NORTH SYDNEY BOYS HIGH SCHOOL

2015 YEAR 10 YEARLY EXAMINATION

STAGE 5.3 MATHEMATICS

Student Name:_____

Class: _____ **Teacher**_____

Number of pages used : _____(Fill at the end of exam)

General Instructions

- Working time – **110 minutes**
- Write using blue or black pen.
- **Board approved** calculators are allowed.
- All necessary working should be shown in every question.
- Each new **Section** is to be started on a **new page.**
- Marks may not be awarded for carelessly or badly arranged work.

(To be used by the exam markers only.)

Section	A	B	C	D	E	F	G	Total	Total
Mark	/17	/21	/24	/7	/15	/11	/12	/107	/100
Working out grade					Legibility grade				

Section A (17 marks)

Question **Marks**

1. Evaluate correct to 3 significant figures. **1**

$$\frac{7.42 \times \sqrt[3]{47.3}}{41.23^3 - 43.1}$$

2. Rationalise the denominator of **2**

$$\frac{2-\sqrt{3}}{4+\sqrt{5}}$$

3. Factorise $49 - 16y^2$. **1**

4. Solve $4x^2 = 28x$. **2**

5. $108 000 was invested for five years at the rate of 6% p.a. compounded annually.

i) Find the interest earned in five years. **2**

ii) What is the percentage increase of his original investment at the end of five years? **1**

6. Find the vertex of $y = x^2 - 4x + 9$. **2**

7. Find the exact value of *tan* 210°. **1**

8. For three points *A* (2, 1), *B* (4, 3), and *C* (***p***, 0), find the value of ***p*** so ΔABC becomes an isosceles triangle with base AB. **2**

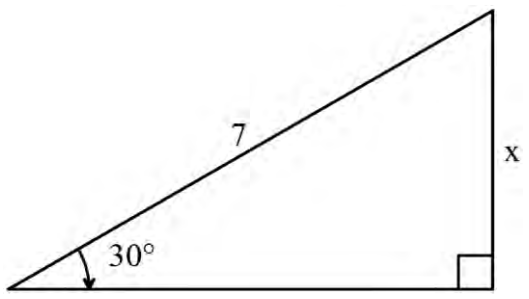
9. Find, and describe geometrically, the equation of the locus of point ***P***(*x, y*) which moves so that its distance from the point *A* (7, 2) is always twice its distance from *B* (4, 2) **3**

Section B (21 marks)

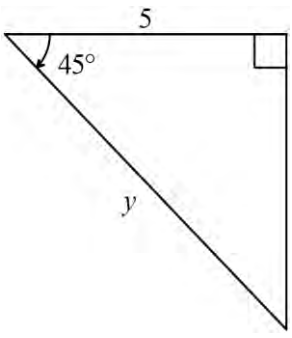
Question **Marks**

1. Find the exact values of the pronumerals. **1 mark each**

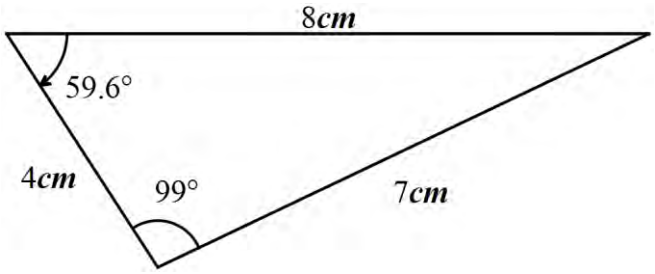
a)



b)



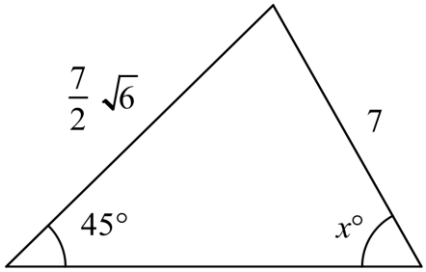
2. Find the area of the following triangle correct to 1 decimal place. **2**



3. Sketch the following graph for $-180° \leq \theta \leq 180°$. **3**

$$y = 2\cos(3\theta)$$

4. Find $x°$ in the given triangle. **3**



5. Graph the following quadratic functions, showing all important features

a) $y = 2x^2 - 18$ **2**

b) $y = -x^2 + 7x - 12$ **3**

6. Find the value of b, given that $x^2 - 6x + a = 0$ and $x^2 - ax + b = 0$ are perfect squares. **2**

7. Given that $m \propto \frac{1}{\sqrt{n}}$, What is the effect on

a) m when n is multiplied by 4 **2**

b) n when m is divided by 3 **2**

Section C (24 marks)

Question **Marks**

1. 1. For a function $f(x) = x^2 + 4x$, find:

a) $f(a)$ **1**

b) $f(2c + 1)$ **1**

2. Sketch on separate number planes. **Show all important features**.

a) $y = 3^x$ **2**

b) $y = 2(x-3)^3 + 4$ **3**

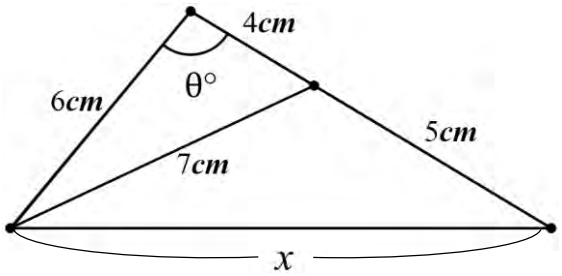
c) $y = \dfrac{3}{x+2}$ **3**

d) $y = \sqrt{9-(x-3)^2}$ **3**

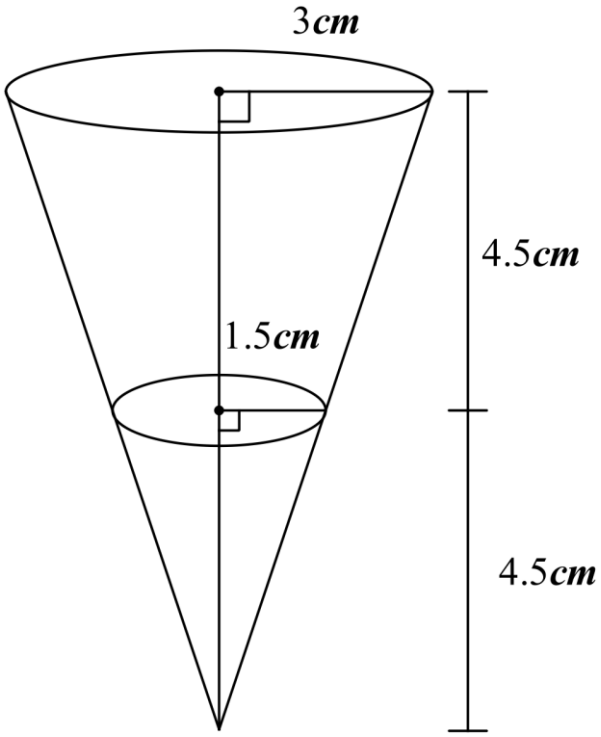
3. a) Sketch the function $f(x) = x^3 + 2$. **2**

b) Find the equation of its inverse and sketch it on the same axis **showing all important features**. **2**

4. Find the value of ***x*** and **θ.** **4**



5. If it takes 5 minutes to fill up the cone with water until it is 4.5cm deep at a constant rate, find how long would it take to fully fill up the entire cone with water. **3**

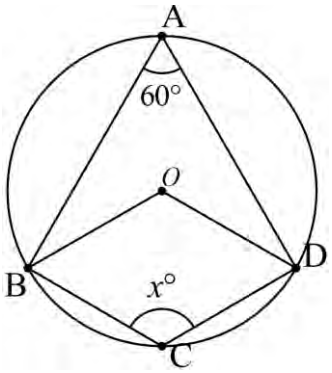


Section D (7 marks)

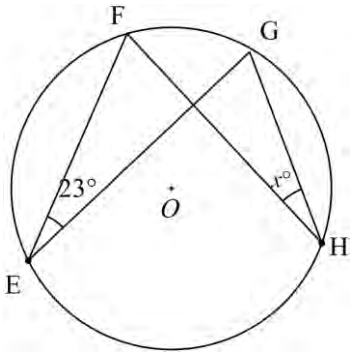
Question **Marks**

1. Calculate the value of the pronumerals**. No reasons required.**

a)

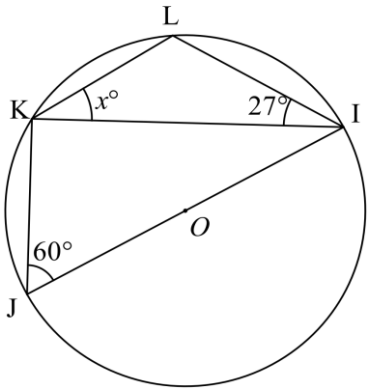


b)

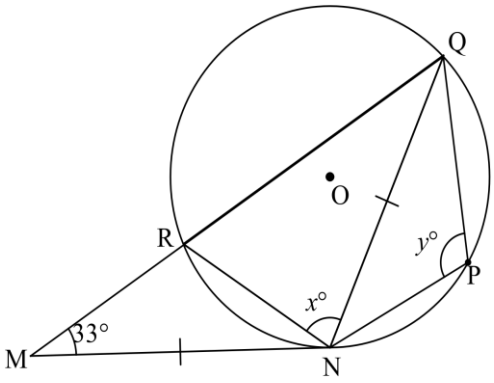


1,1

c)

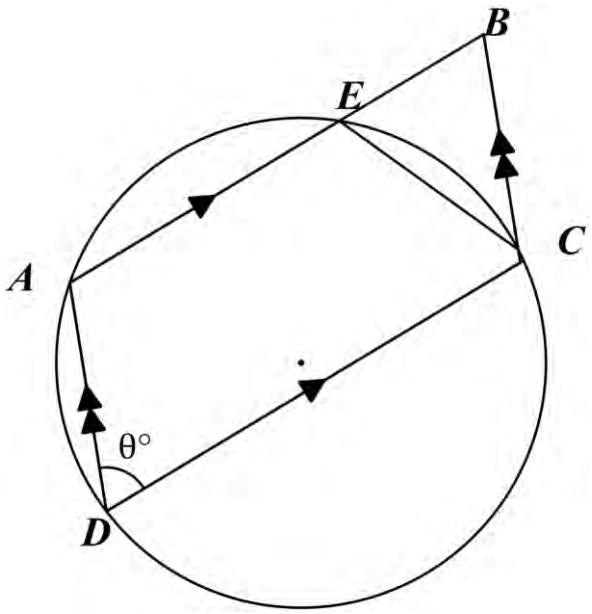


d)



1,2

2. ABCD is a parallelogram. Given $\angle ADC = \theta°$. Show CE = CB. **2**



Section E (15 marks)

Question **Marks**

1. Evaluate the following logarithms

a) $\log_9 729$ **1**

b) $\log_1 1$ **1**

2. Evaluate, **showing relevant working**

$$\frac{3\log_3 243}{\log_2 32}+\frac{5\log_3(\sqrt{3})}{\log_8 8}$$ **3**

3. If $\log_a 3 = 0.8$ and $\log_a 5 = 1.2$, evaluate:

a) $\log_a \frac{1}{9}$ **1**

b) $\log_a \sqrt{45}$ **2**

4. If $0.5^a = 0.0625$ and $3^{1+2b} = 243$, find the value of $\log_b a$. **2**

5. Sketch the following log graphs; showing all important features:

a) $y = 2\log_2 x$ **2**

b) $y = \log_2(x-5)$ **3**

Section F (11 marks)

Question **Marks**

1. Expand and simplify the following polynomial and state the leading coefficient and constant term. **2**

$$(x+2)(2x-7)-(2x+3)(x+5)$$

2. Find the quotient Q(x) and remainder R(x) for the following division **3**

$$(x^4-9x^3+22x^2-5x-25) \div (x-5)$$

3. If x^3-px+q is divisible by both $x-4$ and $x+3$, find the values of p and q. **3**

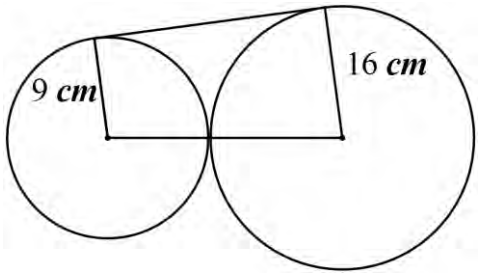
4. Factor the following polynomial. Sketch the graph, showing all intercepts with the axes. **3**

$$P(x)=x^4-x^3-21x^2+45x$$

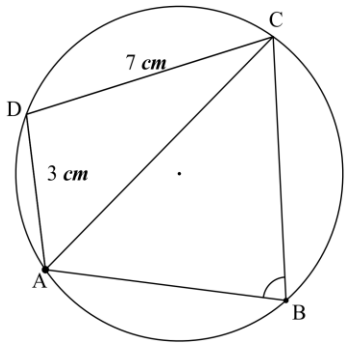
Section G (12 marks)

Question **Marks**

1. Two circles of radii 9cm and 16cm touch externally. Calculate the length of the common tangent shown below, **giving all reasons for your answer.** **3**



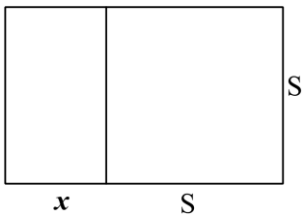
2. On a cyclic quadrilateral ABCD, AD = 3cm, CD = 7cm and cos B = $\frac{1}{9}$



a) Show that cos D = - cos B **1**

b) Find the length of AC **2**

3. 360 metres of fencing is to be used to enclose a rectangular area and divide it into a square and a smaller rectangle. Find the length of the side of the square to get the maximum area of the two areas. **3**



4. If a cubic polynomial satisfies the following conditions,

$P(0) = 3$
$P(x + 1) = P(x) + x^2$

a) Find the value of $P(1)$ **1**

b) Find the remainder of the polynomial when it is divided by $x^2 - 3x + 2$. **2**

End of exam

Section A

1. 0.000383 or 3.83×10^{-4} ✓

2.
$$\frac{2-\sqrt{3}}{4+\sqrt{5}}\times\frac{4-\sqrt{5}}{4-\sqrt{5}}$$
$$=\frac{8-2\sqrt{5}-4\sqrt{3}+\sqrt{15}}{11}$$ ✓✓

3. $(7-4y)(7+4y)$ ✓

4. $4x^2-28x=0$
$4x(x-7)=0 \quad \therefore x=0 \text{ or } x=7$ ✓

5. **a)** P = 108 000
R = 0.06
T = 5
$A=108000(1+0.06)^5=\$36528$
(to the nearest dollar) ✓✓

b) $\frac{36528}{108000}\times 100=34\%$ ✓

6.
$$x=-\frac{b}{2a}=-\frac{-4}{2}=2$$
$y=4-4(2)+9=5 \quad (2,5)$ ✓✓

7. $\frac{1}{\sqrt{3}}$ ✓

8. AC=BC
$$\sqrt{(2-p)^2+(1-0)^2}=\sqrt{(4-p)^2+(3-0)^2}$$
$4p=20 \quad \therefore p=5$ ✓✓

9.
$$\sqrt{(7-x)^2+(2-y)^2}=2\sqrt{(4-x)^2+(2-y)^2}$$
$$3x^2-18x+3y^2-12y+27=0$$
$$(x-3)^2+(y-2)^2=4$$ ✓✓✓
Locus of a circle with centre (3,2) and radius of 2 units.

Section B

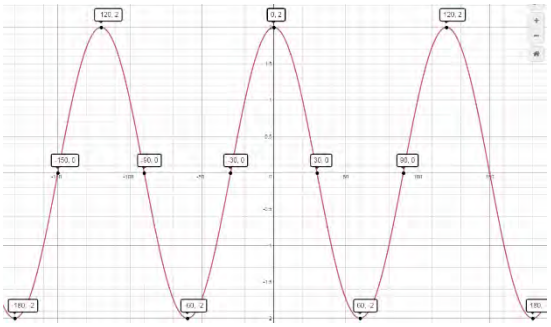
1. a) $\frac{7}{2}$ ✓
b) $5\sqrt{2}$ ✓

2. Area $=\frac{1}{2}ab\,sinC$
$$=\frac{1}{2}\times4\times8\times\sin(59.6)=13.8\,cm^2$$

Or $\frac{1}{2}\times4\times7\times\sin(99)=13.8\text{cm}^2$

(Or $\frac{1}{2}\times7\times8\times\sin(21.6)=10.3\text{cm}^2$)
Use of correct equation
Use of correct figures ✓✓

3.

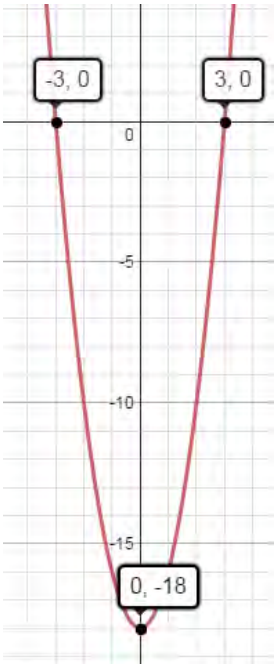


Correct shape
Correct amplitude $-2\le y\le2$
Correct period P = 120° ✓✓✓

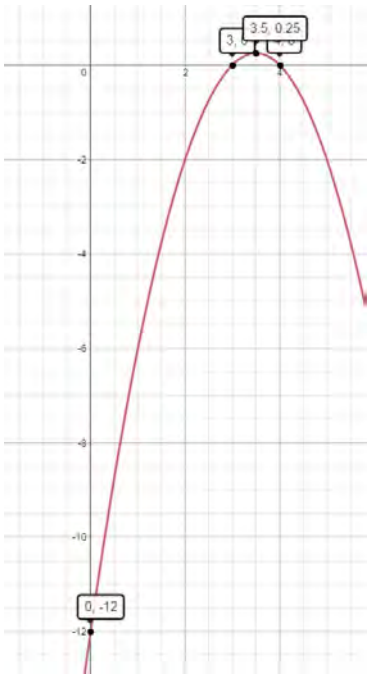
4.
$$\frac{7}{\sin(45)}=\frac{\frac{7}{2}\sqrt{6}}{\sin x}$$
$\sin x=\frac{\sqrt{3}}{2} \quad x=60°$ or $120°$ ✓✓✓

5.

a) Correct shape
Vertex
Intercepts ✓✓



b) v = (3.5 , 0.25)
x-intercepts (3,0), (4,0)
y-intercept (0,-12) ✓✓✓



6. $(x-3)^2 \therefore a = 9$

$\left(x-\frac{9}{2}\right)^2 \therefore b = \frac{81}{4}$ ✓✓

7. **a)** $m = \frac{k}{\sqrt{n}} = \frac{k}{\sqrt{4n}} = \frac{\mathbf{k}}{\mathbf{2\sqrt{n}}}$

m is halved✓✓

b) $n = \frac{k^2}{\left(\frac{1}{3m}\right)^2} = \frac{9k^2}{m^2}$

n is multiplied by 9 ✓✓

Section C

1.

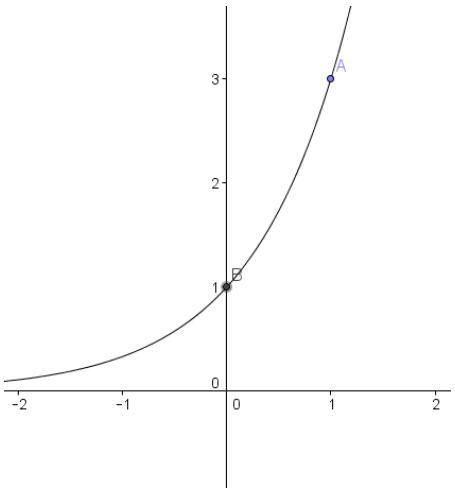
a) $f(a) = a^2 + 4a$ ✓

b) $f(2c+1) = (2c+1)^2 + 4(2c+1)$ ✓

$4c^2 + 12c + 8$

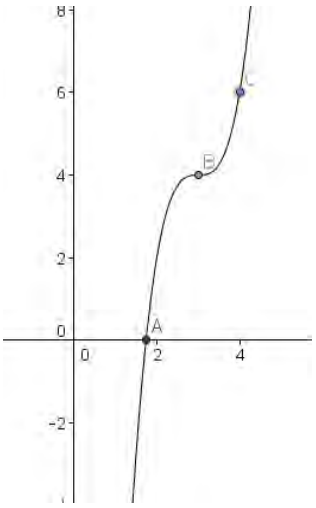
2.

a)



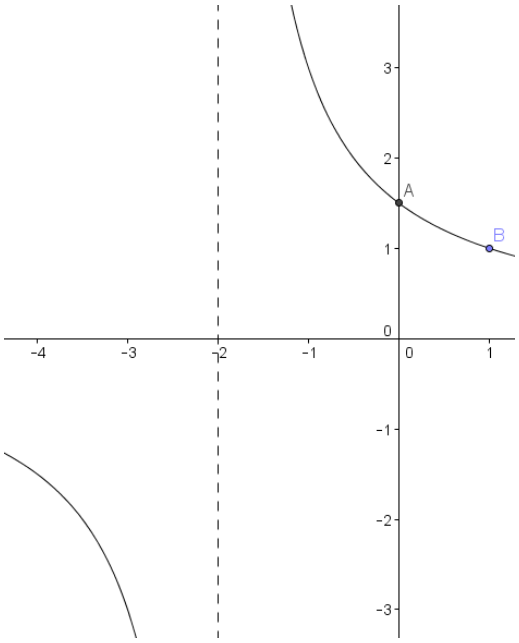
Correct y-intercept (0,1)
Correct shape ✓✓

b)



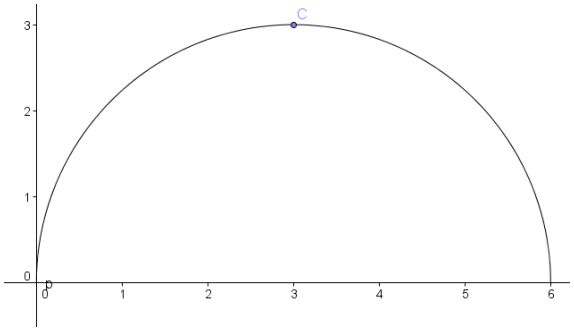
Correct intercepts (≈1.74,0) and (0,-50)
Correct shape ✓✓✓

c)



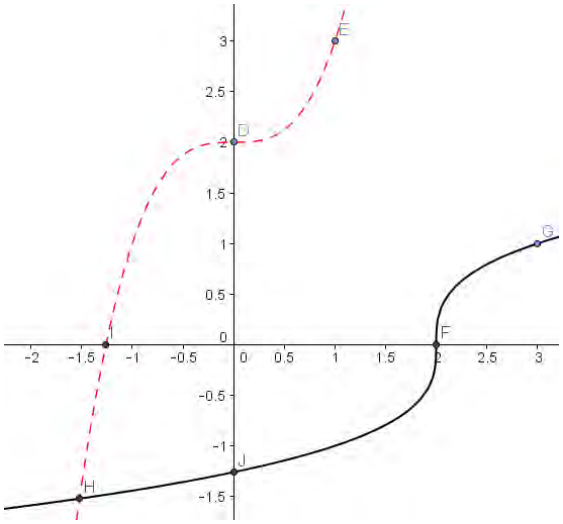
Correct y-intercept (0,1.5)
Correct point on a line and shape
Correct asymptote ✓✓✓

d)



Correct x-intercepts (0,0), (6,0)
Correct shape (3,3)
Correct orientation ✓✓✓

3.



a) Dotted line
Correct shape
Correct intercepts (0,2), (≈-1.26,0) ✓✓

b) Solid line
Correct equation $y = \sqrt[3]{x-2}$
Correct shape
Correct intercepts (2,0), (0,-1.26) ✓✓✓

4.

$$\cos\theta = \frac{4^2 + 6^2 - 7^2}{2 \times 4 \times 6}$$

$\theta = 86°$ (nearest degrees) ✓✓

$x^2 = 6^2 + 9^2 - 2 \times 6 \times 9 \times \cos\theta$

$x = 10.5\text{cm}$ ✓✓

5.
5 min to fill the 4.5 cm depth.

$$V = \frac{4.5}{3}\pi \times 1.5^2 = 1.5^3\,\pi\ \text{cm}^3$$

$$3.375\,\pi\ \text{cm}^3 \text{ per } 5\ min = 0.675\,\pi\ \frac{\text{cm}^3}{min}$$

Volume of the entire cone

$$V = \frac{9}{3}\pi \times 3^2 = 3^3\,\pi\ \text{cm}^3 = 27\pi\ \text{cm}^3$$

Time to fill the entire cone

$$T = \frac{27\,\pi}{0.675\pi} = 40 \text{ minutes}$$ ✓✓✓

Section D

1. a) $x = 120°$✓

b) $x = 23°$✓

c) $x = 33°$ ✓

d) $x = 81°$ ✓

$y = 114°$ ✓

2.
$\angle AEC = 180 - \theta°$
(Opposite angle of a cyclic quadrilateral)

$\therefore\ \angle BEC = 180 - (180 - \theta°) = \theta°$
(supplementary angle)

$\angle ABC = \theta°$

(Opposite angles in a parallelogram)

$\therefore$ CE = BC (The angles opposite the equal sides are also equal) ✓✓

Section E

1.

a) 3 ✓

b) **undefined**

2.

$$\frac{3 \times 5\log_3 3}{5\log_2 2} + \frac{\frac{1}{2} \times 5\log_3 3}{1}$$

$$= \frac{15}{5} + \frac{5}{2} = \frac{55}{10} = \frac{11}{2}$$ ✓✓✓

3.

a) $\log_a 3^{-2} = -2\log_a 3 = -2\times 0.8 = -1.6$ ✓

b) $\log_a 45^{1/2} = \frac{1}{2}(\log_a 9 + \log_a 5)$

$$= \frac{1}{2}(2\log_a 3 + \log_a 5)$$

$= 0.5(2\times 0.8+1.2) = 1.4$ ✓✓

4.

$$\left(\frac{5}{10}\right)^a = \frac{625}{10000}$$

$a=4$

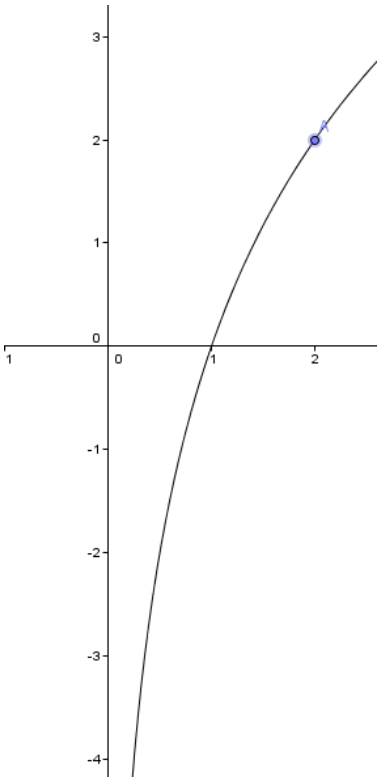
$3^{1+2b} = 243 = 3^5$

$2b + 1 = 5 \quad \therefore\ b=2$

$\log_2 4 = 2$ ✓✓

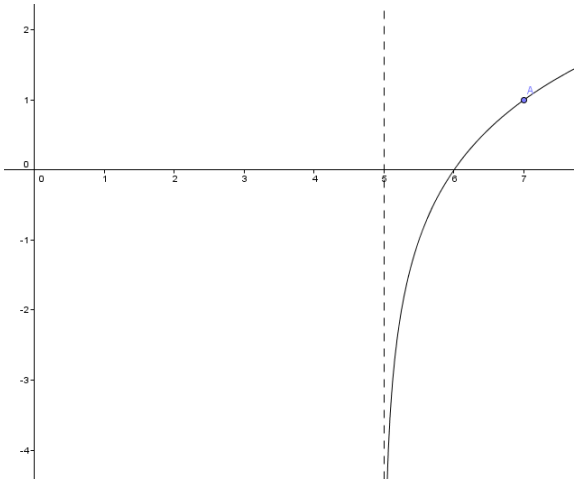
5.

a)



Correct shape and point on the line e.g(2,2)
Correct intercepts (1,0) ✓✓

b)



Correct shape and point on the line e.g(7,1)
Correct intercepts (6,0)
Correct asymptote (x = 5) ✓✓✓

Section F

1. -16x-29 Leading coefficient -16 ✓
Constant term -29 ✓

2.

$x^4 - 9x^3 + 22x^2 - 5x - 25 = (x^3 - 4x^2 + 2x + 5$
+0
Showing long division
Quotient = $x^3 - 4x^2 + 2x + 5$
Remainder = 0✓✓✓

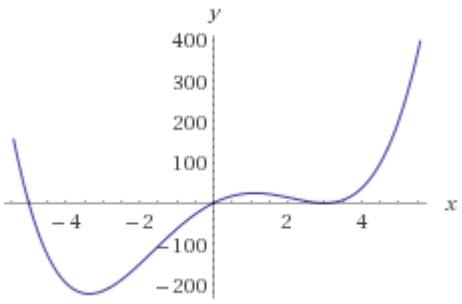
3. P(4) = 0 P(-3) = 0

P(4) = 64 – 4p + q = 0
P(-3)= -27 + 3p + q = 0

91 – 7p =0
p=13
q=-12 ✓✓✓

4. Using factor theorem to find

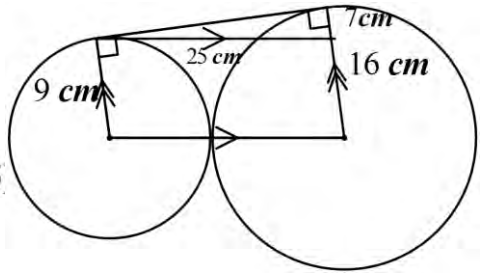
$P(x) = x^4 - x^3 - 21x^2 + 45x = x(x-3)^2(x+5)$
✓



Correct shape
Intercepts ✓✓

Section G

1.



Show that tangent to a circle is perpendicular to the radius at the point of contact for two circles. ✓

Construct a line that is parallel to the line of centres. ✓

Use Pythagoras' theorem to find the length of the common tangent. (24cm) ✓

2.

a)
$\angle D = 180 - \angle B$
(The opposite angles of a cyclic quadrilateral are supplementary)

$\therefore \cos D = \cos(180 - B) = -\cos B$ ✓

b)

$$AC = \sqrt{3^2 + 7^2 - 2 \times 3 \times 7 \times (-\cos B)}$$

$$AC = \sqrt{3^2 + 7^2 - 2 \times 3 \times 7 \times -\frac{1}{9}}$$

$= 7.92\text{cm}\ (2d.p)$ ✓✓

3.

Perimeter = 360 = 5s + 2*x*

$$x = \frac{360 - 5x}{2} = 180 - \frac{5x}{2}$$

$$\text{Area} = s^2 + sx = s^2 + s\left(180 - \frac{5x}{2}\right)$$

$$= s^2 + 180s - \frac{5x^2}{2} = \frac{-3x^2}{2} + 180s$$

Maximum area at the vertex of the parabola

$$s = -\frac{b}{2a} = -\frac{180}{2 \times -\frac{3}{2}} = 60 \quad \checkmark\checkmark\checkmark$$

∴ the area is maximum when side of the square is 60m.

4.

a)

P(1) = P(0+1) = P(0) + 0^2 = 3+0 =3 ✓

b)

$$x^2 - 3x + 2 = (x - 2)(x - 1)$$

Find P(2)

P(2) = P(1+1) = P(1) + 1^2 = 4

P(x) = (x-2)(x-1)Q(x)+a*x*+b

Remainder is linear as the divisor is a quadratic function.

Use remainder theorem to find the remainder a*x*+b

P(1) = a+b=3

P(2) = 2a+b=4

a = 1

b = 2

∴ remainder is *x*+2, when the polynomial is divided by x^2-3*x*+2 ✓✓