



Number of Pages Used \_\_\_\_\_

Teacher \_\_\_\_\_

**NORTH SYDNEY BOYS HIGH SCHOOL**

**2009 YEAR 10  
YEARLY EXAMINATION**

# **Advanced Mathematics**

Student Name: \_\_\_\_\_

## **General Instructions**

- Working time – **120 minutes**
- Write on **one side** of the paper
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Marks may not be awarded for carelessly or badly arranged work

(To be used by the exam markers only.)

<b>Quest</b>	<b>1</b>	<b>2</b>	<b>4</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>	<b>10</b>	<b>11</b>	<b>Total</b>
	/11	/17	/18	/10	/11	/10	/7	/14	/7	/8	/8	/121

**Question 1** ( 11 marks)**Marks**

- a) Evaluate to 3 significant figures  $\sqrt[3]{9.73 \times 10^5}$  **1**
- b) Find the exact value of  $\tan 150^\circ$  **1**
- c) A new Honda cost \$43000 on the road. The buyers' trade-in was valued at \$17000 and the balance was borrowed at 12% p.a. flat, to be paid back monthly over four years.
- i) Calculate the balance. **1**
- ii) Calculate the total interest due. **1**
- iii) Calculate the monthly payment. **1**
- iv) What is the total cost of the car? **1**
- d) i) Using the tax table below, calculate the annual tax due on an income of \$68500. **2**

Taxable income	Tax on this income
\$0 – \$6,000	Nil
\$6,001 – \$35,000	15c for each \$1 over \$6,000
\$35,001 – \$80,000	\$4,350 plus 30c for each \$1 over \$35,000

- ii) If the 1.5% Medicare levy on income is also included, find the total tax bill for the year. **1**
- e) Rationalise the denominator  $\frac{\sqrt{2}}{3 - \sqrt{5}}$  **2**

**Question 2** ( 17 marks)

- a) Solve the following equations
- i)  $4x - 3 = 10x + 11$  **2**
- ii)  $\frac{3x - 1}{2} - \frac{x + 4}{7} = 10$  **3**
- iii)  $4x^2 - 29x - 24 = 0$  **3**
- iv)  $3^{6x} = 27^{x-4}$  **2**

b) Solve simultaneously

i)  $5p + 3q = -6$  and  $3p = 10 - q$  3

ii)  $y = x^2$  and  $y = 20 - x$  4

**Question 3 ( 18 marks)**

a) The heights of 150 Year 10 boys were measured to the nearest 5cm.

Height	145	150	155	160	165	170	175	180
Frequency	2	8	12	35	39	42	9	3

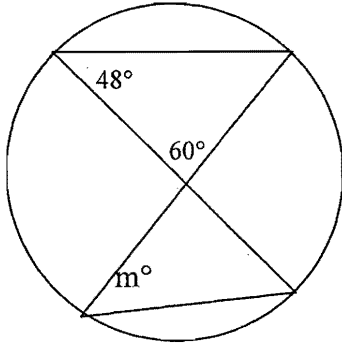
- i) Find the following 4
1. Mode      2. Mean      3. Median      4. Standard deviation
- ii) If a student was picked at random from the group, what is the probability he would be around 155cm tall? 1
- iii) Between what two scores would you find 95% of the scores? 1
- iv) Find the interquartile range. 2
- b) Horatio scored 66% on his Hamlet essay. The mean of the class was 62% and the standard deviation was 6%. This result needed to be standardised to a mean of 70% and a standard deviation of 15%. What is his new mark? 2
- c) Regular NSW number plates come as black on yellow or black on white, both with 4 letters and 2 digits. What is the probability of a plate having 4 letters the same and 2 digits the same? 2
- d) At a recent Sydney Symphony concert 800 people heard a Beethoven concerto and a Dvorak symphony. 30 people did not like either piece, 620 liked the Dvorak and 470 liked the Beethoven. What is the probability of a person only liking the Beethoven? 2
- e) From the four kings (hearts, diamonds, spades and clubs) out of a pack of playing cards, two are chosen at random without replacement.
- i) Draw a tree diagram showing the possible pairs chosen. 1
- ii) Determine the probability of getting: 3
- 1 Two kings the same colour.
- 2 At least one red king.
- 3 The king of spades.

**Question 4** (10 marks)

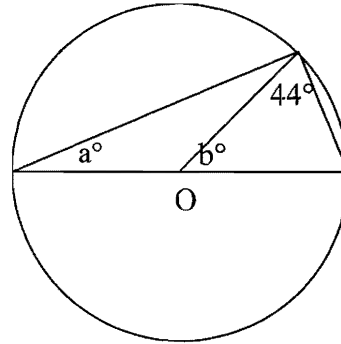
a) Find the value of the pronumerals. (No reasons are required)

1,2,1,2

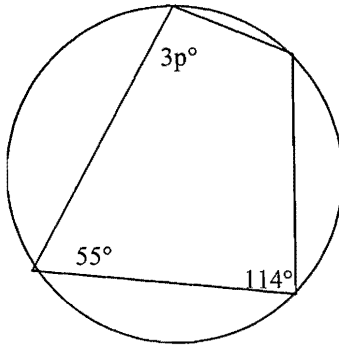
i)



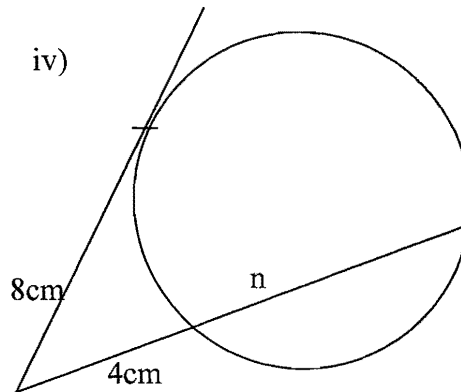
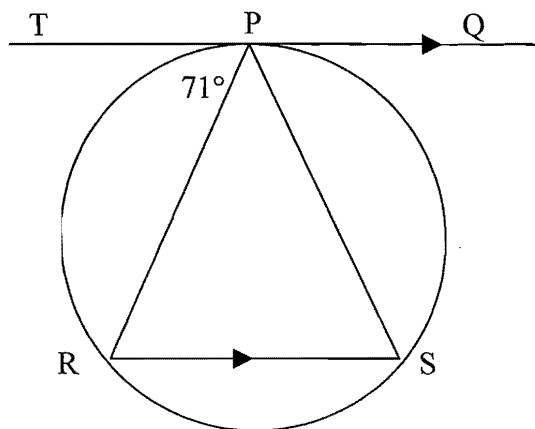
ii)



iii)

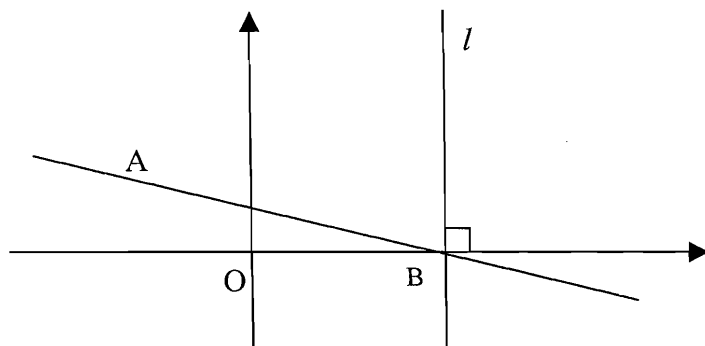


iv)

b) PQ is a tangent parallel to RS. Prove  $PR = PS$ 

**Question 5** (11 marks)

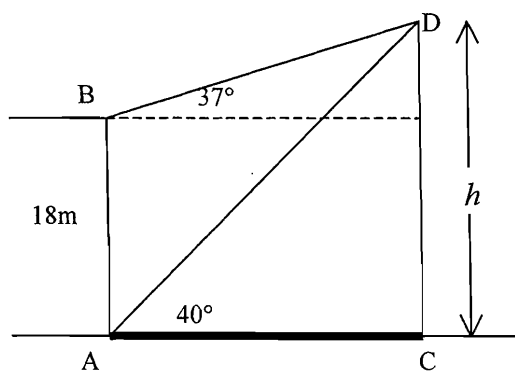
- a) The diagram below shows the points  $A(-1,3)$  and  $B(2,0)$ . The line  $l$  is drawn perpendicular to the  $x$  axis through the point  $B$ .



- |      |  |   |
|------|--|---|
| i)   | Calculate the length of $AB$ .   | 2 |
| ii)  | Find the gradient of $AB$ .  | 2 |
| iii) | What is the size of the acute angle between $AB$ and the $x$ axis?                                       | 1 |
| iv)  | Show that the equation of the line $AB$ is $x + y - 2 = 0$ .   | 2 |
| v)   | Copy the diagram and shade the region defined by $x + y - 2 \leq 0$ .                                    | 1 |
| vi)  | Write down the equation of line $l$ .  | 1 |
| vii) | The point $C$ on the line $l$ is such that $AC$ is perpendicular to $AB$ . Find the coordinates of $C$ . | 2 |

**Question 6** (10 marks)

- a) The angle of elevation from the top of a small building to a near by steeple is  $37^\circ$  and the angle of elevation to the steeple from the road below is  $40^\circ$ .



- |     |   |   |
|-----|---|---|
| i)  | If $AD = x$ metres and $CD = h$ metres, obtain an expression for $x$ and hence show that $h = \frac{18 \sin 127^\circ \sin 40^\circ}{\sin 3^\circ}$ | 3 |
| ii) | Hence determine $h$ to the nearest metre.   | 1 |

- b) Sketch on separate number planes. 4
- i)  $y = 2\cos 3\theta$   $0^\circ \leq \theta \leq 360^\circ$
- ii)  $y = 1 - 2\cos 3\theta$   $0^\circ \leq \theta \leq 360^\circ$
- c) Solve the equation  $\sin \theta = \frac{\sqrt{3}}{2}$   $0^\circ \leq \theta \leq 360^\circ$  2

**Question 7 ( 7 marks)**

- a) The top half (by height) of a cone is sliced off forming a smaller cone. 2  
Find the ratio of the volume of the smaller cone to the original cone.
- b) If a cone with height 12cm and radius 12cm has the top half cut off. 3  
Find the volume of the truncated cone. (Hint: part a above, may be of some help.)
- c) A full spherical water tank develops a leak at the bottom of the tank. 2  
Draw a sketch of the height of the water remaining against time as the water leaks out at a constant rate.

**Question 8 (14 marks)**

- a) On separate number planes sketch the following functions and relations. 3  
Show all the important features, such as: intercepts, axes of symmetry, asymptotes, vertices and centres etc.
- i)  $y = x^2 - 8x + 21$  3
- ii)  $y = \frac{1}{x + 2}$  3
- iii)  $y = 4^x$  2
- iv)  $y = (x + 5)^3$  2
- v)  $(x - 8)^2 + (y - 10)^2 = 16$  2
- b) Find the locus of P(x,y) if P is 2  
4 times the distance from A(-8,0) than from B(7,0)

**Question 9** (7 marks)

- a) Explain in words why  $y = x^2$  is a function and why  $x^2 + y^2 = 1$  is not a function. In your answer you must show a clear understanding of what is/is not a function. 2
- b) In each case, find the inverse function  $f^{-1}(x)$  4
- i)  $f(x) = 4 - 2x$
- ii)  $f(x) = \frac{2 - x}{3 + x}$
- c) Give an example of a function, where  $f(x) = f^{-1}(x)$  1

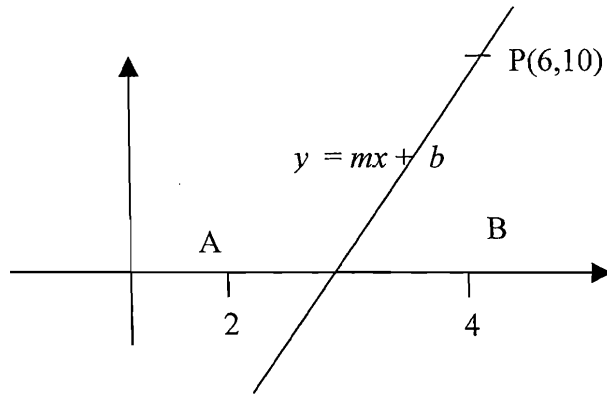
**Question 10** (8 marks)

- a) Find the value of  $x$ .
- i)  $\log_4 64 = x$  1
- ii)  $\log 3x - \log 2 = \log(x + 6)$  2
- iii)  $3^{x+1} = 18$  2
- b) How long will it take \$P to double in value if it is invested at 5.5%p.a. compounding annually. Give your answer to the nearest month. 3

**TURN OVER TO PAGE 7 FOR FURTHER QUESTIONS**

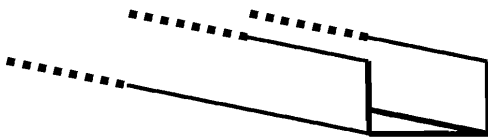
**Question 11** (8 marks)

a)



The line  $y = mx + b$  passes through  $P(6,10)$  and the interval  $AB$ .

- i) Find the minimum and maximum values that  $m$  can take. 2
- ii) Find the minimum and maximum values that  $b$  can take. 2
- b) A sheet of metal, 60cm wide, of indefinite length is bent to form an open, rectangular gutter. Let the height of the gutter be  $x$ .



Find the dimensions of the gutter to give maximum cross-sectional area. 4

END OF EXAMINATION



## Suggested Solutions

### Question 1

(a) (1 mark)

$$\sqrt[3]{9.73 \times 10^5} = 99.09 \dots = 99.1 \text{ (3 s.f.)}$$

(b) (1 mark)

$$\tan 150^\circ = -\frac{1}{\sqrt{3}}$$

(c) i. (1 mark)

$$\$43\,000 - \$17\,000 = \$26\,000$$

ii. (1 mark)

$$\begin{aligned} I &= 26\,000 \times 0.12 \times 4 \\ &= \$12\,480 \end{aligned}$$

iii. (1 mark)

$$\begin{aligned} \text{Payment} &= \frac{26\,000 + 12\,480}{48} \\ &= \$801.67 \end{aligned}$$

iv. (1 mark)

$$C = 43\,000 + 12\,480 = \$55\,480$$

(d) i. (2 marks)

✓ [1] for calculating the cost of the marginal rate of tax.

✓ [1] for final answer.

$$\begin{aligned} \text{Tax} &= 4\,350 + 0.3(68\,500 - 35\,000) \\ &= \$14\,400 \end{aligned}$$

ii. (1 mark)

$$\begin{aligned} \text{Total} &= 14\,400 + (68\,500 \times 0.015) \\ &= 15\,427.50 \end{aligned}$$

(e) (2 marks)

✓ [1] for multiplying by the correct conjugate surd.

✓ [1] for final answer

$$\begin{aligned} \frac{\sqrt{2}}{3 - \sqrt{5}} \times \frac{3 + \sqrt{5}}{3 + \sqrt{5}} &= \frac{3\sqrt{2} + \sqrt{10}}{9 - 5} \\ &= \frac{3\sqrt{2} + \sqrt{10}}{4} \end{aligned}$$

### Question 2

(a) i. (2 marks)

✓ [1] for obtaining  $6x = -14$  (or equivalent).

✓ [1] for final simplified answer.

$$\begin{aligned} \frac{4x}{-4x} - \frac{3}{-11} &= \frac{10x}{-4x} + \frac{11}{-11} \\ 6x &= -14 \\ x &= -\frac{14}{6} = -\frac{7}{3} \end{aligned}$$

ii. (3 marks)

✓ [1] for removing all fractions to obtain  $7(3x - 1) - 2(x + 4) = 140$  (or equivalent).

✓ [1] for obtaining  $19x - 15 = 140$  (or equivalent).

✓ [1] for final simplified answer.

$$\begin{aligned} \frac{3x-1}{2} - \frac{x+4}{7} &= \frac{10}{14} \\ 7(3x-1) - 2(x+4) &= 140 \\ 21x - 7 - 2x - 8 &= 140 \\ 19x - 15 &= 140 \\ 19x &= 155 \\ x &= \frac{155}{19} \end{aligned}$$

iii. (3 marks)

✓ [1] for correct factorisation.

✓ [1] for  $x = -\frac{3}{4}$ .

✓ [1] for  $x = 8$ .

$$\begin{aligned} 4x^2 - 29x - 24 &= 0 \\ (4x + 3)(x - 8) &= 0 \\ x &= -\frac{3}{4}, 8 \end{aligned}$$

iv. (2 marks)

- ✓ [1] for obtaining  $6x = 3x - 12$ .
- ✓ [1] for final answer.

$$\begin{aligned} 3^{6x} &= 27^{x-4} = (3^3)^{x-4} \\ 3^{6x} &= 3^{3x-12} \\ \frac{6x}{-3x} &= \frac{3x-12}{-3x} \\ 3x &= -12 \\ x &= -4 \end{aligned}$$

(b) i. (3 marks)

- ✓ [1] for correct method.
- ✓ [1] for each correct value of  $p$  and  $q$ .

$$\begin{cases} 5p + 3q = -6 & (1) \\ 3p + q = 10 & (2) \end{cases}$$

$$\begin{aligned} (1) - 3 \times (2) : \\ -4p &= -36 \\ p &= 9 & (3) \end{aligned}$$

Substitute (3) to (2)

$$\begin{aligned} 3(9) + q &= 10 \\ 27 + q &= 10 \\ \therefore q &= -17 \end{aligned}$$

ii. (4 marks)

- ✓ [1] for obtaining  $(x-4)(x+5) = 0$ .
- ✓ [3] for correct final answer of (4, 16) and (-5, 25).

$$\begin{cases} y = x^2 \\ y = 20 - x \end{cases}$$

$$\begin{aligned} x^2 &= 20 - x \\ x^2 + x - 20 &= 0 \\ (x-4)(x+5) &= 0 \\ x &= 4, -5 \\ y &= 16, 25 \\ \therefore (4, 16) \text{ or } (-5, 25) \end{aligned}$$

**Question 3**

(a) i. (4 marks)

- Mode = 170.
- $\bar{x} = 164.3 (\approx 164)$ .
- $\tilde{x} = 165$ .
- $\sigma_n = 6.94$ .

ii. (1 mark)

$$P(155) = \frac{12}{150} = \frac{2}{25}$$

iii. (1 mark)

$$\begin{aligned} x_{2\sigma} &= 164 \pm 2\sigma \\ &= 150.13, 177.87 \\ \therefore &150 \text{ cm}, 178 \text{ cm} \end{aligned}$$

iv. (2 marks)

$$\begin{aligned} Q_1 &= 160 \\ Q_2 &= 170 \\ \therefore IQR &= 170 - 160 = 10 \end{aligned}$$

(b) (2 marks)

- ✓ [1] for correct calculation of 66% as the proper  $z$  score.
- ✓ [1] for correct final answer.

$$\begin{aligned} z &= \frac{x - \mu}{\sigma} = \frac{66 - 62}{6} = \frac{2}{3} \\ \frac{2}{3} &= \frac{x - 70}{15} \\ 10 &= x - 70 \\ x &= 80 \end{aligned}$$

(c) (2 marks)

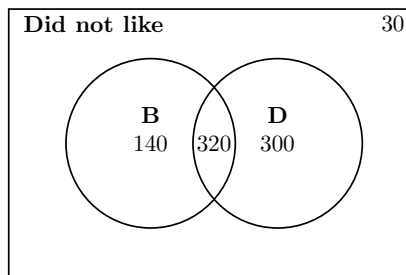
- ✓ [1] for correct numerator.
- ✓ [1] for correct denominator.

Sample space :  $26^4 \times 10 \times 10$ Favourable outcome :  $26 \times 10$ 

$$\begin{aligned} P(1 \text{ letter \& } 1 \text{ number}) &= \frac{26 \times 10}{26^4 \times 100} \\ &= \frac{1}{26^3 \times 10} \\ &= \frac{1}{17576 \times 10} \\ &= \frac{1}{175760} \end{aligned}$$

(d) (2 marks)

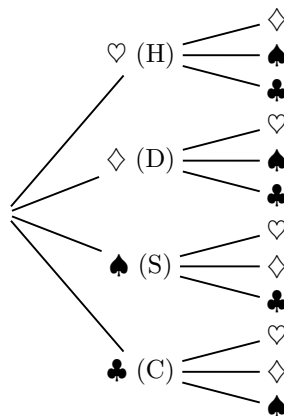
- ✓ [1] for method.
- ✓ [1] for final answer.



$$P(B) = \frac{150}{800} = \frac{3}{16}$$

(e) i. (1 mark)

- ✓ [1] for the list of possible pairs.
- Note that the tree diagram produces duplicated pairs.



Possible pairs are

- ♥♦ (HD)
- ♥♠ (HS)
- ♥♣ (HC)
- ♦♠ (DS)
- ♦♣ (DC)
- ♠♣ (SC)

ii. 1./ (1 mark)

$$P(\text{same colour}) = \frac{2}{6} = \frac{1}{3}$$

2./ (1 mark)

$$\begin{aligned} P(\text{at least 1 red}) &= 1 - P(\text{no red}) \\ &= 1 - \frac{1}{6} = \frac{5}{6} \end{aligned}$$

3./ (1 mark)

$$P(\spadesuit) = \frac{3}{6} = \frac{1}{2}$$

### Question 4

(a) i. (1 mark)

$$m = 48^\circ$$

ii. (2 marks)

$$a = 46^\circ \quad \left| \begin{aligned} b &= 180^\circ - 2 \times 44^\circ \\ &= 92^\circ \end{aligned} \right.$$

iii. (1 mark)

$$p = \frac{180 - 144}{3} = 22^\circ$$

iv. (2 marks)

- ✓ [1] for correct application of tangent/intercept rule to obtain  $4(n + 4) = 8^2$ .
- ✓ [1] for final answer.

$$\begin{aligned} 4(n + 4) &= 8^2 \\ \div 4 & \quad \div 4 \\ n + 4 &= 16 \\ n &= 12 \end{aligned}$$

(b) (4 marks)

- ✓ [1] for each correct bullet point.
- $\angle TPR = \angle PRS = 71^\circ$   
(alternate angles on  $\parallel$  lines  $TP, RS$ )
- $\angle TPR = \angle PSR = 71^\circ$   
(angles in the alternate segment)
- $\therefore \triangle PRS$  has equal base angles,  $\triangle PRS$  is isosceles.
- $\therefore PR = PS$ .

### Question 5

(a) (2 marks)

- ✓ [1] for correct application of distance formula.
- ✓ [1] for final answer.

$$\begin{aligned} d &= \sqrt{(-1 - 2)^2 + (3 - 0)^2} \\ &= \sqrt{9 + 9} = \sqrt{18} \\ &= 3\sqrt{2} \end{aligned}$$

(b) (2 marks)

$$m = \frac{3 - 0}{-1 - 2} = -1$$

(c) (1 mark)

$$m = \tan \theta = 1 \Rightarrow \theta = 45^\circ$$

(d) (2 marks)

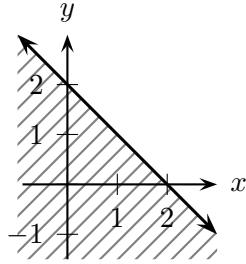
✓ [-1] for each missing line of working.

$$y - 3 = -1(x + 1)$$

$$y - 3 = -x - 1$$

$$x + y - 2 = 0$$

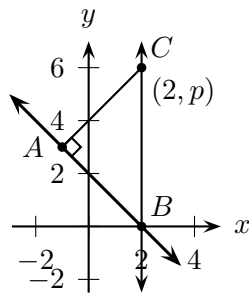
(e) (1 mark)



(f) (1 mark)

$$x = 2$$

(g) (2 marks)



Gradient of  $AC$  is 1 as it is perpendicular to the line  $x + y - 2 = 0$ .

$$\frac{p - 3}{2 + 1} = 1$$

$$p - 3 = 3$$

$$p = 6$$

**Question 6**

(a) i. (3 marks)

✓ [1] for expression in  $x$ .✓ [1] for  $\sin 40^\circ = \frac{h}{x}$ .

✓ [1] for final line.

Let  $AD = x$ .  $\angle ADB = 127^\circ$ ,  
 $\angle BAC = 50^\circ$  and  $\angle ADB = 3^\circ$ .

$$\frac{x}{\sin 127^\circ} = \frac{18}{\sin 3^\circ}$$

$$x = \frac{18 \sin 127^\circ}{\sin 3^\circ}$$

In  $\triangle ADC$ ,

$$\sin 40^\circ = \frac{h}{x}$$

$$h = x \sin 40^\circ$$

$$= \frac{18 \sin 40^\circ \sin 127^\circ}{\sin 3^\circ}$$

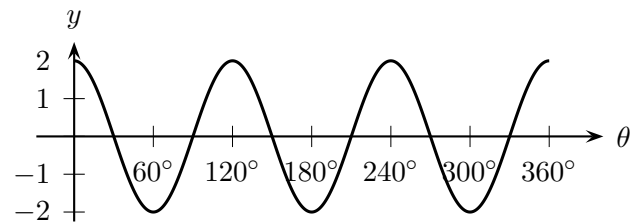
ii. (1 mark)

$$h = 176.6 \dots \approx 177 \text{ m}$$

(b) i. (2 marks)

✓ [1] for correctly sketched amplitude.

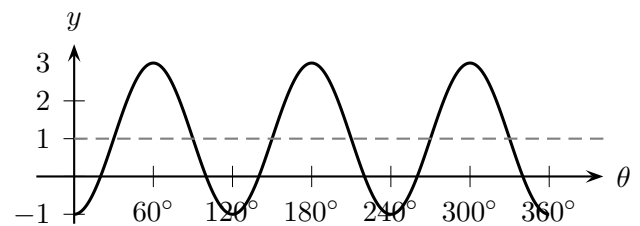
✓ [1] for correctly sketched period.



ii. (2 marks)

✓ [1] for negative "flip".

✓ [1] for correctly upward shift.



(c) (2 marks)

$$\sin \theta = \frac{\sqrt{3}}{2}$$

$$\theta = 60^\circ, 120^\circ$$

**Question 7**

(a) (2 marks)

✓ [1] for each correct line of working.

$$h_1 : h_2 = 1 : 2 \Rightarrow V_1 : V_2 = 1 : 8$$

(b) (3 marks)

✓ [1] for volume of larger cone.

✓ [1] for reasoning.

✓ [1] for final answer.

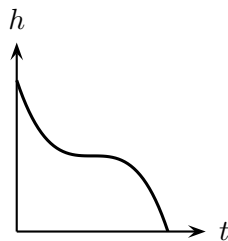
$$V_{\text{larger cone}} = \frac{\pi r^2 h}{3} = \frac{\pi \times 12^2 \times 12}{3} = 576\pi$$

As the larger cone has volume 8 times the smaller cone, then the truncated cone must have  $\frac{7}{8}$  of the volume of the larger cone.

$$V = \frac{7}{8} \times 576\pi = 504\pi \approx 1\,583.4 \text{ cm}^3$$

(c) (2 marks)

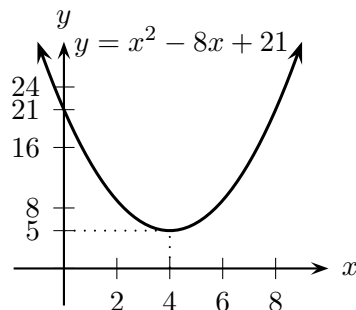
- ✓ [1] for axes.
- ✓ [1] for reasonably correct shape.



**Question 8**

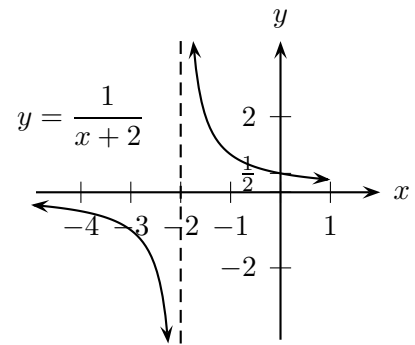
(a) i. (3 marks)

- ✓ [1] for correct shape.
- ✓ [1] for vertex.
- ✓ [1] for  $y$  intercept.



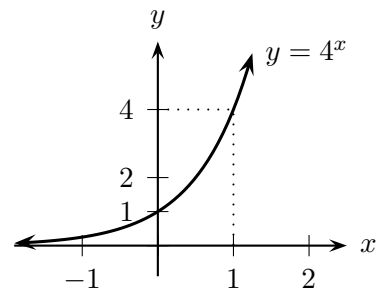
ii. (3 marks)

- ✓ [1] for correct shape.
- ✓ [1] for asymptote.
- ✓ [1] for  $y$  intercept.



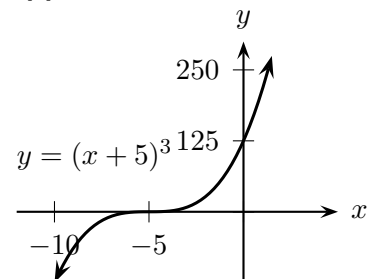
iii. (2 marks)

- ✓ [1] for correct shape & asymptote.
- ✓ [1] for  $y$  intercept.



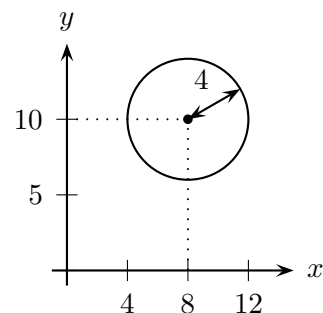
iv. (2 marks)

- ✓ [1] for correct shape & position.
- ✓ [1] for  $x$  and  $y$  intercepts.



v. (2 marks)

- ✓ [1] for correct position including showing the centre (must not cross any axes).
- ✓ [1] for correct radius.



(b) (2 marks)

- ✓ [1] for method.
- ✓ [1] for final answer. If only the final answer is provided, award [2].

$$A(-8, 0) \quad B(7, 0) \quad P(x, y)$$

$$PA = 4PB$$

$$\sqrt{(x+8)^2 + y^2} = 4\sqrt{(x-7)^2 + y^2}$$

$$(x+8)^2 + y^2 = 16((x-7)^2 + y^2)$$

$$(x+8)^2 - 16(x-7)^2 = 16y^2 - y^2$$

$$[(x+8) - 4(x-7)][(x+8) + 4(x-7)] = 15y^2$$

$$[-3x + 36][5x - 20] = 15y^2$$

$$\cancel{3}(x-12)\cancel{5}(x-4) = \cancel{15}y^2$$

$$x^2 - 16x + 48 + y^2 = 0$$

$$x^2 - 16x + 64 - 64 + 48 + y^2 = 0$$

$$(x-8)^2 + y^2 = 16$$

Hence the locus is a circle of radius 4, centre (8, 0). (c) (1 mark)

**Question 9**

(a) (2 marks)

- ✓ [2] for showing understanding of a function.

- ✓ [1] only for stating “vertical line test”.

- A function must only have one corresponding  $y$  value for each  $x$  value in its domain. This is true for  $y = x^2$  but not so for  $x^2 + y^2 = 1$  as  $x = \frac{1}{2}$  would imply  $y = \pm \frac{\sqrt{3}}{2}$ .

(b) i. (2 marks)

- ✓ [1] for interchanging variables.
- ✓ [1] for final answer.

$$y = 4 - 2x$$

Interchange variables,

$$x = 4 - 2y$$

$$2y = 4 - x$$

$$y = 2 - \frac{1}{2}x$$

ii. (2 marks)

- ✓ [1] for obtaining  $xy + y = 2 - 3x$ .
- ✓ [1] for final answer.

$$y = \frac{2-x}{3+x}$$

Interchange variables,

$$x = \frac{2-y}{3+y}$$

$$x(3+y) = 2-y$$

$$\begin{array}{r} 3x + xy = 2 - y \\ -3x \quad +y \quad -3x \quad +y \end{array}$$

$$xy + y = 2 - 3x$$

$$\begin{array}{r} y(x+1) = 2 - 3x \\ \div(x+1) \quad \div(x+1) \end{array}$$

$$y = \frac{2-3x}{x+1}$$

$$y = x \quad y = \frac{1}{x} \quad y = -x \quad \text{etc} \dots$$

**Question 10**

(a) i. (1 mark)

$$x = \log_4 64$$

$$4^x = 64$$

$$x = 3$$

ii. (2 marks)

- ✓ [1] for obtaining  $\frac{3x}{2} = x + 6$ .
- ✓ [1] for final answer.

$$\log 3x - \log 2 = \log(x+6)$$

$$\log \frac{3x}{2} = \log(x+6)$$

$$\frac{3x}{2} = x + 6$$

$$3x = 2x + 12$$

$$x = 12$$

iii. (2 marks)

✓ [1] for obtaining  $x + 1 = \frac{\log 18}{\log 3}$ .

✓ [1] for final answer.

$$3^{x+1} = 18$$

$$(x + 1) \log 3 = \log 18$$

$$x + 1 = \frac{\log 18}{\log 3}$$

$$x = \frac{\log 18}{\log 3} - 1$$

$$\approx 1.63$$

(b) (3 marks)

✓ [1] for obtaining  $2P = P(1 + 0.055)^n$ .

✓ [1] for obtaining  $n = \frac{\log 2}{\log 1.055}$ .

✓ [1] for final answer.

$$A = P(1 + r)^n$$

$$A = 2P, r = 0.055, n = ?$$

$$2P = P(1 + 0.055)^n$$

$$2 = 1.055^n$$

$$n \log 1.055 = \log 2$$

$$n = \frac{\log 2}{\log 1.055}$$

$$= 12.946 \dots$$

$$= 12 \text{ yrs } 11 \text{ mths}$$

**Question 11**

(a) i. (2 marks)

$$m_{AP} = \frac{10 - 0}{6 - 2} \quad \left| \quad m_{BP} = \frac{10 - 0}{6 - 4} \right.$$

$$= \frac{10}{4} = \frac{5}{2} \quad \left| \quad = \frac{10}{2} = 5 \right.$$

$$\therefore m_{\min} = \frac{5}{2} \quad m_{\max} = 5$$

ii. (2 marks)

• When  $m = \frac{5}{2}$ ,

$$y = \frac{5}{2}x + b \Big|_{\substack{x=6 \\ y=10}}$$

$$10 = \frac{5}{2} \times 6 + b$$

$$10 = 15 + b$$

$$b = -5$$

• When  $m = 5$ ,

$$y = 5x + b \Big|_{\substack{x=6 \\ y=10}}$$

$$10 = 5(6) + b$$

$$b = -20$$

Hence  $b_{\max} = -5, b_{\min} = -20$ .

(b) (4 marks)

✓ [1] for diagram.

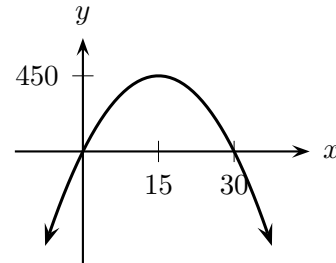
✓ [1] for obtaining an expression of  $A$  in terms of  $x$ .

✓ [1] for concluding maximum  $A = 15$ .

✓ [1] for final dimensions.



$$A = x(60 - 2x) = 60x - 2x^2$$



Maximum value of  $A = 2x(30 - x)$  occurs when

$$x = -\frac{b}{2a}$$

$$= -\frac{60}{2 \times (-2)} = 15$$

Hence the dimensions that give this maximum cross sectional area is

$$x = 15 \quad 60 - 2x = 30$$

i.e. 15 cm  $\times$  30 cm.