NORTH SYDNEY BOYS HIGH SCHOOL

STAGE 5.3 MATHEMATICS

2016 YEAR 10 YEARLY EXAMINATION

Wednesday, 9 November 2016

General instructions

- Working time – **100 minutes**.
- Write using **blue or black pen**. Where diagrams are to be sketched, these may be done in pencil.
- **Board approved** calculators may be used.
- Attempt all questions.
- Commence each new question on a **new booklet. Write on both sides of the paper**.
- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.

STUDENT NAME: ..... # BOOKLETS USED: .....

Class: (please ✓)

- ○ Mr. Berry
- ○ Dr. Jomaa
- ○ Ms. Lee
- ○ Mr. Hwang
- ○ Ms. Gibbs

Marker's use only

SECTION	A	B	C	D	E	F	G	Total	Total
MARKS	___/12	___/16	___/18	___/19	___/15	___/13	___/11	___/104	___/100

Section A (12 marks)

Question **Marks**

1. Evaluate correct to 4 significant figures **2**

$$\sqrt{\frac{657}{8\pi}}$$

2. Rationalise and simplify **2**

$$\frac{1}{2\sqrt{3}-\sqrt{5}}$$

3. Fully simplify **1**

$$\frac{n^2-36}{n+6}$$

4. Solve **2**

$$x^2 = 6x$$

5. Write down the equation of the circle with centre (-2, 4) and radius 3. (Do **NOT** write in expanded form) **1**

6. Fully simplify into a single fraction **3**

$$\frac{3}{x^2-6x+9} - \frac{x}{x^2-x-6}$$

7. The cost of Virtual Reality gear is \$979. This includes a 10% tax on the original price. Calculate the original price of the Virtual Reality gear. **1**

Section B (16 marks)

Question **Marks**

1. Write the exact value of the following:

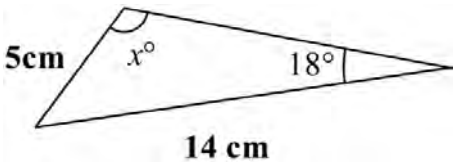
a) $\operatorname{cosec} 60°$ **1**

b) $\sin 240°$ **1**

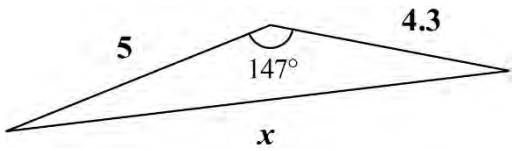
2. If $\sin A = \frac{2}{5}$ and $\tan A < 0$, find the exact value of $\cos A$. **2**

3. Find the value of x. (Diagrams not to scale)

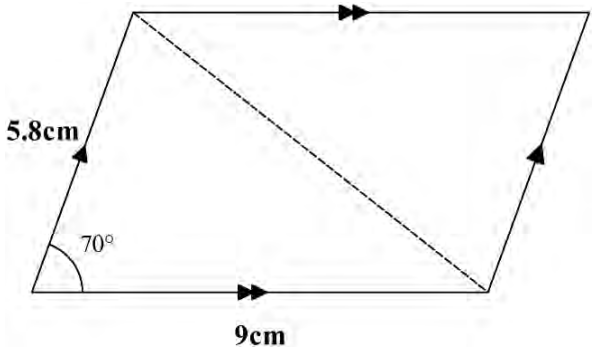
a) **2**



b) **2**



4. Calculate the area of the parallelogram, correct to the nearest cm. **2**



5. Sketch $y = 3 - 3\cos 2x$, for $0° \leq x \leq 360°$ **3**

6. A point $P\ (x, y)$ moves so that the sum of the squares of its distance from each of the points $A\ (-1,0)$ and $B\ (3,0)$ is equal to 40. **3**
Show that the locus of $P\ (x, y)$ is a circle, and state its radius and centre.

Section C (18 marks)

Question **Marks**

1. Solve $3x^2 - 2x - 7 = 0$ **2**

2. For $2x^2 - 3x - 10$, express in the form of $a(x - h)^2 + k$ by completing the square

a) Find the value of a, h and k. **3**

b) Hence or otherwise, find the minimum value of $2x^2 - 3x - 10$ **1**

3. a) Sketch the function $y = -2x^3 + 2$. **2**

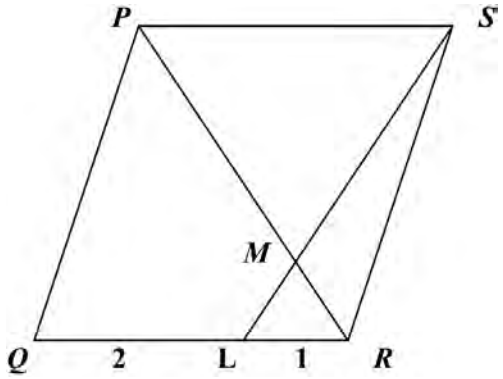
b) Find the equation of its inverse and sketch it on the same axis showing all important features. **3**

4. Sketch these graphs on the separate number planes, showing all important features.

a) $y = -\sqrt{9 - x^2}$ **2**

b) $y = 4 - \dfrac{1}{x + 2}$ **3**

5. $PQRS$ is parallelogram and L is a point on the side QR such that $LR = 1$ unit and $LQ = 2$. M is the point of intersection of PR and LS. Find the ratio of the areas of $\Delta LMR : \Delta SMP$. **2**



Section D (19 marks)

Question **Marks**

1. Jason is the goalkeeper for the soccer team. The probability that Jason can stop a penalty shot at goal is 0.6. During a particular match the opposing team had three penalty shots at goal. Using a tree diagram or otherwise, find the probability that:

a) The goalkeeper will stop all shots at goal **2**

b) The goalkeeper will stop at least 1 shot at goal **2**

2. a) Marks for a Mathematics test had a mean of 70% and Standard Deviation of 12. The top student was away on the day and sat the same test the day after and scored 93%. What effect would this have on the mean and standard deviation? **2**

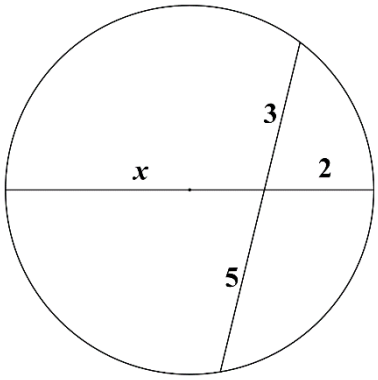
b) In the Yearly Examination, Michael's marks are as follows **3**

	Mark	$\bar{x}$	σ_n
English	82	85	6
Mathematics	70	74	10

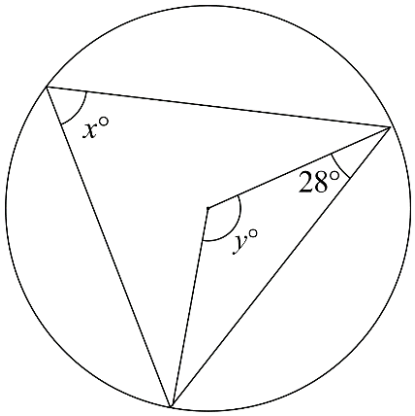
Determine which is the better result (Show all necessary working)

3. Find the value of the pronumerals**. No reasons required.**

a)

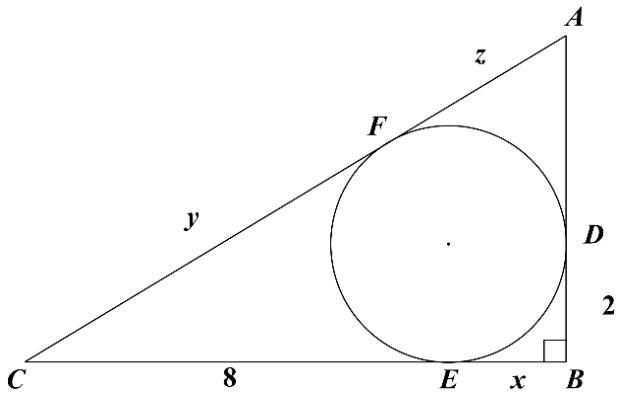


b)

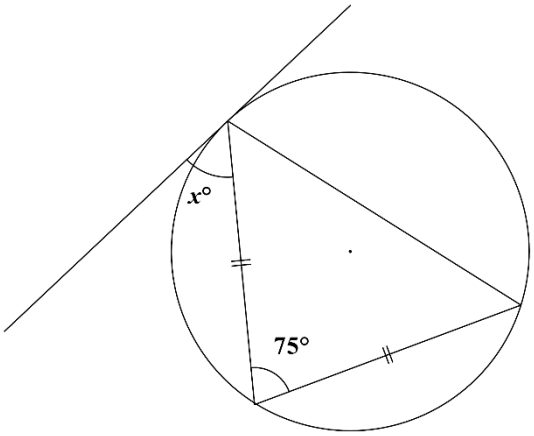


1 mark per pronumeral

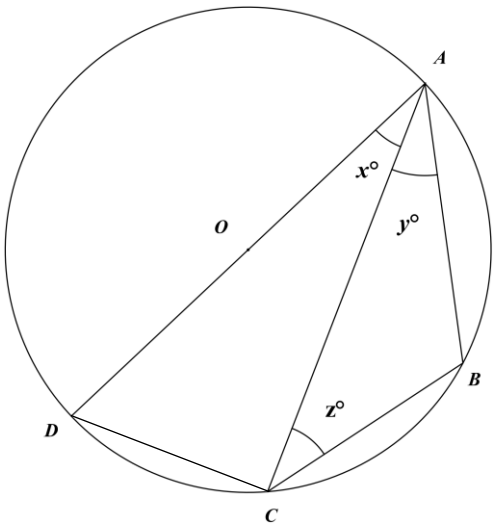
c)



d)



4. The diagram shows points *B* and *C* on a circle with centre *O* and diameter *AD*.



3

Given that $\angle DAC = x, \angle BAC = y$ and $\angle ACB = z$,
Find the value of $x + y + z$, giving geometric reasons

Section E (15 marks)

Question **Marks**

1. Evaluate the following logarithms

a) $\log_5 625$ **1**

b) $\log_{11} \frac{1}{11}$ **1**

c) $\log_3 \sqrt{27}$ **1**

2. Express $\log_5 24$, in terms of a and b, when $a = \log_5 2$ and $b = \log_5 3$ **3**

3. Solve for x, correct to three significant figures.

a) $x = \log_4 17$ **1**

b) $(0.01)^x = 6$ **2**

c) $3^{3x-2} = 8^{x+2}$ **3**

4. Sketch the following logarithmic graph; showing all important features:

$$y = \log_2(x-1)$$ **3**

Section F (13 marks)

Question **Marks**

1. If $P(x) = x^3 - 3x + 1$ and $Q(x) = -x^2 + x + 1$, find the **leading coefficient** and **the constant term** of the following **without evaluating**

a) $P(x) - 3Q(x)$ **2**

b) $2P(x) \times Q(x)$ **2**

2. Perform the division. Express the result in the form

$$dividend = divisor \times quotient + remainder$$
$$(x^3 + 2x^2 - x + 6) \div (x + 2)$$ **2**

3. Use the remainder theorem to find the remainder of

$$(2x^4 - x^3 + 11x - 2) \div (x + 3)$$ **2**

4. a) Fully factorise $y = x^3 + x^2 - 8x - 12$. **2**

b) Sketch the given polynomial, showing all important features **3**

Section G (11 marks)

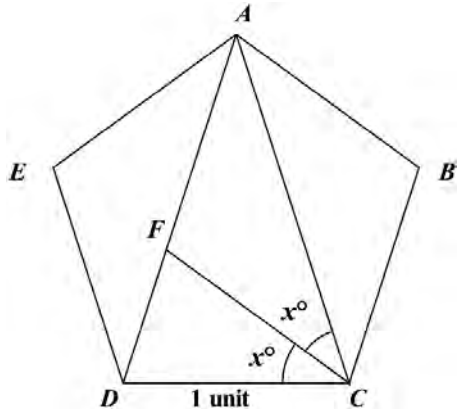
Question **Marks**

1. a) Factorise $a^3 - 1$ **1**

b) Hence or otherwise, when $a = \dfrac{2}{\sqrt{3}-1}$, find the value of **3**

$\log_3(a^3 - 1) - \log_3(a^2 + a + 1)$.

2. In a regular pentagon *ABCDE*, diagonal *AD* and *AC* are drawn to form a triangle *ACD* when point *F* on *AD* bisects the angle *ACD*.



a) Show that the value of $\angle AED = 108°$ **1**

b) Show that $x° = 36°$ **2**

c) Show that triangle *CDF* is an isosceles triangle **1**

d) Show that ΔCFD is similar to ΔACD **1**

e) Hence, find the exact length of *AD* in the form of $a + b\sqrt{c}$. (hint: let $AC = p$) **2**
(where a, b, and c are integers)

END OF EXAMINATION

Section A

Question **Marks**

1. $\sqrt{\frac{657}{8\pi}} = 5.112846507$ ✓ ≈ 5.113 (4.s.f) ✓ **2**

2. $\frac{1}{2\sqrt{3}-\sqrt{5}} = \frac{1}{2\sqrt{3}-\sqrt{5}} \times \frac{2\sqrt{3}+\sqrt{5}}{2\sqrt{3}+\sqrt{5}}$ ✓ $= \frac{2\sqrt{3}+\sqrt{5}}{4\times3-5} = \frac{2\sqrt{3}+\sqrt{5}}{7}$ ✓ **2**

3. $\frac{n^2-36}{n+6} = \frac{\cancel{(n+6)}(n-6)}{\cancel{n+6}} = n-6$ ✓ **1**

4. $x^2 = 6x$
$x^2 - 6x = 0$ ✓
$x(x-6) = 0$
$\therefore x = 0 \text{ or } x = 6$ ✓ **2**

5. $(x+2)^2 + (y-4)^2 = 9$ ✓ **1**

6.
$$\frac{3}{x^2-6x+9} - \frac{x}{x^2-x-6} = \frac{3}{(x-3)^2} - \frac{x}{(x-3)(x+2)} \checkmark = \frac{3(x+2)-x(x-3)}{(x-3)^2(x+2)} \checkmark$$

$$\frac{3x+6-x^2+3x}{(x-3)^2(x+2)} = \frac{-x^2+6x+6}{(x-3)^2(x+2)} \checkmark$$

3

7. $979 \div 1.1 = \$890$ ✓ **1**

Section B

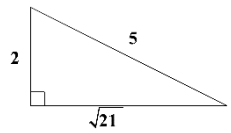
Question **Marks**

1.
i) $\text{cosec } 60° = \frac{1}{\sin 60} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} \text{ or } \frac{2\sqrt{3}}{3}$ ✓ **1**

ii) $\sin 240° = \sin(180+60) = -\sin 60 = -\frac{\sqrt{3}}{2}$ ✓ **1**

2. $\sin A = \frac{2}{5}$ and $\tan A < 0$ **2**

2nd quadrant hence cos A is negative



✓

$$\cos A = -\frac{\sqrt{21}}{5} \checkmark$$

3. a) $\frac{\sin 18}{5} = \frac{\sin x}{14}$, $\sin x = \frac{14 \sin 18}{5}$ ✓ **2**

$x = 60° \text{ or } 120° \text{ (to nearest degrees)}$ ✓

b) $x^2 = 4.3^2 + 5^2 - 2 \times 4.3 \times 5 \times \cos 147$ ✓ **2**

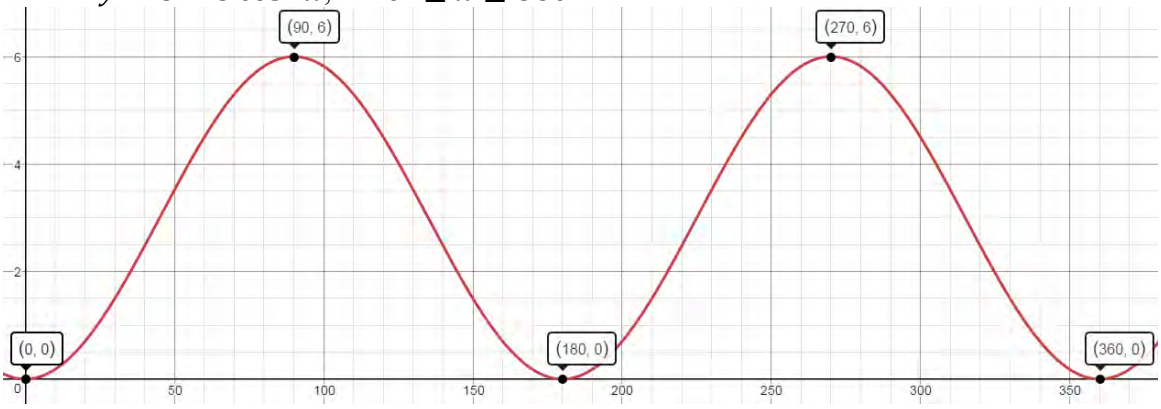
$x = \sqrt{79.553\ldots}$=8.92units (2 d.p) ✓

4.

$$Area\ of\ triangle = \frac{1}{2} \times 5.8 \times 9 \times \sin 70 = 24.526\ldots \checkmark$$ **2**

$the\ congruent\ triangle\ on\ the\ other\ side: 2 \times 24.526 = 49\ cm^2\ (nearest\ unit)$ ✓

5. Sketch $y = 3 - 3\cos 2x$, for $0° \leq x \leq 360°$ **3**



Correct range & translation ✓, correct period ✓, correct shape ✓

6. **3**

$$(\sqrt{(x+1)^2 + (y)^2})^2 + (\sqrt{(x-3)^2 + (y)^2})^2 = 40 \checkmark$$
$$(x+1)^2 + (y)^2 + (x-3)^2 + (y)^2 = 40$$
$$x^2 + 2x + 1 + y^2 + x^2 - 6x + 9 + y^2 = 40$$
$$2x^2 - 4x + 10 + 2y^2 = 40$$
$$x^2 - 2x + 5 + y^2 = 20$$
$$(x^2 - 2x + 1) + y^2 = 16 \checkmark$$
$$(x-1)^2 + y^2 = 4^2$$
$$\therefore Centre\ of\ circle = (1,0)\ and\ radius = 4\ units \checkmark$$

Section C

Question **Marks**

1. **2**

$$x = \frac{2 \pm \sqrt{2^2 - 4 \times 3 \times (-7)}}{2 \times 3} = \frac{2 \pm \sqrt{88}}{6}\ or\ \frac{1 \pm \sqrt{22}}{3}$$ ✓✓

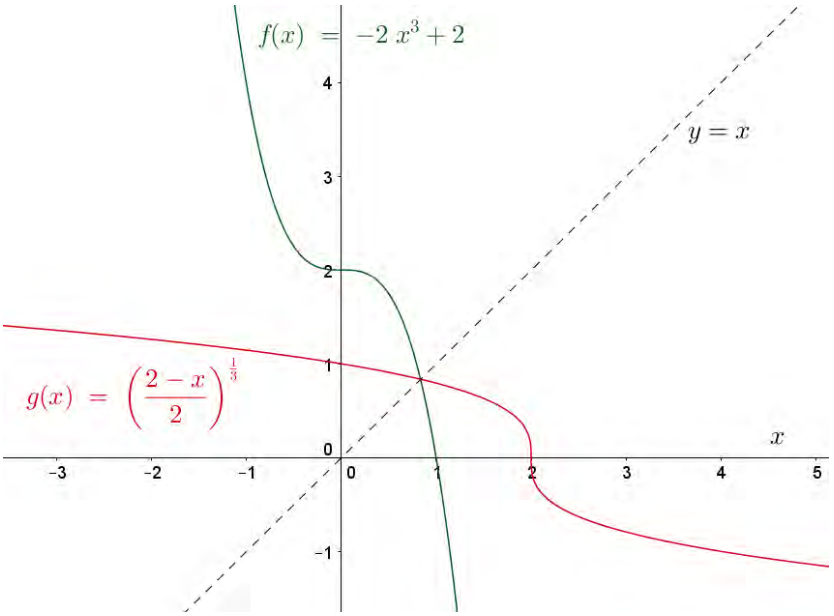
2.

$$2x^2 - 3x - 10 = 2\left(x^2 - \frac{3}{2}x\right) - 10 = 2\left(x^2 - \frac{3}{2}x + \frac{9}{16}\right) - \frac{9}{8} - 10 = 2\left(x - \frac{3}{4}\right)^2 - \frac{89}{8}$$

a) $a = 2, h = \frac{3}{4}\ and\ k = -\frac{89}{8}$ ✓✓✓ **3**

b) minimum value $= -\frac{89}{8}$ ✓ **1**

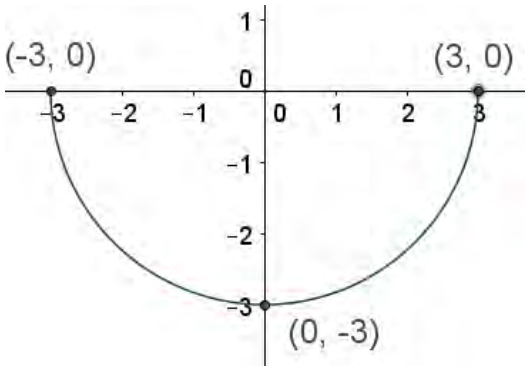
3. **2**



3

correct shape, correct $x-$ intercept and 1 extra point, correct inverse function

4. a) b)



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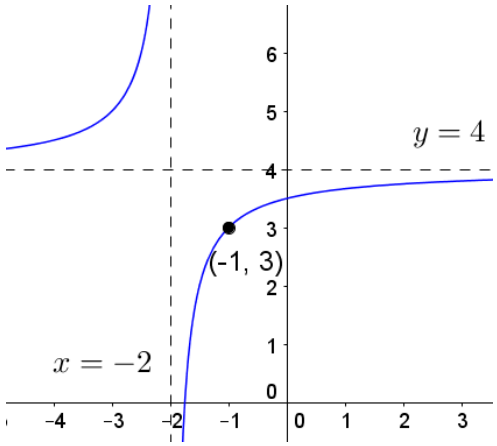


3

Correct shape, correct intercepts

Correct shape, correct asymptotes, one extra point

5. **2**

$\angle LMR = \angle PMS$ *(vertically opposite angles),*
$\angle MLR = \angle MSR$ *or* $\angle MRL = \angle MPS$ *(alternate angles on parallel lines)*
$\therefore \Delta MLR$ *is similar to* ΔMSP *(Equiangular)* ✓
the ratio of sides between the triangles are $1:3$
Area ratio $= 1^2 : 3^2 = 1:9$ ✓

Section D

Question **Marks**

1. **2**

a) $0.6 \times 0.6 \times 0.6$ ✓ $= 0.216 = \frac{27}{125}$ ✓

2

b) *complementary event of stopping all shots*✓ $= 1 - \frac{8}{125} = 0.936 = \frac{117}{125}$ ✓

2. **2**

a) *Increased the mean and standard deviation* ✓✓

3

b) $English = \frac{z-\bar{x}}{\sigma_n} = \frac{82-85}{6} = -\frac{1}{2}$✓, $Maths = \frac{70-74}{10} = -\frac{2}{5}$ ✓

he achieved better marks relative to the year group in maths ✓

3. a)

$$3 \times 5 = 2 \times x$$
$$x = \frac{15}{2} \checkmark$$

b)

$y = an\ isosceles\ triangle, angle\ at\ the\ center\ is\ 124° \checkmark.$
$x = 62° \checkmark$ ($the\ angle\ at\ the\ circumference\ of\ a\ circle\ is\ half\ the\ angle$ $at\ the\ centre\ standing\ on\ the\ same\ arc.$)

c)

$$x = 2\checkmark, y = 8\checkmark$$

$(Tangents\ to\ a\ circle from\ an\ external\ point\ are\ equal)$
$(8+z)^2 = 10^2 + (2+z)^2 (pythagoras' theorem),$
$z^2 + 16z + 64 = 100 + z^2 + 4z + 4$

$$12z = 40, \quad \therefore z = \frac{10}{3} \ \checkmark$$

d)

$looking\ at\ the\ isosceles\ triangle\ the\ base\ angle\ equals\ to\ x$ (the angle between a tangetn and a chord equals the angle at the circumference in the alternate segment)
$the\ base\ angles\ of\ the\ triangles\ are = \frac{180-75}{2} = 52.5° \ \checkmark$

4.

$$\angle ACD = 90 \checkmark$$
$$(the\ angle\ at\ the\ circumference\ in\ a\ semi-circle\ is\ 90°)$$
$$x + y + z + 90 = 180° \ \checkmark$$
$$(the\ opposite\ angles\ of\ a\ cyclic\ quadrilateral\ equals\ to\ 180°)$$
$$x + y + z = 90° \checkmark$$

3

Section E

Question **Marks**

1. $a)\ 5^x = 625 = 5^4$ **1**
$\therefore x = 4 \ \checkmark$

$b)\ 11^x = 11^{-1} \checkmark$ **1**
$x = -1 \checkmark$

$c) 3^x = (3^3)^{\frac{1}{2}}$ **1**

$$\therefore x = \frac{3}{2}$$

2.

$$\log_5 24 = \log_5 (8 \times 3) = \log_5 2^3 + \log_5 3 \ \checkmark$$
$$= 3\log_5 2 + \log_5 3 \ \checkmark$$
$$= 3a + b \checkmark$$

3 **3.**

$$a) \frac{\log 17}{\log 4} = 2.04\ (3\ s.f) \checkmark$$

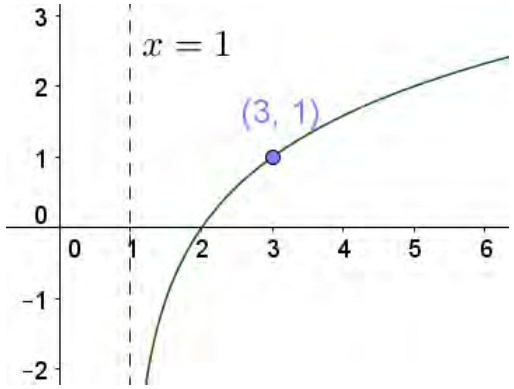
1

b) $\frac{\log 6}{\log 0.01} \checkmark = -0.389\ (3\ s.f) \checkmark \ \checkmark$ **2**

c) $(3x-2)\log 3 = (x+2)\log 8 \ \checkmark$ **3**
$3\log 3 \times x - 2\log 3 = \log 8 \times x + 2\log 8 \ \checkmark$
$(3\log 3 - \log 8)\, x = 2\log 8 + 2\log 3$

$$x = \frac{2\log 8 + 2\log 3}{3\log 3 - \log 8} = 5.23\ (3\ s.f) \ \checkmark$$

4.



Correct asymptote, correct extra point & correct shape **3**

Section F

Question **Marks**

1. a) $P(x) - 3Q(x)$ *leading coefficient* $= 1$✓ *, constant term* $= -2$✓ **2**

b) $2P(x) \times Q(x)$ *leading coefficient* $= -2$✓ *, constant term* $= 2$ ✓ **2**

2. $(x^3 + 2x^2 - x + 6) = (x^2 - 1)(x + 2) + 8$ ✓✓ **2**

3. $P(-3$ ✓$) = 154$✓ **2**

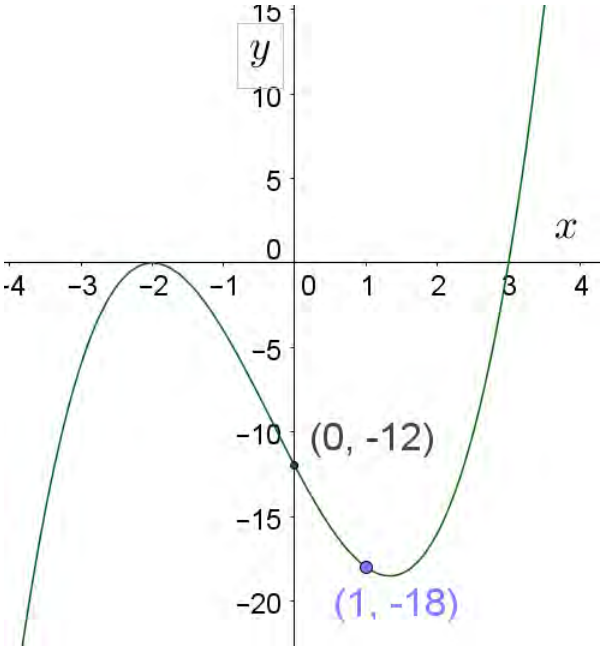
4. *Use factor theorem to find roots* **2**

$P(x) = (x + 2)^2(x - 3)$ ✓✓

Correct shape of the polynomial ✓

all required points shown ✓✓

**x and y axis on different scales*



3

Section G

Question **Marks**

1. *a)* $a^3 - 1 = (a-1)(a^2+a+1)$ ✓ **1**

b) $a = \frac{2}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{2\sqrt{3}+2}{2} = \sqrt{3}+1$

$$\log_3(a^3-1) - \log_3(a^2+a+1).$$
$$= \log_3(a-1)(a^2+a+1) - \log_3(a^2+a+1)$$
$$= \log_3(a-1) + \cancel{\log_3(a^2+a+1) - \log_3(a^2+a+1)} \checkmark$$
$$= \log_3\left(\sqrt{3}+1\right) = \log_3 3^{\frac{1}{2}} + \log_3 1$$
$$= \frac{1}{2}\log_3 3 + \log_3 1 = \frac{1}{2} \times 1 + 0 \checkmark$$
$$\therefore \frac{1}{2} \checkmark$$

3

2. a) Show that the value of $\angle AED = 108°$

$$an\ internal\ angle\ of\ regular\ pentagon = \frac{180(n-2)}{n} = \frac{540}{5} = 108° \checkmark$$

1

b) Find the value of $x°$

$$\angle BCA = (180-108) \div 2 = 36° \checkmark$$
$$\angle DCB = 108 = 36 + 2x$$
$$\therefore x = 36° \checkmark$$

2

c) Prove that triangle *CDF* is an isosceles triangle

$$\angle CAF = 108 - 2 \times 36° = 36°$$
$$\therefore \angle AFC = 108°$$
$$\angle CFD = 72°$$
$$For\ triangle\ CDF\ \angle CDF = 180 - 72 - 36 = 72°$$
$$\therefore \Delta CFD\ is\ an\ isosceles\ triangle\ (two\ base\ angles\ are\ equal) \checkmark$$

1

d) $\angle ADC = \angle ACD = 72° = \angle CFD = \angle CDF$

$$\Delta ACD\ is\ similar\ to\ \Delta CFD\ (equiangular) \checkmark$$

1

e) $let\ AC = p\ then\ FD = p-1$

$$\frac{AC}{CF} = \frac{CD}{DF}$$
$$\frac{p}{1} = \frac{1}{p-1} \checkmark$$
$$p(p-1) = 1$$
$$p^2 - p - 1 = 0$$
$$p = \frac{1 \pm \sqrt{1^2+4}}{2} = \frac{1}{2} + \frac{\sqrt{5}}{2}\ or \frac{1+\sqrt{5}}{2}\ (only\ positive\ for\ measurement) \checkmark$$

2