Name:	
Class:	

YEARLY EXAMINATION

YEAR 10 2012

MATHEMATICS

Time Allowed – 120 minutes

(Plus 5 minutes Reading time)

INSTRUCTIONS:

- All questions may be attempted
- Write your **name** and **Maths class** at the top of *each page*.
- **Write** in **Pen** and draw diagrams in **Pencil**
- Answers to Multiple choice Questions 1-30 are to be entered onto the answer sheet provided
- Answers to Questions 31-34 are to be returned in separate bundles
- Department of Education approved calculators and templates are permitted
- Show all necessary working
- Marks may not be awarded for untidy or carelessly arranged work
- No grid paper is to be used unless provided with the examination paper

SECTION A (1 Mark Each)

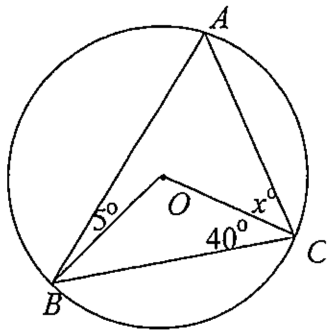
(1) The degree of the polynomial $P(x) = 6x^4 - 4x^3 + 3x^5 - x + 1$ is

(A) 3 (B) 4 (C) 5 (D) 6

(2) The point Y is on a bearing of 065°T from point X and point Z is on a bearing of 165 °T from Y. The size of $\angle XYZ$ is

(A) 65° (B) 80° (C) 115° (D) 280°

(3) Given the chord BC of a circle, centre O, where A is a point on the major arc with $\angle OBA = 5°$ and $\angle OCB = 40°$ as shown in the diagram below.



NOT TO SCALE

The value of x is:

(A) 50 (B) 35 (C) 40 (D) 45

(4) A parabola with a vertical axis of symmetry has its vertex at (0, 8) and an x intercept of 2. If the parabola passes through the point $(1, a)$, then a is

(A) 5 (B) 5·5 (C) 6 (D) 6·5

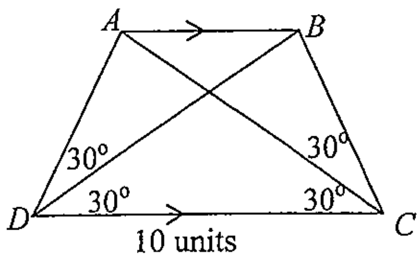
(5) An energy-conscious homeowner makes three successive improvements that save, in turn, 20%, 25% and 55% on the heating costs of the house. The overall percentage saved is

(A) $33\frac{1}{3}$ (B) 27 (C) 73 (D) $66\frac{2}{3}$

(6) If $G = H + \sqrt{\frac{4}{L}}$ and $L > 0$ and $G > H$ then L equals

(A) $\frac{4}{(G-H)^2}$ (B) $4(G-H)^2$ (C) $\frac{4}{G^2-H^2}$ (D) $4(G^2-H^2)$

(7) The diagonals of a trapezium $ABCD$ make angles of 30° as shown in the diagram. The base CD has length of 10 units.



NOT TO SCALE

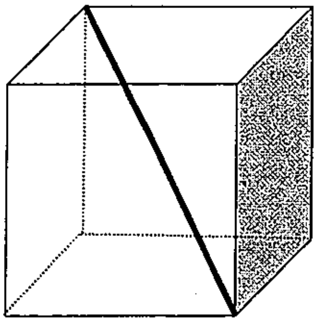
The perimeter of the trapezium is

(A) 30 units (B) $10\sqrt{3}$ units (C) 25 units (D) $15\sqrt{3}$ units

(8) The value of x which satisfies the equation $\frac{3}{\left(2-\frac{x}{2}\right)}=2$ is

(A) 2 (B) 1 (C) -1 (D) $\frac{1}{2}$

(9) The diagram below shows one diagonal of a cube.



If the length of the diagonal is $\sqrt{12}$ units, then the volume of the cube is

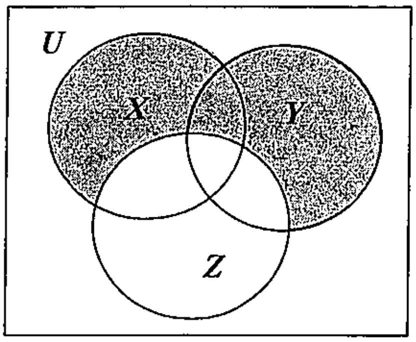
(A) 8 units^3 (B) 12 units^3 (C) 24 units^3 (D) $12\sqrt{12}$ units^3

(10) Which of the following values of x satisfies the equation below?

$$25^{-2}=\frac{5^{\frac{48}{x}}}{\left(5^{\frac{26}{x}}\right)\times\left(25^{\frac{17}{x}}\right)}$$

(A). 2 (B) 3 (C) 5 (D) 6

(11) Sets ***X***, ***Y*** and ***Z*** are shown in the Venn Diagram below.



The shaded area represents the set

(A) $\{(X\cup Y)\cap\tilde{Z}\}$ (B) $\{(X\cup Y)\cup\tilde{Z}\}$ (C) $\{(\tilde{X}\cup\tilde{Y})\cup\tilde{Z}\}$ (D) $\{(X\cap\tilde{Z})\cap(Y\cap\tilde{Z})\}$.

(12) $\cos[(90+\alpha)°]-\cos[(90-\alpha)°]=$

(A) $2\sin(\alpha°)$ (B) $-2\sin(\alpha°)$ (C) $2\cos(\alpha°)$ (D) $-2\cos(\alpha°)$

(13) If $f(x) = x^2 - 9x + 14$, then the asymptotes of the curve with the equation $y = \frac{1}{f(x)}$ are

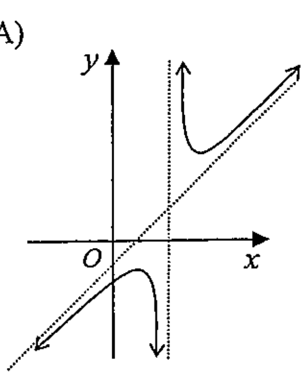
(A) $x = 2, x = 7$ and $y = \frac{1}{14}$ (B) $x = 2, x = 7$ and $y = 0$

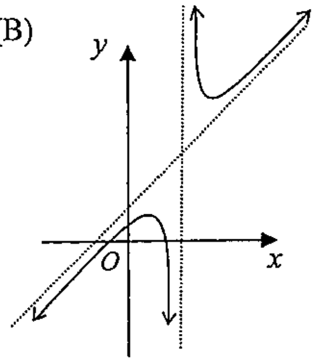
(C) $x = -2, x = -7$ and $y = \frac{1}{14}$ (D) $x = -2, x = -7$ and $y = 0$

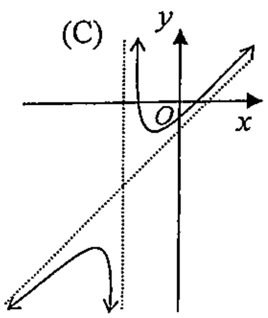
(14) A candle is initially 30 cm long. After burning for three hours its length is 24 cm.
If the length of the candle (L cm) is linearly related to the burning time (t hours) then the rule relating L and t is

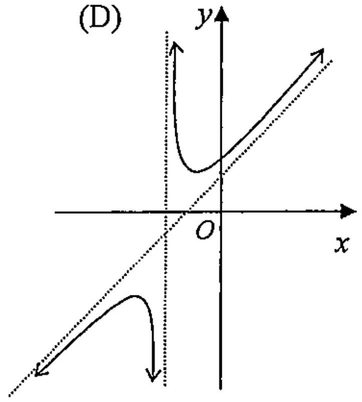
(A) $L = 30 - 12t$ (B) $L = 30 - 8t$ (C) $L = 30 - 6t$ (D) $L = 30 - 2t$

(15) Which of the following could be the graph of the function $y = \frac{x^2 - 3x + 4}{x - 2}$?

(A)


(B)


(C)


(D)


(16) A cube has a volume of 2 400 000 cm^3. This is equivalent to

(A) $2{\cdot}4$ m^3 (B) 240 m^3 (C) 2 400 m^3 (D) 240 000 m^3

(17) In a particular town, the age of the population is normally distributed.
The mean age is 43 years and the standard deviation is 14 years.
The percentage of the population between the ages of 43 and 71 is closest to

(A) 34 (B) 47·5 (C) 68 (D) 95

(18) If $-2{\cdot}4 < x < -1{\cdot}5$ and $0 < p < 2$ then

(A) $-3{\cdot}0 < px < 0$ (B) $-4{\cdot}8 < px < -3{\cdot}0$ (C) $4{\cdot}8 < px < 3{\cdot}0$ (D) $-4{\cdot}8 < px < 0$

(19) One litre of water was used to make exactly 64 ice cubes of a certain size. It was decided that these were too small, so larger ones with double the dimensions were made.
How many of the larger ice-cube can be made using one litre of water?

(A) 4 (B) 8 (C) 16 (D) 32

(20) The mean, median, mode and range of a collection of eight integers are all equal to 8.
The largest possible integer that can be an element of this collection is

(A) 11 (B) 12 (C) 13 (D) 14

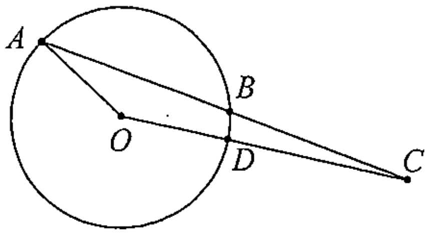
(21) The minimum value of the function $f(x,y)=x^2+y^2-2x+6y+3$

(A) -10 (B) -7 (C) 3 (D) 13

(22) If $0<x<1$, which of the following is true?

(A) $x>\frac{1}{\sqrt{x}}$ (B) $x^2>x$ (C) $\sqrt{x}<\frac{1}{x}$ (D) $\frac{1}{x^2}<\frac{1}{\sqrt{x}}$

(23) Points *A, B* and *D* are on the circle centre *O* and point *C* is outside the circle.



NOT TO SCALE

Which of the following statements is correct?

(A) $BC.AB=CO^2-DO^2$ (B) $BC.AC=CO^2-AO^2$
(C) $BC.AB=CO.DO$ (D) $BC.AC=CD.AO$

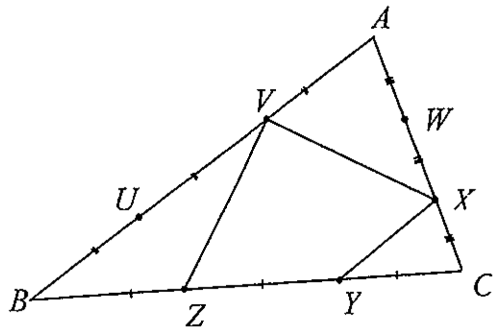
(24) Given the formula $P=ab^2$. If $a=0{\cdot}05$, $b>0$ and $P=4{\cdot}5\times10^{15}$, then the value of b is

(A) $1{\cdot}5\times10^7$ (B) $3{\cdot}0\times10^8$ (C) $1{\cdot}0\times10^{15}$ (D) $2{\cdot}0\times10^{16}$

(25) The graph of $y=2(2^{x+1})-8$ has x and y intercepts with the coordinate axes at

(A) $x=1$ and $y=-6$ (B) $x=-1$ and $y=4$ (C) $x=1$ and $y=-4$ (D) $x=\frac{1}{2}$ and $y=-4$

(26) *ABC* is a triangle. Points *U, V, W, X, Y* and *Z* trisect the sides, as shown in the diagram below.



NOT TO SCALE

The ratio of the area of the quadrilateral *ZVXY* to the area of ΔABC is

(A) $\frac{2}{3}$ (B) $\frac{5}{9}$ (C) $\frac{1}{2}$ (D) $\frac{4}{9}$

(27) If $x > 0$ which of the following must always be less than one?

(A) $\frac{1}{x}$ (B) $\frac{1+x}{x}$ (C) $\frac{1-x}{x}$ (D) $\frac{x}{1+x}$

(28) Coin A is flipped 3 times and coin B is flipped 4 times. What is the probability that the number of heads obtained from flipping the two coins is the same?

(A) $\frac{19}{128}$ (B) $\frac{23}{128}$ (C) $\frac{35}{128}$ (D) $\frac{1}{2}$

(29) Tim decided to invest his savings in the following accounts.

Half of his savings were invested at 12% pa
One third of his savings were invested at 9% pa
The remainder was invested at 6% pa.

The overall percentage interest rate per annum on his total savings is

(A) 8 (B) 9 (C) 10 (D) 11

(30) If $n \neq 0$, then $\sqrt{\frac{2^{2n+4}+2^{2n+2}}{20}}$ equals

(A) $\frac{2^n}{\sqrt{5}}$ (B) $\frac{n}{4}$ (C) $\sqrt{\frac{2^n}{5}}$ (D) 2^n

END OF SECTION A

SECTION B

Question 31 (20 marks) START A NEW PAGE **Marks**

(a) Simplify: $\dfrac{9^n - 1}{3^n - 1}$, $n \neq 0$. **1**

(b) Evaluate: $x^2 - 3x$ when $x = \sqrt{2} - 3$. **2**

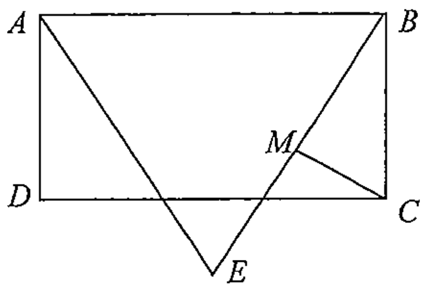
(c) Solve the equation for x: $(2x^2 - x)^2 = 6(2x^2 - x)$. **3**

(d) Simplify the expression: $\dfrac{(2x-3)^2 - 4}{2x^2 + x - 15}$. **2**

(e) Show that the line $3x - 4y + 3 = 0$ is tangent to the circle which has its centre at $(1, -1)$ and has a radius of 2 units. **2**

(f) Find the range of the function $f(x) = x^2 - 4x + 3$. **3**

(g) $ABCD$ is a rectangle in which AB is twice as long as BC. E is a point such that ΔABE is equilateral. M is the midpoint of BE.



Copy the diagram and find the size of $\angle CMB$, giving reasons. **4**

(h) The quantity r varies directly as the square root of v and inversely with the square root of h.

(i) Write an equation linking r, v and h. **1**

(ii) By what percentage will r change if h increases by 40% and v decreases by 65%. **2**

Question 32 (20 marks) START A NEW PAGE

(a) ABC is a triangle such that $AB = AC = 8$ cm and $BC = 10$ cm. Using a ruler and compasses only

(i) Construct ΔABC **on the attached work sheet.** **1**

(ii) Find by construction the points which are 6 cm from point C and equidistant from points A and B. Label the points on your diagram. **(Show all construction lines and arcs)** **2**

(b) Solve the equation $\cos^2 x = \sin^2 x$ for $0° \leq x \leq 360°$. **3**

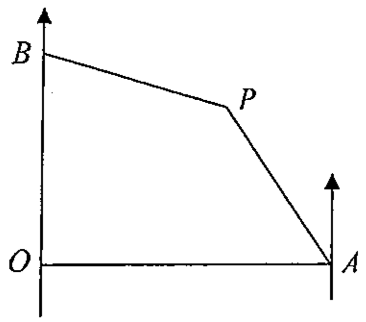
Question 32 continued over page

(c) A box contains exactly five balls, three are red and two are white.
Balls are randomly removed, one at a time without replacement.

(i) Calculate the probability that the second ball drawn is white. **1**

(ii) If the balls are continually removed until all the red or all the white balls are removed, what is the probability that the last ball drawn is red? **3**

(d) Given A is a point due East of O and B is a point due North of O. P is the point on a bearing of 120°T from B and 345°T from A. $AP = 3$ km and $BP = 5$ km, and the bearing of P from O is θ °T.



(i) **Copy the diagram** and show all the given information. **2**

(ii) Show that $OP = \dfrac{5}{2\sin\theta}$. **2**

(iii) Find θ to the nearest degree. **3**

(e) Sketch the graph of $y = 1 - 2\cos\left(\dfrac{x}{3}\right)$ for $0° \le x \le 720°$, showing all intercepts with the coordinate axes. **3**
Use a horizontal scale of $1\text{ cm} = 90°$ and a vertical scale of $2\text{ cm} = 1$ unit.

Question 33 (20 marks) START A NEW PAGE

(a) Given A is the point $(-2,0)$ and $T(t,t^2)$ is a point on the parabola $y = x^2$.
The point $P(x,y)$ divides the interval AT in the ratio 3:1.

(i) Draw a diagram to show the given information. **1**

(ii) Show that $x = \dfrac{3t-2}{4}$ and $y = \dfrac{3t^2}{4}$. **2**

(iii) If P lies on the line $x + y = 4$, find the possible coordinates of P. **3**

(b) Prove that $\dfrac{\sin^3 A}{\cos A} - \dfrac{\cos^3 A}{\sin A} = \tan A - \cot A$ **2**

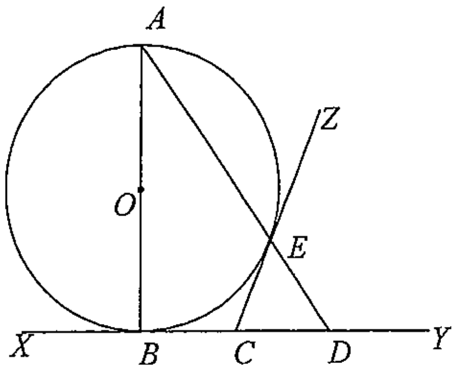
(c) Given α and β are roots of $ax^2 + bx + c = 0$ and γ and δ are the roots of $cx^2 + ax + b = 0$.

(i) Show that $\alpha\beta(\gamma + \delta) = -1$. **2**

(ii) Evaluate $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \gamma\delta$. **2**

Question 33 continued over page

(d) Given XY and ZC are tangents at points B and E respectively to a circle with centre O. AB is a diameter and points A, E and D are collinear.



NOT TO SCALE

Copy the diagram and prove that ΔECD is isosceles, giving reasons. **3**

(e) The formula for the population standard deviation of a set of scores is $\sigma_n = \sqrt{\dfrac{\Sigma(x-\bar{x})^2}{n}}$.

(i) Show that this formula can be rewritten as $\sigma_n = \sqrt{\dfrac{\Sigma(x^2)}{n} - (\bar{x})^2}$. **2**

(ii) Given the mean mark and its standard deviation for a group of 22 students in a Mathematics examination are 62 and 25·20 (2dp) respectively. Hence find the value of Σx^2. **1**

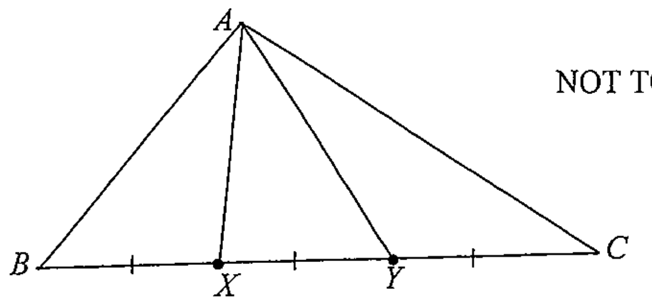
(iii) The lowest 5 marks were 12, 14, 16, 21 and 23. Find the new mean and standard deviation when these lowest 5 marks are removed from the set of scores. Give your answers to 1 decimal place. **2**

Question 34 (20 marks) START A NEW PAGE

(a) Factorise the expression $x^2 - a^2 + 4y^2 + 4xy - b^2 + 2ab$ into the product of two factors. **2**

(b) Find the equation of the straight line which passes through the point of intersection of the two lines $x - y = 1$ and $3x - 2y - 1 = 0$ and makes an angle of 135° with the positive direction of the x axis. **2**

(c) ABC is a triangle. Side BC is trisected at points X and Y.



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(i) **Copy the diagram** and write a formula for $\cos\angle AXB$ in terms of the lengths of AB, AX and BC. **2**

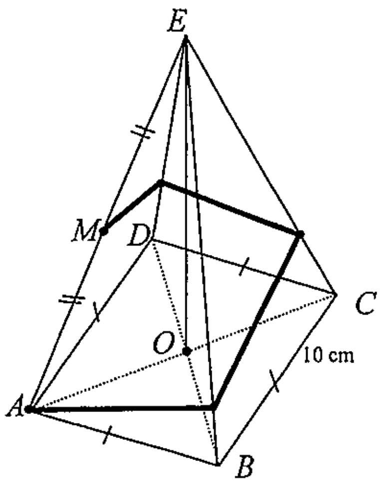
(ii) Hence, or otherwise show that $AB^2 - AC^2 = 3(AX^2 - AY^2)$, giving reasons. **3**

Question 34 continued over page

(d) Two standard 6 sided dice are rolled. The number on the upper face of the first die determines the coefficient b and the number on the upper face of the second die determines the value of c in the quadratic equation $x^2 + bx + c = 0$. **4**

With the aid of a diagram or otherwise determine the probability that the first quadratic equation will have two different real roots.

(e) *ABCDE* is a pyramid with a square base of side length 10 cm and height 12 cm, as shown in the diagram below.



NOT TO SCALE

$OE = 12$ cm

With the use of labelled diagrams:

(i) Find the length of edge *AE*. **2**

(ii) Calculate the size of $\angle AEB$ to the nearest minute. **2**

(iii) A piece of string is attached to point *A* and wound around the pyramid to connect to the midpoint *M*, of edge *AE*. **3**
Find the length of the shortest possible piece of string.
Give your answer to 2 decimal places.

END OF EXAMINATION

YEAR 10 YEARLY
2012 EXAMINATION
ANSWER SHEET

SECTION A: 30 QUESTIONS (1 MARK EACH)

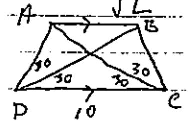
Name:	
Class:	

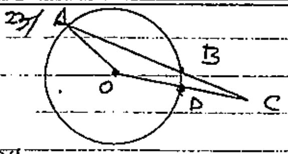
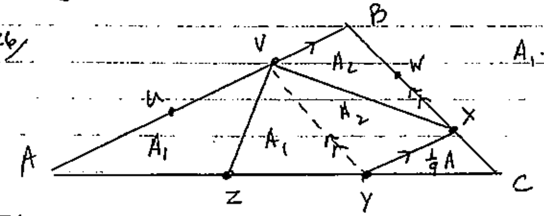
Mark the appropriate answer with a cross X

1	A	B	X (C)	D
2	A	X (B)	C	D
3	A	B	C	X (D)
4	A	B	X (C)	D
5	A	B	X (C)	D
6	X (A)	B	C	D
7	A	B	X (C)	D
8	A	X (B)	C	D
9	X (A)	B	C	D
10	A	X (B)	C	D
11	X (A)	B	C	D
12	A	X (B)	C	D
13	A	X (B)	C	D
14	A	B	C	X (D)
15	X (A)	B	C	D
16	X (A)	B	C	D
17	A	X (B)	C	D
18	A	B	C	X (D)
19	A	X (B)	C	D
20	A	B	C	X (D)
21	A	X (B)	C	D
22	A	B	X (C)	D
23	A	X (B)	C	D
24	A	X (B)	C	D
25	A	B	X (C)	D
26	A	B	C	X (D)
27	A	B	C	X (D)
28	A	B	X (C)	D
29	A	B	X (C)	D
30	A	B	C	X (D)

Question	Mark
Section A	
1 – 30	/ 30
Section B	
31	/ 20
32	/ 20
33	/ 20
34	/ 20
TOTAL	/ 110

HAND IN SEPARATELY AT THE END OF EXAM

1	C	1/ Leading term is $3x^5$ ∴ degree = 5.
2	B	2/  $\angle XYZ = 65° + 180° - 165° = 80°$
3	D	3/ $\angle BOC = 100°$ $\angle OBC = 40°$ $\angle BAC = 50°$ ∴ $5+40+50+x+40 = 180$, $x = 45$
4	C	4/  $y = A(x^2-4)$ sub (0,8) $A = -2$ $y = -2(x^2-4)$; sub (1,a) $a = -2(-3)$ ∴ $a = 6$
5	C	5/ Energy used $= .8 \times .75 \times .45 = 0.27$, used 27% of original ∴ % saved = 73%
6	A	6/ $C = H + \sqrt{\frac{L}{L}}$ ⇒ $(C-H)^2 = \frac{4}{L}$ ∴ $L = \frac{4}{(C-H)^2}$
7	C	7/  $\angle DBC = 90°$ ∴ $BC = 10\cos 60° = 5 = AD$; $AB = 10 - BC\cos 60° - AD\cos 60° = 10 - 2\frac{1}{2} - 2\frac{1}{2} = 5$ ∴ Perimeter $= 10+5+5+5 = 25$ units
8	B	8/ $\frac{3}{2 - x/2} = 2$ ∴ $3 = 4 - x$ ∴ $x = 1$
9	A	9/ $3s = 12$ $s^2 = 4$ ∴ $s = 2$ ∴ $s^3 = 8u^3$
10	B	10/ $5^{-4} = 5^{\frac{48}{x} - \frac{26}{x} - \frac{34}{x}}$ ∴ $-4 = -12/x$ ∴ $x = 3$
11	A	11/ $\{X \cup Y \cap \bar{Z}\}$
12	B	12/ $\cos(90+\alpha) - \cos(90-\alpha) = \cos(180-(90-\alpha)) - \cos(90-\alpha) = -\cos(90-\alpha) - \cos(90-\alpha) = -2\sin\alpha$
13	B	13/ $x^2 - 9x + 14 = (x-7)(x-2)$ ∴ VA $x = 7, x = 2$ HA ⇒ $y = 0$
14	D	14/ $L = mt + b$ $b = 30$ $m = \frac{24-30}{3} = \frac{-6}{3} = -2$. $L = 30 - 2t$
15	A	15/ $y = \frac{x^2-3x+4}{x-2} = (x-1) + \frac{2}{x-2}$; $x-2 \overline{) x^2 - 3x + 4}$ quotient $x-1$: x^2-2x, $-x+4$, $-x+2$, remainder 2. Oblique asymptote $y = x - 1$, VA $x = 2$.
16	A	16/ $2400\,000 \div (100)^3 = 2.4$.
17	B	17/ $71 - 43 = 28 = 28D$ ∴ % $= \frac{95}{2} = 47\frac{1}{2}\%$
18	D	18/ $-4.8 < px < 0$
19	B	19/ $\frac{V_1}{V_2} = \frac{1}{8}$ ∴ $\frac{64}{8} = 8$ new ice cubes/L.
20	D	20/ 6,6,6,8,8,8,8,14 integers, mode = 8, median = 8, mean $= 64/8 = 8$ ∴ 14
21	B	21/ $x^2 + y^2 - 2x + 6y + 3 = x^2 - 2x + 1 + y^2 + 6y + 9 - 7 = (x-1)^2 + (y+3)^2 - 7$ ∴ min value $= -7$
22	C	22/ Test $\frac{1}{2} = x$ $\frac{1}{\sqrt{2}} < 2$

23	B	23/  $AC \cdot BC = DC \cdot (CO + AO)$ $= (CO - DO)(CO + AO)$ $= (CO - AO)(CO + AO)$ $= CO^2 - AO^2$
24	B	24/ $P = ab^2$ $a = 0{\cdot}05$ $P = 4{\cdot}5\times10^{15}$ $b = \sqrt{4{\cdot}5\times10^{15}/0{\cdot}05}$ $b = 3\times10^8$
25	C	25/ $y = 2(2^{x+1}) - 8$ $x=0$ $y = 4-8 = -4$ $y=0$ $8 = 2^{x+2}$ $\therefore x+2=3$, $x=1$ y intercept $y=-4$ x intercept $x=1$
26	D	26/  $A_1 + A_2 = \frac{1}{2}\times\frac{8}{9}A$ $= \frac{4}{9}A$
27	D	27/ $\frac{x}{1+x} < 1$ for $x>0$ as $x < 1+x$.
28	C	28/ (A) H / Prob: 0 1/8, 1 3/8, 2 3/8, 3 1/8. (B) H / Prob: 0 1/16, 1 4/16, 2 6/16, 3 4/16, 4 1/16. $P(0,0)+P(1,1)+P(2,2)+P(3,3)$ $= \frac{1}{8}\times\frac{1}{16}+\frac{3}{8}\times\frac{4}{16}+\frac{3}{8}\times\frac{6}{16}+\frac{1}{8}\times\frac{4}{16}$ $= \frac{35}{128}$
29	C	29/ Assume invest \$100. Interest $= 50\times{\cdot}12 + \frac{100}{3}\times{\cdot}09 + \frac{50}{3}\times{\cdot}06 = 10$ $\therefore$ % interest $= 10\%$
30	D	30/ $\sqrt{\frac{2^{2n+4}+2^{2n+2}}{20}} = \sqrt{\frac{2^{2n}(2^4+2^2)}{20}} = \sqrt{2^{2n}\frac{16+4}{20}}$ $= \sqrt{2^{2n}} = 2^n$

YEAR 10 YEARLY 2012 Qn 31

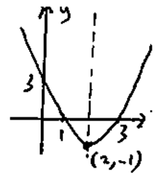
a) $\frac{9^n-1}{3^n-1} = \frac{3^{2n}-1}{3^n-1}$
$= \frac{(3^n)^2-1}{3^n-1}$
$= \frac{(3^n-1)(3^n+1)}{(3^n-1)}$
$= 3^n+1$ 1

b) $(\sqrt{2}-3)^2 - 3(\sqrt{2}-3)$
$= 2+9-6\sqrt{2}-3\sqrt{2}+9$
$= 20-9\sqrt{2}$ 2

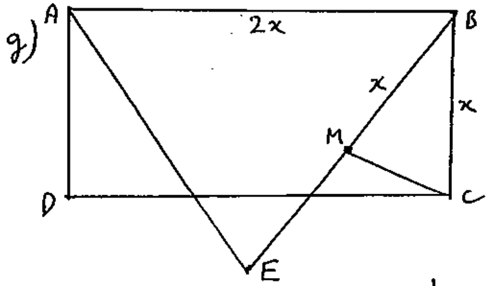
c) Let $u = 2x^2 - x$
$\therefore u^2 = 6u$
$u(u-6)=0$
$u=0$ or $u=6$
$2x^2-x=0$ $\quad$ $2x^2-x=6$
$x(2x-1)=0$ $\quad$ $2x^2-x-6=0$
$(2x+3)(x-2)=0$
$\therefore x = 0,\ \frac{1}{2},\ -\frac{3}{2}$ or 2 3

d) $\frac{(2x-3)^2-4}{2x^2+x-15} = \frac{((2x-3)-2)((2x-3)+2)}{(2x-5)(x+3)}$
$= \frac{(2x-5)(2x-1)}{(2x-5)(x+3)}$
$= \frac{2x-1}{x+3}$ 2

e) Distance of line from centre,
$d = \left|\frac{3\times1 - 4\times(-1)+3}{\sqrt{3^2+4^2}}\right|$
$= \frac{10}{5} = 2$ (units) 2
As this equals the radius of the circle, the line is a tangent to the circle.

f) Consider curve


$y = x^2-4x+3$
$= (x-3)(x-1)$
$\therefore$ Axis of symmetry $x=2$
$\therefore$ Vertex is $(2,-1)$
Parabola is concave up ($a>0$)
$\therefore$ Range is $\{y : y \geq -1\}$ 3

g) 

Let long sides of rectangle be $2x$ and shorter sides x. (given information)
$\therefore BE = 2x$ (ΔABE equilateral – given)
$\therefore BM = x$ (M midpoint – given)
① $\therefore \angle BMC = \angle BCM$ (Equal angles opposite equal sides in ΔBCM)
But $\angle ABE = 60°$ (Angles in equilateral triangle add to 180°)
and $\angle ABC = 90°$ (Rectangle has 90° corners)
$\therefore \angle MBC = 30°$ ($\angle MBC + \angle MBA = 90°$)
② $\therefore \angle BMC + \angle BCM = 150°$ (Angles in ΔBMC add to 180°)
$\therefore \angle BMC = \angle CMB = 75°$ 4
(Combining ① and ②)

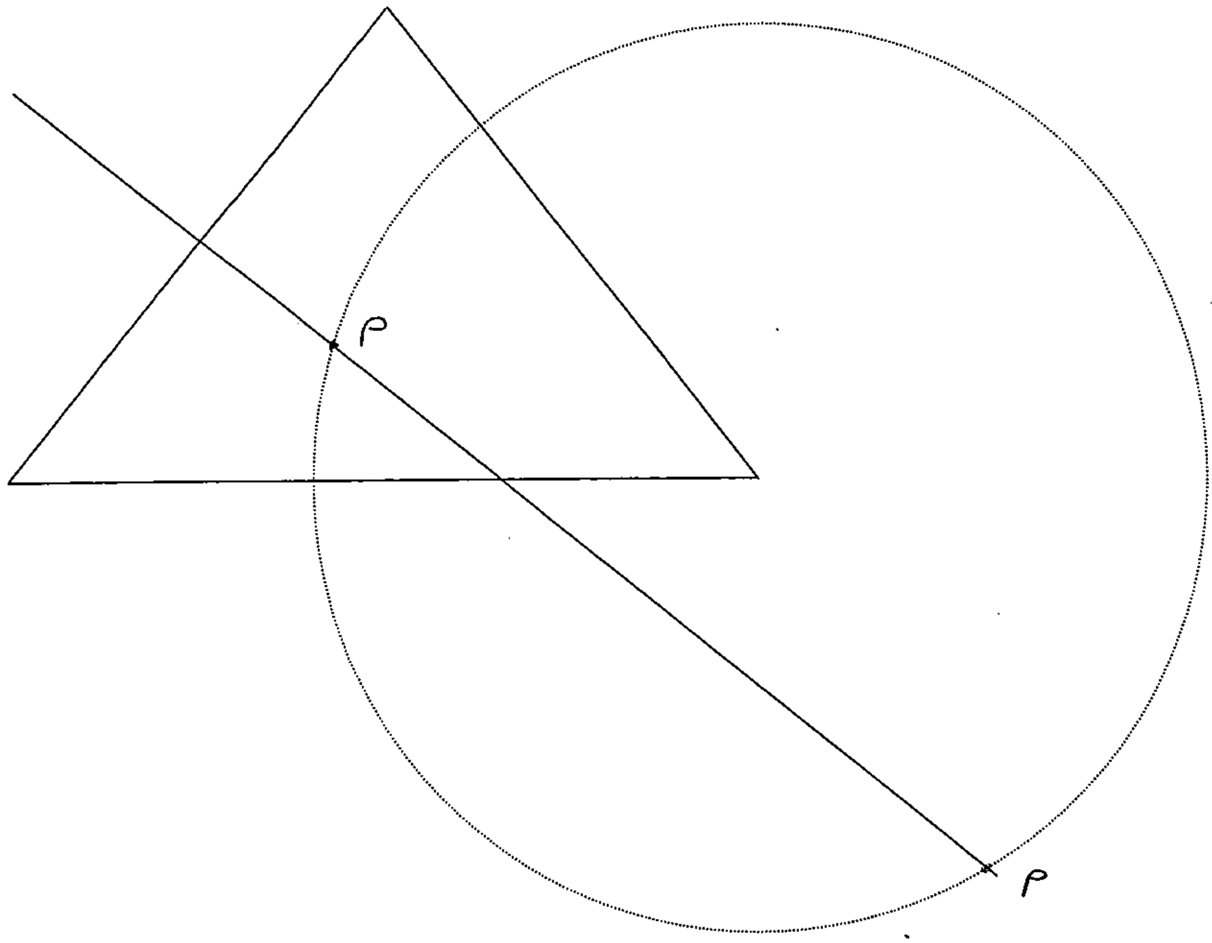
h) i) $r = k\sqrt{\frac{V}{h}}$ 1

ii) $r_2 = k\sqrt{\frac{0.35V}{1.4h}}$
$= \sqrt{\frac{0.35}{1.4}}\ k\sqrt{\frac{V}{h}} = \sqrt{\frac{0.35}{1.4}}\ r_1 = \frac{r_1}{2}$
$\therefore r$ is reduced by 50% 2

ABC is a triangle such that $AB = AC = 8$ cm and $BC = 10$ cm. Using a ruler and compasses only

(i) Construct ΔABC with the base *BC* horizontal.
(ii) Find and label the point(s) which is/are 6 cm from *C* and equidistant from *A* and *B*, using construction. **(Show all construction lines)**

Q32(a).



(f) Construction

(See attached & overlay) — ① — ① method & construction lines

(i) Triangle.

(ii) 2 points - Must construct not just measure. — ② — ½ (2 points). ½ method & construction lines

* no marks allocated in part (a) for accuracy, so no carried forward marks in part (b) if part (a) is inaccurate. — ① Accuracy.

(b) — ③

$\cos^2 x = \sin^2 x$

$1 = \dfrac{\sin^2 x}{\cos^2 x}$

$\tan^2 x = 1$ — ① $\tan^2 x = 1$.

$\tan x = \pm 1$

$x = 45^\circ, 135^\circ, 225^\circ \text{ or } 315^\circ$ — ½ each answer

(c) (i) $P(-,W) = P(RW) + P(WW)$

$= \frac{3}{5}\times\frac{2}{4} + \frac{2}{5}\times\frac{1}{4}$

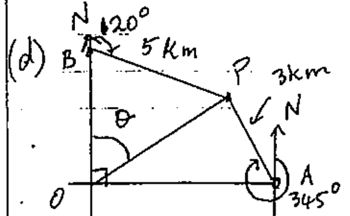
$= \frac{8}{20} = \frac{2}{5}$

(ii) $P(\text{last Red}) = P(RRR) + P(RRWR) + P(RWRR) + P(WRRR)$

$= \frac{3}{5}\times\frac{2}{4}\times\frac{1}{3} + \frac{3}{5}\times\frac{2}{4}\times\frac{2}{3}\times\frac{1}{2} + \frac{3}{5}\times\frac{2}{4}\times\frac{2}{3}\times\frac{1}{2} + \frac{2}{5}\times\frac{3}{4}\times\frac{2}{3}\times\frac{1}{2}$ — ½ Each probability

$= \frac{6}{60} + 3\times\frac{12}{120} = \frac{2}{5}$ — ① answer.

(d) — ② complete



$\angle PBO = 180 - 120$

$= 60^\circ$

angle sum of straight angle. — ① $\angle PBO = 60^\circ$

$\dfrac{OP}{\sin 60^\circ} = \dfrac{5}{\sin\theta}$ (Sine Rule in $\triangle OBP$) — ½ Sine Rule

$\therefore OP = \dfrac{5\sin 60}{\sin\theta}$

$= \dfrac{5\sqrt{3}}{2\sin\theta}$ — ½ Answer.

Question 32.

(iii) In $\triangle OPA$

$\angle PAO = 345^\circ - 270^\circ$ (OA is west).

$= 75^\circ$ — ½ $\angle PAO$ 75°.

$\dfrac{3}{\sin\angle POA} = \dfrac{OP}{\sin\angle PAO}$

$\dfrac{3}{\sin(90-\theta)} = \dfrac{5\sqrt{3}}{2\sin\theta \cdot \sin 75^\circ}$ — ① sine rule

$2\sin\theta \cdot \sin 75 = \cos\theta$

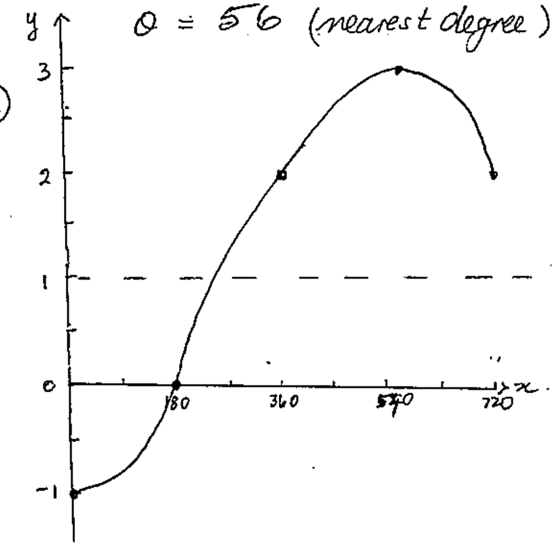
$\tan\theta = \dfrac{5\sqrt{3}}{6\sin 75^\circ}$

$= 1.494292454$ — ① $\tan\theta$

$\theta = 56.2090456$

$\theta = 56$ (nearest degree) — ½ θ nearest degree.

(e)



① end points
½ x intercept

½ axes

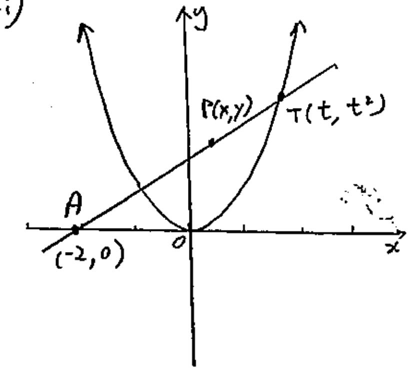
½ max point
½ shape

Yr10 2012 yrly

Yr 10 **MATHEMATICS: Question** 33 P. 1

Suggested Solutions	Marks	Marker's Comments

a) i)



Marks: 1m

ii) For P: $x = \frac{3t-2}{4}$, $y = \frac{3t^2}{4}$ — 1+1 m

iii) $\frac{3t-2}{4} + \frac{3t^2}{4} = 4$

$3t - 2 + 3t^2 = 16$

$3t^2 + 3t - 18 = 0$

$t^2 + t - 6 = 0$ — 1 m

$(t+3)(t-2) = 0 \quad \therefore t = 2$ or -3

when $t = 2$, $x = \frac{6-2}{4} = 1$, $y = \frac{3\cdot 4}{4} = 3$ — 1 m

$t = -3$, $x = \frac{-9-2}{4} = -\frac{11}{4}$, $y = \frac{27}{4}$ — 1 m

$P = \left(-\frac{11}{4}, \frac{27}{4}\right)$ or $(1, 3)$

Marker's comments: many students solve $t = -2$ or 3 so $P = (-2, 3)$ or $\left(\frac{7}{4}, \frac{27}{4}\right)$ max 2m.

b) LHS $= \frac{\sin^4 A - \cos^4 A}{\sin A \cos A}$ $(\frac{1}{2})$ $= \frac{(\sin^2 A + \cos^2 A)(\sin^2 A - \cos^2 A)}{\sin A \cos A}$ $(\frac{1}{2})$

$= \frac{\sin^2 A - \cos^2 A}{\sin A \cos A}$ $(\frac{1}{2})$ $= \frac{\sin A}{\sin A}\cdot\frac{\sin A}{\cos A} - \frac{\cos A}{\cos A}\cdot\frac{\cos A}{\sin A}$ $(\frac{1}{2})$

$= \tan A - \cot A = \text{RHS}$

Marks: 2m. Marker's comments: well done.

MATHEMATICS: Question 33 P. 2

Suggested Solutions	Marks	Marker's Comments

c) $ax^2 + bx + c = 0 \quad \alpha\beta = \frac{c}{a} \quad \alpha + \beta = \frac{-b}{a}$

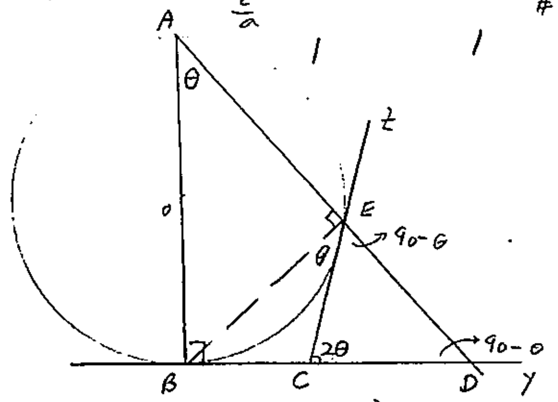
$cx^2 + ax + b = 0 \quad \gamma\delta = \frac{b}{c} \quad \gamma + \delta = \frac{-a}{c}$

i) $(\alpha\beta)(\gamma+\delta) = \frac{c}{a}\left(-\frac{a}{c}\right) = -1$ # — 1+1

ii) $\frac{\alpha+\beta}{\alpha\beta} + \gamma\delta = \frac{-\frac{b}{a}}{\frac{c}{a}} + \frac{b}{c} = -\frac{b}{c} + \frac{b}{c} = 0$ # — 1+1

Marker's comments: well done; no marks for $\frac{\alpha+\beta}{\alpha\beta} + \alpha\beta$ w/o substitution

d)



Marker's comments: without diagram $-\frac{1}{2}$m

Proof AB is diameter (given)

$\therefore \angle ABC = 90°$ (line from centre is perpendicular to tangent at point of contact) — $\frac{1}{2}$

Let $\angle BAD = \theta$, $\therefore \angle ADB = 90 - \theta$ — $\frac{1}{2}$

$\angle AEB = 90°$ (angle in semi-circle is 90°) — $\frac{1}{2}$

$\therefore \angle BED = 180° - 90° = 90°$ (angle sum of straight angle is 180°)

$\therefore \angle BAD = \angle BEC$ (angle between tangent & chord equals angle at circumference in alternate segment) — $\frac{1}{2}$

$\therefore \angle BEC = \theta$

$\therefore \angle DEC = 90 - \theta = \angle ADB$ (or $\angle EDC$) — $\frac{1}{2}$

$\therefore EC = CD$ (sides opposite equal angles equal)

Marker's comments: Several ways to prove. Students not successfully prove $\angle DEC = \angle EDC$ MAX 2m. v. poorly done.

Suggested Solutions	Marks	Marker's Comments
∴ △ECD is isosceles (2 triangles with equal sides	$\frac{1}{2}$m	
e) $\sigma_n = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{\Sigma(x^2-2x\bar{x}+\bar{x}^2)}{n}}$	$\frac{1}{2}$m	v. poorly done
$\sigma_n = \sqrt{\frac{\Sigma(x^2)}{n} - 2\bar{x}\left(\frac{\Sigma x}{n}\right) + \frac{n\bar{x}^2}{n}}$	$\frac{1}{2}$m + $\frac{1}{2}$m	
$\sigma_n = \sqrt{\frac{\Sigma(x^2)}{n} - 2\bar{x}.\bar{x} + \bar{x}^2}$	$\frac{1}{2}$m	
$= \sqrt{\frac{\Sigma(x^2)}{n} - (\bar{x})^2}$ #		
ii) $25.2 = \sqrt{\frac{\Sigma(x^2)}{22} - (62)^2}$		
$635.04 = \frac{\Sigma(x^2)}{22} - 3844$	$\frac{1}{2}$m	
$\Sigma(x^2) = 98538.88$	$\frac{1}{2}$m	
iii) New mean $= \frac{22\times 62 - 86}{17} = \frac{1278}{17}$	$\frac{1}{2}$m	
$= 75.1764\ldots = 75.2$ (1dp)	$\frac{1}{2}$m	→ must show more dp before rounding up
New $\Sigma x^2 = \sqrt{\frac{98538.88 - (75.2)^2}{17}}$	$\frac{1}{2}$m	
$= 7.0176\ldots = 7.0$ (1dp)	$\frac{1}{2}$m	
✓ new $\sigma = \sqrt{\frac{98538.88}{17} - \left(\frac{1278}{17}\right)^2} = 7.263\ldots = 7.3$ (1dp)		

Question 34

a) $x^2 - a^2 + 4y^2 + 4xy - b^2 + 2ab$ — $\frac{1}{2}$ Some grouping or correct factorisation

$= x^2 + 4xy + 4y^2 - (a^2 - 2ab + b^2)$

$= (x+2y)^2 - (a-b)^2$ — $\frac{1}{2}$ Correct GROUPING.

$= \{x+2y-(a-b)\}\{x+2y+a-b\}$

$= (x+2y-a+b)(x+2y+a-b)$ — 1 Correct Result.

b) $x - y = 1 \quad 3x - 2y - 1 = 0$

$\left.\begin{array}{l} 3x - 2y = 1 \\ x - y = 1 \end{array}\right\} \Rightarrow \begin{array}{l} 3x - 2y = 1 \\ 2x - 2y = 2 \end{array}$

Subtract $x = -1$

Substitute -1 for x $\therefore -1 - y = 1$
$-2 = y$

Point of intersection is $(-1, -2)$ — 1 Point of intersection

$m = \tan 135^\circ$
$= -\tan 45^\circ$
$= -1$ — $\frac{1}{2}$ Correct Gradient.

$\therefore$ Required Equation is
$y - (-2) = -1(x - -1)$
$y + 2 = -x - 1$
$x + y + 3 = 0$ — $\frac{1}{2}$ Correct Equation

$3x - 2y - 1 + k(x - y - 1) = 0$ — $\frac{1}{2}$ Correct Family of Lines
$3x - 2y - 1 + kx - ky - k = 0$
$(3+k)x - (2+k)y - 1 - k = 0$
$(3+k)x - 1 - k = (2+k)y$

$\therefore m = \frac{3+k}{2+k}$ — $\frac{1}{2}$ ① to obtain

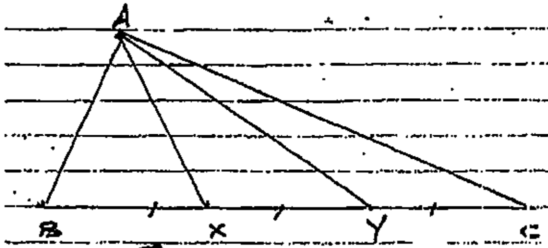
$m = \tan 135^\circ \Rightarrow m = -1$ — $\frac{1}{2}$ Correct k value

$\therefore \frac{3+k}{2+k} = -1$

$3 + k = -2 - k \Rightarrow 2k = -5$
$k = -\frac{5}{2}$

If k value incorrect but is a negative fraction must get appropriate (correct) answer MAX $1\frac{1}{2}$

$\therefore 3x - 2y - 1 - \frac{5}{2}(x - y - 1) = 0$
$6x - 4y - 2 - 5x + 5y + 5 = 0$
$x + y + 3 = 0$ — $\frac{1}{2}$ Correct Answer.



Recognising that
$BX = \frac{1}{3} BC$ — $\frac{1}{2}$ Stated or in solution.

In $\triangle ABX$, $\cos A\hat{X}B = \frac{AX^2 + BX^2 - AB^2}{2AX \cdot BX}$ — $\frac{1}{2}$ Correct Statement of Cosine Rule

$\therefore \cos\angle AXB = \frac{(AX)^2 + \left(\frac{BC}{3}\right)^2 - AB^2}{2AX \cdot \frac{BC}{3}}$ — 1 Correct Result.

Similarly in:
In $\triangle AXY$
$\cos(180^\circ - \angle AXB) = \frac{AX^2 + \left(\frac{BC}{3}\right)^2 - AY^2}{2AX \cdot \left(\frac{BC}{3}\right)}$

Note $\cos(180^\circ - \angle AXB) = -\cos\angle AXB$ — 1 mark stated or implied

$\therefore \frac{AX^2 + \frac{BC^2}{9} - AB^2}{\frac{2}{3}AX \cdot BC} = \frac{-(AX^2 + \left(\frac{BC}{3}\right)^2 - AY^2)}{\frac{2}{3}AX \cdot BC}$ — $\frac{1}{2}$ Recognise Equation and Simplify

$\therefore AX^2 + \frac{BC^2}{9} - AB^2 = -AX^2 - \frac{BC^2}{9} + AY^2$

i.e $2AX^2 + \frac{2}{9}BC^2 = AB^2 + AY^2 \quad \ldots 1)$

Similarly in $\triangle$ AXY and $\triangle$ AYC — $\frac{1}{2}$ Recognise Similarity
$2AY^2 + \frac{2BC^2}{9} = AC^2 + AX^2 \quad \ldots 2)$

Subtracting 2 from 1 — $\frac{1}{2}$ Correct Result.
$2AX^2 - 2AY^2 = AB^2 + AY^2 - AC^2 - AX^2$ OR EQUIVALENT
$3AX^2 - \quad = AB^2 - AC^2$

c)

Second Die

	1	2	3	4	5	6
FIRST DIE 1						
2						
3	✓	✓				
4	✓	✓	✓	=		
5	✓	✓	✓	✓	✓	✓
6	✓	✓	✓	✓	✓	✓

1 for correct S
1 for some explanation
1 for b^2-4ac i.e. $b^2-4c>0$ OR equivalent $b^2>4c$

$P(x^2+bx+c=0$ has different roots$) = \frac{17}{36}$

$\frac{1}{2}$ for $n(E)=17$
$\frac{1}{2}$ for $n(S)=36$

d) $AC^2 = AB^2+BC^2$ (Pythagoras)
$= 10^2+10^2$
$AC=\sqrt{200} \Rightarrow AC=10\sqrt{2}$ cm
$OA = \frac{1}{2}AC \Rightarrow OA = 5\sqrt{2}$ cm
In $\triangle AOE$
$AE^2 = AO^2+OE^2$ (Pythagoras)
$=(5\sqrt{2})^2+12^2$
$= 50+144$
$AE = \sqrt{194}$

Allow 1 mark for Error in finding OA but proceeds to find their correct AE i.e. CFPA.

(2) for correct AE.

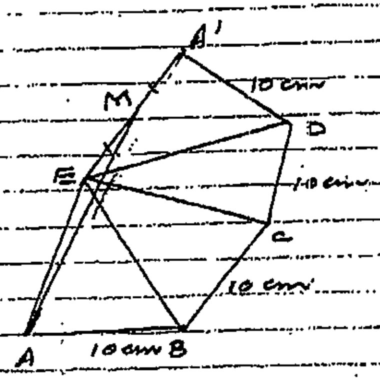
(ii) In $\triangle ABE$, $AE=EB=\sqrt{194}$
$AB = 10$ cm
$\cos\angle AEB = \dfrac{(\sqrt{194})^2+(\sqrt{194})^2-10^2}{2\times\sqrt{194}\times\sqrt{194}}$

Correct (1) Cosine Rule with correct or their AE from (i)

$\angle AEB = 42.07502205$
$= 42°5'$ (nearest minute)

(1) Correct Answer Rounded to nearest minute.

d (iii)



1 Diagram + Method OR Idea

$\angle AEM = 4\times\angle AEB$
$= 4\times 42.07502205°$
$= 168.3000882°$
$= 168°18'$

$(\frac{1}{2})$ Correct $\angle AEM$.

$EM = \frac{1}{2}EA' = \frac{1}{2}\sqrt{194}$

$(\frac{1}{2})$ Uses Cosine Rule

$AM^2 = (\sqrt{194})^2 + (\frac{1}{2}\sqrt{194})^2 - 2\sqrt{194}\times\frac{1}{2}\sqrt{194}\cos 168°18'$
$= 432.4692859$

$AM = 20.79589589$
$= 20.80$ (2 dec. pl.)

1 mark for answer. No mark off for rounding

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