NORTH SYDNEY GIRLS HIGH SCHOOL

Year 10 Yearly Examination 2006

Mathematics

Name:_____ Class: _____

Teacher:_____

Time Allowed: **1 _ hours + 5 minutes reading time**

Instructions:

- Answer Part A, the multiple choice questions, on the answer sheet provided.
- Answer Part B on the paper provided.
- Start each part on a new page.
- Attempt every question.
- Show all necessary working.
- Marks may be deducted for incomplete or poorly arranged work.
- Write on one side of the page only.
- Do not use correcting tape or liquid paper.
- Diagrams are not drawn to scale.

At the end of the examination, staple this question paper to the front of your solutions and hand in one bundle.

Part A: 30 Marks			
A1 (9)	**A2(17)**	**A3(2)**	**A4(2)**
Question 1: 19 Marks			
Question 2: 12 Marks			
Question 3: 14 Marks			
Question 4: 12 Marks			
Question 5: 13 Marks			
Total: 100 Marks			

Part A

Answer all the questions on the answer sheet provided. All questions are 1 mark.

1. The number 0·03075 written correct to three significant figures.

A. 0·03 B. 0·031 C. 0·0307 D. 0·0308

2. $\frac{a}{4}+\frac{2a}{3}=$

A. $\frac{3a}{12}$ B. $\frac{3a}{7}$ C. $\frac{11a}{12}$ D. $\frac{2a^2}{12}$

3. Evaluate $\frac{1}{2\cdot 5^2-1\cdot 3^2}$ correct to four decimal places.

A. 0·2193 B. 0·5917 C. 0·6944 D. 4·56

4. The exact value of $\sqrt{128}$ in simplest form is:

A. $8\sqrt{2}$ B. $2\sqrt{32}$ C. 11.313708 D. 64

5. a% of \$$b$ is:

A. \$$ab$ B. $\$\frac{a+b}{100}$ C. \$100ab D. $\$\frac{ab}{100}$

6. A car purchased for \$18 000 depreciates at the rate of 24% per annum. How many years will it take for its value fall below \$10 000?

A. 6 B. 5 C. 4 D. 3

7. In an election there were only two candidates. The winner won by 120 votes and received 60% of the votes cast. How many people voted?

A. 7200 B. 1200 C. 600 D. None of these

8. $\left(2+\sqrt{3}\right)^2=$

A. 4 + 3 B. $5+2\sqrt{3}$ C. $7+2\sqrt{3}$ D. $7+4\sqrt{3}$

9. With a rational denominator $\frac{1}{3-\sqrt{7}}$ is:

A. $\frac{3-\sqrt{7}}{2}$ B. $\frac{3+\sqrt{7}}{2}$ C. $\frac{3+\sqrt{7}}{-4}$ D. $\frac{3+\sqrt{7}}{16}$

10. One of the factors of $6x^2+x-15$ is:

A. $3x+5$ B. $3x-5$ C. $5-3x$ D. $-3x-5$

11. If $3n^2=9$ then n = ?

A. $-\sqrt{3}$ B. $\sqrt{3}$ C. $\pm\sqrt{3}$ D. ± 3

12. If a = _ 1 and b = 2, then the value of $a^b - b^a$ is:

A. $\frac{1}{2}$ B. $1\frac{1}{2}$ C. 3 D. $_1\frac{1}{2}$

13. $3a^2-(3a)^2=$

A. 0 B. $-3a^2$ C. $-6a^2$ D. $12a^2$

14. If $3x^2-7x-1=0$ then x = ?

A. $\frac{-7\pm\sqrt{37}}{6}$ B. $\frac{-7\pm\sqrt{61}}{6}$ C. $\frac{7\pm\sqrt{37}}{6}$ D. $\frac{7\pm\sqrt{61}}{6}$

15. Which expression is equivalent to $\left(\frac{1}{x}\right)^{-\frac{1}{2}}$?

A. $\sqrt{x}$ B. $\frac{1}{\sqrt{x}}$ C. $-\sqrt{x}$ D. $-\frac{1}{\sqrt{x}}$

16. Which one of the following is **not** equal to $16a^6$?

A. $(4a^3)^2$ B. $48a^7 \div 3a$ C. $8a^6+8a^6$ D. $2a^3\times 8a^2$

17. Change the subject of the formula $V=\frac{4}{3}\pi r^3$ to r.

A. $\sqrt[3]{\frac{4V}{3\pi}}$ B. $\sqrt[3]{\frac{3V}{4\pi}}$ C. $\sqrt[3]{\frac{3\pi V}{4}}$ D. $\left(\frac{3V}{4\pi}\right)^3$

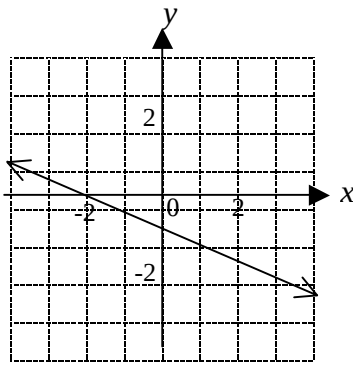
18. When $3x - y = 6$ and $3y - x = 6$ are solved simultaneously, x = ?

A. 0 B. 1 C. 3 D. 6

19. The minimum value of $(x-3)^2+8$ is:

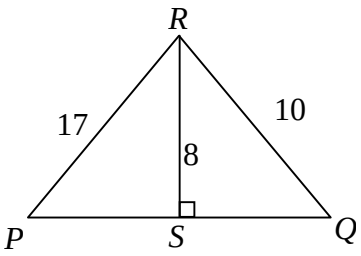
A. 8 B. 0 C. –3 D. 5

20. What is the equation of the following line?



A. $y=-2x-1$ B. $y=-\frac{1}{2}x-1$ C. $y=-\frac{1}{2}x-2$ D. $y=-2x-2$

21. In the diagram the length *PQ* is:



A. $\sqrt{389}$ cm B. 18 cm C. 21 cm D. 25 cm

22. If sin $x°$ = cos 29° and $0 < x < 90$ the value of x is:

A. 29 B. 61 C. 0·8746 D. 0·4848

23. Which one of the lines, described by the equations below, cuts the graph of $y = x^2$ at two distinct points?

A. $x = 0$ B. $y = 7$ C. $x = \_\ 4$ D. $y = \_\ 4$

24. One side of a square, of length x cm, is lengthened by 4cm and the adjacent side is decreased by 1 cm. The area of the resulting rectangle exceeds the area of the original square by 14 cm^2. An algebraic expression which could be solved to find the length of the square?

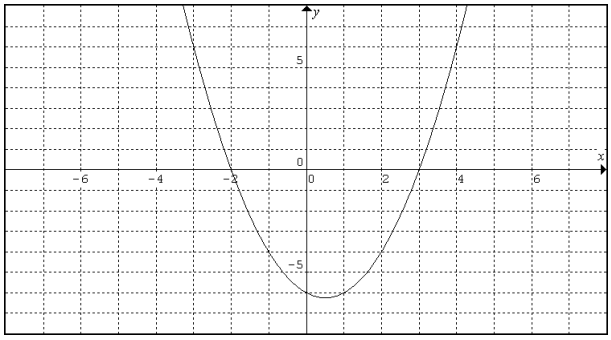
A. $(x+4)(x-1)=14$ B. $x^2-(x+4)(x-1)=14$

C. $x^2+14=(x+4)(x-1)$ D. $(x+4)(x-1)=14-x^2$

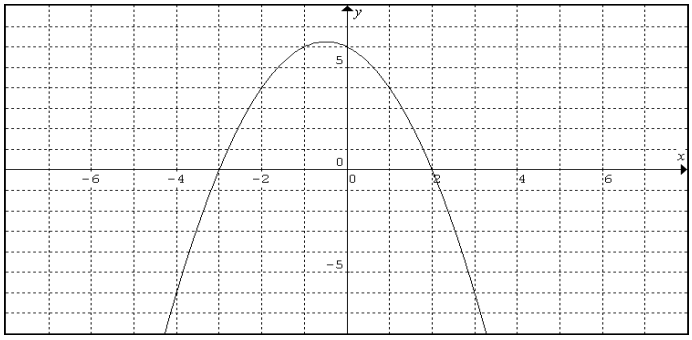
25. Which one of the following sketches could represent the graph of the function

$y = x^2 + x - 6$?

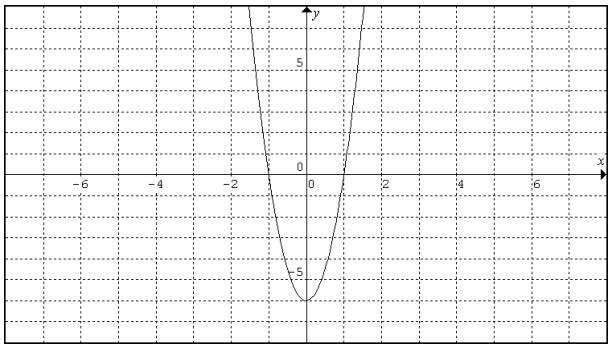
A.



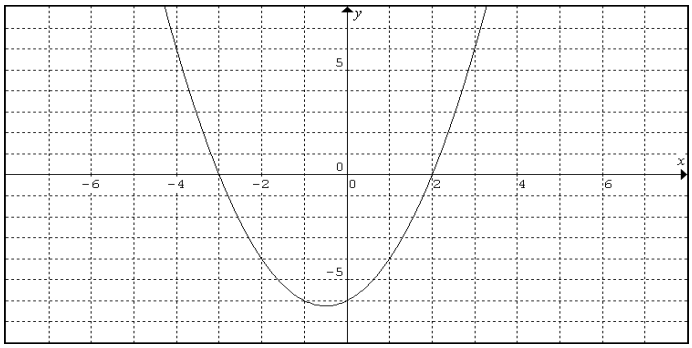
B.



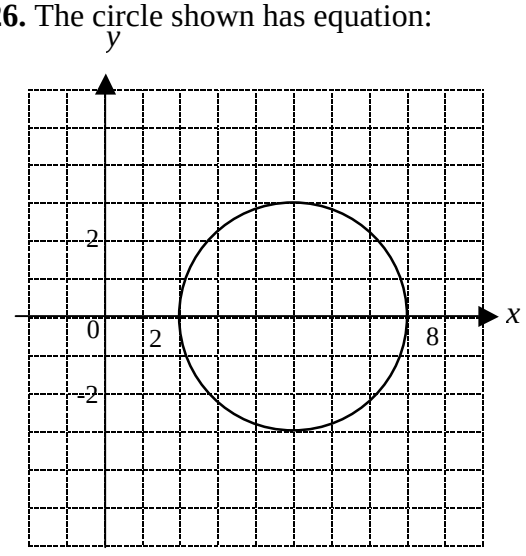
C.



D.



26. The circle shown has equation:



A. $x^2 + (y-5)^2 = 9$

B. $(x-5)^2 + y^2 = 9$

C. $(x-5)^2 + y^2 = 36$

D. $x^2 + (y-5)^2 = 36$

27. In three of the following figures, the ratio of adjacent sides is always 1:1. In which of the following is this **not** true?

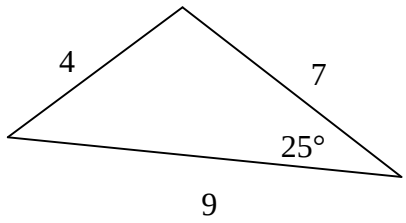
A. Parallelogram

B. Regular Pentagon

C. Equilateral Triangle

D. Rhombus

28. The area of the triangle is given by the expression



A. $\frac{1}{2}\times 4\times 7$

B. $\frac{1}{2}\times 7\times 4\times \sin 25°$

C. $\frac{1}{2}\times 4\times 9\times \sin 25°$

D. $\frac{1}{2}\times 7\times 9\times \sin 25°$

29. The mean for the following set of scores is:

Score	Frequency
6	3
7	3
8	4
9	4
10	6

A. 9

B. 8

C. 8·35

D. 8·5

30. The median for the set of scores in question 29 is:

A. 8

B. 9

C. 8·5

D. 10

Part B – Answer these questions on the paper provided

Question 1 [19 Marks] **Marks**

a) Find an expression for $3y^2+5y$ if $y=at^2$. **1**

b) Factorise fully x^4-16. **2**

c) Change the subject of the formula $x=\frac{\sqrt{a+2}}{3}$ to a. **3**

d) By using a suitable substitution, solve $3^{2x}+2(3^x)-15=0$. **3**

e) Given that $x=\sqrt{2}-1$ express $x-\frac{1}{x}$ as a surd with a rational denominator. **3**

f) Sketch the curve $y=x^3+2$ **2**

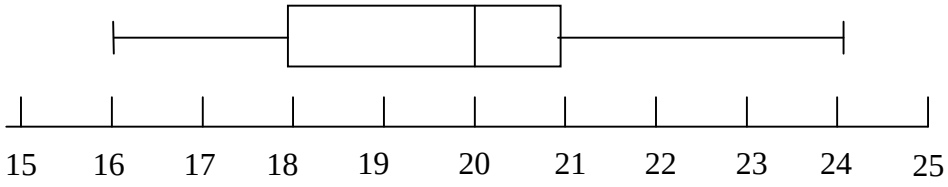
g) For the curves $y=2x$ and $y=x^2-2x$

i) Sketch both graphs on the same axes showing the important features **3**

ii) Hence, or otherwise, find the coordinates of the point(s) of intersection between the curve and the line. **2**

Question 2 [12 Marks] Start a new page

a) This box and whisker plot summarises the results of a class test out of 25.



i) What is the median score on the test? **2**

ii) What is the lower quartile? **1**

iii) What is the interquartile range? **2**

iv) What percentage of students scored more than 18? **1**

b) For a class of 20 students, two tests were given. In the Latin test, the mean was 69·5% and the standard deviation was 14·2. In the French test, the percentage marks were:

59,	75,	61,	52,	60,	51,	71,	66,	68,	72,
53,	39,	59,	45,	61,	79,	63,	48,	59,	46

i) Calculate, for French, the mean and standard deviation correct to one decimal place. **2**

ii) In Latin, a student's result lies within one standard deviation from the mean. Between which two scores could the result lie? **2**

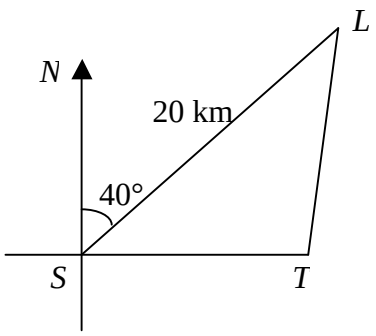
iii) Which is the better score, a score of 70 in French or a score of 80 in Latin? Justify your answer. **2**

Question 3 [14 Marks] Start a new page **Marks**

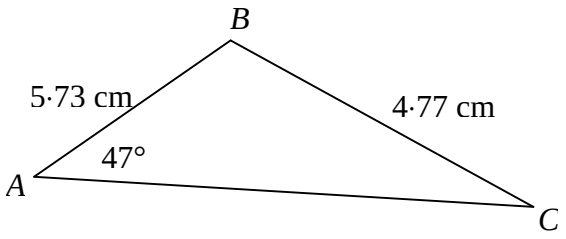
a) The diagram shows that the bearing of a lighthouse L from a ship S, 20 km away, is 040°. The ship travels east at a speed of 24 km/h for 45 minutes to the point T.

i) Show that $ST = 18$km **1**

ii) Find the distance of L from T **2**

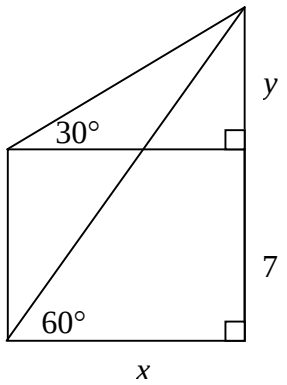


b) In the following diagram, find $\angle BCA$. **3**



c) Draw a suitable graph or diagram to explain why $\cos x + 2 = 0$ has no solutions. **2**

d)



i) Use the two right-angled triangles to write down two different equations involving x and y. [Write your final expressions with y as the subject]. **2**

ii) By solving the equations simultaneously, show that

$$x = \frac{7}{\tan 60° - \tan 30°}$$ **2**

iii) Hence find the exact value of x. **2**

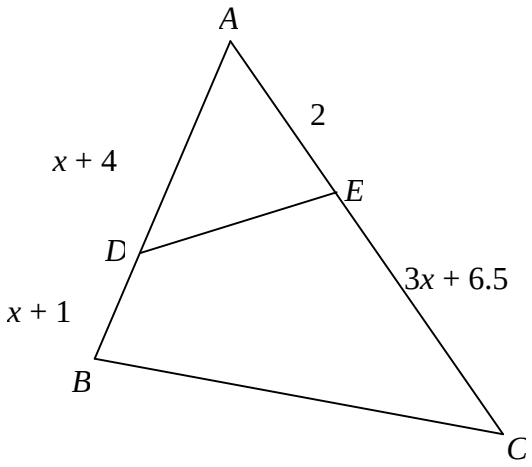
Question 4 [12 Marks] Start a new page **Marks**

a) The volumes of two similar square pyramids are 640 cm^3 and 80 cm^3. Find the ratio of the surface area of the larger pyramid to that of the smaller pyramid. **2**

b) In the diagram shown, $\angle ADE = \angle ACB$.

i) Prove $\Delta ABC \;|||\; \Delta AED$ **3**

ii) Hence, find *x*, with reasons. **4**



c) A closed cylinder with radius *r* and height *h* will hold a volume of 250π cm^3.
Show that the surface area is given by the equation $S = 2\pi r^2 + \frac{500\pi}{r}$ **3**

Question 5 [13 Marks] Start a new page

a) For the circle with equation $x^2 + y^2 + 6x - 8y = 0$

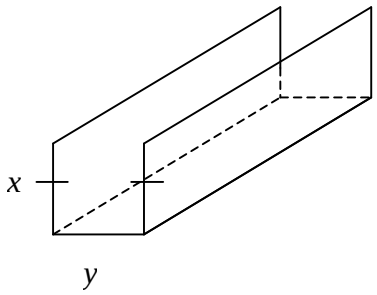
i) Find the centre of the circle. **3**

ii) Show that the origin is on the circle. **1**

iii) If the origin is one end of a diameter of the circle, find the coordinates of the other end point. **2**

Question 5 (Continued) [13 Marks] **Marks**

b) A home guttering company makes metal gutters from material which is 36 cm wide. The gutter is open at the top and it has a rectangular cross section.



i) Show that $y=36-2x$ **1**

ii) Show that the area A cm^2, of the rectangular cross section is given by $A=36x-2x^2$ **1**

iii) Find the value of x for which A is a maximum value, state why this is a maximum value. **2**

iv) Find the maximum area. **1**

c) If $y=\sqrt{\frac{x}{1-x}}+\sqrt{\frac{1-x}{x}}$ then find the simplest expression for y^2. **2**

END OF TEST

Name:_____ **Class:**_____
Teacher:_____

Part A: Multiple choice answer sheet.
Completely colour the circle representing your answer.
Use pencil only.

1.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	16.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
2.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	17.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
3.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	18.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
4.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	19.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
5.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	20.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
6.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	21.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
7.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	22.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
8.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	23.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
9.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	24.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
10.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	25.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
11.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	26.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
12.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	27.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
13.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	28.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
14.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	29.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
15.	Ⓐ	Ⓑ	Ⓒ	Ⓓ	30.	Ⓐ	Ⓑ	Ⓒ	Ⓓ

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Solutions.

Part B.

Question 1

a) $3y^2 + 5y = 3(at^2)^2 + 5(at^2)$
$= 3a^2t^4 + 5at^2$

b) $x^4 - 16 = (x^2 - 4)(x^2 + 4)$
$= (x+2)(x-2)(x^2+4)$

c) $x = \dfrac{\sqrt{a+2}}{3}$.

$3x = \sqrt{a+2}$

$(3x)^2 = a + 2$.

$a = 9x^2 - 2$

d) $3^{2x} + 2(3^x) - 15 = 0$

let $y = 3^x$

$y^2 + 2y - 15 = 0$

$(y-3)(y+5) = 0$

$y = 3$ or $y = -5$.

$3^x = 3$ or $3^x = -5$ but $3^x \neq 0$

$\therefore x = 1$

Question 1 (Continued)

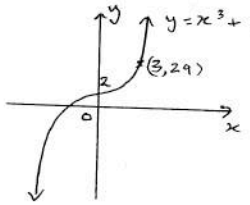
e) $x - \dfrac{1}{x} = \sqrt{2} - 1 - \dfrac{1}{\sqrt{2}-1}$

$= \sqrt{2} - 1 - \dfrac{1}{\sqrt{2}-1} \times \dfrac{\sqrt{2}+1}{\sqrt{2}+1}$

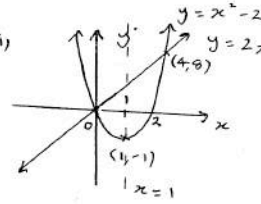
$= \sqrt{2} - 1 - (\sqrt{2}+1)$

$= -2$

f)



g) i)



ii) $x^2 - 2x = 2x$.

$x^2 - 4x = 0$.

$x(x-4) = 0$

$\therefore x = 0$ or $x = 4$

points of intersection (0,0) (4,8)

a) i) median = 20

ii) lower quartile = 18

iii) inter quartile range = 3.

iv) 75%

b) i) mean ($\bar{x}$) = 59.4.

standard deviation (σ) = 10.4.

ii) highest score = 69.5 + 14.2
= 83.7

lowest score = 69.5 - 14.2
= 55.3.

$\therefore$ score lies between 55.3 and 83.7

iii) A score of 70 in French is more than 1 standard deviation above the mean (1.02 to 2dp). A score of 80 in Latin is less than one standard deviations above the mean (0.74 to 2dp). Therefore a score of 70 in French is better as it was achieved by fewer people.

a) i) $5 = \dfrac{b}{T}$

$24 = \dfrac{5T}{0.75}$

$\therefore 5T = 18$.

ii) $\angle LST = 50°$.

$LT^2 = 20^2 + 18^2 - 2(20)(18)\cos 50°$
$= 261.19\ldots$

$LT = 16.16\ldots$

LT is 16 km to the nearest km

b) $\dfrac{\sin \angle BCA}{5.73} = \dfrac{\sin 47°}{4.77}$

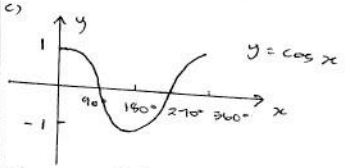
$\sin \angle BCA = \dfrac{5.73 \sin 47°}{4.77}$.

$\angle BCA = 61°28'$ or $118°32'$
(to the nearest minute).

test: $118°32' + 47° = 165°32'$
$< 180°$.

$\therefore$ both solutions are possible.

c)

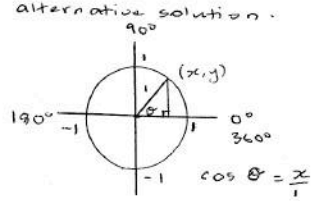


since $-1 \leq \cos x \leq 1$
$\cos x = -2$ has no solutions.

Solutions

Question 3 (continued)

c) alternative solution.



$\therefore x = \cos\theta$

since $-1 \leq x \leq 1$

$-1 \leq \cos\theta \leq 1$

$\cos x = -2$ has no solutions.

d) i) $\tan 30° = \frac{y}{x}$

$y = x \tan 30°$ ①

$\tan 60° = \frac{y+7}{x}$

$y + 7 = x \tan 60°$

$y = x \tan 60° - 7$ ②

ii) Substituting ① in ② for y

$x \tan 30° = x \tan 60° - 7$.

$x \tan 60° - x \tan 30° = 7$

$x(\tan 60° - \tan 30°) = 7$

$x = \frac{7}{\tan 60° - \tan 30°}$

iii) $x = \frac{7}{\tan 60° - \tan 30°}$

$= \frac{7}{\sqrt{3} - \frac{1}{\sqrt{3}}}$.

$= \frac{7}{\frac{3}{\sqrt{3}} - \frac{1}{\sqrt{3}}}$.

$= \frac{7}{\frac{2}{\sqrt{3}}}$.

$= \frac{7\sqrt{3}}{2}$.

Question 4.

a) $\frac{SA_L}{SA_S} = \frac{(\sqrt[3]{640})^2}{(\sqrt[3]{80})^2}$.

$= \frac{4}{1}$.

$= 4$.

b) i) In $\triangle ABC$ and $\triangle AED$

$\angle ADE = \angle ACB$ (given)

$\angle A$ is common.

$\therefore \triangle ABC \sim \triangle AED$ (equiangular)

ii) $\frac{AE}{AB} = \frac{AD}{AC} = \frac{ED}{BC}$ (corresponding sides in the same ratio)

$\frac{2}{2x+5} = \frac{x+4}{3x+8.5}$

[illegible] continued)

$2(3x+8.5) = (2x+5)(x+4)$

$6x + 17 = 2x^2 + 8x + 5x + 20$

$2x^2 + 7x + 3 = 0$

$2x \quad 1$

$x \quad 3$

$(2x+1)(x+3) = 0$

$x = -\frac{1}{2}$ or $x = -3$.

as distance can not be negative the only possible solution is $x = -\frac{1}{2}$.

c) $V = 250\pi$

$\pi r^2 h = 250\pi$

$r^2 h = 250$

$\therefore h = \frac{250}{r^2}$ ①

$S = 2\pi r^2 + 2\pi rh$

$= 2\pi r^2 + 2\pi r\left(\frac{250}{r^2}\right)$ from ①

$= 2\pi r^2 + \frac{500\pi}{r}$

as required.

Question 5.

a) i) $x^2 + y^2 + 6x - 8y = 0$

$x^2 + 6x + 9 + y^2 - 8y + 16 = 25$

$(x+3)^2 + (y-4)^2 = 25$

$\therefore$ the centre is $(-3, 4)$

ii) If $x = 0, y = 0$

LHS $= 0^2 + 0^2 + 6(0) - 8(0)$

$= 0$

$=$ RHS.

$\therefore$ the origin is on the circle.

iii) centre $(-3, 4)$

using the mid point formula.

$\frac{x+0}{2} = -3 \qquad \frac{y+0}{2} = 4$.

$x = -6 \qquad y = 8$.

$\therefore$ the end point is $(-6, 8)$

b) i) $2x + y = 36$

$\therefore y = 36 - 2x$ ①

ii) $A = xy$.

$= x(36 - 2x)$ from ①

$= 36x - 2x^2$

as required.

Question 5 (Continued)

ii) As the coefficient of x^2 is negative, a maximum value will occur at the vertex.

Axis of symmetry occurs if

$x = \frac{36}{4}$.

$x = 9$.

∴ maximum occurs when $x = 9$.

v) If $x = 9$.

$A = 36(9) - 2(9)^2$

$= 162$.

∴ maximum area is $162\ cm^2$

) $y = \sqrt{\frac{x}{1-x}} + \sqrt{\frac{1-x}{x}}$

$y^2 = \frac{x}{1-x} + \frac{1-x}{x} + 2\sqrt{\frac{(x)(1-x)}{(1-x)(x)}}$

$= \frac{x^2 + (1-x)^2 + 2x(1-x)}{x(1-x)}$

$= \frac{x^2 + 1 + x^2 - 2x + 2x - 2x^2}{x - x^2}$

$= \frac{1}{x - x^2}$

Year 12 Yearly Examination 2006

Name: SOLUTIONS **Class:**_____

Teacher:_____

Part A: Multiple choice answer sheet.
Completely colour the circle representing your answer.
Use pencil only.

(A1)

1.	Ⓐ	Ⓑ	Ⓒ	●	16.	Ⓐ	Ⓑ	Ⓒ	●
2.	Ⓐ	Ⓑ	●	Ⓓ	17.	Ⓐ	●	Ⓒ	Ⓓ
3.	●	Ⓑ	Ⓒ	Ⓓ	18.	Ⓐ	Ⓑ	●	Ⓓ
4.	●	Ⓑ	Ⓒ	Ⓓ	19.	●	Ⓑ	Ⓒ	Ⓓ
5.	Ⓐ	Ⓑ	Ⓒ	●	20.	Ⓐ	●	Ⓒ	Ⓓ
6.	Ⓐ	Ⓑ	Ⓒ	●	21.	Ⓐ	Ⓑ	●	Ⓓ
7.	Ⓐ	Ⓑ	●	Ⓓ	22.	Ⓐ	●	Ⓒ	Ⓓ
8.	Ⓐ	Ⓑ	Ⓒ	●	23.	Ⓐ	●	Ⓒ	Ⓓ
9.	Ⓐ	●	Ⓒ	Ⓓ	24.	Ⓐ	Ⓑ	●	Ⓓ

(A2)

10.	●	Ⓑ	Ⓒ	Ⓓ	25.	Ⓐ	Ⓑ	Ⓒ	●
11.	Ⓐ	Ⓑ	●	Ⓓ	26.	Ⓐ	●	Ⓒ	Ⓓ
12.	●	Ⓑ	Ⓒ	Ⓓ					
13.	Ⓐ	Ⓑ	●	Ⓓ					
14.	Ⓐ	Ⓑ	Ⓒ	●					
15.	●	Ⓑ	Ⓒ	Ⓓ					

(A3)

27.	●	Ⓑ	Ⓒ	Ⓓ
28.	Ⓐ	Ⓑ	Ⓒ	●

(A4)

29.	Ⓐ	Ⓑ	●	Ⓓ
30.	Ⓐ	Ⓑ	●	Ⓓ