YEARLY EXAMINATION

YEAR 10 2011

MATHEMATICS

Time Allowed – 120 minutes

(Plus 5 minutes Reading time)

INSTRUCTIONS:

- All questions may be attempted
- Write your **name** and **Maths class** at the top of *each page*.
- **Write** in **Pen** and draw diagrams in **Pencil**
- Answers to Multiple choice Questions 1-30 are to be entered onto the answer sheet provided
- Answers to Questions 31-34 are to be returned in separate bundles
- Department of Education approved calculators and templates are permitted
- Show all necessary working
- Marks may not be awarded for untidy or carelessly arranged work
- No grid paper is to be used unless provided with the examination paper

SECTION A (1 Mark Each)

(1) $\dfrac{1}{2x^3} =$

(A) $2x^{\frac{1}{3}}$ (B) $2x^{-3}$ (C) $\dfrac{1}{2}x^{\frac{1}{3}}$ (D) $\dfrac{1}{2}x^{-3}$.

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(2) Which of the following linear polynomials is a divisor of $x^{17} - 4x^{15} - x^3 + 4$.

(A) $x-1$ (B) $x-2$ (C) $x+1$ (D) $x+2$.

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(3) $\dfrac{1}{1-\dfrac{1}{1-2^{-1}}} + \dfrac{1}{1+\dfrac{1}{1+2^{-1}}} =$

(A) -1 (B) $-\dfrac{2}{5}$ (C) 0 (D) $\dfrac{1}{3}$.

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(4) The length of a rectangle is increased by 25%. By what percentage should its width be decreased in order that the area remains the same?

(A) $12 \cdot 5$ (B) 20 (C) 25 (D) 50.

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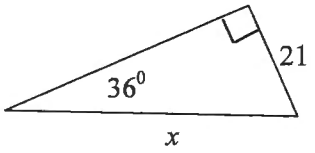
(5) For the given diagram, the correct expression for x is



(A) $21\sin 36^0$ (B) $21\cos 36^0$ (C) $\dfrac{21}{\sin 36^0}$ (D) $\dfrac{21}{\cos 36^0}$.

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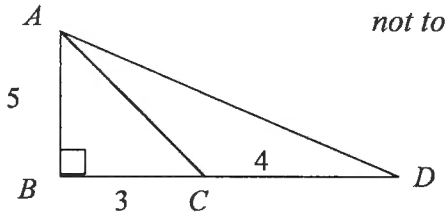
(6) For the given diagram, find angle CAD (to the nearest minute).



(A) $23^0 30'$ (B) $30^0 58'$ (C) $54^0 28'$ (D) $66^0 30'$.

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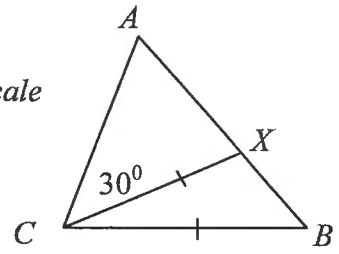
(7) Soo Lon completed four assessment tests. From the statistical data table, which test is the best relative performance from Soo Lon?

Subject	Mark	Mean	SD
Biology	77	70	8
Economics	73	62	10
English	85	80	5
Mathematics	94	90	5

(A) Biology (B) Economics (C) English (D) Mathematics.

(8) In the diagram $AB = AC$, $BC = CX$ and $\angle XCA = 30^0$, find $\angle ACB$.

Not to scale



(A) 75^0 (B) 70^0 (C) 65^0 (D) 60^0.

---

(9) The solutions for x in the quadratic equation: $x^2 + kx + 10 = 0$ are integers. The total number of possible values for k is

(A) 2 (B) 4 (C) 6 (D) 8.

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(10) Find the acute angle formed by the hour and the minute hand of a clock at 6 h 22 min.

(A) 36^0 (B) 44^0 (C) 45^0 (D) 59^0.

---

(11) A car travels at a constant speed of 45 km/h from A to B, and returns from B to A with a constant speed of 90 km/h. What is the average speed of the car

(A) 50 km/h (B) 57·5 km/h (C) 60 km/h (D) 67·5 km/h.

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(12) Which one of the following has the greatest volume?

(A) A cube of edge length x cm

(B) A sphere of diameter x cm

(C) A right cylinder radius x cm and height x cm

(D) A right cone radius x cm and height x cm.

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(13) Given that the bearing of Y from X is S 50^0 E, the bearing of point X from position Y is

(A) $140^0\ T$ (B) $130^0\ T$ (C) $310^0\ T$ (D) $320^0\ T$.

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(14) If $(2x + P)^2 = 4x^2 - 20x + Q$, then

(A) $P = 5$ and $Q = 5$ (B) $P = -5$ and $Q = -5$

(C) $P = 5$ and $Q = 25$ (D) $P = -5$ and $Q = 25$.

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(15) Asif and Gromit are waiting in a queue. There are x people behind Asif, who is y places in front of Gromit. If there are n people in front of Gromit, how many people are in the queue?

(A) $n + x - y + 1$ (B) $n + x - y$ (C) $n - x + y - 1$ (D) $n - x + y - 2$.

(16) The Stem and Leaf plot shows the test marks of a class of Year 9 students.

Stem	Leaf
4	7 8 9
5	0 4 6 6 8
6	2 3 5 6 6 6 7 8 8
7	1 2 4 4 5 9
8	3 7 7 8 8 9

What is the median mark?

(A) 66 (B) 67 (C) 67½ (D) 68.

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(17) For the two sets of marks: **Set A**: 8 12 14 34 36 40
Set B: 4 16 18 30 32 44

Consider the two statements
I: Both sets have the same mean.
II: Both sets have the same standard deviation.
Which is True?

(A) **I** only (B) **II** only (C) both **I** and **II** (D) neither **I** nor **II**.

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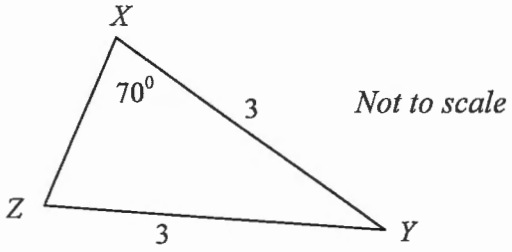
(18) The expression for the area of ΔXYZ is



(A) $\frac{1}{2}\times 3\times 3\times \sin 70^0$ (B) $\frac{1}{2}\times 3\times 3\times \sin 40^0$

(C) $\frac{1}{2}\times 3\times 3\times \cos 70^0$ (D) $\frac{1}{2}\times 3\times 3\times \cos 40^0$.

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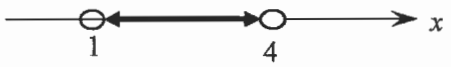
(19) Which of the following graphs represents the solution set for the inequation:

$$x^2 - 5x + 4 \leq 0.$$

(A)


(B)


(C)


(D)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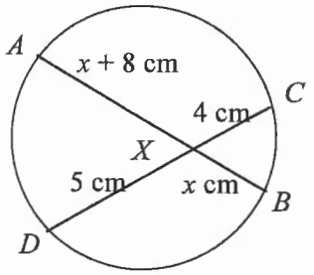
(20) The letters of the word '**EXAMINATION**' are placed in a hat.
One letter is chosen at random from the hat. What is the probability that it is a vowel?

(A) $\frac{4}{11}$ (B) $\frac{5}{11}$ (C) $\frac{6}{11}$ (D) $\frac{7}{11}$.

(21) For this diagram, the length of AX is?



(A) 2 cm (B) 9 cm (C) 10 cm (D) 12 cm.

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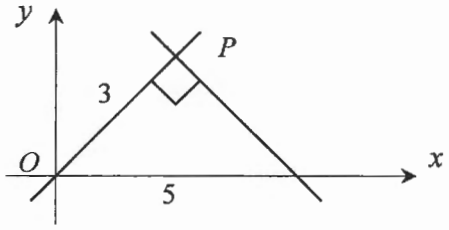
(22) The slope of OP is



not to scale

(A) $\frac{3}{5}$ (B) $\frac{3}{4}$ (C) $\frac{4}{3}$ (D) $\frac{5}{3}$.

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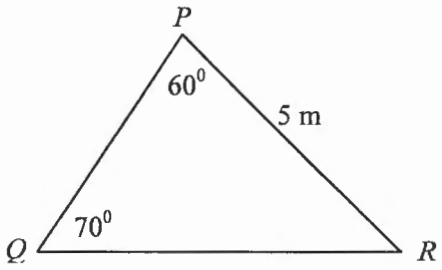
(23) In ΔPQR,



not to scale

$PQ = ?$

(A) $\frac{5}{\sin 70^0}$ m (B) $\frac{5\sin 70^0}{\sin 50^0}$ m

(C) $\frac{5}{\sin 50^0 \sin 70^0}$ m (D) $\frac{5\sin 50^0}{\sin 70^0}$ m.

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(24) The line which is perpendicular to the line $y = -\frac{1}{3}x + 1$ passing through the point $(0, -4)$ has the equation:

(A) $x - 3y - 4 = 0$ (B) $3x + 4y - 4 = 0$

(C) $3x - y - 4 = 0$ (D) $4x + y - 3 = 0$.

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(25) Which of the following pair of inequations is satisfied by the point (2, 3)?

(A) $y > x$ and $y < -2x$ (B) $y > -x$ and $y < 2x$

(C) $y > 2x$ and $y > x$ (D) $y < 2x$ and $y < -x$.

(26) How many solutions exist for n when $n^2 = 2^n$?

(A) 1 (B) 2 (C) 3 (D) 4.

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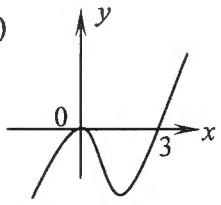
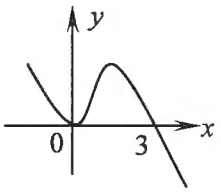
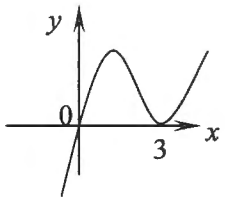
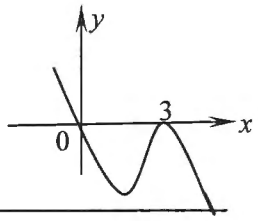
(27) Which of the following sketches best represent the graph of $y = 2x^2(3-x)$?

(A) 

(B) 

(C) 

(D) 

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(28) A bag contains identical balls of which w are white and b are black.
Two balls are removed randomly without replacement.
The probability of one ball of each colour is chosen is?

(A) $\dfrac{wb}{(w+b)(w+b-1)}$ (B) $\dfrac{wb}{(w+b)^2}$

(C) $\dfrac{2wb}{(w+b-1)^2}$ (D) $\dfrac{2wb}{(w+b)(w+b-1)}$.

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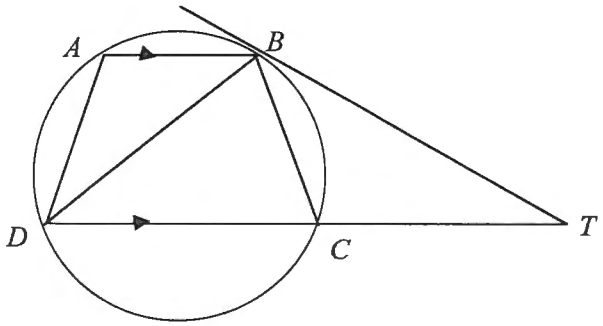
(29) In the diagram, $AB \parallel DT$, $AB \neq DC$ and BT is a tangent to the circle at B. Which triangles are similar?



(A) ΔCDB and ΔABD (B) ΔCDB and ΔCBT

(C) ΔABD and ΔBDC (D) ΔABD and ΔCBT.

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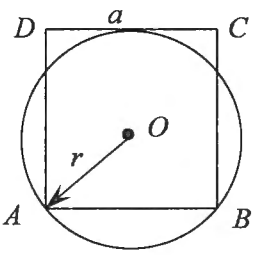
(30) A circle centre O of radius r is tangent to side CD of length a of the square $ABCD$ and it passes through A and B, as shown.



Not to scale

The area S of the square $ABCD$ in terms of radius r is?

(A) $S = \dfrac{8}{5}r^2$ (B) $S = 2r^2$ (C) $S = \dfrac{64}{25}r^2$ (D) $S = \dfrac{8}{3}r^2$.

Question 31 (20 marks) — Marks

(a) Fully factorize $2^{3x}-1$. **2**

(b) Rationalize the denominator : $\dfrac{11}{3-2\sqrt{5}}$. **2**

(c) Solve for x : $9^x-10.3^x+9=0$ **3**

(d) Simplify: $\dfrac{a^2-b^2}{a^{-2}-b^{-2}}$. **2**

(e) If $5^k\left(5^2\right)^3=1$, find k. **2**

(f) $ABCD$ is a parallelogram. E is a point on AB and F is a point on CD such that $AE=CF$. Draw the diagram showing the above information and prove that EF bisects BD. **4**

(g) (i) Given that $\tan(x+y)=\dfrac{\tan x+\tan y}{1-\tan x\tan y}$, Express $\tan 2x$ in terms of $\tan x$. **1**

(ii) Hence show that: $\tan^2 15^0+2\sqrt{3}\tan 15^0-1=0$. **2**

(iii). Hence find the exact value of $\tan 15^0$. **2**

Question 32 (20 marks) [Start a new Page]

(a) Sketch the graph of $y=3\sin 2x$ for $0^o\le x\le 360^0$. **2**

(b) A ship leaves port P on a bearing of 125^0T for 25km. It then changes course to S10^oW for 10 km.

(i) Sketch the diagram showing the above information. **1**

(ii) Find the distance of the ship from the Port P correct to 2 decimal places. **2**

(iii) Find the bearing of the ship from Port P to the nearest degree. **3**

(c) Line l has equation $x+2y-10=0$ and the line k, perpendicular to line l, passes through the origin.

(i) If the line l crosses the x-axes at A and the y-axes at B. Find the co-ordinates of the points A and B. **1**

(ii) Find the co-ordinates of Q which divides AB in the ratio of 2:3 **2**

(iii) Prove that the equation of the line k is $y=2x$. **2**

(iv) Find the area enclosed by: $x+2y-10\le 0$, $y-2x\ge 0$ and the y-axis. **2**

(d) A six-sided die is biased so that the number 3 occurs twice as often as any other number.

(i) The die is rolled once. Show that the probability that an odd number occurs is $\dfrac{4}{7}$. **1**

(ii) If the biased die is rolled twice, find the probability of the sum of the uppermost numbers being six. **2**

Q32 (d) (iii) is continued over the page.

Question 32(d) (iii) (continued) **Marks**

(iii) This biased die is now rolled together with TWO fair six-sided dice. What is the probability that at least two odd numbers are uppermost? **2**

Question 33(20marks) **[Start a new Page]**

(a) The line $y = x+3$ is tangent to circle $(x-1)^2+(y+2)^2 = R^2$. Find the exact area of the circle. **3**

(b) Solve for θ: $2\cos^2\theta + 7\sin\theta - 5 = 0$ for $0^0 \le \theta \le 720^0$ **4**

(c)

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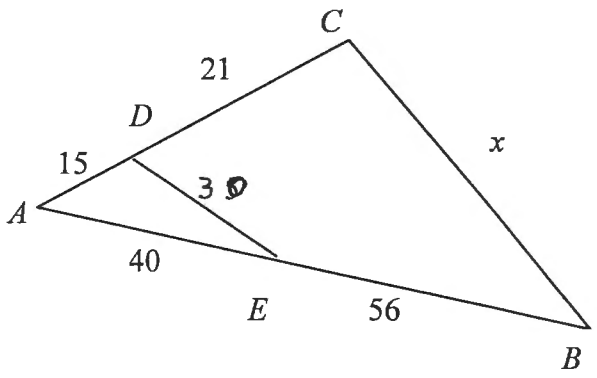


Given the diagram, find the value of x, give full reasons. **4**

(d) The time to plough a field varies directly as the area to be ploughed and inversely as the number of workers. If 8 workers can plough a 5 ha field in 6 hours, how long will it take to plough a 10 ha field with 12 workers working at the same rate? **3**

(e) Prove that : $\dfrac{\cos\theta}{1-\tan\theta} + \dfrac{\sin\theta}{1-\cot\theta} = \sin\theta + \cos\theta$. **4**

(f) State the domain of the following: $y = \dfrac{\sqrt{1-x^2}}{x}$. **2**

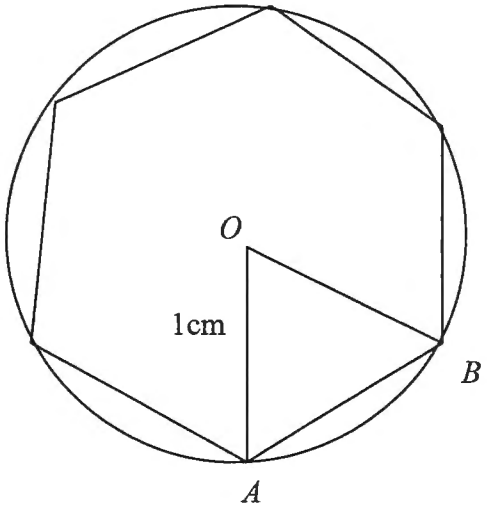
Question 34(20 marks) [Start a new Page] **Marks**

(a) If $\sin\theta = -\frac{13}{15}$ and $\cos\theta > 0$, find the exact value of $\tan\theta$. **2**

(b) Sketch the graph of $y = \frac{x(x+1)}{x^2-4}$, showing all the x and y-intercepts and asymptotes. **4**

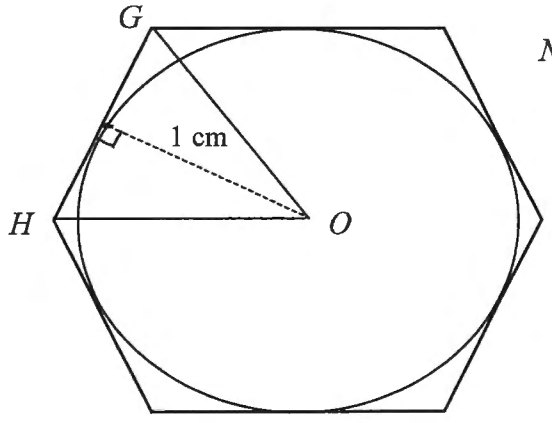
(c) A regular hexagon is drawn inside a circle with centre O so that its vertices lie on the circumference, as shown in the diagram. He circle has radius 1cm.

Not to scale



(i) Find the area of ΔABO and hence find the area of the hexagon. Leave your answer in surd form. **3**

(ii) Another regular hexagon is drawn outside the circle, as shown. The altitude of ΔOGH is 1 cm. **3**



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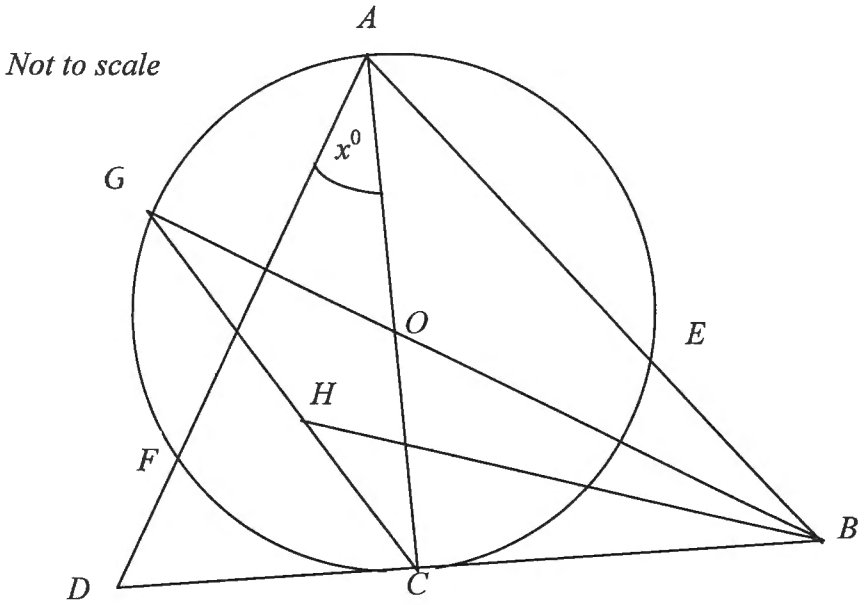
Find the area of ΔOGH and hence find the area of the hexagon.
Leave your answer in surd form

Q34 (c) (iii) continued over the page.

(iii) By considering the results in (ii) and (iii), show that : $\frac{3\sqrt{3}}{2} \le \pi \le 2\sqrt{3}$. **1**

(d)

In the diagram below, AC is the diameter of circle with centre O and BD is a tangent to the circle at C. Let $\angle DAC = x^0$.



(i) Copy the diagram onto your answer sheet and explain why $\angle FEC = x^0$. **1**

(ii) Prove that $DFEB$ is a cyclic quadrilateral. **2**

(iii) If $\angle BHC = \frac{\angle GHB}{3}$, prove that $\angle HBG = \angle HBC$ **4**

END OF PAPER

1. D
2. A
3. B
4. B
5. C
6. A
7. B
8. B
9. B
10. D
11. C
12. C
13. C
14. D
15. A
16. B
17. C
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