NORTH SYDNEY BOYS HIGH SCHOOL

2007 YEAR 10
YEARLY EXAMINATION

Mathematics

Examiner: R Lowe

General Instructions

- Working time – 120 minutes
- Write on one side of the paper
- Write using blue or black pen
- All necessary working should be shown in every question
- Board approved calculators may be used
- Write all attempted solutions neatly on your own paper

Total Marks (117)

- Attempt questions
- **Class Teacher: ☐ Mr Lowe**

☐ Mr Fletcher,
☐ Mr Barrett,
☐ Mr Weiss,
☐ Mr Ireland

Student Name :_____

Number of working pages used = (fill this in at the end of the exam)

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Total	%
/20	/13	/16	/13	/22	/21	/12	/117	

Question 1) **20 marks**

a) **(1)** Write 0.54 as a fraction in simplest form.

b) **(1)** Find the simple interest on \$980 at11% p.a over 5 years

c) **(2)** Find the exact value of

i) $\sin 60^0$

ii) $\tan 135^0$

d) **(1)** If θ is reflex and $\cos\theta = \frac{1}{2}$ what is the size of θ?

e) **(1)** If $\sqrt{a} = 2\sqrt{3}$ find the value of a

f) **(2)** Evaluate correct to 3 decimal places $\sqrt{\frac{2.691^3}{4.6+7.3}}$

g) **(2)** Factorise fully $5x^2 - 245$

h) **(2)** How many 3 digit numbers can be formed from the numbers 5, 4,3,2,1?

i) **(2)** Change the subject of the following formula form C to F

$C = \frac{5}{9}(F-32)$. For what value is C = F?

j) **(2)** Solve $x^2 + 10x - 24 = 0$

k) **(2)** Simplify $\sqrt{75} - \sqrt{12}$

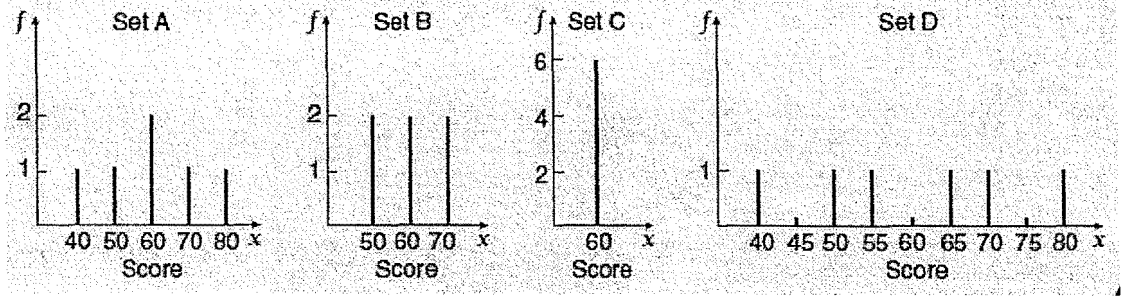
l) **(2)** If a motorist left Sydney at 6 a.m and travelled 880 km to Melbourne at an average speed of 64 km/hr, at what time did he arrive? Answer in 12 hour time

Question2) **13 marks**

a)**(1)** When shooting at a target in archery the probability of Bob hitting the bullseye on the first throw is $\frac{1}{4}$ and for Ted on the first throw it is $\frac{1}{5}$. Find the probability that both Bob and Ted hit it on the first throw?

b)**(1)**

Each of the sets of scores A to D has a mean of 60. Without calculating the standard deviations, arrange the sets in order of increasing standard deviation.



c) **(3)** A die is thrown twice, what is the probability:

i) Of throwing a double (the same number twice)?

ii) That the sum of the numbers thrown is 8?

d) **(4)** A regular coin is tossed 3 times, what is the probability

i) Of throwing 3 heads?

ii) Of throwing at least 1 tail?

iii) Of throwing 2 heads and a tail?

e) **(2)** The mean life expectancy of wombats is 12 years with a standard deviation of $2\frac{1}{2}$ years. What percentages of wombats die between the age range of 7 and 17 years?

f) **(2)** After completing 4 tests Willson has a mean of 60%. If there are 2 more tests out of 100, what is the maximum mean mark that Willson can achieve?

Question 3) **16 marks**

a) **(3)** In each of the following, state whether the set of ordered pairs represents a function or not.

i) (1, 5), (2, 6), (3, 7), (4, 8) ii) (8, 4), (7, 5), (6, 6), (5, 4)

iii) (1, 3), (1, 4), (0, 3), (2, 4)

b) **(2)** If $f(x) = 2x - 5$, find:

i) $f(0)$

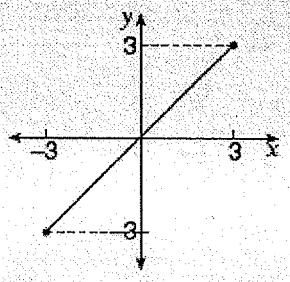
ii) $f(2)$

c) **(2)** If $F(p) = p^2$, find an expression for $\dfrac{F(p+h) - F(p)}{h}$

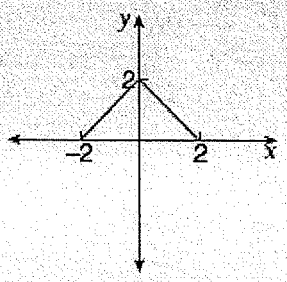
d) **(3)**

Which of the following are graphs of functions?

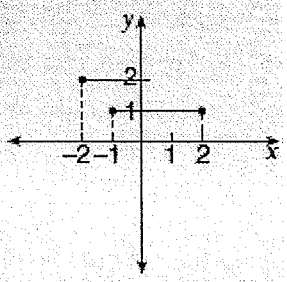
a



b



c



e) **(3)** Find the inverse function, $f^{-1}(x)$, for each of these linear functions.

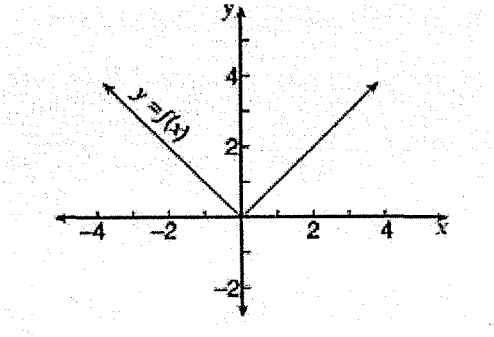
i) $y=3x+5$ ii) $y=\dfrac{x-1}{x-2}$

f) **(3)** Use the given graph of y = f(x) to sketch the following functions

i) $y=f(x)+1$

ii) $y=f(x-1)$

iii) $y=-f(x)$



Question 4) **13 marks**

a) **(1)** How many square centimetres in a square metre?

b) **(1)** The escape velocity of a space shuttle to leave the Earth is 112 m/s. Express this in km/h

c) **(1)** If a cube has volume 15625cm^3, what is the length of each side?

d) **(1)** Find the ratio of the length of the sides of 2 squares if their areas are 81cm^2 and $256\ \text{cm}^2$

e) **(1)** The radius of the Earth is approximately 6400 km. What is the circumference of the Earth at the Equator?

f) **(2)** A tap is dripping at the rate of 1 drop every 5 seconds. How much water is wasted if the tap is left to drip for 18 hours, if 10 drops of water make 1 mL ?

g) **(3)** Find the TOTAL surface area of a cone with height 12cm and slant height 13cm.

h) **(3)** A swimming pool is 8 m long and 3 m wide. It is 2.1 m at the deep end and 1.3 m at the shallow end, with a constant fall from the shallow end to the deep end. If the pool is full how much water will it hold (m^3)? What is the capacity of the pool in litres?

Question 5) **22 marks**

a) **(5)** Simplify

i) $3x - 2(x-5)$

ii) $\dfrac{x+2}{3} - \dfrac{x}{2}$

iii) $\dfrac{x^4 - x^2}{x^2 + x}$

b) **(5)** Solve simultaneously

i) $2x + y - 5 = 0$ *and* $3x - 4y - 24 = 0$

ii) $y = \dfrac{3}{x}$ *and* $x + y = 0$ .What does your solution mean ?

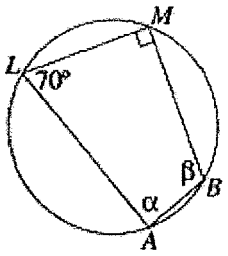
c) **(3)** Three towns A, B and C are connected by straight roads, where AB = 25 km, AC = 40 km and $\angle BAC = 20^0$. What distance is saved by going directly from A to C, instead of via B?

d) **(2)** For the circle $(x+8)^2 + y^2 = 64$ what are the coordinates of the centre and the length of the radius?

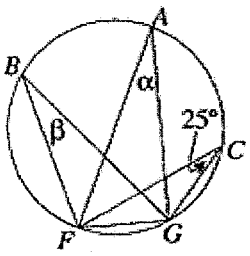
e) **(3)** Two parallel chords in a circle of diameter 50 cm have lengths of 14 cm and 48 cm. What is a possible distance between the chords?

f) **(4)** Find the value of α and β, no reasons are required

i)



ii)



Question 6) **21 marks**

a) **(7)** Consider the points A (-2, 7) and B (3, 6)

Find:

i) The distance AB

ii) The gradient of AB

iii) The mid point of AB

iv) The equation of the line AB

v) The equation of a line that is perpendicular to AB and passes through the origin

b) **(6)** Sketch graphs of the following, showing all intercepts

i) $y = 3^x$

ii) $y = \dfrac{1}{x+1}$

iii) $y = (4-x)(x+3)$

c) **(4)** Given the curve $y = 3\sin 4\theta$

i) What is the amplitude of the curve?

ii) What is the period of the curve?

Sketch the curve where $0^0 \leq \theta \leq 180^0$

d) **(4)** Simplify

i) $\left(3x^3\right)^2$

ii) $\dfrac{2a^2 \times 3ab^2}{12ab^3}$

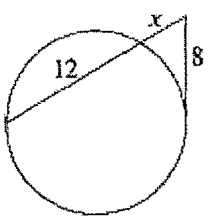
Question 7) **12 marks**

a)**(3)** Solve

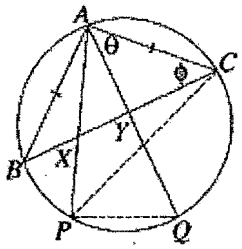
i) $9^x = 3^5$

ii) $2^{3x+1} = 32^x$

b)**(2)** Solve for x



c)(5)



In the diagram above, $AB = AC$.

(i) Prove that $\angle CPQ = \theta$.

(ii) Prove that $\angle CPA = \phi$.

(iii) Hence prove that $PQYX$ is cyclic.

d) (2)

I bought two trays of mangoes. In the first tray a quarter of them were bad. In the second tray, which contained four less mangoes, only a fifth of them were bad. If I had a total of 34 good mangoes, how many were in each tray?

Suggested Sol^{ns} Year 10 Yearly

a) $\frac{27}{50}$ — 1

b) \$539 — 1

c) i) $\frac{\sqrt{3}}{2}$ — 1

ii) -1 — 1

d) 240° — 1

e) $a = 12$ — 1

f) 1.280 — 2 (1 if not rounded)

g) $5(x^2 - 49)$ — 1

$5(x-7)(x+7)$ — 1

h) $5 \times 4 \times 3$

$= 60$ — 2

i) $F = \frac{9C}{5} + 32$ — 1

$F = C$ Then

$C = \frac{9C}{5} + 32$

$5C = 9C + 160$

$C = -40$ — 1

j) $(x+12)(x-2) = 0$ — 1

$x = 2$ or -1 — 1

k) $5\sqrt{3} - 2\sqrt{3}$ — 1

$3\sqrt{3}$ — 1

l) $\frac{880}{64} = 13\frac{3}{4}$

∴ Arrive at 19:45 — 1

or 7:45 pm — 1

(2) a) $\frac{1}{4} \times \frac{1}{5} = \frac{1}{20}$ — 1

b) C B A D — 1

c) i) $\frac{6}{36} = \frac{1}{6}$ — 1

ii) (3,5), (5,3), (4,4), (6,2), (2,6)

$\frac{5}{36}$ — 2

d) i) $\frac{1}{8}$ — 1

ii) $1 - \frac{1}{8}$

$= \frac{7}{8}$ — 1

iii) HHT, HTH, THH

$\frac{3}{8}$ — 2

e) 2 standard deviations

95% — 2

c) $240 + 200$

$= 440$ — 1

$\frac{440}{6} = 73\frac{1}{3}\%$ — 1

(3) a) i) Yes — 1

ii) Yes — 1

iii) No — 1

b) i) $f(0) = -5$ — 1

ii) $f(2) = -1$ — 1

c) $\frac{(p+h)^2 - p^2}{h}$ — 1

$= \frac{p^2 + 2ph + h^2 - p^2}{h}$

$= \frac{2p+h}{}$ — 1

d) a) Yes — 1

b) Yes — 1

c) No — 1

If answer just a + b give 3 marks

e) i) Inverse is

$x = 3y + 5$

$y = \frac{x-5}{3}$ — 1

$f^{-1}(x) = \frac{x-5}{3}$

ii) Inverse is

$x = \frac{y-1}{y-2}$ — 1

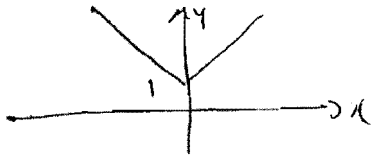
$x(y-2x-y = -1$

$y(x-1) = 2x-1$

$y = \frac{2x-1}{x-1}$

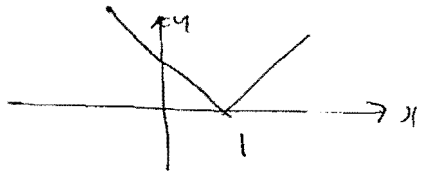
$f^{-1}(x) = \frac{2x-1}{x-1}$ — 1

f) i) $y = f(x)$ M



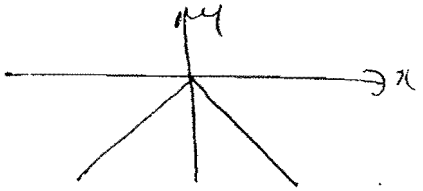
— 1

ii) $y = f(x-1)$



— 1

iii) $y = -f(x)$



— 1

(4) a) 10 000 — 1

b) $\frac{112 \times 60 \times 60}{1000}$

$= 403.2$ km/hr — 1

c) $\sqrt[3]{15625}$

$= 25$ cm — 1

d) 9 : 16 — 1

e) $2 \times \pi \times 6400$

12800π km

(40212.4) } — 1

f) 12 | min

12×60 | hr

$12 \times 60 \times 18 = 12960$ drops — 1

ii) 1296 mL

1.296 L — 1

g) $\pi r^2 + \frac{1}{3}\pi r L$ — 1

$= 25\pi + \frac{65\pi}{3}$ $(L = 13 - 1)$

$= \frac{140\pi}{3}$ cm^2

(146.6) cm^2 — 1

h) holds $\frac{8}{2}(1.3 + 2.1) \times 3$ — 1

$= 40.8$ m^3 — 1

$= 40800$ L — 1

(5) a) i) $3x - 2x + 10$

$x + 10$ — 1

ii) $\frac{2(x+2) - 3x}{6}$ — 1

$= \frac{4 - x}{6}$ — 1

iii) $\frac{x^2(x^2-1)}{x(x+1)}$ — 1

$= x(x-1)$ — 1

b) i) $2x + y - 5 = 0$ — (1)

$3x - 4y - 24 = 0$ — (2)

4 × (1) $8x + 4y - 20 = 0$

$11x = 44$

$x = 4$

$8 + y - 5 = 0$

$y = -3$

$(x, y) = (4, -3)$ — 2

ii) $y = \frac{3}{x}$ — (1)

$x + y = 0$ — (2)

(1) → (2) $\frac{3}{x} + x = 0$

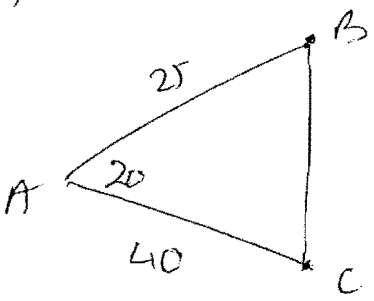
$3 + x^2 = 0$

$x^2 = -3$ — 2

∴ no solns

iii) no point of intersects between graphs — 1

c)



$BC^2 = 25^2 + 40^2 - 2 \times 25 \times 40 \cos 20$

$= 345.614$ — 1

$BC = \sqrt{\quad}$

$= 18.6$ km — 1

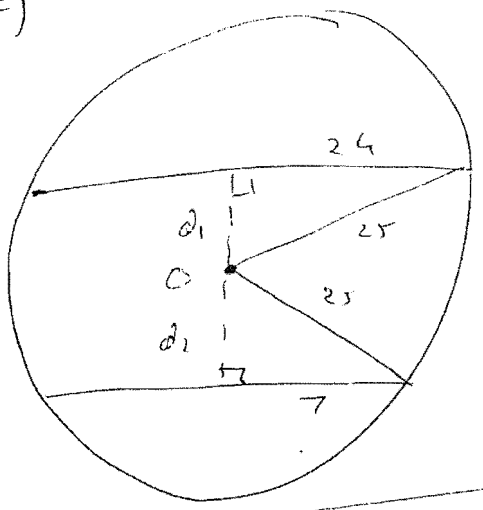
∴ Distance saved is

$25 + 18.6 - 40 = 3.6$ km — 1

d) Centre $(-8, c)$ — 1

radius 8 — 1

e)



— 1

$d_1 = 7$ $d_2 = 24$ — 1

∴ distance can be 24+7 (opposites)
$= 31$ cm
or $24 - 7 = 17$ cm (same side) — 1

f) i $\alpha = 90°$ — 1

$\beta = 110°$ — 1

ii $\alpha = 25°$ — 1

$\beta = 25°$ — 1

6 a) i $d = \sqrt{5^2 + (-1)^2}$

$= \sqrt{26}$ — 1

ii $M_{AB} = \frac{6-7}{3-(-2)} = \frac{-1}{5}$ — 1

iii Mid pt $\left(\frac{3-2}{2}, \frac{6+7}{2}\right)$

$= \left(\frac{1}{2}, \frac{13}{2}\right)$ — 1

iv) AB has eqn

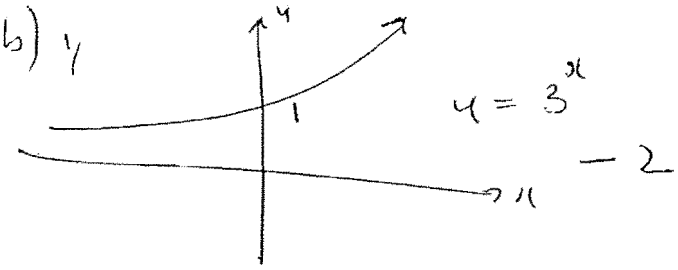
$y - 6 = -\frac{1}{5}(x - 5)$ — 1

$5y - 30 = -x + 3$

$x + 5y = 33$ — 1

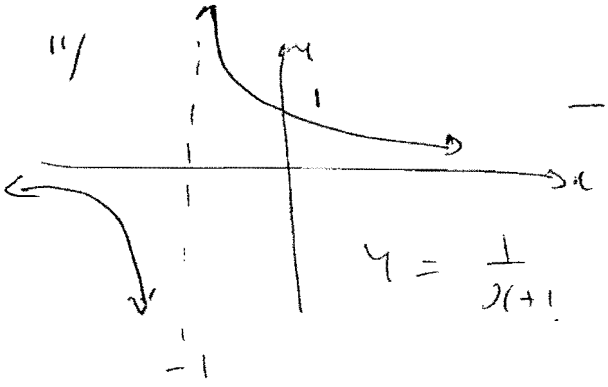
v) $y = 5x$ — 2

b) i



— 2

ii



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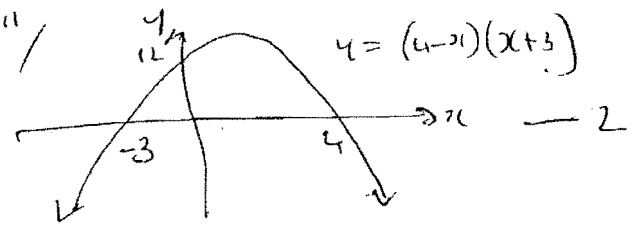
iii



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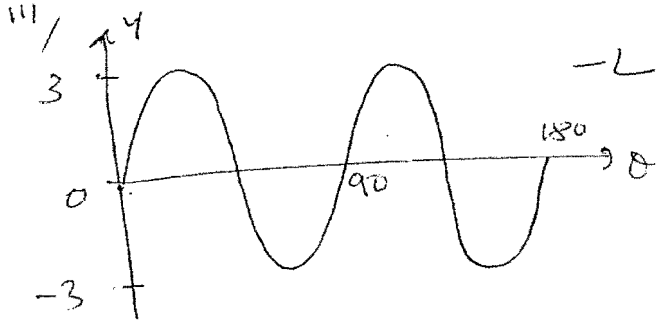
c) $y = 3 \sin 4\theta$

i $A = 3$ — 1

ii $P = \frac{360}{4} = 90°$ — 1

iii



— 2

7) i) $9x^6$ — 2

ii) $\frac{a^2}{2b}$ — 2

7) a) i) $3^{2x} = 3^5$

$2x = 5$

$x = 5/2$ — 1

ii) $2^{3x+1} = 2^{5x}$ — 1

$3x+1 = 5x$

$2x = 1$

$x = \frac{1}{2}$ — 1

b) $x(12+x) = 8^2$ — 1

$12x + x^2 = 64$

$x^2 + 12x - 64 = 0$

$(x-4)(x+16) = 0$

$x = 4 \quad x \neq -16$ — 1

c) i) $\angle CPB = \angle CAB$ (same arc)

$\therefore \angle CPB = \theta$ — 1

ii) $\angle ACB = \angle ABC$ (isos $\triangle ABC$)

$= \phi$

$\angle ABC = \angle APC$ (same arc)

$= \phi$ — 2

iii) $\angle AYB = \phi + \theta$ (ext $\angle$ of $\triangle$)

$\angle XAP$ ...

$\angle XAP = \angle APC + \angle CPB$

$= \theta + \phi$

$\therefore$ XYBP is cyclic

(opp $\angle$s supplementary) — 2

d) Let mangoes be M

$\frac{3M}{4} + \frac{4(M-4)}{5} = 34$ — 1

$15M + 16M - 64 = 680$

$31M = 744$

$M = 24$

4 I have 24 mangoes — 1