NORTH SYDNEY BOYS HIGH SCHOOL

STAGE 5.1-5.3 MATHEMATICS

2010 Year 10 Final Examination

General instructions

- Working time – 120 minutes.
- Marks may be deducted for careless or poorly arranged work.
- Commence each new question on a new sheet.
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- Board approved calculators may be used.
- All necessary working should be shown in every question.
- Attempt **all** questions.
- At the conclusion of the examination, bundle the sheets used in the correct order within this paper and hand to examination supervisors.

Class (please ✔)

- ◯ 10M1 – Mr Berry/Mr Weiss
- ◯ 10M2 – Mr Ireland
- ◯ 10M3 – Mr Lam/Mr Fletcher
- ◯ 10M4 – Mr Barrett
- ◯ 10M5 – Mr Lowe

NAME: ..... **# SHEETS USED**: .....

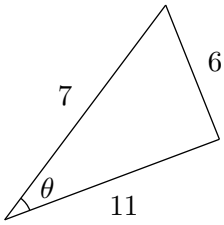
Marker's use only.

QUESTION	1	2	3	4	5	6	7	8	9	10	11	**Total**	%
MARKS	/12	/12	/12	/11	/13	/13	/12	/12	/16	/8	/11	/132	

Question 1 (12 Marks) Commence a NEW page. **Marks**

(a) Write the exact value of $\tan 150°$. **2**

(b) Find the value of θ to the nearest minute. **2**



(c) Given that $y = 3\sin 4\theta$,

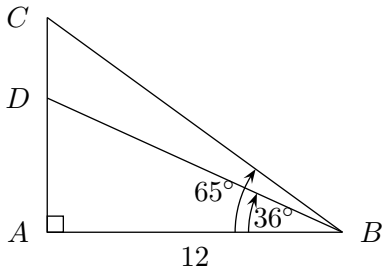
i. What is the amplitude of the curve? **1**

ii. What is the period of the curve? **1**

iii. Sketch the curve between $0° \leq \theta \leq 180°$. **1**

(d) If θ is an acute angle and $\cos\theta = \frac{1}{3}$, find the exact values of $\tan\theta$ and $\sin\theta$. **3**

(e) Find CD correct to 2 decimal places. **2**



Question 2 (12 Marks) Commence a NEW page. **Marks**

(a) If $\left(a+\sqrt{2}\right)^2 = m + 6\sqrt{2}$, find the value of a and m. **2**

(b) Solve $(x-1)^2 = 36$. **2**

(c) Simplify:

i. $5\sqrt{12} \times 3\sqrt{3}$. **1**

ii. $2\sqrt{8} + 5\sqrt{18} - 3\sqrt{50}$. **2**

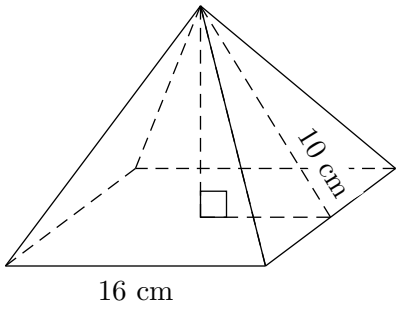
(d) Solve $x^2 - 6x + 6 = 0$, leaving your solution as exact values. **2**

(e) Rationalise $\dfrac{1}{5-2\sqrt{6}}$ and write in simplest form. **1**

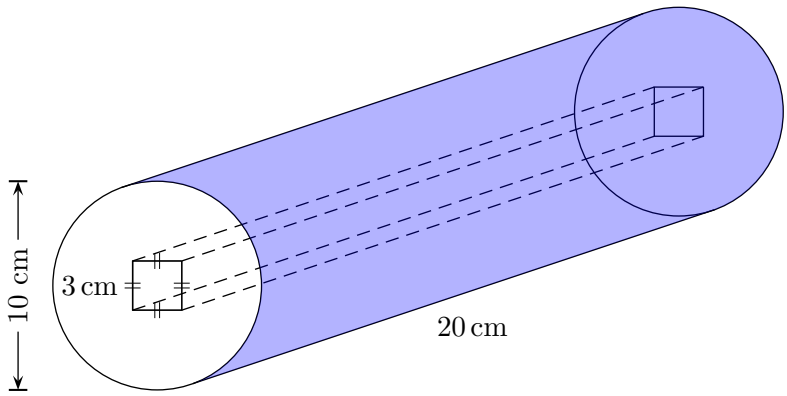
(f) What is the minimum value of $x^2 + 6x + 13$? **2**

Question 3 (12 Marks) Commence a NEW page. **Marks**

(a) Find the volume of the square pyramid. **2**



(b) Find the surface area of the cylinder. **3**
Note: the hole goes through the entire cylinder.

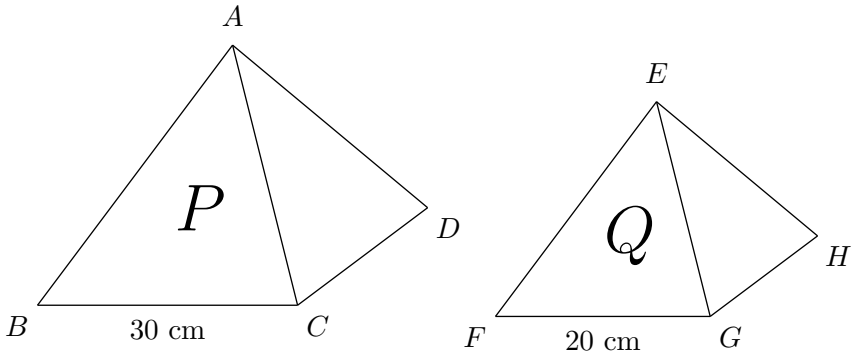


(c) Calculate the compound interest on \$3 000 invested at 6% p.a. for 5 years, with interest calculated annually. **2**

(d) After 10% GST is added, a TV costs \$605. **1**

What is the cost of the TV before tax?

(e) Solids P and Q are similar. Find:



i. Area of $\triangle ABC$: Area of $\triangle EFG$. **1**

ii. Volume of P : Volume of Q. **1**

iii. Volume of P if the volume of Q is $2\,000\,\text{m}^3$. **2**

Question 4 (11 Marks) Commence a NEW page. **Marks**

(a) This unordered stem and leaf plot represents the marks of 18 students in an examination.

4	5 1 3
5	0 8 1 2 4
6	8 9 2
7	3 2 4 3 9
8	9 6

i. Find the median mark. **1**

ii. Find the interquartile range. **2**

iii. Draw the box and whisker plot. **2**

(b) A student's report mark is to be made from two tests which will be equally weighed.

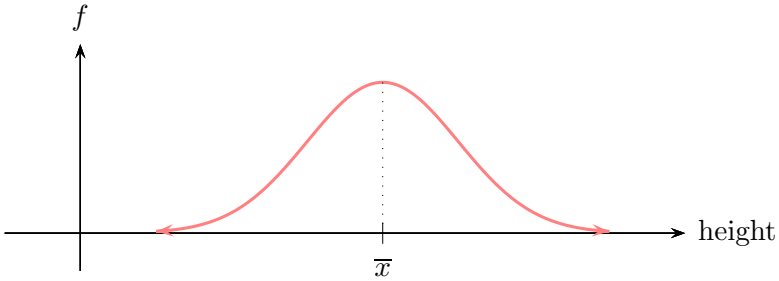
	Test 1	Test 2
Mean	55	70
Standard deviation	10	8
Student's mark	60	78

i. Which test did the student perform better in? Justify your answer by a reason. **2**

ii. Find the equivalent mark for both tests with a new mean of 65 and standard deviation of 12. **2**

(c) The following frequency distribution shows the height of a large group of people. **2**

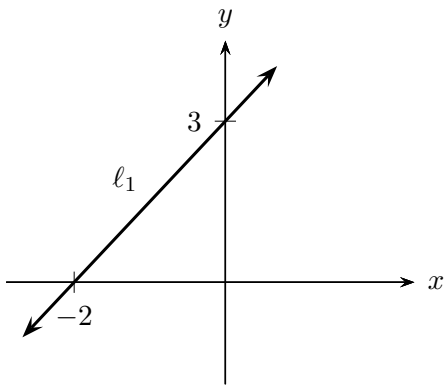
The mean $\overline{x}$ is 155 cm and standard deviation is 11.2 cm.



Between what two values would 95% of heights from the mean be found?

Question 5 (13 Marks) Commence a NEW page. **Marks**

(a) Refer to the following diagram.



i. What is the gradient of the line ℓ_1? **1**

ii. Show that the equation of the line ℓ_2 passing through $(0, 3)$ and perpendicular to ℓ_1 is $2x + 3y - 9 = 0$. **2**

iii. Reproduce the diagram on your own paper, and sketch the line ℓ_2, showing its x intercept. **2**

iv. Shade the region bounded by the lines ℓ_1, ℓ_2 and the x axis. **1**

v. Calculate the area of the shaded region. **1**

vi. Write down a system of inequalities which defines the shaded region. **3**

(b) Find the points of intersection of the curves **3**

$$\begin{cases} y = x^2 + 5 \\ y = 4x + 50 \end{cases}$$

Question 6 (13 Marks) Commence a NEW page. **Marks**

(a) On separate diagrams, sketch the graph of

i. $y = x^2 - 1$. **2**

ii. $y = -\dfrac{1}{x}$. **2**

iii. $x^2 + y^2 = 9$. **2**

iv. $y = 2^x$. **2**

(b) Explain how, without using a table of values, it is possible to use the graph of $y = 2^x$ to assist drawing the graph of $y = 2^x + 2$. **1**

(c) i. Show that $1 - \dfrac{1}{x+1} = \dfrac{x}{x+1}$. **1**

ii. Hence or otherwise, sketch $y = \dfrac{x}{x+1}$. **3**

Question 7 (12 Marks) Commence a NEW page. **Marks**

(a) A bag contains 3 blue and 4 red marbles. Two marbles are drawn without replacement.

i. What is the probability that the first marble drawn is red. **1**

ii. By drawing a probability tree, find the probability that one marble of each colour is chosen. **2**

(b) A captain and a vice-captain are to be selected from a team of 12 players. **2**

What is the probability of 2 particular players being chosen?

(c) Three dice are thrown. What is the probability of

i. all three numbers are even? **1**

ii. all three numbers are odd? **1**

iii. one even and two odd numbers? **2**

(d) The probability that it will rain on any given day in Sunnyville is 0.4. The Zhang family spend a three day holiday in Sunnyville. Find the probability that

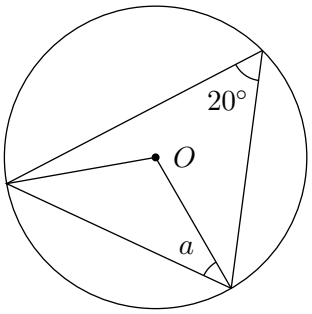
i. There will be three fine days. **1**

ii. There will be at least one day when it will rain. **2**

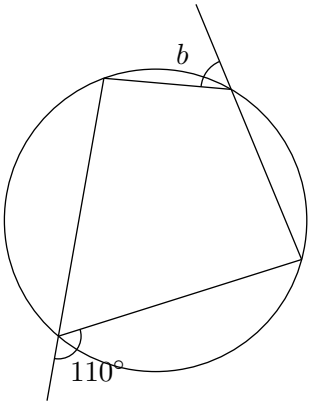
Question 8 (12 Marks) Commence a NEW page. **Marks**

(a) Find the value of each pronumeral. No reasons are required. **7**

i.



ii.



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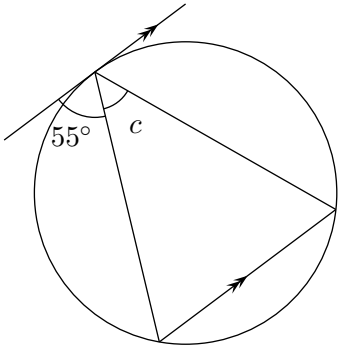

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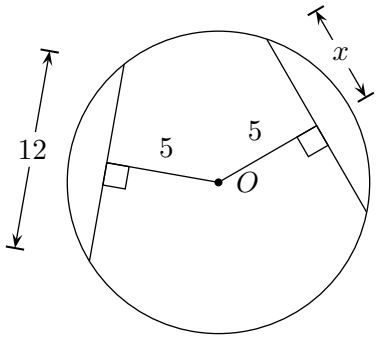

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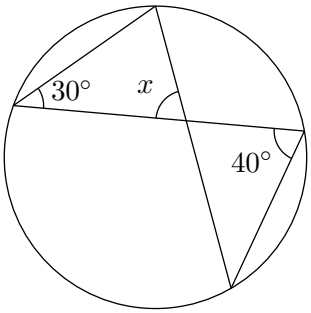
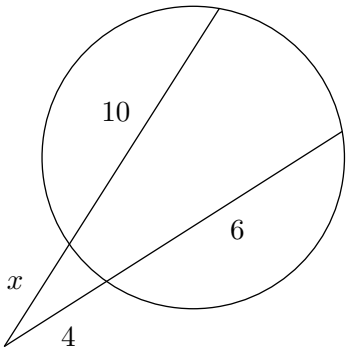
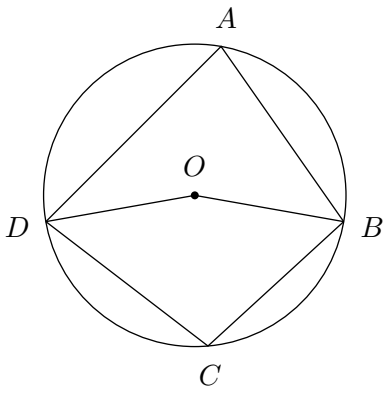


vi.



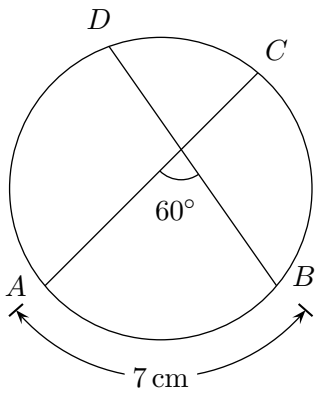
(b) $ABCD$ is a cyclic quadrilateral. O is the centre of the circle.



i. Prove that $\angle DAB + \angle DCB = 180°$. **2**

ii. If $\angle DAB = x$ and $\angle DOB = \angle DCB$, find the value of x. **2**

(c) If arc $AB = 7\,\text{cm}$, which of the following statements is true? **1**



Statement 1: arc $DC = 7\,\text{cm}$.

Statement 2: The circumference of the circle is $44\,\text{cm}$.

(A) Statement 1 only.

(B) Statement 2 only.

(C) Both Statement 1 and Statement 2.

(D) Neither Statement 1 or Statement 2.

Question 9 (16 Marks) Commence a NEW page. **Marks**

(a) State the domain and range of

i. $\{(1,2),(2,3),(3,8),(3,9)\}$. **2**

ii. $y = x^2$. **2**

(b) i. Is $(x-3)^2 + y^2 = 16$ the graph of a relation or function? **1**

ii. Why? **1**

(c) Find the equation of the inverse function of $y = 3x - 2$. **2**

(d) i. Draw a graph of $y = x^2$, $x \leq 0$. **1**

ii. Draw the inverse function $f^{-1}(x)$ on the same graph. **1**

iii. Write the equation of the inverse function $f^{-1}(x)$. **2**

(e) If $F(x) = \dfrac{1}{x} + \dfrac{1}{x^2} - \dfrac{1}{x^3}$, find the following values in simplest form.

i. $F(1)$. **1**

ii. $F\left(\dfrac{1}{\sqrt{2}}\right)$. **2**

iii. $F\left(x^2\right)$. **1**

Question 10 (8 Marks) Commence a NEW page. **Marks**

(a) Simplify:

i. $\dfrac{x^5y^2}{y^3} \times \dfrac{xy}{x^6}$. **1**

ii. Express without a negative index: $\left(\dfrac{5}{3}\right)^{-2}$. **1**

(b) Solve $8^{x-3} = 16^{3-x}$. **2**

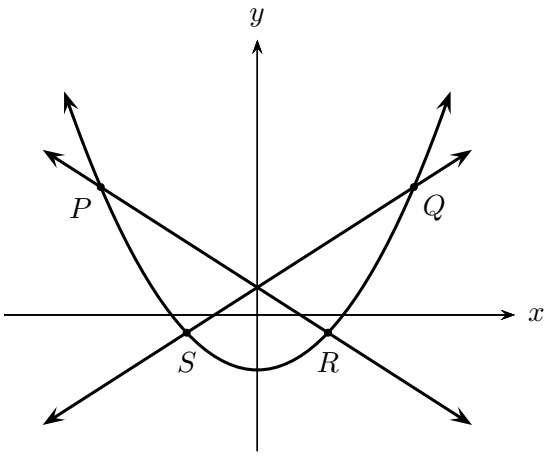
(c) Express $25^n \times 5^{n+3}$ as a power of 5. **1**

(d) Simplify $\left(x^{-1} + y^{-1}\right)^{-1}$ fully, expressing the answer as a fraction. **3**

Exam continues overleaf ...

Question 11 (11 Marks) Commence a NEW page. **Marks**

(a) On the number plane given, the graphs of $y = x^2 - 4$, $y = 2x + 2$ and $y = -2x + 2$ are shown. **2**



The solutions of $x^2 + 2x - 6 = 0$ are given by the x coordinates of which of the following points P, Q, R or S? Show working.

(b) Solve for x and y if: **3**

$$5^{x+y} = \frac{1}{5} \quad \text{and} \quad 5^{3x+2y} = 1$$

(c) A ship sails 50 km from port A to port B on a bearing of 63°, then sails 130 km from port B to port C on a bearing of 296°.

i. Sketch a diagram representing the above information. **1**

ii. Show that $\angle ABC = 53°$. **1**

iii. Find, to the nearest km, the distance of port A from port C. **2**

iv. Find $\angle ACB$ and hence find the bearing of port A from port C. **2**

End of paper.

Suggested Solutions

Question 1 (Commences on page 2)

(a) (2 marks)

✓ [1] for correct numerical value.
✓ [1] for sign.

$$\tan 150^\circ = -\frac{1}{\sqrt{3}}$$

(b) (2 marks)

✓ [1] for correct substitution of values into cosine rule.
✓ [1] for correct final answer.

$$\cos\theta = \frac{7^2 + 11^2 - 6^2}{2 \times 7 \times 11} = \frac{67}{77}$$
$$\therefore \theta = 29^\circ 32'$$

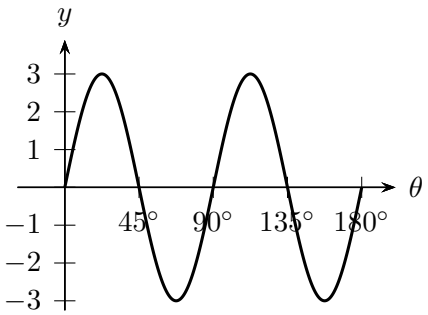
(c) i. (1 mark)

$$a = 3$$

ii. (1 mark)

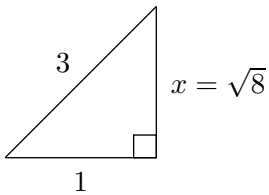
$$T = 90^\circ$$

iii. (1 mark)



(d) (3 marks)

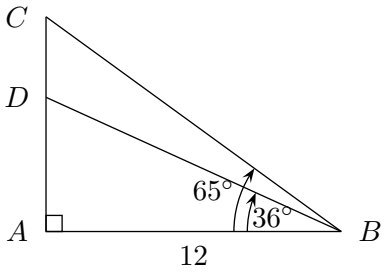
✓ [1] for correct right angle triangle.
✓ [1] for $\tan\theta$.
✓ [1] for $\sin\theta$.



$$1^2 + x^2 = 3^2$$
$$x^2 = 8$$
$$x = \sqrt{8}$$
$$\therefore \sin\theta = \frac{\sqrt{8}}{3} \qquad \tan\theta = \sqrt{8}$$

(e) (2 marks)

✓ [1] for $\frac{AD}{12} = \tan 36^\circ$.
✓ [1] for $CD = 17.02$ (2 d.p.)



$$\frac{AC}{12} = \tan 65^\circ \Rightarrow AC = 12\tan 65^\circ$$
$$\frac{AD}{12} = \tan 36^\circ \Rightarrow AD = 12\tan 36^\circ$$
$$CD = AC - AD = 12\tan 65^\circ - 12\tan 36^\circ$$
$$= 17.02 \text{ (2 d.p.)}$$

Question 2 (Commences on page 2)

(a) (2 marks)

✓ [1] for $a = 3$.
✓ [1] for $m = 11$.

$$\begin{aligned}\left(a+\sqrt{2}\right)^2 &= a^2 + 2a\sqrt{2} + 2 \\ &\equiv m + 6\sqrt{2}\end{aligned}$$

i.e. $a^2 + 2 = m$, $2a = 6$

$$\begin{aligned}\therefore a &= 3 \\ \therefore a^2 + 2 &= 3^2 + 2 = m = 11\end{aligned}$$

(b) (2 marks)

✓ [1] for $x - 1 = \pm 6$.
✓ [1] for $x = -5, 7$.

$$\begin{aligned}(x-1)^2 &= 36 \\ x - 1 &= \pm 6 \\ x &= 1 \pm 6 \\ \therefore x &= -5, 7\end{aligned}$$

(c) i. (1 mark)

$$\begin{aligned}5\sqrt{12} \times 3\sqrt{3} &= 5 \times 2\sqrt{3} \times 3\sqrt{3} \\ &= 30 \times 3 = 90\end{aligned}$$

ii. (2 marks)

✓ [1] for simplifying surds to multiples of $\sqrt{2}$.
✓ [1] for final answer.

$$\begin{aligned}&2\sqrt{8} + 5\sqrt{18} - 3\sqrt{50} \\ &= 2 \times 2\sqrt{2} + \cancel{5 \times 3\sqrt{2}} - \cancel{3 \times 5\sqrt{2}} \\ &= 4\sqrt{2}\end{aligned}$$

(d) (2 marks)

✓ [1] for correctly substituting into quadratic formula.
✓ [1] for correct final answer.

$$\begin{aligned}x^2 - 6x + 6 &= 0 \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{6 \pm \sqrt{36 - 4 \times 1 \times 6}}{2 \times 1} \\ &= \frac{6 \pm \sqrt{12}}{2} = \frac{\cancel{6}^{3} \pm \cancel{2}\sqrt{3}}{\cancel{2}} \\ &= 3 \pm \sqrt{3}\end{aligned}$$

(e) (1 mark)

$$\begin{aligned}\frac{1}{5-2\sqrt{6}} \times \tfrac{5+2\sqrt{6}}{5+2\sqrt{6}} &= \frac{5+2\sqrt{6}}{25-24} \\ &= 5 + 2\sqrt{6}\end{aligned}$$

(f) (2 marks)

✓ [1] for correctly converting expression to vertex form/finding axis of symmetry.
✓ [1] for final answer.

$$\begin{aligned}x^2 + 6x + 13 &= x^2 + 6x + 9 - 9 + 13 \\ &= (x+3)^2 + 4\end{aligned}$$

Minimum value is 4.

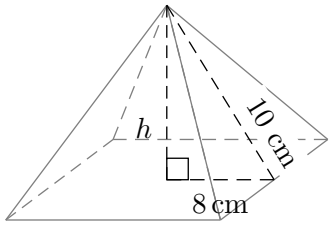
Alternatively find the axis of symmetry via $x = -\frac{b}{2a}$ and substitute back into expression to obtain 4.

Question 3 (Commences on page 3)

(a) (2 marks)

✓ [1] for finding the missing perpendicular height ($h = 6$)

✓ [1] for final answer.



$$\begin{aligned} h^2 + 8^2 &= 10^2 \\ \therefore h &= 6 \\ V &= \frac{1}{3} \times Ah \\ &= \frac{1}{3} \times 16^2 \times 6 \\ &= 512\,\text{cm}^3 \end{aligned}$$

(b) (3 marks)

✓ [1] for correct outer surface area.

✓ [1] for correct inner surface area.

✓ [1] for final answer.

$$\begin{aligned} SA_{\text{outer}} &= 2\pi r^2 + 2\pi rh \\ &= 2\pi r\,(r + h) \\ &= 2 \times \pi \times 5(5 + 20) \\ &= 250\pi \end{aligned}$$

The "inner" surface area contains four rectangles, but subtracts the areas of the squares at the front and back

$$\begin{aligned} SA_{\text{inner}} &= 4 \times (20 \times 3) - 2 \times 3^2 \\ &= 240 - 18 = 222 \\ \therefore SA_{\text{total}} &= 250\pi - 222 \approx 1\,007.4\,\text{cm}^2 \end{aligned}$$

(c) (2 marks)

✓ [1] for total amount at the end of 5 years.

✓ [1] for correct amount of interest.

$$\begin{aligned} A &= P(1 + r)^n \\ &= 3\,000 \times 1.06^5 = 4\,014.68 \\ I &= A - P = \$1\,014.68 \end{aligned}$$

(d) (1 mark)

$$\begin{aligned} \underset{\div 1.1}{1.1x} &= \underset{\div 1.1}{\$605} \\ x &= \frac{605}{1.1} = \$550. \end{aligned}$$

(e) i. (1 mark)

$$A_{\triangle ABC} : A_{\triangle EFG} = 3^2 : 2^2 = 9 : 4$$

ii. (1 mark)

$$V_P : V_Q = 3^3 : 2^3 = 27 : 8$$

iii. (2 marks)

✓ [1] for correctly setting up equation.

✓ [1] for final answer.

$$\begin{aligned} \frac{V_P}{V_Q} &= \frac{V_P}{2\,000} = \frac{27}{8} \\ \therefore V_P &= \frac{2\,000 \times 27}{8} = 6\,750\,\text{m}^3 \end{aligned}$$

Question 4 (Commences on page 4)

(a) i. (1 mark)

$$\tilde{x} = 65$$

ii. (2 marks)

✓ [1] for correct values of Q_1 and Q_3.

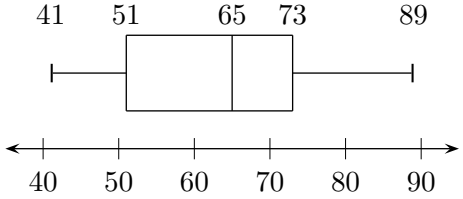
✓ [1] for final answer.

$$IQR = 73 - 51 = 22$$

iii. (2 marks)

✓ [1] for correct median sketched.

✓ [1] for correct maximum sketched.



(b) i. (2 marks)

- Performed better in test 2.

- Student's score in Test 1 is only 0.5σ above μ. Score in test 2 is 1σ above μ – i.e. 0.5σ above the mean compared to test 1.

ii. (2 marks)

✓ [1] for each correct value of the scaled result.

$$z = \frac{x-\mu}{\sigma}$$

$$z_1 = \frac{60-55}{10} \quad \Big| \quad z_2 = \frac{78-70}{8}$$
$$= 0.5 \quad \Big| \quad = 1$$

Rescale to $\mu = 65$ and $\sigma = 12$:

$$z = \frac{x_1-\mu}{\sigma} \quad \Big| \quad z = \frac{x_2-\mu}{\sigma}$$
$$0.5 = \frac{x_1-65}{12} \quad \Big| \quad 1 = \frac{x_2-65}{12}$$
$$6 = x_1 - 65 \quad \Big| \quad 12 = x_2 - 65$$
$$\therefore x_1 = 71 \quad \Big| \quad \therefore x_2 = 77$$

(Alternatively, $z_1 = 0.5$ and $z_2 = 1$. Hence $x_1 = \mu + 0.5\sigma = 71$ and $x_2 = \mu + 1\sigma = 77$)

(c) (2 marks)

✓ [1] for each correct value.

$$x_L = \mu - 2\sigma \quad \Big| \quad x_U = \mu + 2\sigma$$
$$= 155 - 2(11.2) \quad \Big| \quad = 155 + 2(11.2)$$
$$= 132.6 \quad \Big| \quad = 177.4$$

Question 5 (Commences on page 5)

(a) i. (1 mark)

$$m_1 = \frac{3}{2}$$

ii. (2 marks)

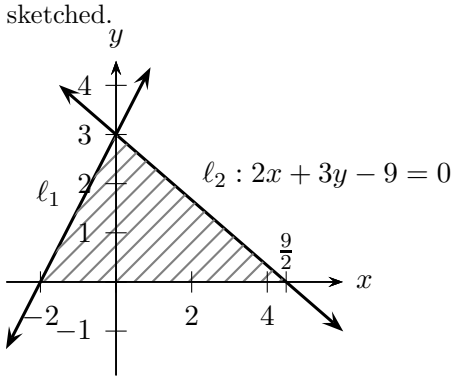
$$m_\perp = -\frac{2}{3}$$

Passes through $(0, 3)$. Hence $b = 3$.

$$\therefore \underset{\times 3}{y} = \underbrace{-\frac{2}{3}x + 3}_{\times 3}$$
$$3y = -2x + 9$$
$$2x + 3y - 9 = 0$$

iii. (2 marks)

✓ [1] for each correct intercept sketched.



iv. (1 mark) See shading.

v. (1 mark)

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2} \times \frac{13}{2} \times 3 = \frac{39}{4}$$

vi. (3 marks)

✓ [1] for each correct inequality.

$$\ell_1 : \underset{\times 2}{y} = \underbrace{\frac{3}{2}x + 3}_{\times 2}$$
$$\therefore 2y = 3x + 6$$
$$3x - 2y + 6 = 0$$

The inequalities defining the shaded region is

$$y \geq 0$$
$$2x + 3y - 9 \leq 0 \quad \left(y \leq -\frac{2}{3}x + 3\right)$$
$$3x - 2y + 6 \geq 0 \quad \left(y \leq \frac{3}{2}x + 3\right)$$

(b) (3 marks)

✓ [1] for equating both equations.
✓ [1] for correct x values.
✓ [1] for correct points of intersection.

$$\begin{cases} y = x^2 + 5 & \text{(A)} \\ y = 4x + 50 & \text{(B)} \end{cases}$$

Equating both equations

$$x^2 + 5 = 4x + 50$$
$$x^2 - 4x - 45 = 0$$
$$(x-9)(x+5) = 0$$
$$x = 9, -5$$

Substitute into (A)

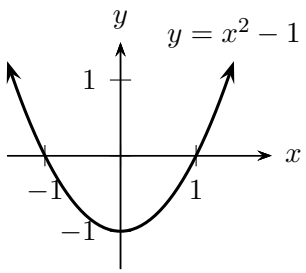
$$y = 81 + 5 = 86$$
$$y = 25 + 5 = 30$$

The pts of intersection are $(9, 86)$ and $(-5, 30)$.

Question 6 (Commences on page 6)

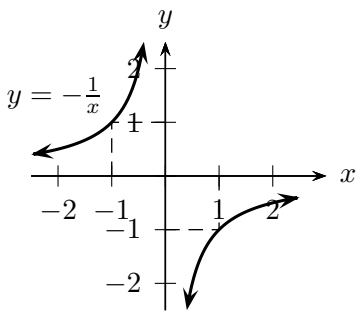
(a) i. (2 marks)

✓ [1] for intercepts.
✓ [1] for curve.



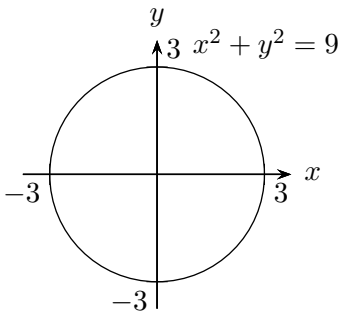
ii. (2 marks)

✓ [1] for points shown on curve.
✓ [1] for curve.



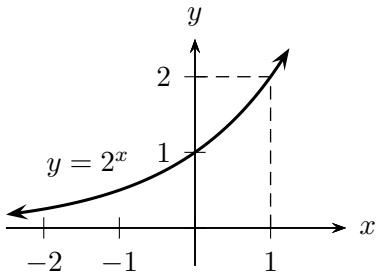
iii. (2 marks)

✓ [1] for intercepts.
✓ [1] for curve.



iv. (2 marks)

✓ [1] for intercepts
✓ [1] for curve.



(b) (1 mark) By shifting 2 units up.

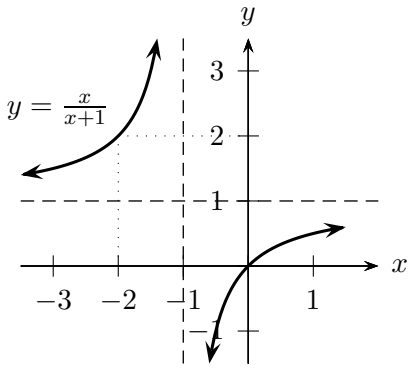
(c) i. (1 mark)

$$1 - \frac{1}{x+1} = \frac{x+1}{x+1} - \frac{1}{x+1}$$
$$= \frac{x}{x+1}$$

ii. (3 marks)

✓ [1] for intercepts.
✓ [1] for curve.
✓ [1] for asymptotes.

$$y = \frac{x}{x+1} = -\frac{1}{x+1} + 1$$



Question 7 (Commences on page 6)

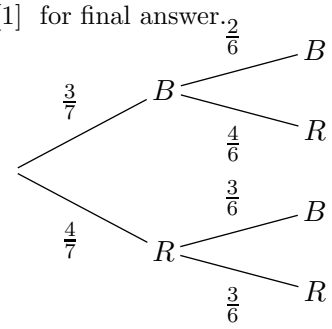
(a) i. (1 mark)

$$P(R) = \frac{4}{7}$$

ii. (2 marks)

✓ [1] for tree diagram.

✓ [1] for final answer.



$$\begin{aligned} &P(RB) + P(BR) \\ &= \left(\frac{4}{7} \times \frac{3}{6}\right) + \left(\frac{3}{7} \times \frac{4}{6}\right) \\ &= \frac{24}{42} = \frac{4}{7} \end{aligned}$$

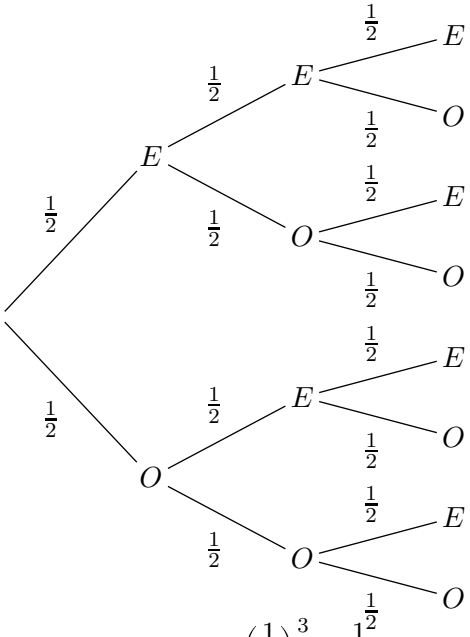
(b) (2 marks)

✓ [1] for correct expression.

✓ [1] for correct final answer.

$$\frac{1}{12} \times \frac{1}{11} = \frac{1}{132}$$

(c) i. (1 mark)



$$P(EEE) = \left(\frac{1}{2}\right)^3 = \frac{1^2}{8}$$

ii. (1 mark)

$$P(OOO) = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

iii. (2 marks)

$$P(1E, 2O) = 3 \times \left(\frac{1}{2}\right)^3 = \frac{3}{8}$$

(d) i. (1 mark)

$$P(\text{all days sunny}) = \left(\frac{3}{5}\right)^3 = \frac{27}{125}$$

ii. (2 marks)

✓ [1] for using the complement.

✓ [1] for final answer.

$$\begin{aligned} P(\text{at least 1 rainy day}) &= 1 - P(\text{no rain}) \\ &= 1 - \frac{27}{125} \\ &= \frac{98}{125} \end{aligned}$$

Question 8 (Commences on page 7)

(a) i. (1 mark)

$$a = 70^\circ$$

ii. (1 mark)

$$b = 70^\circ$$

iii. (1 mark)

$$c = 70^\circ$$

iv. (1 mark)

$$x = 6$$

v. (1 mark)

$$x = 110^\circ$$

vi. (2 marks)

$$\begin{aligned} x(x+10) &= 4 \times 10 \\ x^2 + 10x &= 40 \\ x^2 + 10x - 40 &= 0 \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-10 \pm \sqrt{100 + 160}}{2} \\ &= -5 \pm \sqrt{65} \end{aligned}$$

As x is a length, the negative solution of x is invalid.

$$\therefore x = -5 + \sqrt{65} \approx 3.06 \text{ (2 d.p.)}$$

(b) i. (2 marks)

✓ [0] for stating the theorem for opposite $\angle$ in cyclic quad.

✓ [1] for using $\angle$ at the centre is double the $\angle$ at circumference.

✓ [1] for final successful proof.

- Let $\angle DAB = x$.
- $\therefore \angle DOB = 2x$ (angle at the centre is double the angle at the circumference subtended by the same arc)
- $\therefore$ reflex $\angle DOB = 360^\circ - 2x$
- $\therefore \angle DCB = \frac{1}{2}(360 - 2x)$
  $= 180^\circ - x$
  (angle at the centre is double the angle at the circumference subtended by the same arc)

$$\begin{aligned}\therefore \angle DAB + \angle DCB &= x + (180^\circ - x) \\ &= 180^\circ\end{aligned}$$

ii. (2 marks)

✓ [1] for $\angle DOB = 2x$.

✓ [1] for $x = 60^\circ$.

- $\angle DAB = x$.
- $\angle DCB = 180^\circ - x$.
- $\therefore \angle DOB = 2x$.
- $\angle DCB = \angle DOB$:

$$\begin{aligned}180^\circ - x &= 2x \\ 3x &= 180^\circ \\ \therefore x &= 60^\circ\end{aligned}$$

(c) (1 mark) – (D)

Question 9 (Commences on page 9)

(a) i. (2 marks)

$$D = \{1, 2, 3\} \qquad R = \{2, 3, 8, 9\}$$

ii. (2 marks)

$$D = \{x : x \in \mathbb{R}\} \qquad R = \{y : y \geq 0\}$$

(b) i. (1 mark) – relation.

ii. (1 mark)
More than one y value per x value.

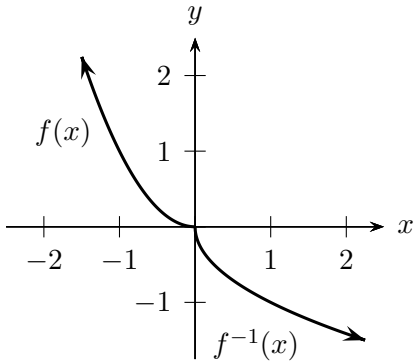
(c) (2 marks)

✓ [1] for interchanging x and y.

✓ [1] for correct final answer.

$$\begin{aligned}y &= 3x - 2 \\ x &= 3y - 2 \\ x + 2 &= 3y \\ \therefore y &= \frac{x+2}{3}\end{aligned}$$

(d) i. (1 mark)



ii. (1 mark) – see above.

iii. (2 marks)

✓ [1] for $x = y^2$.

✓ [1] for $y = -\sqrt{x}$.

$$y = x^2 \qquad x \leq 0$$

Interchanging variables,

$$\begin{aligned}x &= y^2 \qquad y \leq 0 \\ \therefore y &= -\sqrt{x}\end{aligned}$$

(e) i. (1 mark)

$$\begin{aligned}F(x) &= \frac{1}{x} + \frac{1}{x^2} - \frac{1}{x^3} \\ F(1) &= 1 + 1 - 1 = 1\end{aligned}$$

ii. (2 marks)

$$\begin{aligned}F\left(\frac{1}{\sqrt{2}}\right) &= \sqrt{2} + 2 - 2\sqrt{2} \\ &= 2 - \sqrt{2}\end{aligned}$$

iii. (1 mark)

$$\begin{aligned}F\left(x^2\right) &= \frac{1}{x^2} + \frac{1}{x^4} - \frac{1}{x^6} \\ &= \frac{x^4 + x^2 - 1}{x^6}\end{aligned}$$

Question 10 (Commences on page 9)

(a) i. (1 mark)

$$\frac{x^5y^2}{y^3} \times \frac{xy}{x^6} = 1$$

ii. (1 mark)

$$\left(\frac{5}{3}\right)^{-2} = \frac{9}{25}$$

(b) (2 marks)

✓ [1] for obtaining $3x - 9 = 12 - 4x$.
✓ [1] for final answer.

$$\begin{aligned} 8^{x-3} &= 16^{3-x} \\ 2^{3x-9} &= 2^{12-4x} \\ 3x - 9 &= 12 - 4x \\ 7x &= 21 \\ x &= 3 \end{aligned}$$

(c) (1 mark)

$$25^n \times 5^{n+3} = 5^{2n} \times 5^{n+3} = 5^{3n+3}$$

(d) (3 marks)

✓ [1] for removing negative indices within the parentheses.
✓ [1] for forming common denominator.
✓ [1] for final answer.

$$\begin{aligned} \left(x^{-1} + y^{-1}\right)^{-1} &= \left(\frac{1}{x} + \frac{1}{y}\right)^{-1} \\ &= \left(\frac{x+y}{xy}\right)^{-1} = \frac{xy}{x+y} \end{aligned}$$

Question 11 (Commences on page 10)

(a) (2 marks)

$$\begin{cases} y = x^2 - 4 \\ y = -2x + 2 \end{cases}$$

$$\begin{aligned} x^2 - 4 &= -2x + 2 \\ x^2 + 2x - 6 &= 0 \end{aligned}$$

Hence P and R are give the correct solutions.

(b) (3 marks)

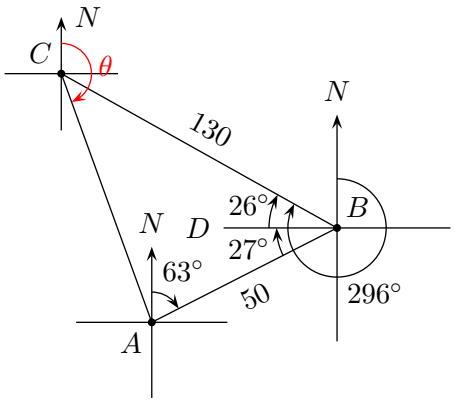
✓ [1] for converting to 5^{-1} and 5^0.
✓ [2] for solving simultaneous equations correctly.

$$\begin{cases} 5^{x+y} = \frac{1}{5} = 5^{-1} \\ 5^{3x+2y} = 5^0 \end{cases} \quad \Rightarrow \quad \begin{cases} x + y = -1 & (1) \\ 2x + 2y = -2 & (1a) \\ 3x + 2y = 0 & (2) \end{cases}$$

(2) − (1a)

$$\therefore x = 2 \quad y = -3$$

(c) i. (1 mark)



ii. (1 mark)

- $\angle ABD = 90° - 63° = 27°$ ($\angle$ sum of $\triangle ABD$)
- $\angle CBD = 296° - 270° = 26°$

$$\begin{aligned} \angle ABC &= \angle ABD + \angle CBD \\ &= 27° + 26° = 53° \end{aligned}$$

iii. (2 marks)

✓ [1] for applying the cosine rule.
✓ [1] for final answer.

$$\begin{aligned} AC^2 &= 50^2 + 130^2 - 2(50)(130)\cos 53° \\ &= 11\,576.40\cdots \\ AC &= 108\,\text{km} \end{aligned}$$

iv. (2 marks)

$$\cos\angle ACB = \frac{130^2 + AC^2 - 50^2}{2 \times 130 \times AC} = 0.9285\cdots$$

$$\therefore \angle ACB = 21.78°$$

Hence the bearing of A from C (shown as θ) is

$$180° - (360° - 296°) + \angle ACB = 138°$$