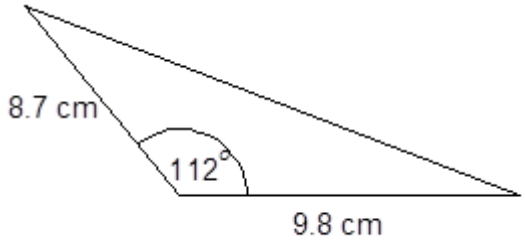
Question 1 (9 Marks) Commence a NEW page. **Marks**

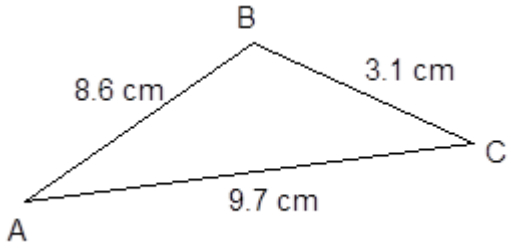
(a) Find the exact value of $\cos 150°$ **2**

(b) Find the area of the following triangle correct to 1 decimal place. **2**



NOT TO SCALE

(c) In ΔABC, $AB = 8.6cm$, $BC = 3.1cm$, and $AC = 9.7cm$. Find the size of $\angle ABC$ to the nearest minute. **2**



NOT TO SCALE

(d) Given θ is obtuse and $\sin\theta = \frac{1}{5}$, find the exact value of $\tan\theta$. **3**

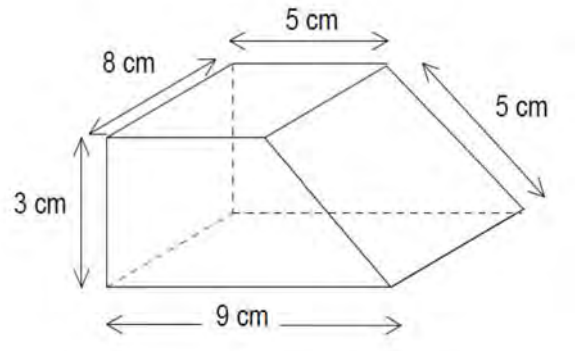
Question 2 (7 Marks) Commence a NEW page. **Marks**

(a) The diagram shows two bottles of Silvo Shampoo. **2**

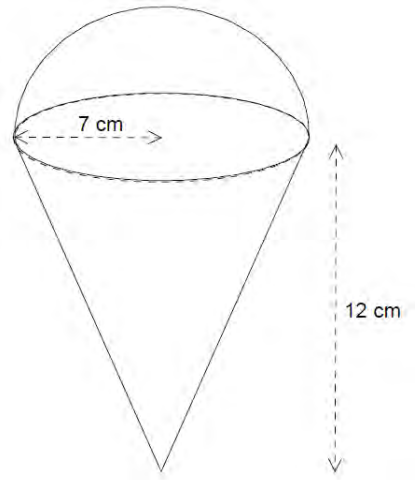
The two bottles are mathematically similar, and the cost of the shampoo depends only on the volume of the liquid in the bottle.

If the small one cost \$1.10, what should the large one cost (to the nearest cent)?

(b) A prism has a cross-section in the shape of a trapezium. Calculate the surface area of the prism. **2**

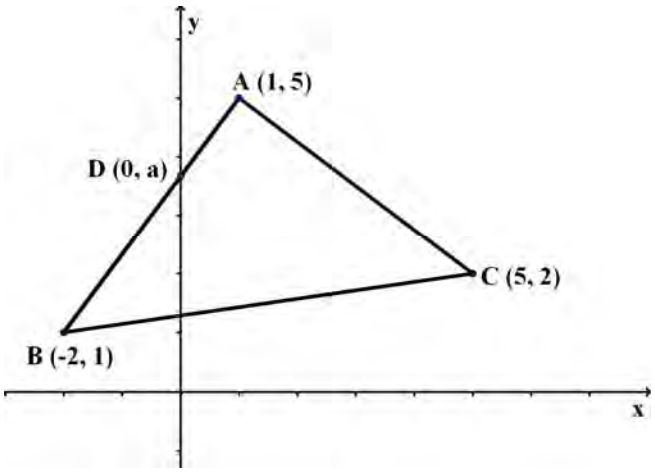


(c)



The diagram is formed from a hemisphere and a cone. Calculate the volume of the entire object. Give your answer correct to 2 decimal places. **3**

Question 3 (9 Marks) Commence a NEW page. **Marks**



The triangle ABC has vertices $A(1,5), B(-2,1)$ and $C(5,2)$ as shown.

D lies on AB and has the coordinates of $(0, a)$

(i) Show that the gradient of AB is $\frac{4}{3}$ **1**

(ii) Show that $\angle BAC$ is a right angle **2**

(iii) Show that the equation of the line AB is given by $4x - 3y + 11 = 0$ **2**

(iv) Determine the coordinates of D. **1**

(v) Given that the length of AB is 5 units, show that ΔABC is isosceles **2**

(vi) Find the area of ΔABC **1**

Question 4 (8 Marks) Commence a NEW page. **Marks**

(a) Solve $x^2 + 4x - 10 = 0$ **2**

(b) It is given that $x^2 - 6x + 13 = (x - a)^2 + b$, by completing the square,

(i) Find the values of a and b. **2**

(ii) Hence find the minimum value of $x^2 - 6x + 13$ **1**

(c) Sketch the graph of $y = 7 + 5x - 2x^2$ showing all important features. **3**

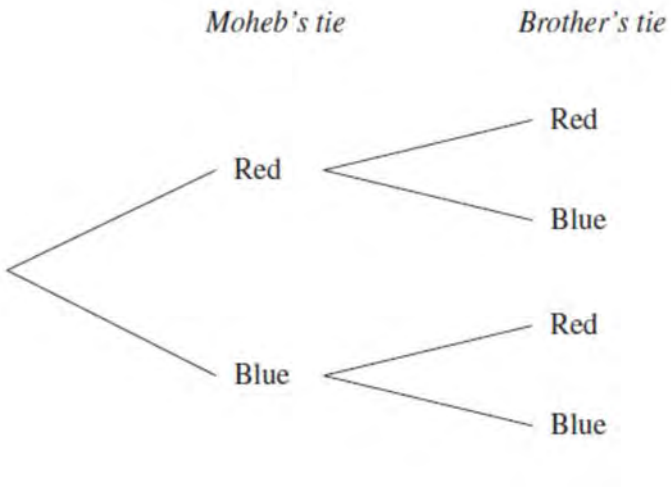
Question 5 (5 Marks) Commence a NEW page. **Marks**

Moheb owns five red and seven blue ties. He chooses a tie at random for himself and puts it on. He then chooses another tie at random, from the remaining ties, and gives it to his brother.

(i) What is the probability that Moheb chooses a red tie for himself? **1**

(ii) **Copy the tree diagram into your writing booklet**. **2**

Complete your tree diagram by writing the correct probability on each branch



(iii) Calculate the probability that both of the ties are the same colour. **2**

Question 6 (7 Marks) Commence a NEW page. **Marks**

(a) The ages of nine students in a team were recorded.

Ages		
12	11	16
14	16	15
14	15	14

(i) What is the standard deviation, correct to two decimal places? **1**

(ii) Briefly explain what is meant by the term *standard deviation.* **1**

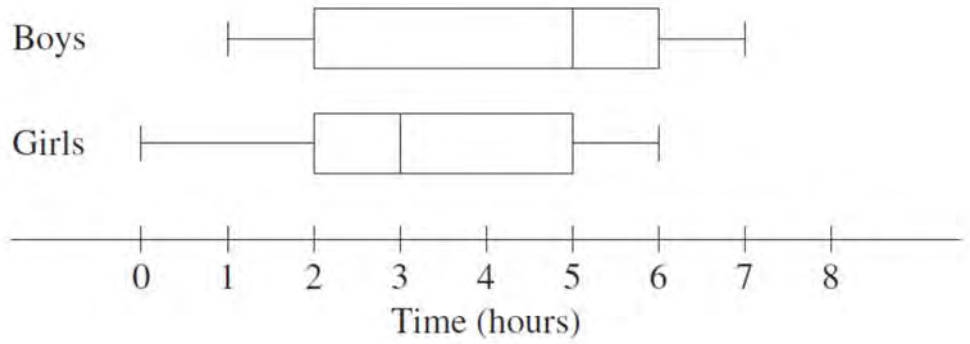
(b) The diagram below shows a stem-and-leaf plot for 22 scores.

2	3	5	9		
3	1	4	7	9	
4	2	4	4	5	7
5	1	2	4		
6	2	3	7		
7	5	8	8	8	

(i) What is the mode (s) for this data? **1**

(ii) What is the median for this data? **1**

(c) In a school, boys and girls were surveyed about the time they usually spend on the internet over a weekend. These results were displayed in box-and-whisker plots as shown below.



(i) Find the interquartile range for boys. **1**

(ii) What percentage of girls usually spend 5 or less hours on the internet over a weekend? **1**

(iii) Jenny said that the graph shows that the same number of boys as girls usually spend between 5 and 6 hours on the internet over a weekend. **1**
Under what circumstances would this statement be true?

Question 7 (6 Marks) Commence a NEW page. **Marks**

Jason made a deposit of $P in an investment account 5 years ago.

Jason's investment earned compound interest at the rate of 4.76% p.a. paid quarterly over the first 2 years and then increased to 4.84%p.a paid quarterly for the remaining 3 years.
At the end of the first 2 years, Jason had $11850 in his investment account.

(i) Calculate the amount of Jason's initial investment ($P) to the nearest dollar. **3**

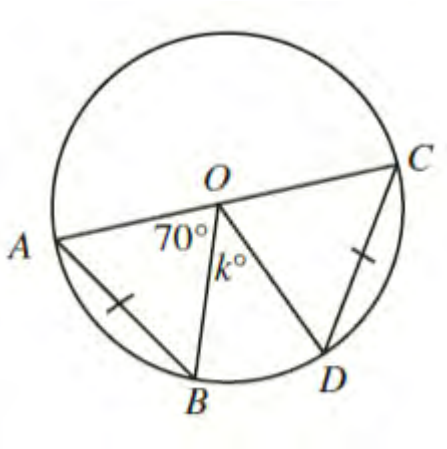
(ii) Calculate the amount (to the nearest dollar) Jason will have in his account at the end of the 5 year period. **2**

(iii) Determine the total amount of interest Jason earned on his investment. **1**

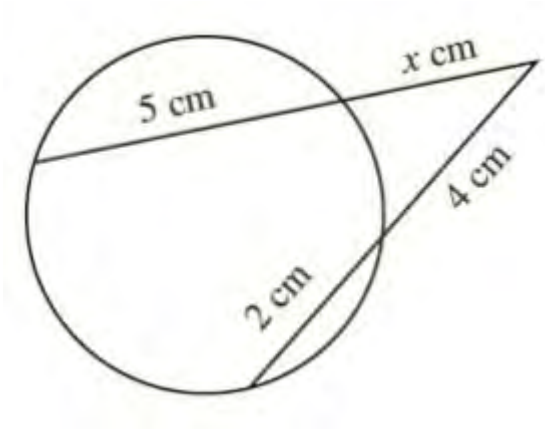
EXAMINATION CONTINUES ON NEXT PAGE

Question 8 (11 Marks) Commence a NEW page. **Marks**

(a) AC is the diameter and $AB = CD$. Find the value of k. (Show **all** working but geometrical reasoning are **not** required) **2**

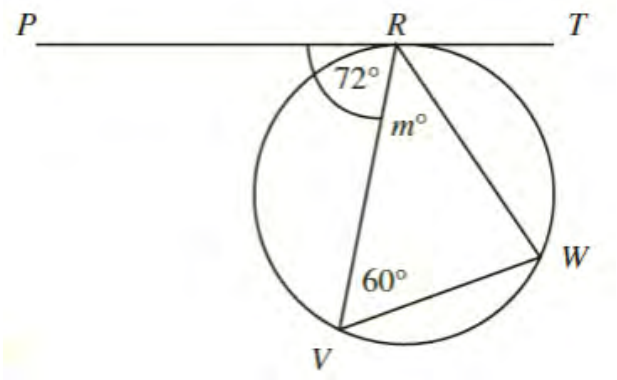


(b) Find the value of x. (Show **all** working but geometrical reasoning are **not** required) **3**



Find the value of m if PT is a tangent to the circle. (Show **all** working but geometrical reasoning are **not** required)

(c) **2**



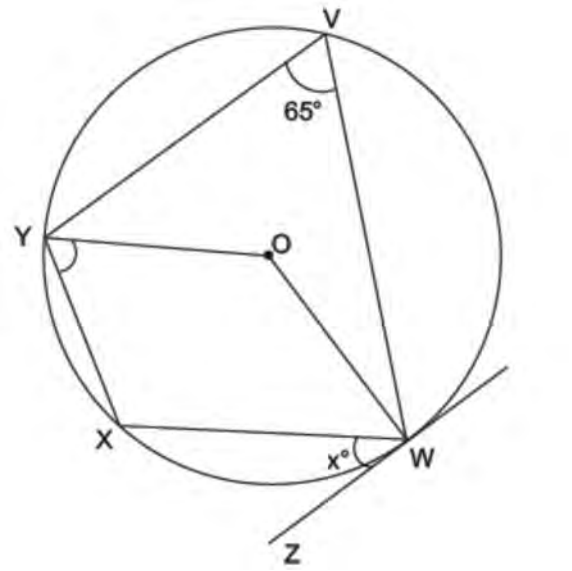
Find the value of m if PT is a tangent to the circle. (Show **all** working but geometrical reasoning are **not** required)

(d) In the diagram, V, W, X and Y lie on a circle, centre O. **4**

ZW is a tangent to the circle at W.

$$\angle YVW = 65°$$

$\angle XWZ = x°$

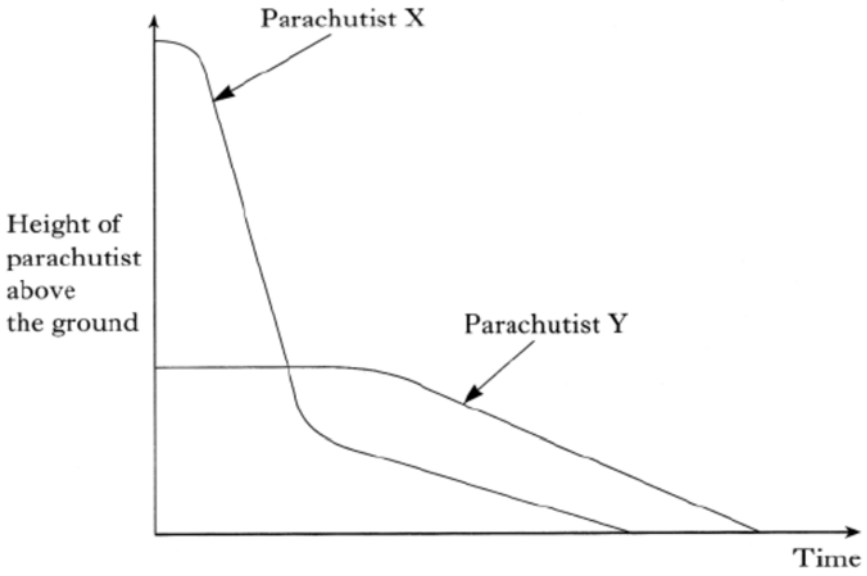


Find the size of $\angle OYX$ **in terms of** $\boldsymbol{x}$. Showing **all** geometric reasoning.

Question 9 (10 Marks) Commence a NEW page. **Marks**

(a) Two parachutists, X and Y, jump from two separate aircrafts at different times.

The graph shows how their height above the ground changes over a period of time.



(i) Which parachutist jumped first? **1**

(ii) Which parachutist did not open his parachute immediately after jumping? Explain your answer clearly. **2**

(b) On separate number planes sketch the graph of

(i) $y = 2\cos\frac{1}{2}\theta \quad -180° \leq \theta \leq 180°$ **3**

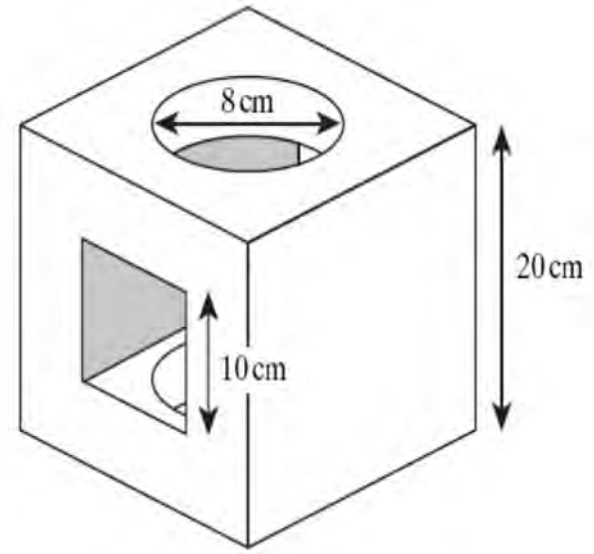
(ii) $(x-2)^2 + (y+3)^2 = 4$ **2**

(iii) $y = 2^{-(x-1)}$ **2**

EXAMINATION CONTINUES ON NEXT PAGE

Question 10 (9 Marks) Commence a NEW page. **Marks**

(a) A solid cube has a square hole cut through horizontally and a circular hole cut through vertically. **3**



Both holes are cut centrally in the appropriate faces.

The dimensions of the cube and the hole are shown in the diagram.

Calculate the volume remaining after the holes have been cut (to two decimal place).

(b) A box of chocolate has 6 chocolates: 3 milk, 2 white and 1 dark. Three friends, Alex, Ben and Chris will take two chocolates each at random, with Alex choosing first, then Ben, then Chris. What is the probability that each person receives a milk chocolate? **2**

(c) A man observes the top of a distant peak with an angle of elevation of $24°$ from a point A. After advancing a distance of 2 kilometres up a path inclined at $8°$ to the horizontal directly towards the peak, he arrives at point B. He finds the angle of elevation from point B to the peak is now $28°$.

(i) Sketch a diagram to represent the above information **1**

(ii) Find the height of the peak above point A. **3**

END OF EXAMINATION

Year 10 Yearly Solutions 2013

Question 1

N.B Rounding in questions is to make marking easier. Don't need to take marks off

a) $\cos 150^\circ = -\cos 30^\circ$ [1] Identity negative

$= -\frac{\sqrt{3}}{2}$ [1] Correct exact value

b) Area $= \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 8.7 \times 9.8 \times \sin 112^\circ$ [1]

$\doteq 39.5\,cm^2$ (to 1 d.p.) [1]

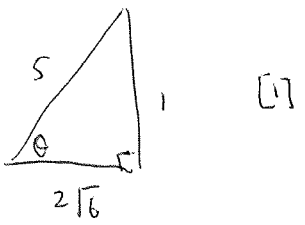
c) $\cos B = \frac{8.6^2 + 3.1^2 - 9.7^2}{2 \times 8.6 \times 3.1}$ [1]

$\doteq -0.197$

$\therefore \angle ABC \doteq 101^\circ 23'$ [1]

d) θ is in 2nd quadrant

$\therefore \tan\theta$ is negative [1]



[1]

$\therefore \tan\theta = -\frac{1}{2\sqrt{6}}$ [1]

/8

Question 2

a) $18 : 27 \equiv 2 : 3$

$\therefore$ volume is in ratio $2^3 : 3^3$

$8 : 27$ [1]

$\therefore$ cost of larger bottle is

$\$1.10 \times \frac{27}{8}$ [1]

$= \$3.71$

b) $SA = 2 \times \underbrace{\left(\frac{1}{2} \times (5+9) \times 3\right)}_{\text{trapezium faces}} + \underbrace{(3+9+5+5) \times 8}_{\text{perimeter} \times \text{length}}$ [1] [1]

$= 42 + 176$

$= 218\,cm^2$

-1 for no units.

c) $V_{Total} = V_{hemisphere} + V_{cone}$

$= \frac{1}{2} \times \frac{4}{3} \times \pi \times 7^3 + \frac{1}{3} \times \pi \times 7^2 \times 12$ [1] [1]

$= \frac{1274}{3}\pi$

$\doteq 1334.13\,cm^3$ [1]

/7

Question 3

N.B be careful of "fudging" here

i) $m_{AB} = \frac{5-1}{(1-(-2))}$ [1]

$= \frac{4}{3}$

ii) $m_{AC} = \frac{5-2}{1-5}$

$= \frac{3}{-4}$ [1]

$\therefore m_{AB} \times m_{AC} = \frac{4}{3} \times -\frac{3}{4}$

$= -1$ [1]

$\therefore AB \perp AC$

$\therefore \angle BAC$ is a right angle

iii) $y - 5 = \frac{4}{3}(x-1)$ [1]

$3y - 15 = 4x - 4$ [1] rearranging

$4x - 3y + 11 = 0$

iv) when $x = 0$

$-3y + 11 = 0$

$3y = 11$

$y = \frac{11}{3}$

$\therefore D\ (0, \frac{11}{3})$ [1]

v) $AC = \sqrt{(5-2)^2 + (1-5)^2}$

$= \sqrt{9+16}$

$= \sqrt{25}$

$= 5$

$\therefore AC = AB$

$\therefore \triangle ABC$ is isosceles

vi) Area $= \frac{1}{2} \times 5 \times 5$

$= \frac{25}{2}$ units^2

/9

Question 4.

a) $x^2 + 4x - 10 = 0$ [1] for correct method

$x^2 + 4x = 10$

$x^2 + 4x + 4 = 14$

$(x+2)^2 = 14$

$x + 2 = \pm\sqrt{14}$

$x = -2 \pm \sqrt{14}$ [1].

b) (i) $x^2 - 6x + 13$

$= (x^2 - 6x + 9) + 4$

$= (x-3)^2 + 4$ [1]

$\therefore a = 3 \quad b = 4$ [1]

(i) minimum is 4.

c)

$y = 7 + 5x - 2x$

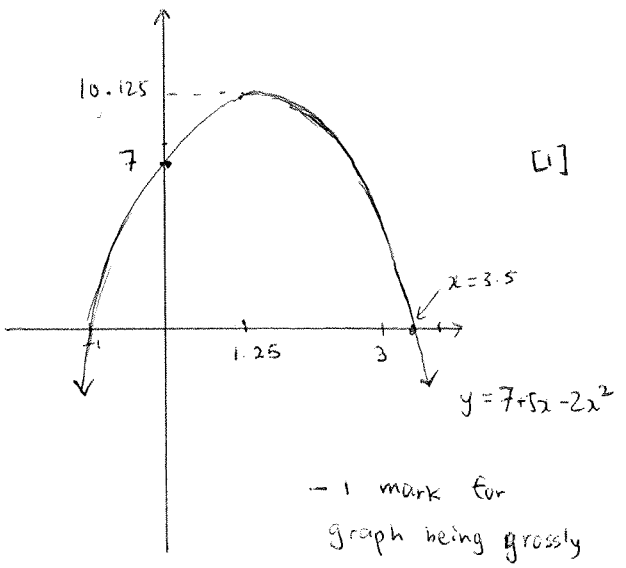
$= (7 - 2x)(1 + x)$

x int: $y = 0 \Rightarrow x = \frac{7}{2} \quad x = -1$ [2]

y int: $x = 0 \Rightarrow y = 7$

vertex at $x = -\frac{5}{4} \quad y = 10.125$

-1 for each incorrect feature



[1]

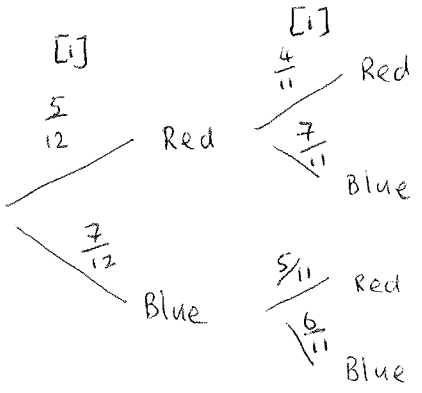
-1 mark for graph being grossly not to scale.

/8

Question 5

i) $\frac{5}{12}$ [1]

ii) [1] [1]



iii) $P(RR) + P(BB)$ [1]

$= \frac{5}{12} \times \frac{4}{11} + \frac{7}{12} \times \frac{6}{11}$

$= \frac{20+42}{132}$

$= \frac{62}{132}$ [1]

$= \frac{31}{66}$

max ½ if only 1 colour tie

/5

Question 6

a) i) 1.59 [1]

ii) Standard deviation is a measure of spread around the mean. (Root of the variance) [1]

Key words: spread, mean.

b) i) 78 [1]

ii) $\frac{45+47}{2}$

$= 46$ [1]

c) i) $IQR = Q_3 - Q_1$

$= 6 - 2$

$= 4$ [1]

ii) 75% [1]

iii) Same number of boys as girls. [1]

/7

Question 7

i) $A = P(1+r)^n$

$A = \$11850$ $r = \frac{4.76}{4} = 1.19$ $n = 4 \times 2 = 8$ [1]

$\therefore 11850 = P(1.0119)^8$ [1]

$\therefore P = \frac{11850}{1.0119^8}$

$= \$10780$ [1]

ii) $P = \$11850$ $r = 4.84 \div 4 = 1.21$ $n = 3 \times 4 = 12$ [1]

$\therefore A = 11850\ (1.0121)^{12}$

$= \$13690$ [1]

iii) Interest earned $= \$13690 - \$10780$

$= \$2910$. [1]

/6

Question 8

a) $\angle AOC = 180°$

$\angle AOB = \angle COD = 70°$ [1]

$\therefore k = 180 - 70 - 70$

$= 40$ [1]

b) $x(x+5) = 4 \times 6$ [1]

$x^2 + 5x = 24$

$x^2 + 5x - 24 = 0$ [1]

$(x+8)(x-3) = 0$

$\therefore x = 3$ or $x = -8$

$\therefore x = 3$

Since $x > 0$

[1] for negation

c) $\angle RWV = 72°$ [1]

$\therefore m = 180 - 60 - 72$

$= 48°$ [1]

d) $\angle YOW = 130°$ (angle at centre is twice angle at circumference standing on same arc YW) [1]

$\angle YXW = 115°$ (opposite angles in cyclic quadrilateral XWVY are supplementary) [1]

$\angle OWX = 90 - x$ (radius meets tangent at 90°, and $\angle OWX$ is adjacent to $\angle XWZ$) [1]

$\therefore \angle OYX = 360 - (130 + 115 + 90 - x)$

$= 25 + x$ [angle sum of quadrilateral is equal to 360°] [1]

- [1] mark for incorrect reasonings up to 2 marks. i.e. no reasoning maximum mark obtainable is 2/4 up to marker on detail of reasonings.

11

Question 9

a) i) X [1]

ii) X, he has a steeper gradient at the start indicating free fall, after some time, he opens his parachute which is indicated by a change in gradient.

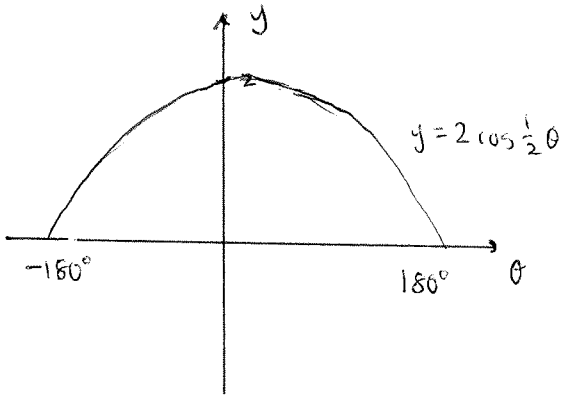
[1] 1 for X

[1] if they show understanding of rates of change / gradients.

b) i) $y = 2\cos\frac{1}{2}\theta$

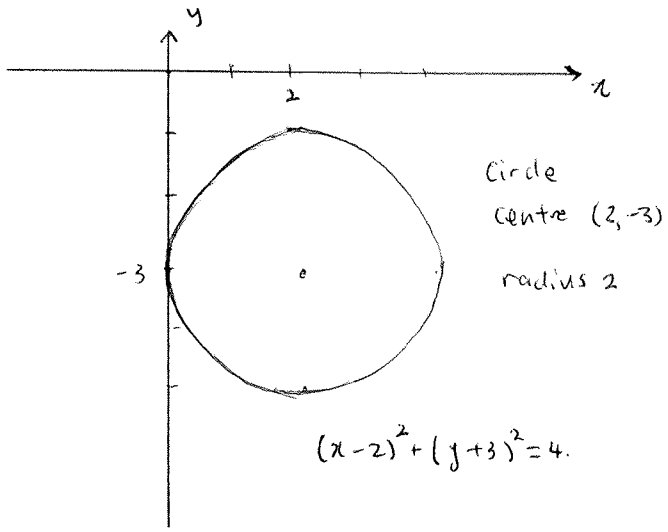
amplitude = 2. [1]

period = 720° [1]



[1]

ii)



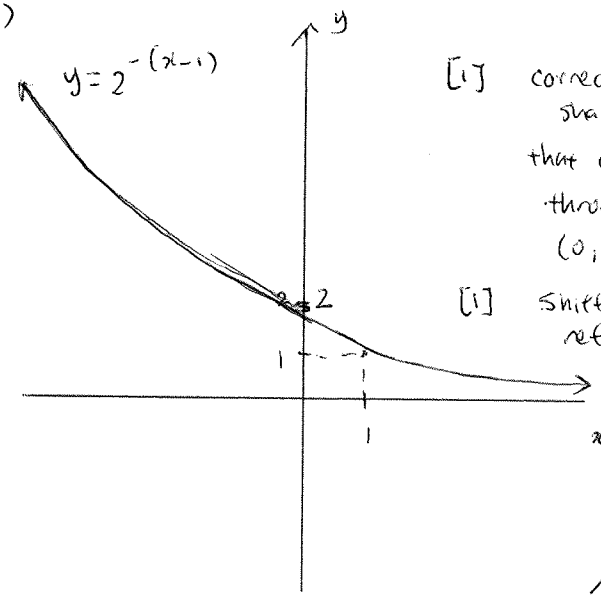
Circle
centre (2, -3)
radius 2

$(x-2)^2 + (y+3)^2 = 4$.

[1] correct centre

[1] correct radius

iii)



[1] correct shape that goes through (0, 2)

[1] Shifted and reflected.

/10

N.B

-1 mark for each mistake on graphs.

Question 10

a) $V = \underbrace{20^3 - (10^2 \times 20)}_{\text{rectangular prism}} - \underbrace{2 \times (\pi \times 4^2 \times 5)}_{\text{the two extra cylinders}}$ [1] [1]

$\doteq 5497.35\ cm^3$ (to 2d) [1]

b) P(1 milk chocolate each) $= 2\left(\frac{3}{6} \times \frac{3}{5}\right) \times 2\left(\frac{2}{4} \times \frac{2}{3}\right) \times 2\left(\frac{1}{2} \times \frac{1}{1}\right)$

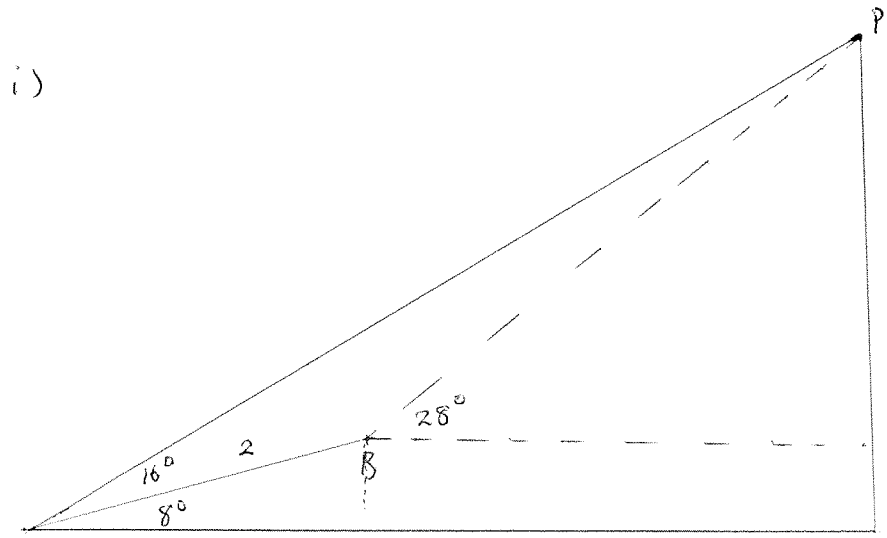
$= \frac{2}{5}$

$= 40\%$

2/2 for correct answer

1/2 if recognise dependancy and use multiplication principle.

c) i)



[1]

ii) $\angle ABP = 160^\circ$

$\therefore \angle APB = 4^\circ$

} [1]

$\frac{BP}{\sin 16^\circ} = \frac{2}{\sin 4^\circ}$ [1]

$\therefore BP = 7.9\text{km}$

$\frac{AP}{\sin 160^\circ} = \frac{2}{\sin 4^\circ}$

$\therefore AP = \frac{2 \sin 160^\circ}{\sin 4^\circ}$

$= 9.81$

$\therefore h = 9.81 \times \sin 24^\circ$

$= 3.99\text{ km}$ [1]

- 1 mark for obtaining expressions that are solvable
- 1 mark for significant process on solving
- last mark for correct answer

/9