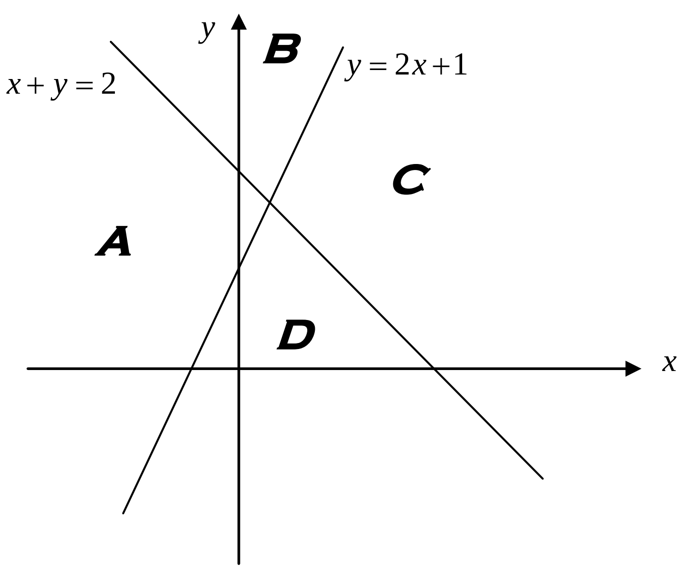
14. The experimental data obtained by a laboratory test are recorded in the table below.

P	5	10	25	30	40
Q	4	16	100	144	256

What is the relationship between *P* and *Q*?

A $P \propto Q$ B $P \propto Q^2$ C $P \propto \sqrt{Q}$ D $P \propto \dfrac{1}{Q^2}$

15. The region representing the solution to $\{(x, y) : y \geq 2x + 1\} \cap \{(x, y) : y + x \leq 2\}$ is



not to scale

A ***A*** B ***B*** C ***C*** D ***D***

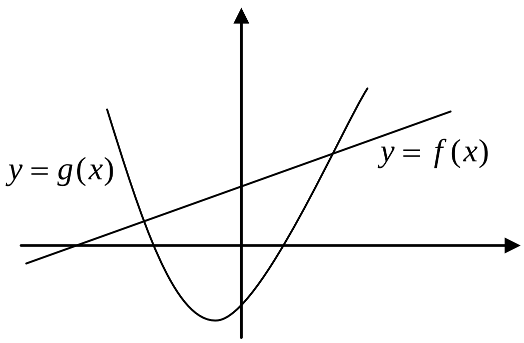
16. The graphs of $y = 3 + 4x - x^2$ and $y = K$ have only one point in common.
The value of K is

A -1 B 1 C 4 D 7

17. $\sin 120^{o} + \cos 210^{o} =$

A $\sqrt{3}$ B $\frac{\sqrt{3}}{2} - \frac{1}{2}$ C 0 D $\frac{1-\sqrt{2}}{2}$

18. The following graphs represent functions $y = f(x)$ and $y = g(x)$.



The equation $f(x) = g(x)$ has

A Two positive solutions.

B Two positive solutions and one negative solution.

C Two negative solutions.

D One positive and one negative solution.

19. Which of the following equations describes a circle?

A $x^2 + 4x + y^2 + 9 = 0$

B $x^2 + 4x + y^2 - 9 = 0$

C $x^2 + 4x - y^2 + 9 = 0$

D $x^2 - 4y + y^2 + 9 = 0$

20. The time taken (T weeks) to build a road is directly proportional to the length of the road (L metres) but inversely proportional to the number of men (M) working on it. Which of the following formulae is correct where k and c are constants?

A $T = kLM$ B $T = k\frac{M}{L}$ C $T = kL + cM$ D $T = k\frac{L}{M}$

21. $\frac{x - y^{-1}}{x^{-1} - y} =$

A $\frac{-x}{y}$ B $\frac{y}{x}$ C $\frac{x}{y}$ D $\frac{-y}{x}$

22. The same class sat for three tests in English, Mathematics and Science.
Jill's results are shown in the table below.

SUBJECT	**CLASS MEAN**	**CLASS STANDARD DEVIATION**	**JILL'S MARK**
ENGLISH	65	10	80
MATHEMATICS	70	5	80
SCIENCE	55	20	85

Compared to the rest of the class Jill performed better in

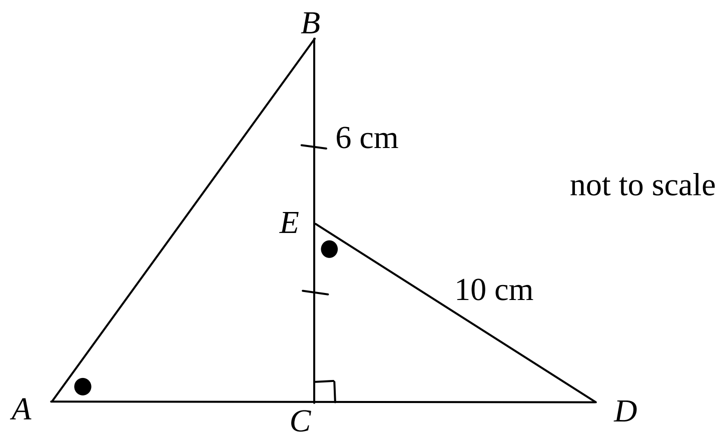
A Mathematics than English.

B Science than English.

C Science than Mathematics.

D English than Science.

23. In the diagram below $BE = CE = 6\text{ cm}$ and $\angle BAC = \angle CED$.



The length of AD is

A 9 cm

B 17 cm

C 20 cm

D 24 cm

24. A new model of a car costs \$$P$ each to manufacture. It is sold for \$$S$. So far a total of \$$W$ has been spent on the advertising campaign and N cars have been sold. The percentage profit overall so far is

A $\dfrac{NS}{W + NP} \times 100$

B $\dfrac{NS - 1}{W - NP} \times 100$

C $\dfrac{NS - W - NP}{W - NP} \times 100$

D $\left(\dfrac{NS}{W + NP} - 1\right) \times 100$

25. The graph of the function $y = b^{-x}$ passes through the point $\left(2, \frac{1}{4}\right)$. The value of b is

A $\frac{1}{4}$ B $\frac{1}{2}$ C 2 D 4

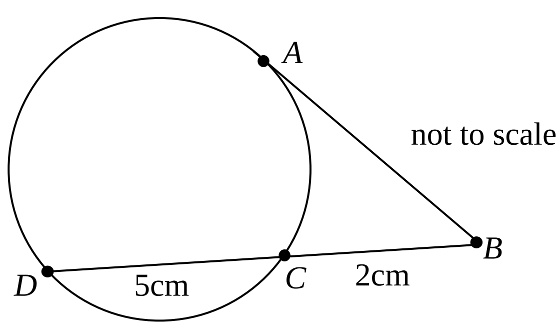
26. The number of solutions to the pair of simultaneous equations $(x+2)^2 + (y-3)^2 = 4$ and $2x - y - 2 = 0$ is

A 0 B 1 C 2 D 4

27. $\dfrac{\left(2^{-x} \times 4^{2x}\right)}{16^{\frac{-x}{4}}} =$

A 2^{-2x^2} B 2^{2x^2} C 2^{-4x} D 2^{4x}

28. AB is a tangent to a circle as shown in the diagram below. BC = 2 cm and CD = 5 cm.



The length of AB in centimetres is

A $\sqrt{10}$ B $\sqrt{14}$ C $\sqrt{20}$ D $\sqrt{35}$

29. The equations of two lines are $y = 3x + 5$ and $6x - 2y + 7 = 0$.
The two lines are:

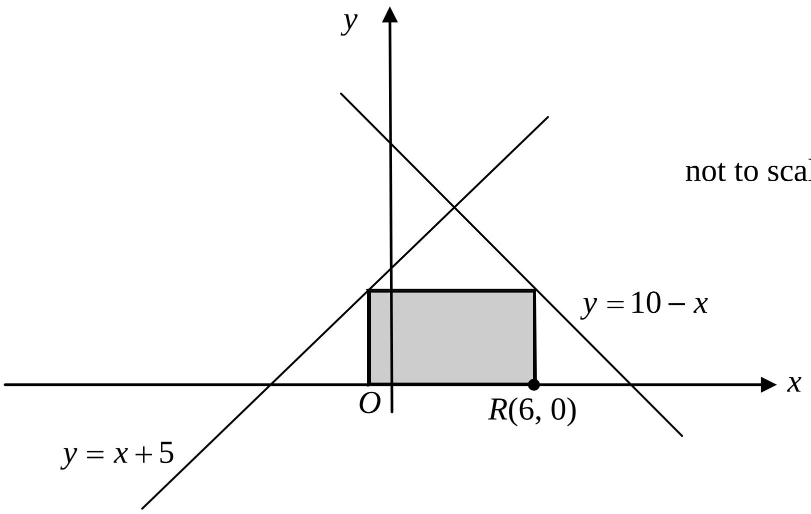
A Perpendicular to each other.

B Parallel but not the same line.

C The same line.

D Neither parallel nor perpendicular.

30. *R* is the point with coordinates (6, 0).



The area of the shaded rectangle in square units is

A 20

B 24

C 28

D 60

END OF SECTION A

QUESTION 31 (20 Marks) START A NEW PAGE **Marks**

a) If $\sqrt{27}+\sqrt{12}=\sqrt{x}$, find the value of x, showing intermediate working. **2**

b) Find all solutions, for $0° \le x° \le 360°$, of $2\sin^2 x + 15\cos x - 9 = 0$. **4**

c) A triangle with two adjacent sides of length 12cm and 14 cm has an area of 70cm^2. What, to the nearest minute, is the angle between the two sides? **2**

d) Find rational numbers a and b if $\dfrac{\sqrt{3}-4}{2+3\sqrt{3}} = a + b\sqrt{3}$. **3**

e) Draw a neat sketch of $y = 2 - \cos 2x°$ for $0° \le x° \le 360°$. **4**

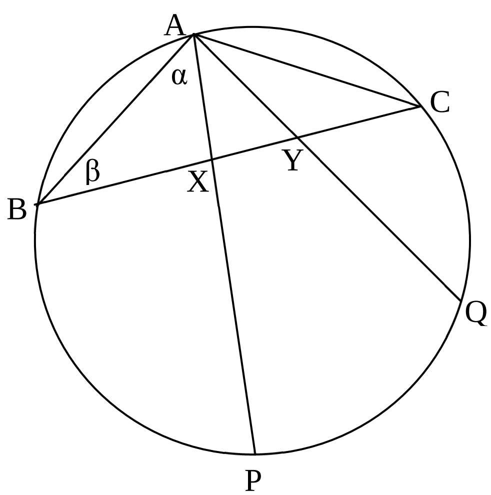
f) Let *ABPQC* be a circle such that *AB*=*AC*, *AP* meets *BC* at *X* and *AQ* meets *BC* at *Y*, as shown in the diagram. Let $\angle BAP = \alpha$ and $\angle ABC = \beta$.

i) Copy the diagram and state why $\angle AXC = \alpha + \beta$. **1**

ii) State why $\angle BQP = \alpha$. **1**

iii) Prove that $\angle BQA = \beta$. **2**

iv) State why *PQYX* is a cyclic quadrilateral. **1**



QUESTION 32 (20 Marks) START A NEW PAGE **Marks**

a) A new car, valued at \$20000, loses 10% of its value on first leaving the car yard and then depreciates by 5% each year. What is the value, to the nearest dollar, of the car after 3 years ? **2**

b) A ship *A*, sailing in a straight line with constant speed, is 10 nautical miles SW of a harbour *H* from which ship *B* is just leaving. *B* sails for two hours at 8 knots (8 nautical miles/hour) in a direction 105°T at which time ships *A* and *B* collide.

i) Draw a diagram with this information shown on it. **2**

ii) Show that the distance travelled by *A* in the two hours is 22.7 n.m.(1DP) **2**

iii) Find the bearing (to the nearest degree) on which ship *A* was travelling? **2**

(Question 32 continued on the next page)

QUESTION 32 (continued)

c) The points A,B and C have coordinates (2,2), (1,10) and (8,6) respectively. The angle between the line AC (extended if necessary) and the x axis is θ.

i)	Draw the points A, B and C on a suitable diagram and find the gradient of the line AC.	**2**
ii)	Calculate the size of angle θ to the nearest minute.	**1**
iii)	Find the equation of the line AC.	**1**
iv)	Find the coordinates of D, the midpoint of AC.	**1**
v)	Show that AC is perpendicular to BD.	**2**
vi)	Find the area of triangle ABC.	**3**
vii)	Write down the coordinates of a point E such that $ABCE$ is a rhombus.	**2**

QUESTION 33 (20 Marks) START A NEW PAGE **Marks**

a) Find the values of x and y if $3^{x+y} = 27$ and $4^{x-y} = 8$ simultaneously. **3**

b) If $\sin\theta = 8/17$, find two possible values of $\tan\theta + \sec\theta$. **3**

c) i) Use long division to divide $P(x) = 3x^3 - 2x^2 - 5x - 1$ by $D(x) = x^2 + 1$ and express your answer in the form

$$P(x) = D(x)Q(x) + R(x)$$ **3**

where *R(x)* is the remainder polynomial.

ii) *F(x)* is a polynomial which gives a remainder of 7 when it is divided by $(x-2)$ and a remainder of 3 when it is divided by $(x+2)$. Find the remainder polynomial when *F(x)* is divided by $x^2 - 4$. **3**

d) Tangents from the origin O touch the circle $(x-4\sqrt{3})^2 + (y-4)^2 = 16$ at two points.

i)	Prove that the x axis is a tangent to the circle and write down the coordinates of A, the point of contact of the circle with the x axis.	**3**
ii)	The other tangent from O touches the circle at B. Show that the angle AOB is 60° and hence that triangle OAB is equilateral. (Any congruences used must be clearly stated but need not be proved.)	**3**
iii)	P is a point on the major arc AB of the circle. Find the size of the angle APB.	**2**

(Question 34 is on the next page)

QUESTION 34 (20 Marks) START A NEW PAGE **Marks**

a) Find the minimum value of $2x^2 - 5x + 3$. **2**

b) If A and B are the points (1,2) and (5,6) respectively, find the point C which divides the interval AB **externally** in the ratio 3:1. Show your answer on a sketch, illustrating the meaning of external division in this case. **3**

c) Prove that $\dfrac{1-\tan^2\theta}{1+\tan^2\theta} = 2\cos^2\theta - 1$ **3**

d) Consider the curve of $y = \dfrac{(1-x)(2x+5)}{(x+1)(x-5)}$.

i) Write down the equation of the horizontal asymptote to the curve and determine any point(s) where the curve crosses this asymptote. **3**

ii) Sketch the curve, clearly showing any asymptotes, intercepts and other point(s) of interest. **4**

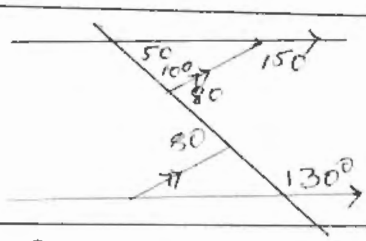
iii) How many solutions are there to the equation $\dfrac{(1-x)(2x+5)}{(x+1)(x-5)} = 2^{-x}$?

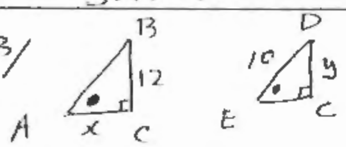
Explain your answer with reference to your sketch. **2**

e) Show that, for n=1,2,3…, the number $n^4 + 2n^3 + 2n^2 + 2n + 1$ can never be the square of an integer. **3**

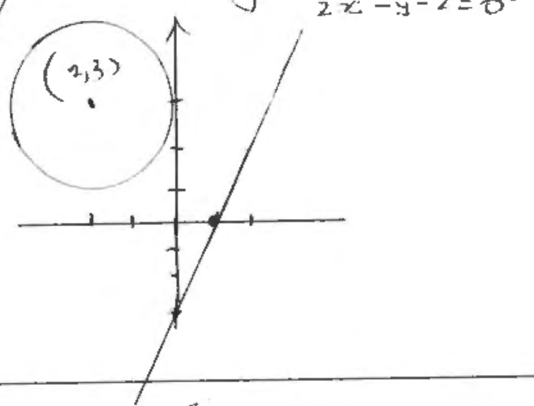
END OF EXAMINATION

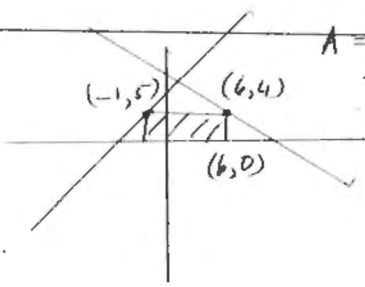
Section A

	$a^2-2ab+b^2-a^2+b^2$ $= 2b^2-2ab = 2b(b-a)$	C
	$5.81987\,6952 = 5.8$ (2dp)	A
	$x(x+4)^2(x-3)(x+3)=0$ $x = 0, -4, 3,$ or -3	A
	$\cos x = 2\times\frac{1}{2} = 1$ $\therefore x = 0$	A
/	$y = x(ax+b)$ concave down $a<0$ x intercept $x=0$ $x = -\frac{b}{a} > 0$ as $a<0$, $b>0$	B
/	$\frac{n}{2} + 12 = n^2 - 5$	A
/	Number of scores is unchanged ∴ median stays same as $a < f$ ∴ $10\% a < 10\% f$ ∴ increase > decrease ∴ mean increases	C
3/	$x^\circ + y^\circ = 90^\circ$ ∴ $y = 90 - x$ ∴ $\cos y = \sin x$	D
/	$2 - x \le \frac{5}{2} - x$ holds for all x	D
0/		D
11/	$\frac{2}{10} = 0.2$	B

12/	Domain = {x values} = {1, 2, 3, 4}	B
13/	$M^2 = \frac{N}{2L^2}$ ... $= N$ $L = \frac{1}{M}\sqrt{\frac{N}{2}}$	B
14/	$Q = \left(\frac{P}{5}\right)^2 \times 4$	C
15/	—	A
16/	 y value of vertex $x = -\frac{b}{2a} = \frac{-4}{-2} = 2$ $\therefore y = 3 + 4(2) - 2^2$ $= 7$	D
17/	$\frac{\sqrt{3}}{2} + \left(-\frac{\sqrt{3}}{2}\right)$	C
18/	—	D
19/	$x^2 + 4x + 4 = 9 + 4$ $(x+2)^2 + y^2 = 13$	B
20/	$T \propto \frac{L}{M}$ $\therefore T = \frac{kL}{M}$	D
21/	$\frac{x - \frac{1}{y}}{\frac{1}{x} - y} = \frac{\frac{xy-1}{y}}{\frac{1-xy}{x}} = \frac{-x}{y}$	A
22/	English +1.5 SD Maths +2.0 SD Science +1.5 SD	A
23/	 ∴ $y = 8$ $\frac{x}{6} = \frac{12}{8}$ $x = 9$ ∴ AD = 17	B
24/	% Profit = $\frac{\text{income} - \text{costs}}{\text{cost price}} \times 100$ $= \left(\frac{NS - W - NP}{W + NP}\right) \times 100$ $= \left[\frac{NS}{W+NP} - 1\right] \times 100$	D
25/	$\frac{1}{4} = b^{-2}$ $\frac{1}{4} = \frac{1}{b^2}$ ∴ $b = 2$	C

26/	Solve graphically.	A
27/	$\dfrac{2^{-x} \times 2^{4x}}{2^{-x}} = 2^{4x}$	D
28/	$AB^2 = 2 \times 7$ $AB = \sqrt{14}$	B
29/	$y = 3x + 7/2$ $m_1 = 3 \quad m_2 = 3$	B
30/	$A = 7 \times 4$ $= 28$	C





YR10 YEARLY 2008

31 a) $\sqrt{27}+\sqrt{12} = 3\sqrt{3}+2\sqrt{3}$

$= 5\sqrt{3}$

$= \sqrt{75}$

$\therefore \underline{x = 75}$ (2)

b) $2(1-\cos^2 x)+15\cos x - 9 = 0$

$2\cos^2 x - 15\cos x + 7 = 0$

$(2\cos x - 1)(\cos x - 7) = 0$

$\cos x = 7$ (impossible) or $\frac{1}{2}$

$\therefore \underline{x^\circ = 60^\circ \text{ or } 300^\circ}$ (4)

c) Area $= \frac{1}{2}ab\sin C$

$\therefore 70 = \frac{1}{2}\,12.\,14.\,\sin C$

$\therefore \sin C = \frac{10}{12} = \frac{5}{6}$

$C = 56^\circ 27'$ or $123^\circ 33'$ (2)

$\therefore \underline{C = 56^\circ 27'}$ (acute)

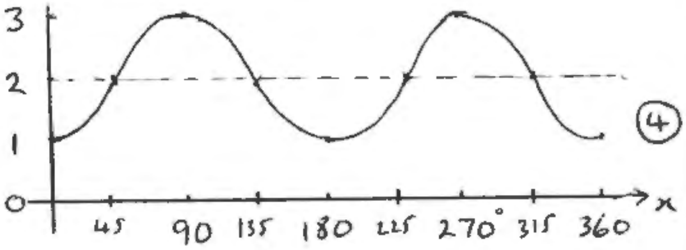
d) $\frac{(\sqrt{3}-4)(2-3\sqrt{3})}{(2+3\sqrt{3})(2-3\sqrt{3})} = \frac{14\sqrt{3}-17}{4-27}$

$= +\frac{17}{23} - \frac{14\sqrt{3}}{23}$

$= a + b\sqrt{3}$

if $\underline{a = \frac{17}{23}}$, $\underline{b = -\frac{14}{23}}$ (3)

e)

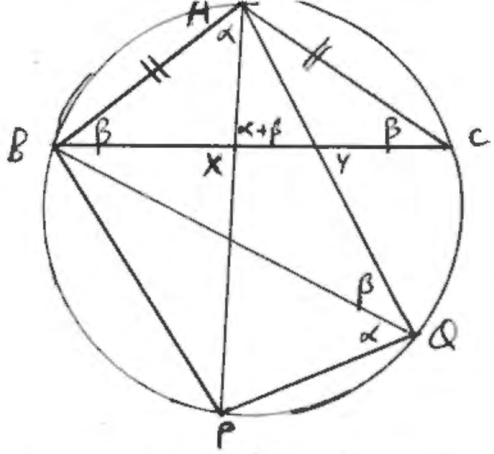


(4)

f) i) $\angle AXC = \alpha+\beta$ (External angle of triangle equals the sum of the two remote interior angles) (1)

ii) $\angle BQP = \angle BAP = \alpha$ (Angles at the circumference standing on the same arc of the circle are equal) (1)

iii) $\angle ACB = \beta$ (Angles opposite equal sides in ΔABC are equal)



$\therefore \underline{\angle AQB = \beta}$ (Angles at the circumference standing on the same arc (AB) are equal.) (2)

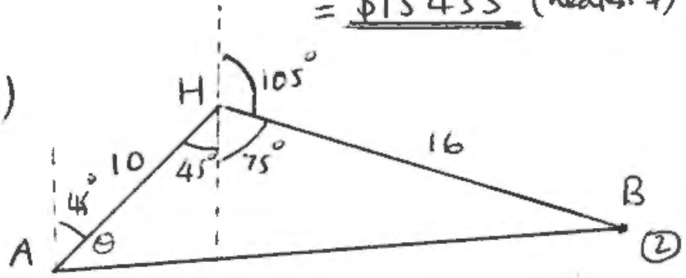
iv) PQYX is cyclic as $\angle YQP = \angle YXA$ (External angle of quadrilateral equals the opposite interior angle) (1)

32 a) After delivery, value = \$18000

after 3 years, value $= \$18000\left(\frac{95}{100}\right)^3$ (2)

$= \underline{\$15433}$ (nearest \$)

b) i)



(2)

ii) Cosine Rule

$AB^2 = 10^2+16^2-2.10.16.\cos 120^\circ$

$= 100+256+160$

$= 516$ (2)

$\therefore AB = \sqrt{516} = \underline{22.7 \text{ (n.m)}}$ (1 DP)

iii) Let $\angle HAB = \theta$

$\frac{\sin\theta}{16} = \frac{\sin 120^\circ}{\sqrt{516}}$

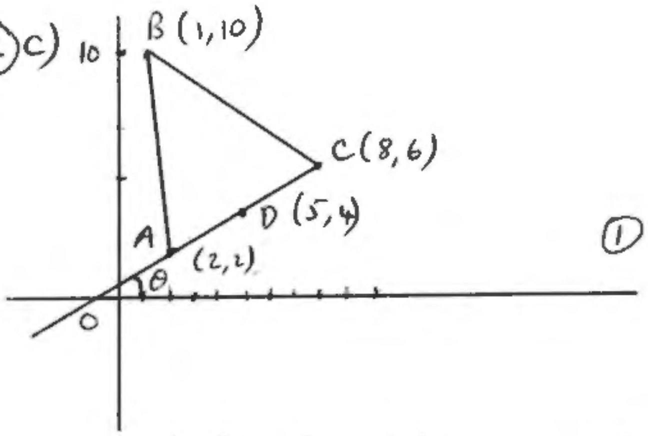
$\sin\theta = 0.6100$

$\theta = 38^\circ$ (nearest degree)

$\therefore$ Bearing is $45^\circ + 38^\circ$

i.e. $\underline{083^\circ T}$ (2)

(32) c)



(1)

i) $m_{AC} = \frac{6-2}{8-2} = \frac{4}{6} = \underline{2/3}$ (1)

ii) $\tan\theta = 2/3 \therefore \theta = \underline{33^\circ 41'}$ (1)

iii) AC is the line

$$y - 2 = \frac{2}{3}(x-2)$$
$$3y - 6 = 2x - 4$$
$$\underline{3y = 2x + 2}$$ (1)

iv) D is $\left(\frac{8+2}{2}, \frac{6+2}{2}\right) = \underline{(5, 4)}$ (1)

v) $m_{BD} = \frac{10-4}{1-5} = -\frac{3}{2}$

$m_{BD}\, m_{AC} = -\frac{3}{2} \cdot \frac{2}{3} = -1$

$\therefore$ $\underline{BD \perp AC}$. (2)

vi) $AC = \sqrt{(8-2)^2 + (6-2)^2} = \sqrt{52}$

$BD = \sqrt{(10-4)^2 + (5-1)^2} = \sqrt{52}$

Area $\Delta ABC = \frac{1}{2} AC \cdot BD = \frac{1}{2}\sqrt{52}\sqrt{52}$

$= \underline{26 \text{ sq units}}$ (3)

vii) D is the midpoint of BE

$\therefore \underline{E = (9, -2)}$ (2)

(33) a) $3^{x+y} = 3^3$ $x + y = 3$ (1)

$2^{2(x-y)} = 2^3$ $2x - 2y = 3$ (2)

$2x + 2y = 6$ (1) × 2 (3)

$4x = 9$ (2) + (3)

$\therefore x = 9/4$

$\therefore y = 3 - 9/4 = 3/4$

$\therefore \underline{x = 9/4, \; y = 3/4}$ (3)

b) $\sin\theta = 8/17$ $\cos^2\theta = 1 - \sin^2\theta$

$= 1 - \frac{64}{289}$

$= \frac{225}{289}$

$\therefore \cos\theta = \pm\frac{15}{17}$

$\therefore \tan\theta + \sec\theta = \pm\frac{8}{15} \pm \frac{17}{15}$

$= \pm\frac{25}{15} = \underline{\pm\frac{5}{3}}$ (3)

c) i)

$$\begin{array}{r|l} & 3x - 2 \\ x^2+1 & 3x^3 - 2x^2 - 5x - 1 \\ & 3x^3 \quad\quad + 3x \\ \hline & -2x^2 - 8x \\ & -2x^2 \quad\quad - 2 \\ \hline & -8x + 1 \end{array}$$ (3)

$\therefore \underline{(3x^3 - 2x^2 - 5x - 1) = (x^2+1)(3x-2) + (-8x+1)}$

$P(x) = D(x)\, Q(x) + R(x)$

ii) By remainder theorem

$F(2) = 7$ and $F(-2) = 3$

Divisor of degree 2 thus remainder could be of degree 1 (or 0). Say $R(x) = ax + b$.

$F(x) = (x^2 - 4)\,Q(x) + ax + b$

$= (x-2)(x+2)\,Q(x) + ax + b$

But

$F(2) = 2a + b = 7$

$F(-2) = -2a + b = 3$

Solving, $b = 5$, $a = 1$

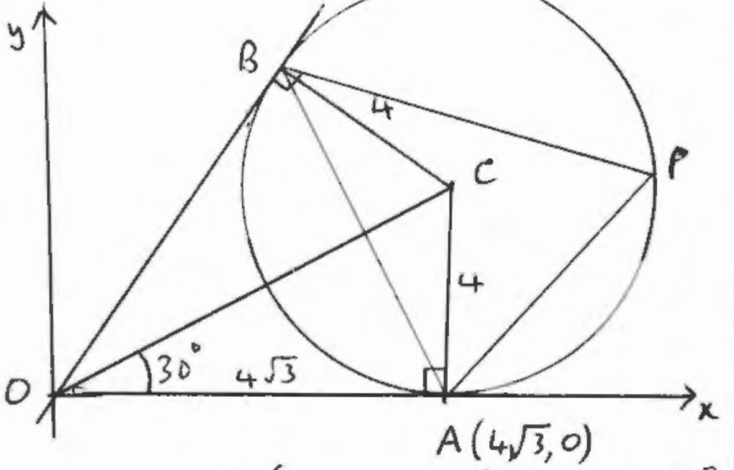
$\therefore$ Remainder is $\underline{x + 5}$ (3)

d) i) Circle has radius 4, centre $(4\sqrt{3}, 4)$

Distance from centre to $y = 0$ is $\frac{4}{\sqrt{1}} = \underline{4}$ which is the radius. Thus $y = 0$ is a tangent to the circle. Point of contact is $y = 0 \Rightarrow$

$x = 4\sqrt{3}$.

$\therefore \underline{A \text{ is } (4\sqrt{3}, 0)}$ (3)



$\angle CAO = 90°$ (Radius forms a 90° with a tangent)

$\therefore \tan COA = \frac{4}{4\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\therefore COA = 30°$

$\Delta OBC \equiv \Delta OAC$ (SSS) (since $OB = OA$ – tangents from any exterior point equal).

$\therefore \angle BOC = 30°$ (Corresponding angles in congruent triangles are equal.

$\therefore \angle BOA = 30° + 30° = \underline{\underline{60°}}$

But $OB = OA$ (above) so ΔOAB is isosceles. (

Thus $\angle OBA = \angle OAB$ (angles opposite equal sides are equal.

But $\angle OAB + \angle OBA = 120°$ ③

$\therefore \angle OAB = \angle OBA = \angle BOA = 60°$

Thus $\underline{\underline{\Delta OAB \text{ is equilateral.}}}$

iii) $\angle BCA = 120°$ (Sum of angles of quadrilateral OACB = 360°)

$\therefore \underline{\underline{\angle APB = 60°}}$ (Angle at the circumference is ½ angle at the centre when standing on the same arc.) ②

34 a) $2x^2 - 5x + 3 = 2(x^2 - \frac{5}{2}x) + 3$

$= 2\left(\left(x - \frac{5}{4}\right)^2 - \frac{25}{16}\right) + 3$

$= 2\left(x - \frac{5}{4}\right)^2 - \frac{1}{8}$

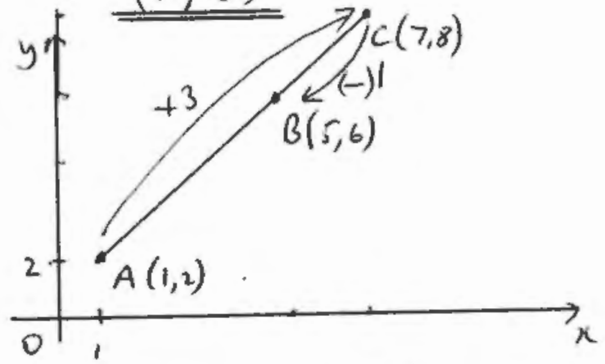
$\therefore$ Minimum value is $\underline{\underline{-\frac{1}{8}}}$ ②

(since $(x - \frac{5}{4})^2 \geq 0$)

b) Use ratio 3:-1 in normal formula:

$C = \left(\frac{3\times5 - 1\times1}{3+-1}, \frac{3\times6 - 1\times2}{3+-1}\right)$

$= \underline{\underline{(7, 8)}}$ ②



①

c) $\frac{1 - \tan^2\theta}{1 + \tan^2\theta} = \frac{1 - \frac{\sin^2\theta}{\cos^2\theta}}{1 + \frac{\sin^2\theta}{\cos^2\theta}}$

$= \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta}$

$= \frac{\cos^2\theta - (1 - \cos^2\theta)}{1}$

$= \underline{\underline{2\cos^2\theta - 1}}$ ③

d) i) Horizontal Asy. $\underline{\underline{y = -2}}$ ①

$\frac{(1-x)(2x+5)}{(x+1)(x-5)} = -2$

$-2x^2 - 3x + 5 = -2(x^2 - 4x - 5)$

$-2x^2 - 3x + 5 = -2x^2 + 8x + 10$

$11x = -5$

Crosses asymptote at $\underline{\underline{(-\frac{5}{11}, -2)}}$ ②

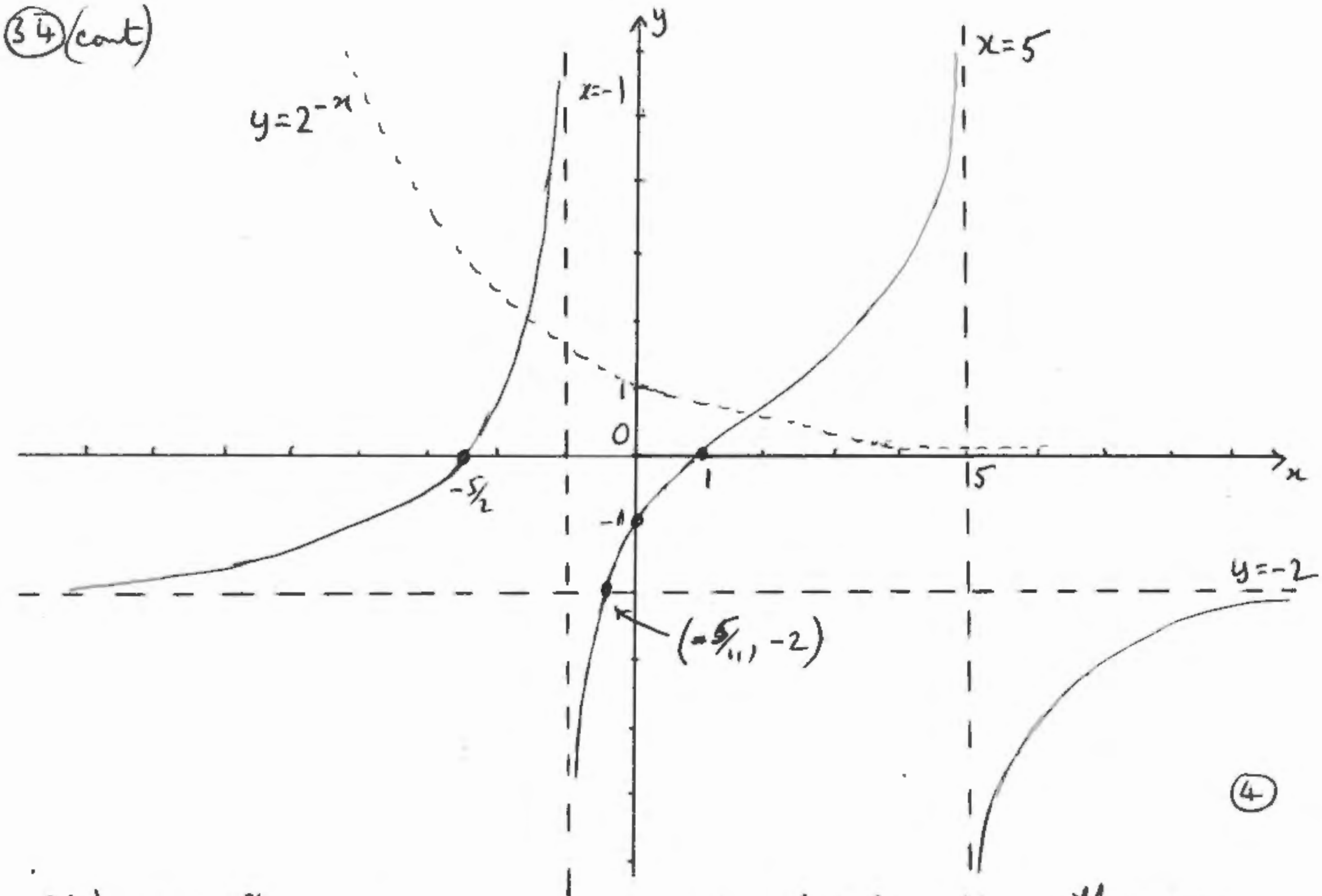
Vertical Asymptotes at $x = -1$, $x = 5$

X intercepts at $(1, 0)$, $(-5/2, 0)$

y intercept at $(0, -1)$

[No other crossings allowed]

(34) (cont)



(4)

iii) $y = 2^{-x}$ is drawn on to the sketch. It will cross original graph at 2 points. Thus there will be 2 solutions to given equation. (Asymptotes ensure that there are no further crossings). (2)

e) $n^4 + 2n^3 + 2n^2 + 2n + 1 = (n^4 + 2n^3 + n^2) + (n^2 + 2n + 1)$

$= (n^2+1)(n^2 + 2n + 1)$

$= (n^2+1)(n+1)^2$

$> n^2(n+1)^2 = [n(n+1)]^2$

So our expression is bigger than the square of $n(n+1)$.

Now consider $(n(n+1)+1)$.

$(n(n+1)+1)^2 = (n^2+n+1)^2$

$= n^4 + 2n^3 + 3n^2 + 2n + 1$

$\therefore n^4 + 2n^3 + 2n^2 + 2n + 1 < (n(n+1)+1)^2$

i.e. our expression is less than the square of $n(n+1)+1$

i.e. it is sandwiched between the squares of two consecutive integers and hence cannot be the square of an integer itself. (3)