



NORTH SYDNEY BOYS HIGH SCHOOL

2014 YEAR 10 YEARLY EXAM STAGE 5.3 MATHEMATICS

Student Name: _____

Class: _____

Teacher _____

General Instructions

- Working time – **90 minutes**.
- Write on **one side** of your paper only
- Write using blue or black **pen**.
- **Board approved** calculators are allowed
- All necessary working should be shown in every question.
- Each new **Section** is to be started on a **new page**
- Marks may not be awarded for carelessly or badly arranged work.

(To be used by the exam markers only.)

Section	A	B	C	D	E	F	Total	%
Mark	<u>24</u>	<u>17</u>	<u>15</u>	<u>26</u>	<u>23</u>	<u>13</u>	<u>118</u>	

Section A

Question

Marks

1. Express as a fraction with a rational denominator.

3

a) $\frac{3}{\sqrt{2}}$

b) $\frac{2}{5-2\sqrt{5}}$

2. Write 0.000 000 286 in scientific notation, correct to two significant figures.

2

3. Simplify fully $\sqrt{5} - \sqrt{45}$

1

4. Express $\frac{x-3}{2} - \frac{1-x}{3}$ with a single denominator.

2

5. Change the subject of $y = \frac{x}{2x-1}$ to x

3

6. a) Find the exact value of $\cos 135^\circ$

9

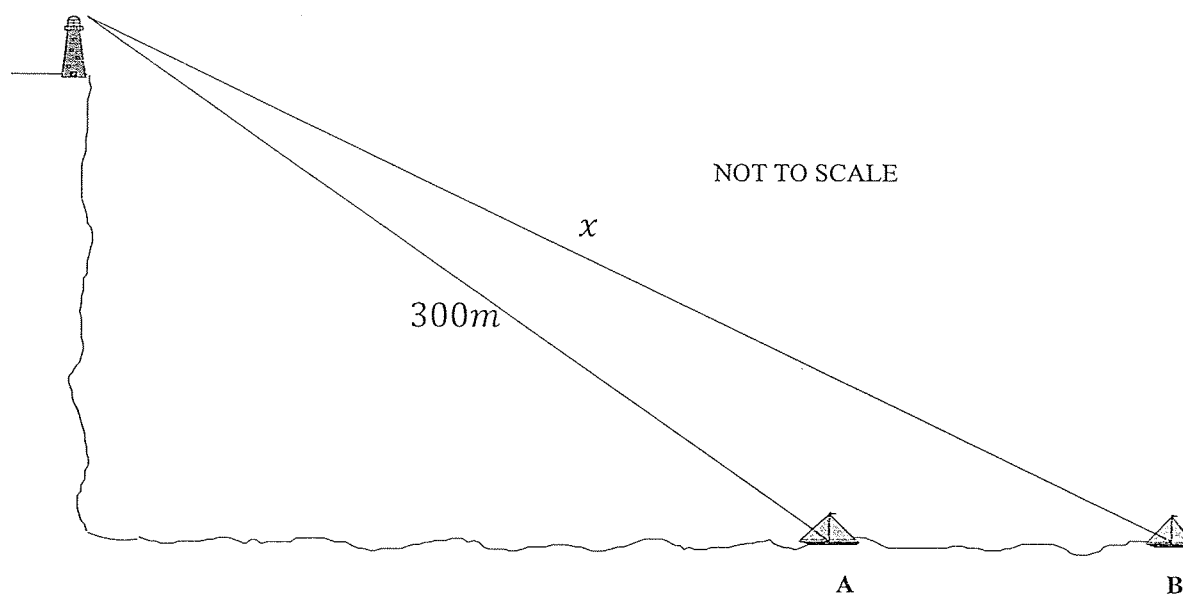
- b) Write $\tan(90^\circ - \theta)$ as a trig ratio in terms of θ .

- c) Solve $\sin^2 \theta = \frac{1}{2}$ for $0^\circ \leq \theta \leq 360^\circ$.

- d) Graph $y = -2 \cos \theta + 1$ for $0^\circ \leq \theta \leq 360^\circ$

7. Two yachts are both due east of a lighthouse, which is on top of a cliff. The angle of elevation from Yacht A is 37° and from Yacht B 23° . It is known that Yacht A is exactly 300m from the lighthouse.

4



Sketch this diagram on your answer sheet.

- a) Find the value of x (to the nearest metre).
 b) Hence or otherwise find the distance between the two ships, correct to the nearest metre.

Section B
Start a new page

Marks

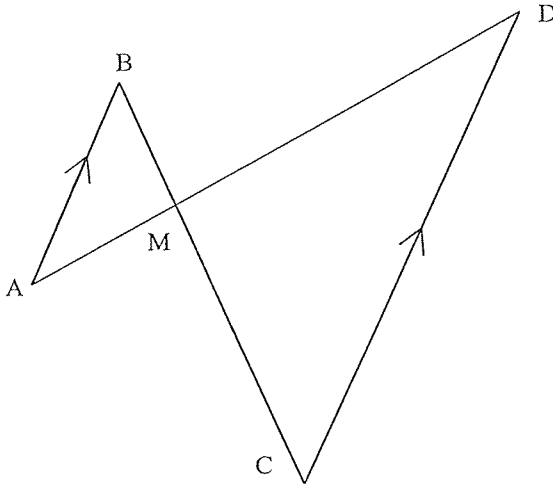
Question

1. In the diagram below, $AB \parallel CD$ and DA intersects BC at M .

4

a) Prove $\triangle AMB \sim \triangle DMC$.

b) If $AB:CD = 2:5$ and $\text{area } \triangle AMB = 10 \text{ unit}^2$, find the area of $\triangle DMC$



2. John bought a motorbike for \$7000. He paid 15% as a deposit and borrowed the remainder from a bank for 5 years at 24% p.a. simple interest. How much is the monthly repayment?

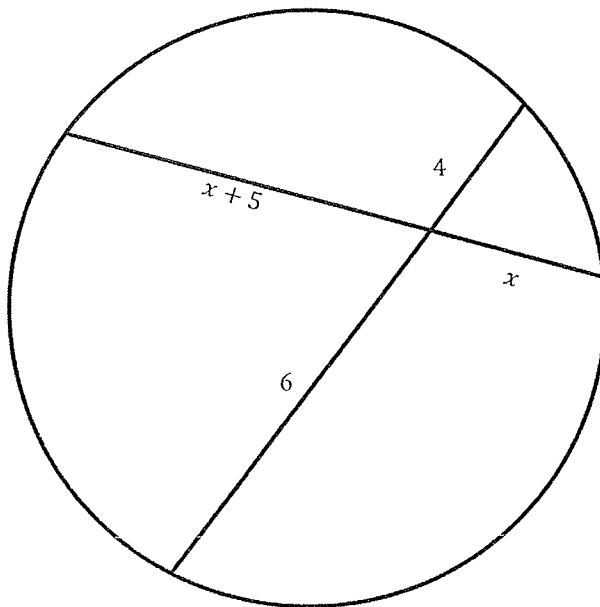
3

3. A photocopier depreciates each year by $r\%$. If it depreciates in value from \$23 000 to \$10 500 in three years, find the value of r .

2

4. Find the value of x . (You do not need to give reasons).

4

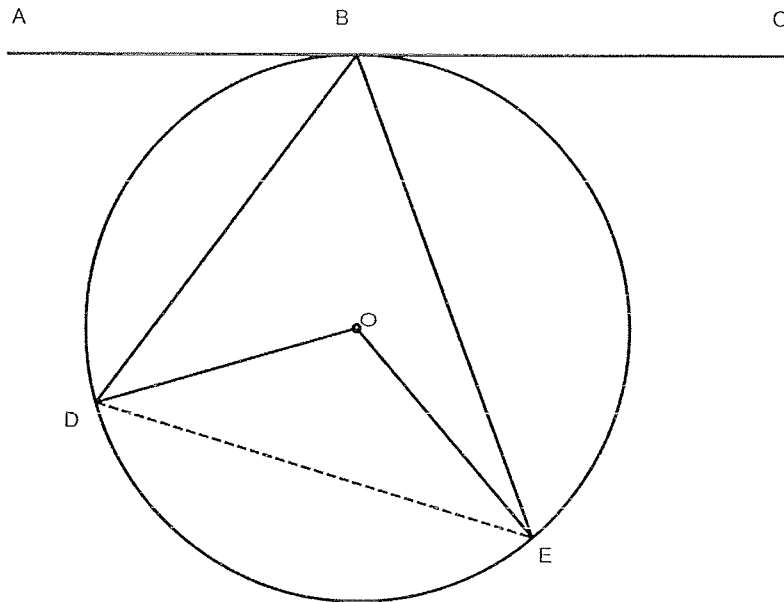


Section B continued**Question****Marks**

5. In the diagram below, O is the centre of the circle, AC is a tangent at B and D and E are points on the circumference. If $\angle ABD = 80^\circ$, and $\angle DBE = 40^\circ$, find the following angles, giving reasons.

4

- a) $\angle BED$
 b) $\angle BEO$



Section C
Start a new page

Question**Marks**

1. a) Solve simultaneously for x and y
 $x - 4y + 12 = 0$ and $y = 3 - 2x$.
- b) Solve $x^2 - 3x - 88 = 0$
2. Write down the coordinates of the vertex of the parabola $y = (x + 1)^2 - 2$
3. a) Find the equation of the axis of symmetry of the parabola $y = x^2 - 2x + 5$
 b) Hence, sketch this parabola, showing the coordinates of the vertex, and the y -intercept.
4. The points $A(-1, -1)$, $B(4, 2)$, $C(3, 6)$ are the vertices of a triangle
- a) Find the equation of the line AB
- b) Find the equation of the line perpendicular to AB and going through C .
- c) Find the acute angle (to the nearest degree) that the line AB makes with the positive x -axis.

4

1

4

6

Section D
Start a new page

Question **Marks**

1. Find the equation of the locus of the point $P(x, y)$ so that its distance from the point $A(0,3)$ is twice its distance from the point $B(4,7)$ 3
2. Draw neat quarter-page sketch of the following functions, clearly labelling all intercepts: 9
 - a) $y = x^3$
 - b) $y = 2^x$
 - c) $y = -\frac{3}{x}$
 - d) $x^2 - 6x + y^2 + 4y = 3$

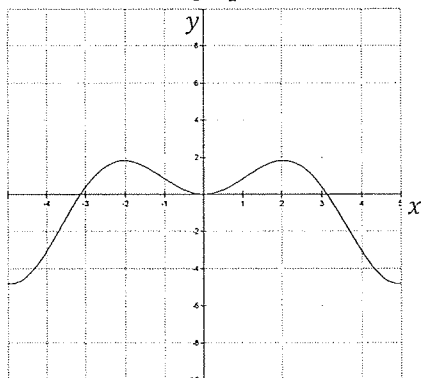
3. The scores for a class test are 6, 4, 9, 6, 5, 7, 2. Calculate the standard deviation for the class. 1

4. The table below compares the mean and standard deviation of the times (in minutes) of male and female athletes in a club on their 20 km training run. The times for both males and females are distributed normally. 4

- (i) Jason runs 20 km in 54.4 minutes. What percentage of males had a lesser time than Jason?
- (ii) Melissa runs 20km in 55.3 minutes. Did Melissa or Jason perform better in comparison to the other members of their respective sex? Explain your answer.

Sex	Mean Time	Standard Deviation
Male	60.2	5.8
Female	62.5	3.6

5. State whether each of the following relations is a function or not. 3
 - i) The set of ordered pairs (1, 2), (1, 3), (2, 3) (3, 4)
 - ii) The relation $x = y^2$
 - iii) The relation whose graph is shown below.



6. For the function $f(x) = x^2 - 6x$: 6
 - (i) Evaluate $f(3)$
 - (ii) Evaluate $f(x + 2)$
 - (iii) Find the inverse **relation** in the form $y = g(x)$
 - (iv) Give a domain for $f(x)$ so that the inverse relation is a function

Section E
Start a new page

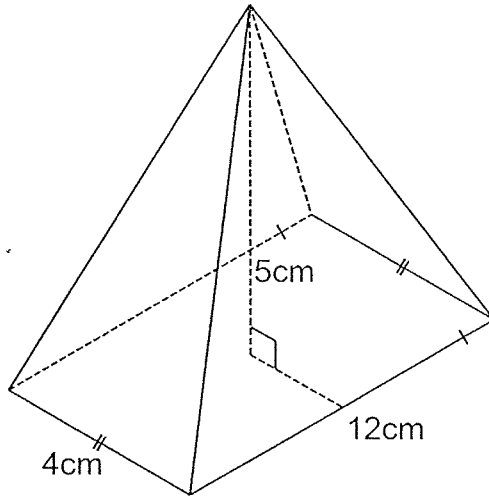
Question	Marks
1. Simplify $\log_3 27 + \log_3 9$	1
2. Given that $\log_a b = 2.75$ and $\log_a c = 0.25$, find the value of: a) $\log_a \left(\frac{b}{c}\right)$ b) $\log_a (bc)^2$	4
3. Solve $\log_2(3x - 4) = 5$	2
4. Simplify $\log(x^2 - x - 2) - \log(x + 1)$	2
5. Sketch the graph of $y = \log_{10}(x - 1)$	3
6. State the coefficient of the leading term in $2x^2 - 3x^4 + 5x$	1
7. Divide $2x^4 + 6x^3 + x + 1$ by $x + 1$	2
8. Andy, Roger, Leyton and Jo turn up for tennis practice. a) How many games will be played if every player is to play a practice game against each of the others? b) If all games are arranged randomly, what is the probability that Andy will play Leyton, then Jo, then Roger, in that order?	3
9. When Lleyton and Bec play tennis, Bec has a 0.1 chance of winning any particular game. On the weekend they intend to play 3 games. Using a tree diagram or otherwise, calculate the probability (as a fraction) that : (i) Bec wins no games. (ii) Bec wins at least one game. (iii) Bec wins 2 or more games.	5

Section F
Start a new page

Question

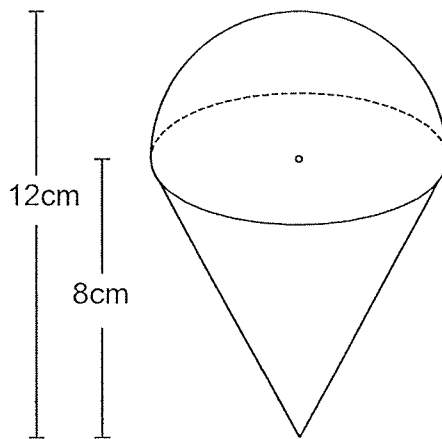
Marks
5

1. Find for the following rectangular pyramid
 - a) the volume
 - b) the surface area



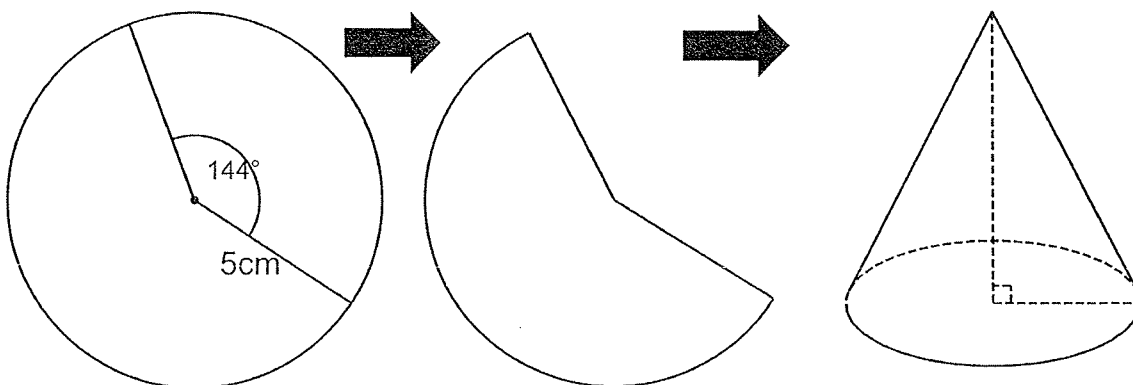
2. Find the volume of the following solid which has a hemisphere stacked on top of a cone.

4



3. A circular piece of paper with radius 5cm has the sector containing an angle of 144° removed. The remaining sector is folded to make a right circular cone. Find the surface area of the enclosed cone created by the sector.

4



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YEAR 10 YEARLY SOLNS - 2014

(1)

Section A

1. a) $\frac{3\sqrt{2}}{2}$

(1)

b) $\frac{2}{5-2\sqrt{5}} \times \frac{5+2\sqrt{5}}{5+2\sqrt{5}}$

(1)

$$= \frac{2(5+2\sqrt{5})}{25-4 \times 5}$$

(1)

$$= \frac{2(5+2\sqrt{5})}{5}$$

2. $2.86 \times 10^{-7} = 2.9 \times 10^{-7}$

(2)

- 1 for 2.9
- 1 for 10^{-7}

3. $\sqrt{5} - 3\sqrt{5} = -2\sqrt{5}$

(1)

4. $\frac{3(x-3) - 2(1-x)}{6}$

$$= \frac{3x - 9 - 2 + 2x}{6}$$

(1)

$$= \frac{5x - 11}{6}$$

(1)

5. $y(2x - 1) = x$

$$2xy - y = x$$

(1)

$$2xy - x = y$$

$$x(2y - 1) = y$$

(1)

$$x = \frac{y}{2y - 1}$$

(1)

6. a) $-\cos 45^\circ = -\frac{1}{\sqrt{2}}$

(2) - 1 mark off for no neg.

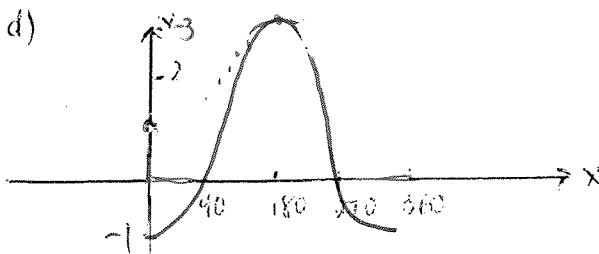
b) $\frac{\sin(90^\circ - \theta)}{\cos(90^\circ - \theta)} = \frac{\cos \theta}{\sin \theta} = \cot \theta$

(1) for $\frac{\cos \theta}{\sin \theta}$ or $\cot \theta$

c) $\sin \theta = \pm \frac{1}{\sqrt{2}}$

$$\theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ$$

(3)

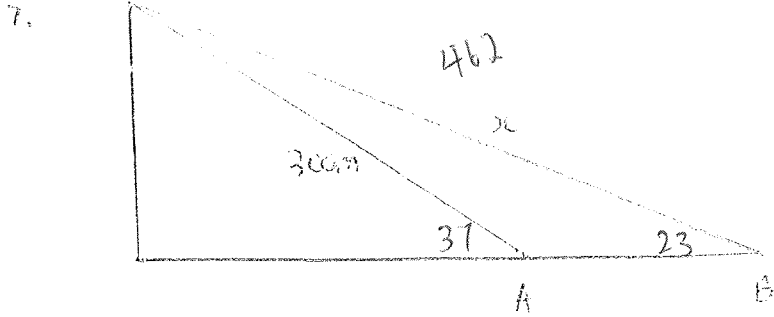


(3)

1 for negative graph

1 for shift up

1 for correct shape of cos not sin.



a) $\frac{x}{\sin 143} = \frac{300}{\sin 23}$

$$x = \frac{300 \sin 143}{\sin 23}$$

$$= 462\text{m}$$

(1) for correct set up

(1) correct answer

b) $y^2 = 300^2 + x^2 - 2(300)(x) \cos 15^\circ$

$$y = \sqrt{300^2 + x^2 - 2(300)(x) \cos 15^\circ}$$

$$= 186.$$

(1) for set up

(1) for correct answer

Section B

1. $\hat{BMA} = \hat{CMD}$ (Vertically opposite)
 $\hat{ABM} = \hat{DCM}$ (Alternate angles,
 $AB \parallel CD$)

(2)

$\therefore \triangle AMB \parallel \triangle DMC$ (Equiangular)

b) $AB : CD$

$2 : 5$

Area $4 : 25$

$10 : x$

$$\frac{x}{25} = \frac{10}{4}$$

(1) for proportion equ'n

$$x = 62.5 \text{ units}^2$$

(1) for answer

$$2. \text{ Deposit} = 0.15 \times 7000$$

$$= \$1050$$

(3)

$$\text{Loan} = 7000 - 1050$$

$$= \$5950 \quad (1) -$$

$$\text{Interest} = 0.24 \times 5950 \times 5$$

$$= \$7140 \quad (1)$$

$$\text{Total} = 7140 + 5950$$

$$= \$13090$$

$$\text{Monthly Repayment} = \frac{13090}{60}$$

$$= 218.17 \quad (1)$$

$$3. \quad 10500 = 23000(1-r)^3$$

$$\sqrt[3]{\frac{10500}{23000}} = 1-r$$

$$r = 1 - \sqrt[3]{\frac{10500}{23000}} \quad (1)$$

$$= 0.324 \dots$$

$$= 32.43\% \text{ (2 d.p.)} \quad (1)$$

$$23\%$$

if they used SI instead of compound will have to still give marks ???

$$4. \quad x(x+5) = 6x+4 \quad (1)$$

$$x^2 + 5x - 24 = 0 \quad (1)$$

$$(x+8)(x-3) = 0$$

$$x = 3, x > 0 \quad (2)$$

a) $\hat{BED} = \hat{ABD} = 80^\circ$
 (Angle between chord & tangent equal to angle in alternate segment)

(1) 1 for answer
 1 for reason

b) $\hat{DOE} = 2\hat{DBE} = 80^\circ$
 (Angle at centre twice angle at circumference, standing on same arc)

(2) need to have 2 reasons

$\hat{OED} = \frac{180 - 80}{2}$ (Equal angles opposite equal sides, DO and EO radii)
 $= 50^\circ$

Section C

1. a) $x - 4y + 12 = 0$ (1)
 $y = 3 - 2x$ (2)

$x - 4(3 - 2x) + 12 = 0$
 $x - 12 + 8x + 12 = 0$
 $9x = 0$
 $x = 0$
 $y = 3$

(1)

(1)

b) $(x - 11)(x + 8) = 0$
 $x = -8, 11$

(2)

2. $(-1, -2)$

(1)

(5)

3.

a) $x = -\frac{b}{2a}$

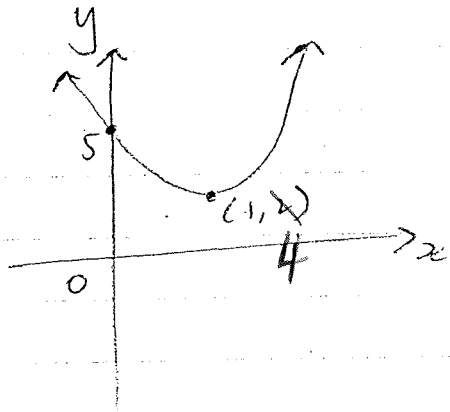
$= -\frac{(-2)}{2}$

$= 1$

_____ (2)

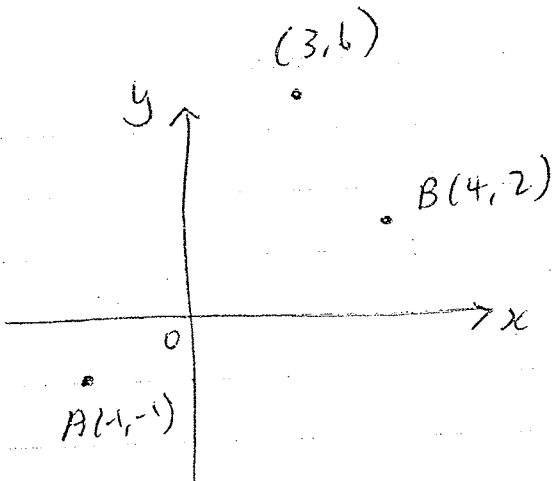
$y = 1 - 2 +$
 $=$

b)



- (2) _____ y int
 _____ Min value
 to scale.
 - label axes
 - shape

4.



a) $m_{AB} = \frac{2+1}{4+1}$

$= \frac{3}{5}$

$y - 6 = \frac{3}{5}(x - 3)$

$5y - 30 = 3x - 9$

$3x - 5y + 21 = 0$

(2)

$$b) m = -\frac{5}{3} \quad C(3, 6)$$

$$y - 6 = -\frac{5}{3}(x - 3)$$

$$3y - 18 = -5x + 15$$

$$5x + 3y - 33 = 0 \quad (2)$$

$$c) \tan \theta = m$$

$$\tan \theta = \frac{3}{5}$$

$$\theta = 31^\circ \quad (2)$$

Section D

$$1. PA = 2PB$$

$$PA^2 = 4PB^2 \quad (1)$$

$$PA^2 = x^2 + (y-3)^2$$

$$PB^2 = (x-4)^2 + (y-7)^2$$

$$\therefore x^2 + (y-3)^2 = 4[(x-4)^2 + (y-7)^2]$$

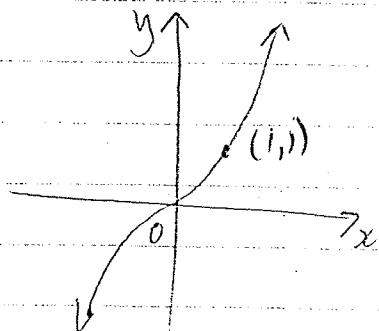
$$x^2 + y^2 - 6y + 9 = 4(x^2 - 8x + 16 + y^2 - 14y + 49) \quad (1)$$

$$x^2 + y^2 - 6y + 9 = 4x^2 - 32x + 64 + 4y^2 - 56y + 196$$

$$3x^2 - 32x + 3y^2 - 50y + 251 = 0 \quad (1)$$

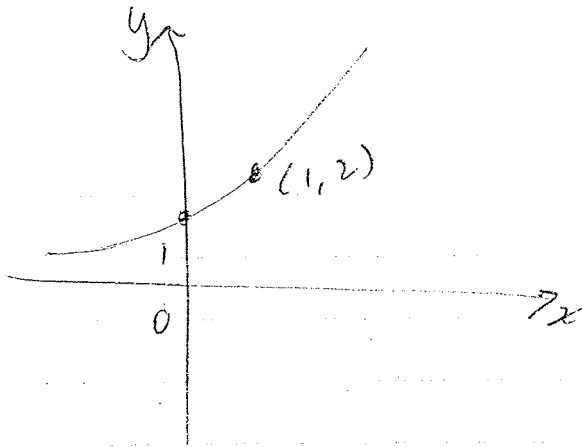
2. a)

$$a) y = x^3$$



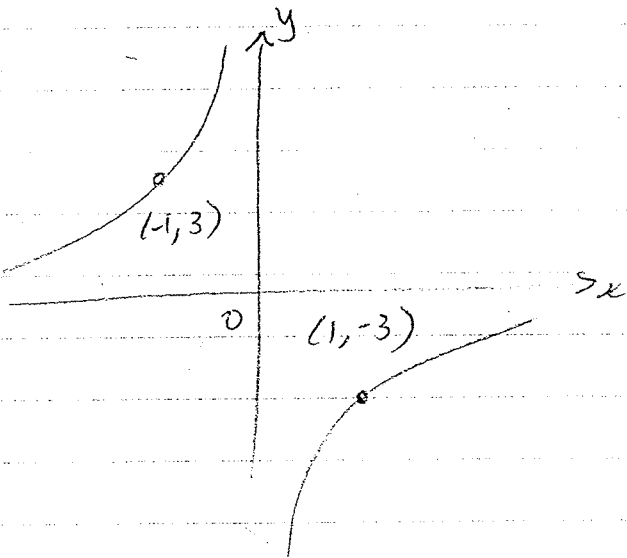
(2) shape point.

b) $y = 2^x$



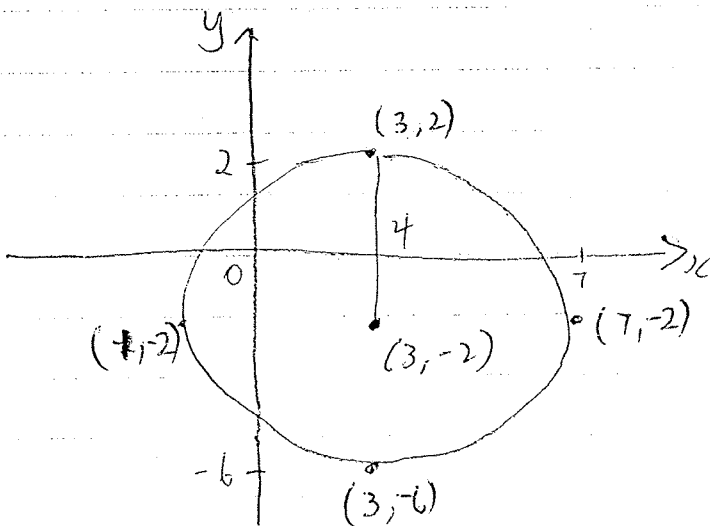
- (2) - Y-int / shape
- point

c) $y = -\frac{3}{x}$



- (2) shape / asymptotes
- point

d) $x^2 - 6x + (-3)^2 + y^2 + 4y + 2^2 = 3 + 9 + 1$
 $(x - 3)^2 + (y + 2)^2 = 16$



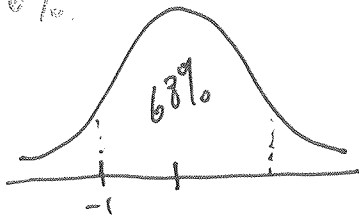
- (3) - centre
- endpoints of circle can be seen clearly
- (radius can be) seen

3 ~~2.3324~~ 2.06

(1)

(8)

4. (i) 16%



(f) — 1 mark for calculating he is 1 standard deviation below mean

— 1 mark for calculating percentage

(ii) Jason did better and Melissa is more stand. dev. away from mean.

Jason's z-score = -1

Melissa did better

Melissa's z-score = -2

(2) — 1 for calculation of z-scores
1 for explanation

5. (i) NO

(1)

(ii) NO.

(1)

(iii) YES

(1)

6. (i) $f(x) = x^2 - 6x$

$f(3) = 3^2 - 6(3)$

$= 9 - 18$

$= -9$

(1)

(ii) $f(x+2) = (x+2)^2 - 6(x+2)$

(1)

$= x^2 + 4x + 4 - 6x - 12$

$= x^2 - 2x - 8$

(1)

(iii) $f^{-1}: x = y^2 - 6y$

(1)

$(-3)^2 + x = y^2 - 6y + (-3)^2$

$x + 9 = (y - 3)^2$

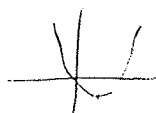
$y - 3 = \pm \sqrt{x + 9}$

$y = \pm \sqrt{x + 9} + 3$

(1)

(iv) $f(x) = x^2 - 6x = x(x - 6)$

D: $x \geq 3$ OR $x \leq -3$



(1) one or the other only not need to have both.

Section E

(9)

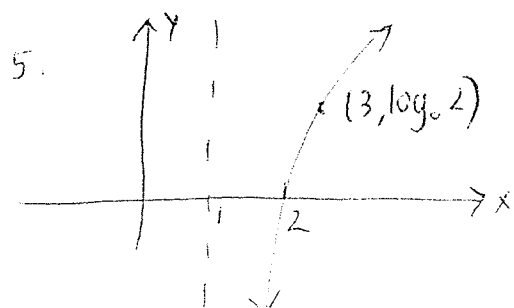
$$\begin{aligned} 1. \quad & \log_3 3^3 + \log_3 3^2 \\ & = 3 \log_3 3 + 2 \log_3 3 \\ & = 5 \end{aligned} \quad (1)$$

$$\begin{aligned} 2. \quad a) \quad & \log_a \left(\frac{b}{c} \right) \\ & = \log_a b - \log_a c \\ & = 2.75 - 0.25 \\ & = 2.5 \end{aligned} \quad (1)$$

$$\begin{aligned} b) \quad & \log_a (bc)^2 \\ & = 2 \log_a bc \\ & = 2 [\log_a b + \log_a c] \\ & = 2 (2.75 + 0.25) \\ & = 6 \end{aligned} \quad (1)$$

$$\begin{aligned} 3. \quad & 2^5 = 3x - 4 \\ & 32 = 3x - 4 \\ & 3x = 36 \\ & x = 12 \end{aligned} \quad (1)$$

$$\begin{aligned} 4. \quad & \log \left(\frac{x^2 - x - 2}{x+1} \right) \\ & = \log \frac{(x-2)(x+1)}{x+1} \\ & = \log(x-2) \end{aligned} \quad (1)$$



(3)

1 mark for x int
1 mark for asymptote
1 mark for p+.

6. -3

(1)

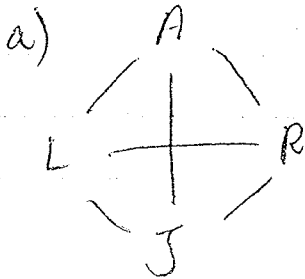
(10)

7

$$\begin{array}{r}
 2x^3 + 4x^2 - 4x + 5 \quad R - 4 \\
 x+1 \overline{) 2x^4 + 6x^3 + 0x^2 + x + 1} \\
 \underline{2x^4 + 2x^3} \\
 4x^3 + 0 \\
 \underline{4x^3 + 4x^2} \\
 -4x^2 + x \\
 \underline{-4x^2 - 4x} \\
 5x + 1 \\
 \underline{5x + 5} \\
 -4
 \end{array}$$

(2) - -1 for error

8.a)



6 games

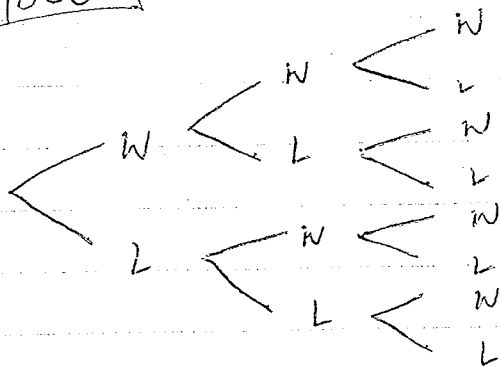
(2)

b) $\frac{1}{6}$

(1)

9.

Bec



NWL
NLW
LWW

(1)

b) i) $P(\text{no win}) = 0.9^3 = 0.729$

(1)

ii) $1 - P(\text{no win})$

$= 1 - 0.729$

$= 0.271$

(1)

$$\text{iii) } P(\geq 2 \text{ wins})$$

$$= P(2 \text{ wins}) + P(3 \text{ wins})$$

$$= P(WNL) + P(NLW) + P(LNW) + P(WNW)$$

$$= (0.1)^2 \times 0.9 + 0.1 \times 0.9 \times 0.1 + 0.1^3 + 0.9 \times (0.1)^2$$

$$= 0.029$$

(1)

Section F

$$1. a) V = \frac{1}{3} Ah$$

$$= \frac{1}{3} \times 4 \times 12 \times 5$$

$$= 120 \text{ cm}^3$$

(1)

$$b) \text{ Slant height}_1 = \sqrt{29} \quad (\text{by Pyth Thm})$$

(1)

$$\text{Slant height}_2 = \sqrt{61}$$

(1)

$$SA = (4 \times 12) + 2 \times \left(\frac{1}{2} \times 4 \times \sqrt{61}\right) + 2 \times \left(\frac{1}{2} \times 12 \times \sqrt{29}\right)$$

(1)

$$= (48 + 4\sqrt{61} + 12\sqrt{29}) \text{ sq. units}$$

$$= 143.863 \text{ sq. units}$$

(1)

$$2. V = \frac{1}{3} \pi r^2 h + \frac{1}{2} \times \frac{4}{3} \pi r^3$$

(1)

$$= \frac{1}{3} \pi \times 16 \times 8 + \frac{2}{3} \pi \times 64$$

(1)

$$= \frac{128\pi}{3} + \frac{128\pi}{3}$$

$$= \frac{256\pi}{3} \text{ sq. units}$$

(1) — exact or approx.

(1) must have units.

$$= 268.08 \text{ c. units}$$

$$3. C = \frac{216 \times \pi \times 10}{300}$$

3 if ~~used~~

$$= \frac{216\pi}{30}$$

$$= 6\pi$$

(1)

$$\therefore SA = \pi r^2 + \pi r l$$

$$= \pi \times 9 + \pi \times 3 \times 5$$

$$= 24\pi \text{ cm}^2$$

$$= 75.4 \text{ cm}^2 \quad (1)$$

$$\therefore \text{radius of cone} = \frac{C}{2\pi} = \frac{6\pi}{2\pi} = 3 \text{ cm} \quad (1)$$

$$\therefore \text{height of cone} = 4 \text{ cm} \quad (\text{Pyth Thm}) \quad (1)$$

tot/4