| Name: |
| --- |
| Maths Class: |

# YEARLY EXAMINATION

# YEAR 10 2013

# MATHEMATICS

*Time Allowed – 120 minutes*

*(Plus 5 minutes Reading time)*

*INSTRUCTIONS:*

- All questions may be attempted
- Write your **name** and **Maths class** at the top of *each page.*
- **Write** in **Pen** and draw diagrams in **Pencil**
- Answers to Multiple choice Questions 1-10 are to be entered onto the answer sheet provided
- Answers to Sections B-F are to be returned in separate bundles
- Department of Education approved calculators and templates are permitted
- Show all necessary working
- Marks may not be awarded for untidy or carelessly arranged work
- No grid paper is to be used unless provided with the examination paper

## Section A (1 mark each)

### Choose the best answer and mark it on the answer sheet provided.

1. The numbers $A = 5 \cdot 6 \times 10^{-2}$, $B = 0 \cdot 93 \times 10^{-1}$ and $C = 71 \times 10^{-3}$, listed in ascending order is:

(A) *A, B, C* (B) *B, A, C* (C) C, *A, B* (D) *A, C, B*

2. Bob, Mary and Tom have a race to reach the canteen at recess time. Bob and Mary are each twice as likely as Tom to be first of the three to get to the canteen.
What is the probability that Tom will be the first of the three to reach the canteen?

(A) $\frac{1}{6}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{1}{3}$

3. $\left(1 \cdot 2 \times 10^{3}\right)^{2} =$

(A) $1 \cdot 44 \times 10^{5}$ (B) $1 \cdot 44 \times 10^{6}$ (C) $1 \cdot 44 \times 10^{8}$ (D) $1 \cdot 44 \times 10^{9}$

4. The arc length of a sector of a circle, of radius 10cm, is 12 cm.
The angle subtended by the arc is closest to:

(A) 47° (B) 48° (C) 68° (D) 69°

5. The roof of a building has $1 \cdot 056$ million tiles. If each tile covers 175 $cm^2$, the area covered by the tiles is:

(A) $184 \cdot 8$ $m^2$ (B) 18 480 $m^2$ (C) 184 800 $m^2$ (D) 1 848 000 $m^2$

6. $\dfrac{\cos^2 x° - 1}{\sin(90° - x°) - 1} =$

(A) $\cos(x°)$ (B) $\cos(x°) - 1$ (C) $\cos(x°) + 1$ (D) $\text{cosec}(x°) + 1$

7. A data set of nine scores has a median of 7. The scores of 6, 6, 12, and 17 are added to this data set.
The median of the data set is now:

(A) 6 (B) 7 (C) 8 (D) 9

8. The number of terms in the series $\sum_{r=28}^{57}(2r+1)$ is

(A) 28 (B) 29 (C) 30 (D) 57

**Section A continued on next page**

**Section A continued**

9. Given that a curve has equation $y = 2 + \dfrac{x^2 + 3x + 2}{x^2 - 3x + 2}$, the asymptotes of this curve are:

(A) $x = 1, x = 2, y = 3$
(B) $x = 1, x = 2, y = 2$
(C) $x = -1, x = -2, y = 3$
(D) $x = -1, x = -2, y = 2$

10. The relationship between two variables is $A = kB^3$.
Which of the following graphs would produce a straight line?

(A) $A^3$ plotted against $B^3$
(B) $A$ plotted against $\sqrt[3]{B}$
(C) $A^3$ plotted against $B$
(D) $\sqrt[3]{A}$ plotted against $B$

## Section B (20 Marks) Start a new page

**Marks**

(a) Expand and simplify **2**

$$\left(3-\sqrt{5}+\sqrt{2}\right)\left(3-\sqrt{5}-\sqrt{2}\right)$$

(b) Make $a$ the subject of the formula: $x = \dfrac{c}{\left[\dfrac{1}{a}+\dfrac{2}{b}\right]}$ **2**

(c) Solve the equation for $x$: $2^{2x+1} + 2^{2x-1} = 20$ **3**

(d) Sketch the graph of $y = (x+1)^3(x-3)$. Label all intercepts. **3**

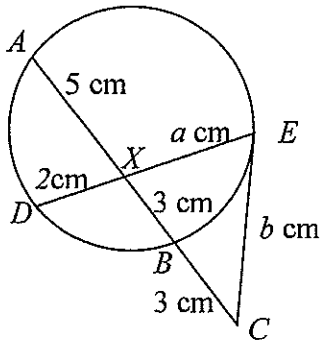
(e) If $\alpha$ and $\beta$ are the roots of the equation $2x^2 + 4x + 1 = 0$, find the values of:

(i) $\alpha^2\beta + \alpha\beta^2$ **2**

(ii) $\dfrac{1}{\alpha\beta^3} + \dfrac{1}{\alpha^3\beta}$ **3**

(f) Four points on the number plane are $A(3, 4)$, $B(5,-5)$, $C(-5, -4)$ and $D(-7,5)$. **3**
Show that $ABCD$ is a parallelogram, giving reasons.

(g) In the diagram below $EC$ is a tangent to the circle. **2**




Not to scale

Calculate the values of $a$ and $b$. (Reasons are not required)

## Section C (20 Marks) Start a new page

**Marks**

(a) Solve the equation $2\cos x° = \cot x°$ for $0 \le x \le 360$ **3**

(b) (i) On the same set of axes, sketch the graphs of the functions **4**

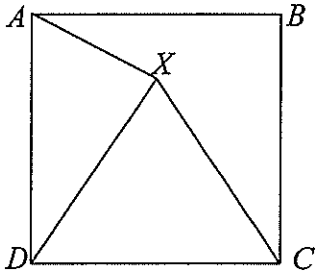
$$y = 3\sin 2x \text{ and } y = \cos^2 x,\ -180° \le x \le 180°$$

(ii) State the number of solutions to the equation $3\sin 2x - \cos^2 x = 0$ for $-180° \le x \le 180°$ **1**

(c) A boat starts at port $A$ and travels in a direction of 222°T until it reaches point $B$. From point $B$ two buoys $X$ and $Y$ are seen in a direction due South of $B$. $Y$ is known to be 200 m due South of $X$. The boat continues on the same course until it reaches point $C$. From point $C$ the bearing of $X$ is 140°T and $Y$ is 165°T.

Draw a diagram showing all of the information given above. **3**

(d) $ABCD$ is a square and $CDX$ is an equilateral triangle as shown in the diagram below. **4**




Not to scale

Copy the diagram and calculate the size of $\angle XAD$, giving reasons.

(e) A sequence is defined by $t_1 = 3$ and $t_{n+1} = \dfrac{1+3t_n}{3+t_n}$ for all integers $n \ge 1$. **2**

Evaluate $t_3$.

(f). Ben is part of a large group of students whose scores on their Mathematics and English tests were normally distributed. Ben's results are shown in the table below.

| Subject | Mean | Standard Deviation | Ben's Mark |
|---|---|---|---|
| English | 50 | 15 | 67 |
| Mathematics | 55 | 12 | 68 |

(i) In which subject did Ben do better? Justify your answer. **2**

(ii) What is the equivalent mark in English to a Mathematics mark of 87? **1**

## Section D (20 Marks) Start a new page

**Marks**

(a) Solve the equation for $x$: **3**

$$\left(\sqrt{x}+1\right)^2-\left(\sqrt{x}+1\right)=6$$

(b) The sum of the first $n$ terms of a sequence is given by the formula

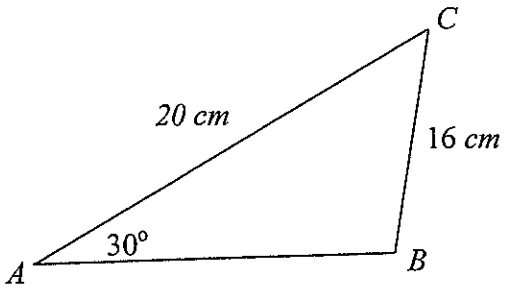
$$S_n=\frac{n}{2(n+2)} \text{ for } n\geq 1.$$

(i) Find the formula for the $n^{th}$ term, ( $t_n$ ), of the sequence. **2**

(ii) Hence or otherwise evaluate: $\sum_{r=1}^{50}\frac{10}{(r^2+3r+2)}$. **2**

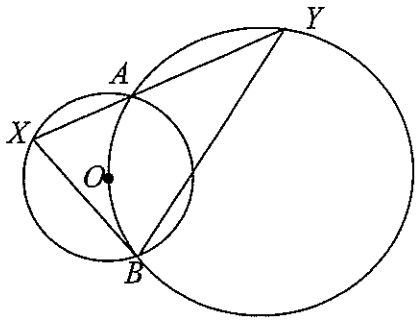
(c) Prove the following identity: $\frac{\sin x}{1+\cos x}+\frac{1+\cos x}{\sin x}=\frac{2}{\sin x}$ **2**

(d) Calculate the size of $\angle ABC$ to the nearest minute, in the triangle below, if it is known that $\angle ACB$ is acute. **3**




Not to scale

(e) $O$ is the centre of the circle through points $A$, $B$ and $X$. $XY$ passes through point $A$, as shown in the diagram below. **4**




Not to scale

Copy the diagram and prove that $BY=XY$, giving reasons.
(Hint: Let $\angle YXB=x°$ ).

(f) (i) Calculate the exact area of an equilateral triangle which has a side length of 3 cm. **2**

(ii) A point is selected at random inside the triangle. **2**
Find the probability that the distance from the point to any vertex is less than 1 cm.

## Section E (20 Marks) Start a new page

**Marks**

(a) The value ($\$V$) of a type of gemstone is proportional to the square of its weight ($w$ carats). A jeweler buys a particular gemstone which is worth \$450 in its original form. However, he accidently breaks it into two pieces with weights in the ratio 1:2. **3**

Calculate the jeweler's total loss when he sells these two gemstones separately.

(b) (i) Line $\ell$ has the equation: $2x-5y+1=0$. What is the gradient of line $\ell$? **1**

(ii) Find the value of $k$ so that the equation $x-4y+7+k(4x+y-10)=0$ describes a line which is parallel to line $\ell$. **2**

(c) (i) The point $P(1, y)$ divides the interval joining $A(-5, 2)$ and $B(3, -1)$ in the ratio $m$:1. **2**

Find the value of $m$.

(ii) What is the $y$ coordinate of $P$? **1**

(d) Solve the following simultaneous equations **4**

$3x-2y=1$ and $2y^2-3x^2=5$

(e) In certain town 40% of the people have brown hair, 25% have brown eyes, and 15% have brown hair and brown eyes.

(i) Draw a Venn diagram to represent this data. **2**

(ii) A person is selected at random from the town. **2**

($\alpha$) If he has brown hair, what is the probability that he also has brown eyes?

($\beta$) What is the probability that he has neither brown hair nor brown eyes?

(f) A tank can be filled by two pipes in 80 minutes. If the two pipes are used separately, the larger pipe will take 120 minutes less to fill it than the smaller pipe. **3**

How long will it take to fill the tank using only the smaller pipe?

## Section F (20 Marks) Start a new page

**Marks**

(a) For a particular pair of six sided dice the probabilities that the numbers 1, 2, 3, 4, 5, 6 appear on the upper most face of each die are in the ratio 1:2:3:4:5:6. What is the probability of rolling a total of 11 on the two dice? **2**

(b) When the polynomial $P(x) = x^3 - ax^2 + x$ is divided by $(x-3)$ the remainder is 12. Find the remainder when $P(x)$ is divided by $(x+1)$. **2**

(c) (i) Find the values of $y$ so that the quadratic equation $x^2 + (y-1)x + y = 0$ has real roots. **3**

(ii) Hence or otherwise find the range of the function $y = \dfrac{x - x^2}{x+1}$. **1**

Give your answer to 1 decimal place.

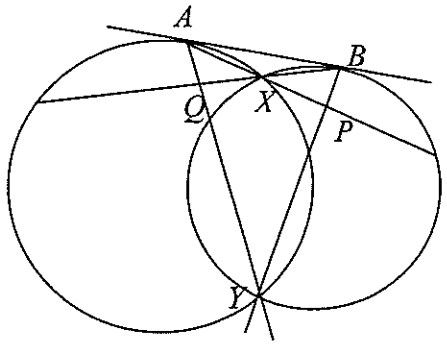
(iii) Find the oblique asymptote for the graph of the function $y = \dfrac{x - x^2}{x+1}$. **2**

(iv) Sketch the graph of $y = \dfrac{x - x^2}{x+1}$ for $-3 \le x \le 4$. **3**

Label all intercepts and asymptotes.

**Use a scale of 1cm = 1 unit for both the $x$ and $y$-axes.**

(d) Two circles intersect at $X$ and $Y$. $AB$ is a common tangent. The lines $AX$ and $BY$ meet at $P$. The lines $AY$ and $BX$ meet at $Q$ as shown in the diagram below.




Not to scale

(i) Use the **attached** diagram and prove that $\angle QXP + \angle QYP = 180°$, giving reasons. **4**
**Attach the diagram to your Section F answers.**

(ii) Give a reason why $QXPY$ is a cyclic quadrilateral. **1**

(iii) Hence or otherwise prove that $QP$ is parallel to $AB$. **2**

# End of Examination

# YEAR 10 YEARLY EXAMINATION /2013

## ANSWER SHEET

| | |
|---|---|
| **Name:** | |
| **Class:** | |

Mark the appropriate answer with a cross X

**SECTION A: 10 QUESTIONS (1 MARK EACH)**

| | | | | |
|---|---|---|---|---|
| 1 | A | B | C | ~~D~~ |
| 2 | A | ~~B~~ | C | D |
| 3 | A | ~~B~~ | C | D |
| 4 | A | B | C | ~~D~~ |
| 5 | A | ~~B~~ | C | D |
| 6 | A | B | ~~C~~ | D |
| 7 | A | ~~B~~ | C | D |
| 8 | ~~A~~ | B | ~~C~~ | D |
| 9 | ~~A~~ | B | C | D |
| 10 | A | B | C | ~~D~~ |

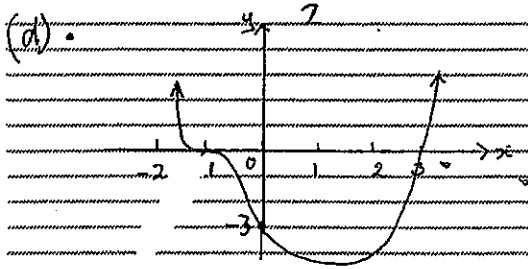
To be completed by teacher

| Outcome | A | B | C | D | E | F | Total |
|---|---|---|---|---|---|---|---|
| Number | | | | | | | |
| Measurement | | | | | | | |
| Algebra | | | | | | | |
| Space | | | | | | | |
| Statistics | | | | | | | |
| | /10 | /20 | /20 | /20 | /20 | /20 | /110 |

**MATHEMATICS : Question A**

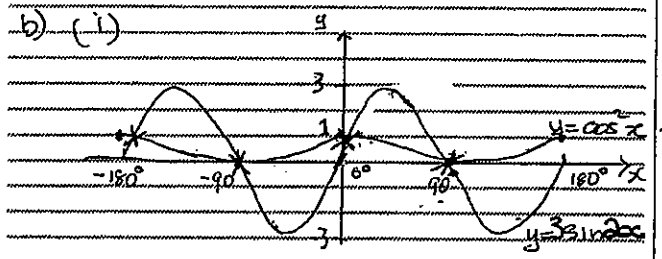
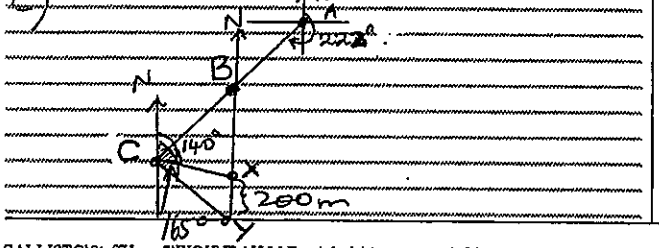
| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| 1 Ascending order: $A\ (6.16\times10^{-25})\ C\ (7.1\times10^{-2})\ B\ (9.1\times10^{7})$ | ANSWER D | |
| 2. $P(T)=x$ $\therefore 2x+2x+x=1$ $\therefore x=\frac{1}{5}$ $P(T)=\frac{1}{5}$ | B | |
| 3. $(1.2\times10^{3})^2 = 1.44\times10^{6}$ | B | |
| 4. $\frac{12}{214\times10}\times360^{\circ} = 68.755\ldots \doteq 69^{\circ}$ | D | |
| 5. $\frac{1.056\times10^{6}}{10000}\times175 = 18480$ | B | |
| 6. $\frac{\cos^2x^{\circ}-1}{\sin(90^{\circ}-x^{\circ})-1} = \frac{(\cos x-1)(\cos x+1)}{\cos x-1}$ $= \cos x^{\circ}+1$ | C | |
| 7. x x x x 6 x x x x $\quad 6\ 6\ 7\ 12\ 17$ Median remains at 7 | B | |
| 8. $\sum_{r=28}^{57}(2r+1)$ no. of terms $= 57-28+1 = 30$ | C | |
| 9. $y = 2+\frac{(x+1)(x+2)}{(x-1)(x-2)}$ Asymptotes $y=3$ $x=1$ $x=2$ | A | |
| 10. $A=RB^3$ $\therefore \sqrt[3]{A}=RB$ | D | |

**MATHEMATICS** — Question: SECTION B

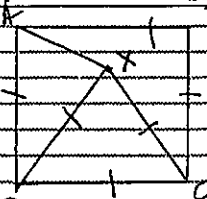
| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| (a) $\frac{(3-\sqrt{5}+\sqrt{2})(3-\sqrt{5}-\sqrt{2})}{(3-\sqrt{5})^2-2} = \frac{9-6\sqrt{5}+5-2}{12-6\sqrt{5}}$ | 2 | ① multiplication<br>① answer |
| (b) $x = \frac{c}{\left[\frac{1}{a}+\frac{2}{b}\right]}$<br>$\frac{x}{a}+\frac{2x}{b} = c$<br>$\frac{x}{a} = \frac{c-2x}{b}$<br>$\frac{x}{a} = \frac{bc-2x}{b}$<br>$\frac{1}{a} = \frac{bc-2x}{xb}$ $\therefore a = \frac{xb}{bc-2x}$ | 2 | ① cross multiply (several methods)<br>① answer. |
| (c) $2^{2x+1} + 2^{2x-1} = 20$<br>$2^{2x}\left[2+\frac{1}{2}\right] = 20$<br>$2^{2x}\left[\frac{5}{2}\right] = 20$<br>$2^{2x} = 8$<br>$x = \frac{3}{2}$ | 3 | ① factorise<br>① simplify<br>① answer |
| (d)  | 3 | ① x intercepts<br>½ y intercept<br>½ shape at $x=-1$<br>½ general shape<br>½ axes + labels. |
| (e) (i) $2x^2+4x+1=0$ $a=2$ $b=4$ $c=1$<br>$\alpha+\beta = -\frac{4}{2} = -2$ $\alpha\beta = \frac{1}{2}$.<br>$\alpha^2\beta+\alpha\beta = \alpha\beta(\alpha+\beta) = -2\times\frac{1}{2} = 1$ | ② | ① $\alpha\beta$, $\alpha+\beta$<br>① answer |
| (ii) $\frac{1}{\alpha\beta^3}+\frac{1}{\alpha^3\beta} = \frac{\alpha^4+\beta^4}{\alpha^3\beta^3} = \frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^3}$<br>$= \frac{(-2)^2-2\times\frac{1}{2}}{(\frac{1}{2})^3} = \frac{4-1}{\frac{1}{8}} = 3\times 8 = 24$ | ③ | ① $\alpha^2+\beta^2=3$<br>① simplifying $\frac{1}{\alpha\beta^3}+\frac{1}{\alpha^3\beta}$.<br>① answer. |



**MATHEMATICS** — Question: Section B

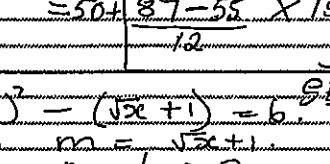
| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| (f) $M_{AC} = \left[\frac{3-5}{2}, \frac{4-4}{2}\right] = (-1,0)$<br>$M_{BD} = \left[\frac{5-7}{2}, \frac{-5+5}{2}\right] = (-1,0)$<br>Diagonals have same midpoint<br>∴ Diagonals bisect each other<br>∴ parallelogram. | 3 | Many methods.<br>② calculations<br>① answer.<br>$-\frac{1}{2}$ for shorthand. |
| (g) (i) $2a = 5\times 3$ $\therefore a = 7\frac{1}{2}$. | 2 | ① answer for a<br>① answer for b.<br>$b=\sqrt{66}$ ½ mark |
| (ii) $b^2 = 11\times 3 = 33$ $b = \sqrt{33}$ $b>0$ | | |
| (a) $2\cos x^\circ = \cot x^\circ$ $0\le x\le 360$<br>$2\cos x^\circ = \frac{\cos x^\circ}{\sin x^\circ}$<br>$\cos x^\circ[2\sin x^\circ - 1] = 0$<br>$\cos x^\circ = 0$ or $\sin x^\circ = \frac{1}{2}$<br>$x = 90^\circ$ or $270^\circ$ or $x = 30^\circ$ or $150^\circ$ | 3 | ① simplify equation<br>① 90°, 270°<br>① 30°, 150°. |
| (b) (i)  | 4 | ② each graph.<br>① x intercepts<br>① y intercepts and shape. } × ② |
| (ii) 4 points of intersection<br>∴ 4 solutions. | 1 | Correct answer or CFE. |
| (c)  | 3 | ① showing 222° and N↑<br>① X, Y South of B and XY = 200<br>① angles 140° and 165° |

**MATHEMATICS** Question: Section C.

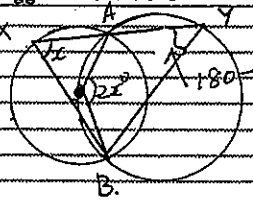
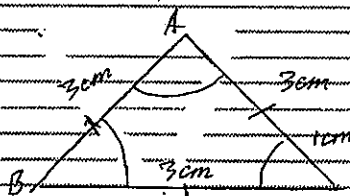
| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| (a)   | (4) | |
| $\angle XDC = 60^\circ$ (angle of equilateral triangle) vertex | | |
| $\angle ADC = 90^\circ$ (vertex angle of square) | | ① |
| $\angle ADX + \angle XDC = 90^\circ$ | | |
| $\therefore \angle ADX = 30^\circ$ | | |
| $\angle AXD = \angle XAD$ (equal angles opposite equal sides in $\triangle AXD$) | | ① |
| $\angle AXD + \angle XAD + 30^\circ = 180^\circ$ (angle sum of $\triangle AXD$ is $180^\circ$) | | |
| $\therefore 2\angle XAD = 150$ | | ① |
| $\angle XAD = 75^\circ$ | | |
| (i) $t_1 = 3$ $t_{n+1} = \frac{1+3t_n}{3+t_n}$ $n \geq 1$ | ② | |
| $t_2 = \frac{1+3t_1}{3+t_1} = \frac{1+9}{3+3} = \frac{10}{6} = \frac{5}{3}$ | | ① |
| $t_3 = \frac{1+3t_2}{3+t_2} = \frac{1+5}{3+5/3} = \frac{18}{14}$ | | ① |
| $= 9/7$ | | |
| (b) (i) $z_{ENGLISH} = \frac{67-50}{15} = \frac{17}{15} = 1.13$ | ② | ½ each calculation |
| $z_{MATHS} = \frac{68-55}{12} = \frac{13}{12} = 1.083$ | | ½ conclusion |
| $\therefore z_{maths} < z_{English}$ | | ½ reason |
| $\therefore$ Ben did better in English | | (Accept SD's above mean instead of z-scores) |
| (ii) English equivalent mark | ① | |
| $= 50 + \frac{87-55}{12} \times 15 = 90$ | | ① correct answer. |
| **SECTION B** | | |
| (a) $(\sqrt{x}+1)^2 - (\sqrt{x}+1) = 6$. | ③ | ① sub + solve quadratic eqn. |
| sub $m = \sqrt{x}+1$. | | |
| $m^2 - m - 6 = 0$ | | ① $x = 4$ |
| $(m-3)(m+2) = 0$ $\therefore m = 3$ or $-2$. | | |
| $\sqrt{x}+1 = 3$ or $\sqrt{x}+1 = -2$ | | ① reject $\sqrt{x} = -3$. |
| $\sqrt{x} = 2$ or $\sqrt{x} = -3$ no solution | | |
| $x = 4$ | | |



**MATHEMATICS** Question: Section D.

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| (b) $S_n = \frac{n}{2(n+2)}$ $n \geq 1$ | ② | |
| (i) $t_n = \frac{n}{2(n+2)} - \frac{(n-1)}{2(n+1)}$ | | ① $t_n = S_n - S_{n-1}$ |
| $= \frac{n(n+1) - (n-1)(n+2)}{2(n+2)(n+1)}$ | | |
| $= \frac{n^2+n-n^2-n+2}{2(n^2+3n+2)}$ | | ① answer |
| $= \frac{1}{n^2+3n+2}$ | | |
| (ii) $\sum_{r=1}^{50} \frac{10}{r^2+3r+2} = 10S_{50}$ | | ① $10S_{50}$ |
| $= \frac{500}{2(52)}$ | | |
| $= \frac{250}{52}$ | | ① Answer |
| (c) $\frac{\sin x}{1+\cos x} + \frac{1+\cos x}{\sin x} = \frac{2}{\sin x}$ | ② | |
| LHS $= \frac{\sin^2 x + (1+\cos x)^2}{\sin x(1+\cos x)}$ | | |
| $= \frac{\sin^2 x + 1 + 2\cos x + \cos^2 x}{\sin x(1+\cos x)}$ | | ① common denominator and multiply out |
| $= \frac{1+1+2\cos x}{\sin x(1+\cos x)}$ | | |
| $= \frac{2+2\cos x}{\sin x(1+\cos x)}$ | | ① complete identity |
| $= \frac{2(1+\cos x)}{\sin x(1+\cos x)} = \frac{2}{\sin x} = $ RHS. | | |
| (d)   | ③ | ① apply sine Rule. |
| $\frac{\sin\theta}{20} = \frac{\sin 30}{16}$ | | |
| $\sin\theta = \frac{20}{32}$ | | ① $30^\circ 41'$ |
| $\theta = 38^\circ 41'$ or $141^\circ 19'$. | | |
| as $\angle ACB < 90^\circ$ $\theta \neq 38^\circ 41'$ | | ① $141^\circ 19'$ as correct answer. |
| $\therefore \angle ABC = 141^\circ 19'$ only | | |

**MATHEMATICS** : Question... Section D.

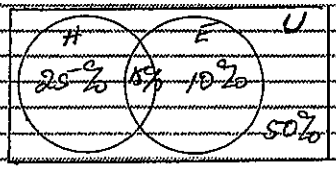
| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| a)   | (4) | + Reasons. |
| Let $\angle YXB = x^\circ$ | | ① $\angle AXB = 2x^\circ$ |
| $\therefore \angle AXB = 2x^\circ$ (angle at centre exhibits twice angle at circumference standing on the same arc) | | ① $\angle AYB = 180 - 2x^\circ$ |
| $\angle AYB + 2x^\circ = 180^\circ$ (angle sum of cyclic quad AOBY is $180^\circ$) | | |
| $\therefore \angle AYB = 180^\circ - 2x^\circ$ | | |
| $x + \angle YBX + 180 - 2x^\circ = 180^\circ$ (angle sum of $\triangle YBX$ is $180^\circ$) | | ① $\angle YBX = x^\circ$ |
| $\therefore \angle YBX = x^\circ$ | | |
| $\therefore \angle XBX = \angle YXB = x^\circ$ | | ① |
| $\therefore BY = XY$ equal angles opposite equal sides in $\triangle AYBY$. | | |
| b) (i)   | (2) | and angles. |
| $AB = BC = AC$ equal sides in equilateral triangle | | ① |
| $\angle ABC = \angle ACB = \angle BAC = 60^\circ$ | | Sine Rule. |
| Area $= \frac{1}{2} \times 3 \times 3 \times \sin 60$ | | ① Answer. |
| $= \frac{1}{2} \times 9 \times \frac{\sqrt{3}}{2} = \frac{9\sqrt{3}}{4}\ cm^2$ | | |
| (ii) Area less than 1cm from vertex $= \frac{1}{2} \pi \times 1^2 = \frac{\pi}{2}$. | (2) | ① area of semicircle |
| $\therefore P(< 1cm \text{ from vertex}) = \frac{\frac{\pi}{2}}{\frac{9\sqrt{3}}{4}} = \frac{2\pi}{9\sqrt{3}}$ | | ① answer. (CFE allowed). |



**MATHEMATICS** : Question... SECTION E

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| (a) $V = kW^2$ | (3) | |
| $450 = kW^2$ | | |
| $\frac{450}{W^2} = k$ | | |
| $V_1 = k\left(\frac{W}{3}\right)^2$ (piece 1) $\frac{1}{3}$ of W | | ① \$50 for $W_1$ |
| $= \frac{450}{W^2} \times \frac{W^2}{9}$ | | |
| $= 50$ $\therefore$ \$50. | | |
| $V_2 = k\left(\frac{2W}{3}\right)^2$ Piece 2 $\frac{2}{3}$ of W | | ① \$200 for $W_2$. |
| $= \frac{450}{W^2} \times \frac{4W^2}{9}$ \$200 | | |
| $= 200$ | | ① \$200 Loss. |
| Total New value = \$250 | | |
| Loss = \$200 | | |
| (b) (i) $2x - 5y + 1 = 0$ | ① | ① $m = \frac{2}{5}$ |
| gradient $= \frac{2}{5}$ | | |
| (ii) $2x = 4y + 7 + k(4x + y - 10) = 0$ | ② | ① equation in general form |
| $x[1+4k] + y[-4+k] + 7 - 10k = 0$ | | |
| gradient $= \frac{-(1+4k)}{-4+k} = \frac{2}{5}$ | | |
| $-5 - 20k = -8 + 2k$ | | |
| $3 = 22k$ | | |
| $k = \frac{3}{22}$ | | ① answer. |
| (c) P(1, y) A(-5, 2) B(3, -1) $m = 1$ | ② | ① sub into formula |
| (i) $1 = \frac{-5(1) + 3(m)}{m+1}$ | | |
| $m + 1 = -5 + 3m$ | | |
| $6 = 2m$ | | ① answer |
| $m = 3$ | | |
| (ii) $y = \frac{2(1) + (-1)(3)}{3+1}$ | ② | ① sub into formula |
| $= \frac{2-3}{4} = \frac{-1}{4}$ | | ① answer (CFE allowed) |

**MATHEMATICS** : Question... Section E

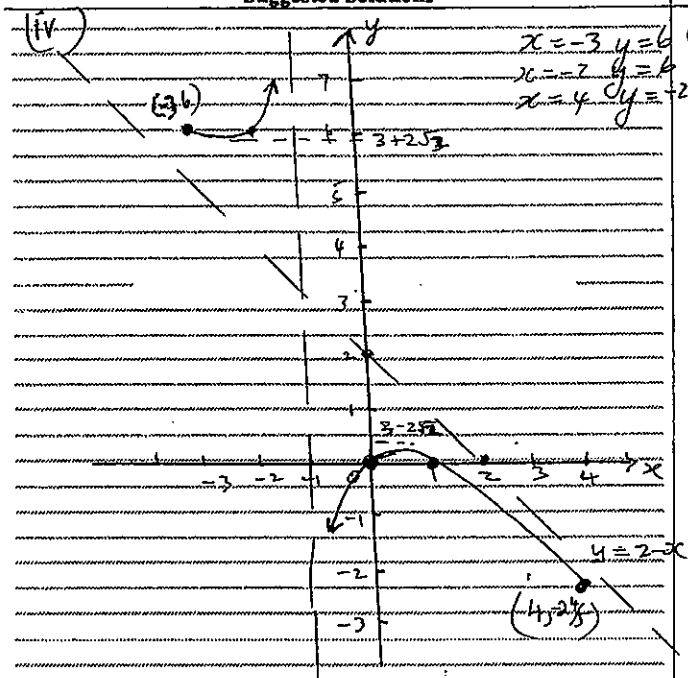
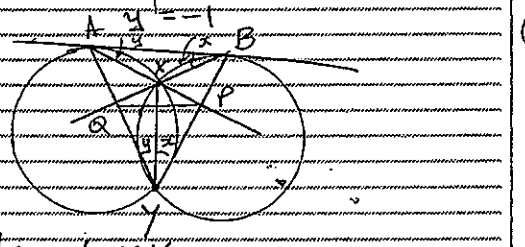
| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| (a) $3x - 2y = 1$ (1) | (4) | |
| $y = \frac{3x-1}{2}$ | | ① y subject |
| $2y^2 - 3x^2 = 5$ | | |
| $2\left[\frac{3x-1}{2}\right]^2 - 3x^2 = 5$ | | |
| $2\left[\frac{9x^2 - 6x + 1}{4}\right] - 3x^2 = 5$ | | |
| $9x^2 - 6x + 1 - 6x^2 = 10$ | | ① Quadratic |
| $3x^2 - 6x - 9 = 0$ | | |
| $x^2 - 2x - 3 = 0$ | | |
| $(x-3)(x+1) = 0$ | | ① x values |
| $\therefore x = 3$ or $x = -1$ | | |
| $y = 4$ $\quad x = -2$ | | ① y values |
| (b) | (2) | |
|   | | ½ for each probability |
| H = Brown hair<br>E = Brown Eyes<br>U = Town's population | | |
| $P(\text{brown eyes} \mid \text{brown hair}) = \frac{15}{40} = \frac{3}{8}$ | | ① Answer. |
| $P(\text{neither brown hair or brown eyes}) = \frac{50}{100} = \frac{1}{2}$ | | ① Answer. |
| (c) Let T = time for small pipe only | (3) | |
| L = rate of flow large pipe. | | |
| S = rate of flow small pipe | | ① equations |
| $80L + 80S (= L(T-120)) = TS$ | | |
| $200L + 80S = LT$ | | |
| $200 + 80\frac{S}{L} = T$ i.e. $80\frac{S}{L} = T - 200$ | | |
| $80\frac{L}{S} + 80 = T$ $\quad 80\frac{L}{S} = T - 80$ | | ① quadratic equation |
| $6400 = (T-200)(T-80)$ | | |
| $T^2 - 280T + 9600 = 0$ | | |
| $(T-40)(T-240) = 0$ but $T > 120$ | | ① answer |
| $T = 40$ or $240$ $\quad \therefore T = 240$ only | | |
| Time = 240 minutes. | | |



**MATHEMATICS** : Question... Section F

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| (a) $P(1) = x$ $P(2) = 2x$ $P(3) = 3x$ | (2) | |
| $P(4) = 4x$ $P(5) = 5x$ $P(6) = 6x$. | | ① $P(5) = \frac{5}{21}$ |
| $21x = 1$ $\quad x = \frac{1}{21}$ | | ① $P(6) = \frac{6}{21}$ |
| $11 = 5 + 6$ or $6 + 5$. | | |
| $P(11) = \frac{5}{21} \times \frac{6}{21} + \frac{6}{21} \times \frac{5}{21} = \frac{60}{441} = \frac{20}{147}$ | | ① Answer. (unsimplified ok) |
| (b) $P(x) = x^3 - ax^2 + x$ | (2) | |
| $P(3) = 12$ | | |
| $12 = 27 - 9a + 3$ | | ① $a = 2$ |
| $9a = 18$ | | |
| $a = 2$. | | ① $R = -4$. |
| $P(-1) = -1 - 2(1) - 1$ | | |
| Remainder $= -4$ | | |
| (c) (i) $2x^2 + (y-1)x + y = 0$ | (3) | |
| real roots $\quad \Delta \geq 0 \quad \Delta = b^2 - 4ac$ | | |
| $(y-1)^2 - 4y \geq 0$ | | ① inequality |
| $y^2 - 2y + 1 - 4y \geq 0$ | | |
| $y^2 - 6y + 1 \geq 0$ | | ① y values |
| For equation. | | |
| $y = \frac{+6 \pm \sqrt{36-4}}{2}$ | | ① solution |
| $= 3 \pm 2\sqrt{2}$ | | |
| $\therefore y \leq 3 - 2\sqrt{2}$ or $y \geq 3 + 2\sqrt{2}$ | | |
| (ii) $y = \frac{x - x^2}{x+1}$ | (1) | } working not necessary |
| $\therefore y(x+1) = x - x^2$ | | |
| $yx + y = x - x^2$ | | ① Only answers required. |
| $x^2 + (y-1)x + y = 0$. | | |
| Range gives real values of $x$. | | |
| $y \leq 0.2$ or $y \geq 5.8$ | | |
| (iii) $-x + 2$. | (2) | |
| $x+1 \overline{)} -x^2 + x + 0$ | | ① Division (or part of). |
| $-x^2 - x$ | | |
| $2x + 0$ | | |
| $2x + 2$ | | |
| $-2$ | | |
| Oblique asymptote is $y = -x + 2$ | | ① Answer. |

**MATHEMATICS** : Question Section F

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| (iv)   | (3) | ① Asymptotes.<br>① Intercepts and endpoints<br>① shape. including turning points (approximately in correct position) No need to label values. |
| d)   | (4) | |
| i) Construct XY<br>Let ∠ABX = x° = ∠XYB (angle between tangent and chord equals angle at circumference in alternate segment) | ① | |
| Similarly:<br>Let ∠BAX = y° = ∠AXY (alternate) | ① | |
| ∠BXP = 3rd ty angle in △ABX equal sum of 2 interior opposite angles | ① | |
| ∠QXP + ∠BXP = 180° (angle sum of straight angle QXB is 180°) | | |
| ∠QYP = x°+y° = ∠BXP. | ① | |
| ∴ ∠QYP + ∠BXP = 180° | | |
| ii) QXPY is a cyclic quad as sum of opposite angles is 180° | ① | |



**MATHEMATICS** : Question Section F

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| d(iii) Construct QP<br>As QXPY is cyclic<br>∠XQP = ∠XYP = x°<br>angles at the circumference are equal standing on the same arc | (2) ① | |
| ∴ ∠ABX = ∠XQP = x°<br>∴ alternate angles are equal | ①. | |
| ∴ AB // QP | | |