NORTH SYDNEY GIRLS HIGH SCHOOL

Year 10 Mathematics Yearly Examination 2008

Name: _____ Class: _____

Teacher: _____

Time Allowed: **1·5 hours + 5 minutes reading time**
Marks: **93**

Instructions:

- Answer Part A, the multiple choice questions, on the answer sheet provided.
- Answer Part B on the paper provided.
- Start each question on a new page.
- Attempt every question.
- Show all necessary working.
- Marks may be deducted for incomplete or poorly arranged work.
- Write on one side of the page only.
- Do NOT use correcting tape or liquid paper.
- Diagrams are not drawn to scale.

At the end of the examination, hand in one STAPLED bundle. Place this question paper on top, followed by your Part B written solutions and your multiple choice sheet LAST.

	Num	PA	Meas	Data	SG	WM	TOTAL
Part B	Q1 /14	Q2 /14	Q3 /12	Q4a /7	Q4bc /7	Q5 /14	/68
M/C	/5	/9	/8	/3			/25
TOTAL							/93

Section A - Each question is worth 1 mark.
Answer on the multiple choice sheet provided.

1. A molecule of hydrogen has a mass of $3 \cdot 32 \times 10^{-24}$ g. What is the mass of 40 billion molecules?

(A) $1 \cdot 328 \times 10^{13}$ g (C) $1 \cdot 328 \times 10^{16}$ g
(B) $1 \cdot 328 \times 10^{-13}$ g (D) $1 \cdot 328 \times 10^{-16}$ g

2. Simplify $9^3 \div 9^{-1}$

(A) 3^{-6} (C) 3^4
(B) 1 (D) 3^8

3. Katherine is about to go on holidays for 4 weeks. Her weekly salary is \$280 and her holiday loading is $17\frac{1}{2}\%$ of four weeks pay. What is her total pay for the four weeks holiday?

(A) \$196 (C) \$329
(B) \$1169 (D) \$1316

4. Blue and yellow paints are mixed in the ratio 5 : 3 to make green paint. In making a batch of green paint, the number of litres of blue paint was 12 more than the number of litres of yellow paint. How much green paint was produced in this batch?

(A) $19 \cdot 2$ L (C) 48 L
(B) 32 L (D) 96 L

5. Each student in a class is given a packet of lollies.
Miss James records the number of red lollies in each packet using a frequency table.

What is the relative frequency of a packet of lollies containing more than three red lollies?

(A) $\frac{4}{19}$ (C) $\frac{4}{15}$
(B) $\frac{11}{19}$ (D) $\frac{11}{15}$

No. of red lollies in a packet	*Frequency*
0	2
1	4
2	2
3	7
4	3
5	1

6. Use the formula $R = \sqrt[3]{\frac{3V}{4\pi}}$ to find R correct to 2 decimal places if $V = 18 \cdot 76$.

(A) $1 \cdot 65$ (C) $2 \cdot 12$
(B) $3 \cdot 54$ (D) $4 \cdot 49$

7. Solve the equation $4x-2(x-3)=14$

(A) $x=4$ (C) $x=5\frac{1}{2}$

(B) $x=8\frac{1}{2}$ (D) $x=10$

8. Nuria started to make the following pattern using matchsticks:

Shape 1 **Shape 2** **Shape 3**

If the pattern was continued, which shape would use exactly 486 matchsticks?

(A) Shape 96 (C) Shape 97

(B) Shape 121 (D) Shape 122

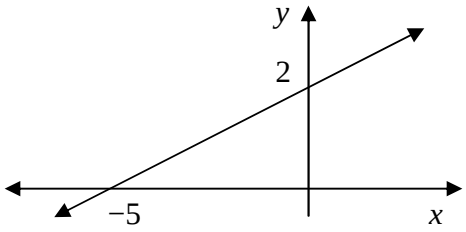
9. The equation of the line in the diagram is:

(A) $y=-\frac{2}{5}x-2$

(B) $y=\frac{2}{5}x-2$

(C) $y=2-\frac{2}{5}x$

(D) $y=2+\frac{2}{5}x$



10. $(2x)^3 \div x^{-2} =$

(A) $2x$ (C) $8x$

(B) $2x^5$ (D) $8x^5$

11. The gradient of the line $3x-2y+6=0$ is

(A) 3 (C) -2

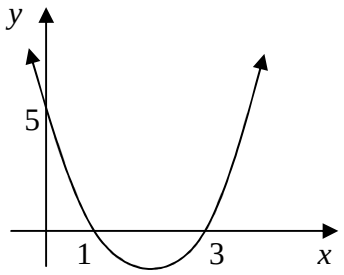
(B) $\frac{3}{2}$ (D) $-\frac{2}{3}$

12. Solve $\frac{1}{P-2}=\frac{5}{3P}$

(A) $P = 2$ (C) $P = 5$
(B) $P = 1$ (D) $P = -1$

13. The parabola in the diagram is described by the rule:

(A) $y=(x+1)(x+3)$
(B) $y=(x-1)(x-3)$
(C) $y=\frac{5}{3}(x+1)(x+3)$
(D) $y=\frac{5}{3}(x-1)(x-3)$



14. Which of the following correctly expresses T as the subject of $B=2\pi\left(R+\frac{T}{2}\right)$?

(A) $T=\frac{B}{\pi}-2R$ (C) $T=2R-\frac{B}{\pi}$
(B) $T=\frac{B}{\pi}-R$ (D) $T=\frac{B}{4\pi}-\frac{R}{2}$

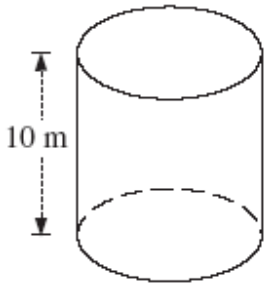
15. A racing car completed a $6\cdot3$ km lap in 2 minutes and 30 seconds. What is the car's average speed in km/h?

(A) 151·2 (C) 164·4
(B) 252 (D) 273·9

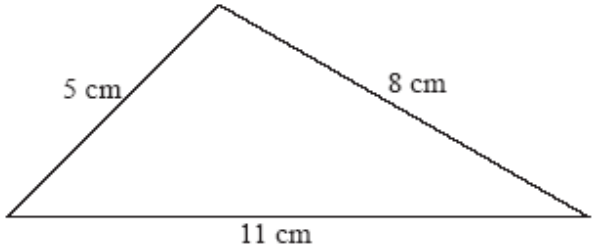
16. The cylindrical tank pictured has diameter 6m and is used to store liquid chemicals.

What is its capacity, to the nearest L?

(A) 283 (C) 282673
(B) 1131 (D) 1130973



17. The largest angle in this triangle is θ.



Which equation would give the correct value for θ ?

(A) $\cos\theta = \dfrac{5^2+8^2-11^2}{2\times5\times8}$

(B) $\cos\theta = \dfrac{5^2+11^2-8^2}{2\times5\times11}$

(C) $\cos\theta = \dfrac{8^2+11^2-5^2}{2\times8\times11}$

(D) $\cos\theta = \dfrac{5^2+8^2-11^2}{2\times8\times11}$

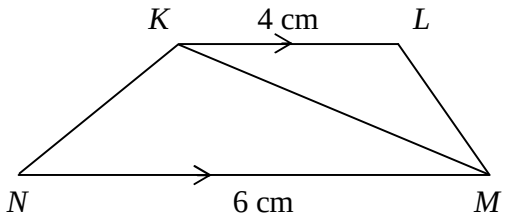
18. Orla used one litre of water to make exactly 64 ice cubes. They were too small, so she decided to make larger cubes by doubling all the dimensions.
How many of these larger cubes can be made using 1 litre of water?

(A) 4
(B) 16
(C) 8
(D) 32

19. A boat sails 3 km west, then 3 km south. What is its bearing from its original position?

(A) 045°T
(B) 315°T
(C) 135°T
(D) 225°T

20. The area of triangle *KLM* is 16 cm^2.



The area of the trapezium *KLMN* is:

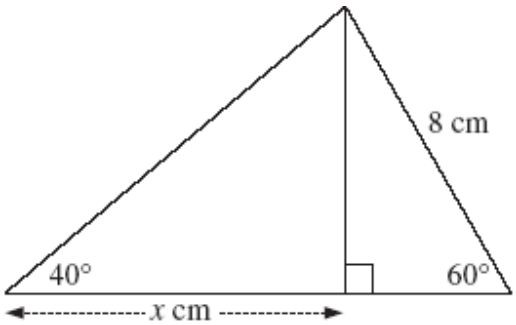
(A) 20 cm^2
(B) 32 cm^2
(C) 40 cm^2
(D) 48 cm^2

21. Nicola has a weekly income of $900 and allocates her money according to the budget shown in the sector graph below:

How long will it take her to save $3600?

(A) 4 weeks
(B) 5 weeks
(C) 16 weeks
(D) 18 weeks

22.



The value of x in the diagram can be calculated using the expression

(A) $\dfrac{8\sin 60°}{\tan 40°}$
(B) $8\sin 60° \tan 40°$
(C) $\dfrac{8\tan 40°}{\sin 60°}$
(D) $\dfrac{8}{\sin 60° \tan 40°}$

23. In which set of scores is the mode greater than the median?

(A) 3, 4, 4, 5, 5, 6, 6, 6, 7
(B) 3, 4, 4, 5, 5, 5, 6, 6, 7
(C) 3, 4, 4, 4, 5, 5, 6, 6, 7
(D) 2, 2, 2, 2, 3, 4, 4, 5, 6

24. After 6 mathematics tests, An's mean score was 71. She hopes to raise her mean to 75 after the next test. Assuming each test is equally weighted, what mark will An need to achieve on her seventh test to reach her goal?

(A) 75
(B) 95
(C) 79
(D) 99

25. The set of data below is ordered from smallest to largest. The range is 6 less than twice the value of x.

5, 6, 11, x, 13, 18, 25.

Which of the following is true?

(A) The median is 12 and the interquartile range is 7.
(B) The median is 12 and the interquartile range is 12.
(C) The median is 13 and the interquartile range is 7.
(D) The median is 13 and the interquartile range is 12.

Section B **Start each question on a new page.** **Marks**

Question 1. (14 marks)

a) Evaluate $\frac{5\cdot 67}{\sqrt{4\cdot 34}}$ to 3 significant figures. 2

b) A packet contains 3 red lollies and 2 green lollies. Jane randomly chooses one, eats it, then selects and eats another one.
   i) Draw a tree diagram showing all possible outcomes. 2
   ii) Find the probability that the two lollies Jane eats are the same colour. 2

c) Jenny borrows $25000 for a new car. She repays the loan in fortnightly installments of $256 over 5 years. Find:
   i) The amount of interest she pays 1
   ii) The flat rate of interest (p.a.) 1

d) Simplify $\frac{6\sqrt{70}}{4\sqrt{5}\times 3\sqrt{7}}$ 2

e) Lavinnia buys a new computer system for $8900. It depreciates in value at a rate of 20% p.a. Find its value after 5 years. 2

f) Carrie wants to borrow $160 000 to buy an apartment. Her bank sends her the following table in an email:

Monthly Repayments

Amount Borrowed	**Term of Loan**				
	10 years	**15 years**	**20 years**	**25 years**	**30 years**
	120 months	**180 months**	**240 months**	**300 months**	**360 months**
$80 000	$970·62	$764·52	$669·15	$617·45	$587·01
$90 000	$1091·95	$860·09	$752·80	$694·63	$660·39
$100 000	$1213·28	$955·65	$836·44	$771·82	$733·76
$110 000	$1334·60	$1051·22	$920·08	$849·00	$807·14
$120 000	$1455·93	$1146·78	$1003·73	$926·18	$880·52
$130 000	$1577·26	$1242·35	$1087·37	$1003·36	$953·89
$140 000	$1698·59	$1337·91	$1171·02	$1080·54	$1027·27
$150 000	$1819·91	$1433·48	$1254·66	$1157·72	$1100·65
$160 000	$1941·24	$1529·04	$1338·30	$1234·91	$1174·02

How much more will she pay if she chooses to pay off the loan over 20 years rather than 15 years? 2

Question 2. (14 Marks) **Start a new page.** **Marks**

a) Factorise $3d^2+5d+2$ 2

b) Solve $(2m+3)^2=49$ 2

c) What are the restrictions on the values can x take in the formula $y=\sqrt{16-x^2}$? 2

d) Solve $-11>1-2x$ 2

e) i) Sketch the graphs of $x+y=2$ and $y=2x-7$ on the same set of axes. 3

ii) Hence, or otherwise, simultaneously solve the pair of equations
$x+y=2$ and
$y=2x-7$ 1

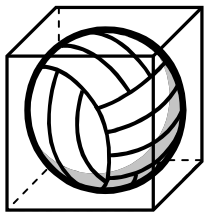
f) Solve $w^2-4w+1=0$ giving your answer in simplest exact form. 2

Question 3. (12 marks) **Start a new page.** **Marks**

a) Find the area of the curved surface of a cone with diameter 12cm and slant height 8cm, to one decimal place. 2

b) Find the exact surface area of an equilateral triangular pyramid where each edge is 5cm. 2

c) A sphere with radius 4cm fits neatly in a cube shaped box as shown below. What percentage of the volume of the box does it occupy? 3

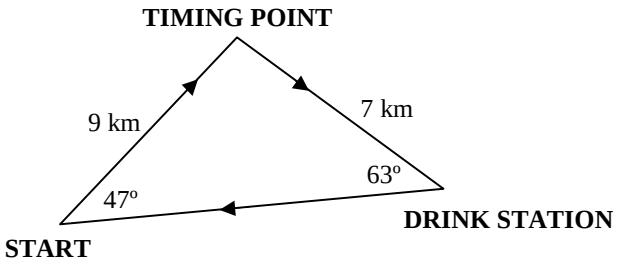


d) Given θ is an acute angle, find θ if $\cos 123^\circ=-\cos\theta$. 1

e) If A is an angle in a triangle, find the possible sizes of A, to the nearest minute, if $\sin A=\frac{4}{7}$. 2

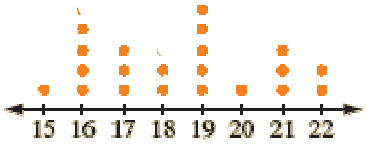
Please turn the page…

f) Rachel is competing in a cross country race that follows the course shown below. Find the total length of the course. 2



Question 4. (14 marks) **Start a new page.** **Marks**

a) i) Construct a box and whisker plot using the data in the dot plot below. 3



ii) State the interquartile range. 1

iii) Find the standard deviation, correct to two decimal places. 1

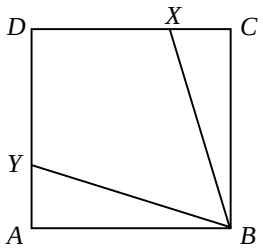
iii) An extra score of 12 is added. Explain how this will affect the BOTH the median and the standard deviation. 2

b) Answer true or false to the following statements:

i) The diagonals of a parallelogram are equal 1

ii) Any two isosceles triangles are similar 1

c) In the diagram below, $ABCD$ is a square and $AY = CX$.



i) Prove $\Delta ABY \equiv \Delta CBX$. 3

ii) Hence prove that $DBXY$ is a kite. 2

Question 5. (14 marks) **Start a new page.** **Marks**

a) The sum, S, of the first n positive integers, $1 + 2 + 3 + 4 + \ldots + n$, is given by $S = \frac{n}{2}(n+1)$. Find the number of positive integers needed to give a sum of 325. 2

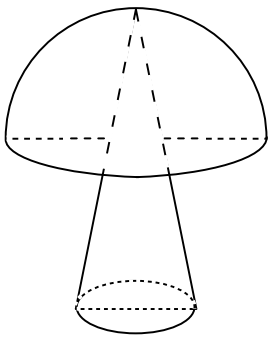
b) The third side of an isosceles triangle is 3 units less than twice the length of the equal sides. If the perimeter of the triangle is less than 48 units, what are the largest possible whole unit lengths of the sides of the triangle? 2

c) A teddy bear is shot out of a stunt cannon and follows a parabolic path given by the equation $y = 20x - \frac{x^2}{2}$, where x and y are measured in metres. A ramp with a gradient of 0·5 begins at the cannon and extends underneath the path of flight.

i) How high off the ground is the teddy bear at the highest point of its flight? 1

ii) The teddy bear lands on the ramp. How high off the ground is the teddy bear when it lands? 3

d) Indonesia has a population of 231 million with a population density of approximately 121 persons/km^2. Australia's population density is about 2·6 persons/km^2 and its population is 20 million. If Australia was as densely populated as Indonesia, what would our population be? 2



e) A mushroom ornament, pictured to the left, is constructed from a hemisphere and a cone. The hemisphere is hollowed out to allow the cone's apex to reach through and sit inside the hemisphere. The ornament is 16 cm tall. If the radius of the cone is half the radius of the sphere, and the ratio of the height of the cone to its diameter is 4:1, find the total volume of the ornament.

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Name: _____ **Class:** _____
Teacher: _____

Part A - Multiple Choice Answer Sheet

Completely colour the circle representing your answer.
Use pencil only.

1.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
2.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
3.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
4.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
5.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
6.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
7.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
8.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
9.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
10.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
11.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
12.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
13.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
14.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
15.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
16.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
17.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
18.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
19.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
20.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
21.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
22.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
23.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
24.	Ⓐ	Ⓑ	Ⓒ	Ⓓ
25.	Ⓐ	Ⓑ	Ⓒ	Ⓓ

Num /5	P&A /9	Meas /8	Data /3	TOTAL /25

Name: YEAR 10 YEAR 11 - 2008 **Class:** _____

Teacher: _____

Part A - Multiple Choice Answer Sheet

Completely colour the circle representing your answer.
Use pencil only.

1.	Ⓐ		Ⓒ	Ⓓ
2.	Ⓐ	Ⓑ	Ⓒ	
3.	Ⓐ	Ⓑ	Ⓒ	
4.	Ⓐ	Ⓑ		Ⓓ
5.		Ⓑ	Ⓒ	Ⓓ
6.		Ⓑ	Ⓒ	Ⓓ
7.		Ⓑ	Ⓒ	Ⓓ
8.	Ⓐ		Ⓒ	Ⓓ
9.	Ⓐ	Ⓑ	Ⓒ	
10.	Ⓐ	Ⓑ	Ⓒ	
11.	Ⓐ		Ⓒ	Ⓓ
12.	Ⓐ	Ⓑ		Ⓓ
13.	Ⓐ	Ⓑ	Ⓒ	
14.		Ⓑ	Ⓒ	Ⓓ
15.		Ⓑ	Ⓒ	Ⓓ
16.	Ⓐ	Ⓑ		Ⓓ
17.		Ⓑ	Ⓒ	Ⓓ
18.	Ⓐ	Ⓑ		Ⓓ
19.	Ⓐ	Ⓑ	Ⓒ	
20.	Ⓐ	Ⓑ		Ⓓ
21.	Ⓐ	Ⓑ	Ⓒ	
22.		Ⓑ	Ⓒ	Ⓓ
23.		Ⓑ	Ⓒ	Ⓓ
24.	Ⓐ	Ⓑ	Ⓒ	
25.	Ⓐ	Ⓑ	Ⓒ	

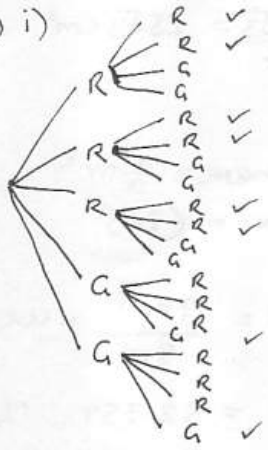
ANSWERS

Num /5	P&A /9	Meas /8	Data /3	TOTAL /25

Question 1

a) 2·72 (3sf)

b) i)



ii) $P(2 \text{ same}) = \frac{8}{20}$

$= \frac{2}{5}$

c) i) Interest $= 256 \times 26 \times 5 - 25000$

$= \$8280$

ii) $I = PRT$

$8280 = 25000 \times R \times 5$

$R = 0.06624$

$= 6.624\%$ p.a.

d) $\frac{\cancel{2}6 \times \cancel{\sqrt{7}} \times \sqrt{2} \times \cancel{\sqrt{5}}}{24 \times \cancel{\sqrt{5}} \times \cancel{3} \times \cancel{\sqrt{7}}} = \frac{\sqrt{2}}{2}$ or $\frac{1}{\sqrt{2}}$

e) $A = P(1-r)^n$

$\therefore$ Value $= 8900(0.8)^5$

$= \$2916.35$ (nearest cent)

f) 20 year loan:

repayment total $= \$1338.30 \times 240$

$= \$321\,192.00$

15 year loan:

repayment total $= \$1529.04 \times 180$

$= \$275\,227.20$

$\therefore$ Carrie pays \$45 964·80 more with the 20 year loan.

Question 2

a) $(3d+2)(d+1)$

b) $(2m+3)^2 = 49$

$2m+3 = \pm 7$

$2m = 4$ or -10

$m = 2$ or -5

c) $16 - x^2 \geq 0$

$\therefore x^2 \leq 16$

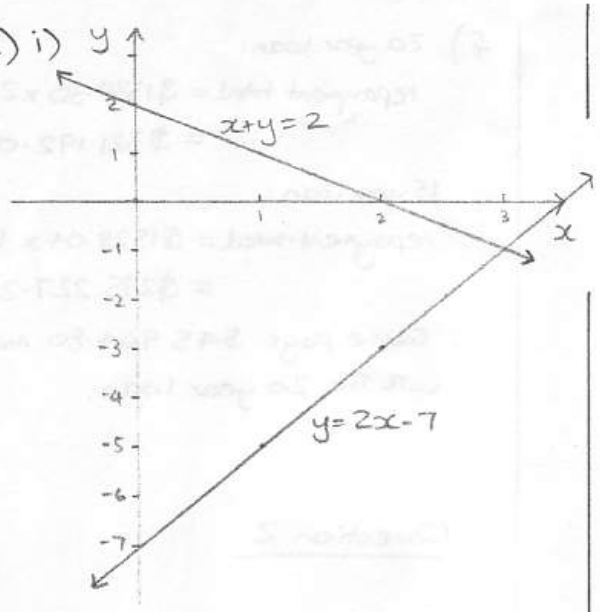
$-4 \leq x \leq 4$

d) $-11 > 1 - 2x$

$2x > 12$

$x > 6$

e) i)



ii) $x=3$ & $y=-1$

f) $w = \frac{4 \pm \sqrt{16-4}}{2}$

$= \frac{4 \pm \sqrt{12}}{2}$

$= 2 \pm \sqrt{3}$

Question 3

a) $A = \pi rs$

$= \pi \times 6 \times 8$

$= 150.796...$

$= 150.8 cm^2$ (1dp)

b)

pyramid has 4 equal faces

$A = \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 5 \times 5 \times \sin 60°$

$= \frac{25\sqrt{3}}{4}$

$\therefore S.A = 4 \times \frac{25\sqrt{3}}{4} = 25\sqrt{3} cm^2$

c) Volume sphere $= \frac{4}{3}\pi r^3$

Volume box $= (2r)^3$

% occupied $= \frac{\frac{4}{3}\pi r^3}{8r^3} \times 100\%$

$= 52.359...\%$

$= 52.4\%$ (1dp).

d) $\theta = 57°$

e) $A = 34°51'$ or $180° - 34°51'$

$= 34°51'$ or $145°9'$

f) missing angle $= 70°$

missing length (x): $\frac{x}{\sin 70°} = \frac{9}{\sin 63°}$

$x = \frac{9\sin 70°}{\sin 63°}$

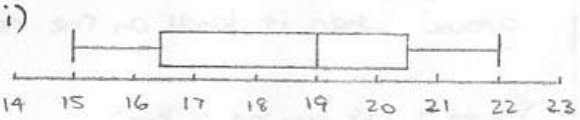
$\therefore$ total length $= \frac{9\sin 70°}{\sin 63°} + 9 + 7$

$= 25.4917...$

$= 25.5 km.$

Question 4

a) i)



ii) $IQR = 20.5 - 16.5$

$= 4$

iii) $\sigma = 2.084\ldots$

$= 2.08$ (2dp)

iv) The median will change to 18.5
(ie decrease by 0.5)
The standard deviation will increase.

b) i) F ii) F

c) In $\triangle ABY$ & $\triangle CBX$:
$AY = CX$ (given)
$AB = BC$ (sides of a square)
$\angle YAB = \angle XCB = 90°$ (angles in a sqaure)
$\therefore \triangle ABY \equiv \triangle CBX$ (SAS)

d) $BY = BX$ (corresponding sides of congruent $\triangle$s)
$AY + YD = DX + XC$ (sides of a square)
$\therefore YD = DX$ since $AY = CX$ (given)
$\therefore$ DBXY is a kite (2 pairs of adjacent sides equal)

Question 5

a) $325 = \frac{n}{2}(n+1)$

$0 = n^2 + n - 650$

$n = \frac{-1 \pm \sqrt{1 + 4(650)}}{2}$

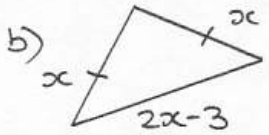
$= \frac{-1 \pm 51}{2}$ but $n > 0$

$\therefore n = \frac{50}{2}$

$= 25$

$\therefore$ need 25 positive integers.

b)



$4x - 3 < 48$

$4x < 51$

$x < 12.75$

$\therefore x = 12$

$\therefore 2x - 3 = 21$

sides are 12, 12 & 21cm.

c) i) max height occurs at the vertex: $x = \frac{-b}{2a}$

$y = -\frac{1}{2}x^2 + 20x$ so

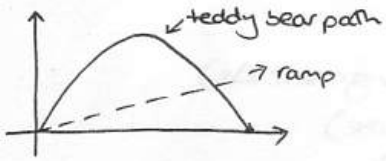
vertex $x = \frac{-20}{-1} = 20$

then $x = 20$, $y = -\frac{1}{2}(20)^2 + 20^2$

$= 200$

$\therefore$ max height = 200m.

ii)



ramp has equation $y = 0.5x$ — ①

solve simultaneously with

$y = 20x - \frac{x^2}{2}$ — ②

sub ① → ②: $0.5x = 20x - \frac{x^2}{2}$

$x^2 - 39x = 0$

$x = 0, 39$

sub into ①: $y = 0.5(39)$

$y = 19.5$

$\therefore$ teddy bear is 19.5m above the ground when it lands on the ramp.

d) Let Aust's area be x km^2

$\frac{20\,000\,000}{x} = \frac{2.6}{1}$

$x = 2 \times 10^7 \div 2.6$ km^2

$\therefore$ new population $= 121 \times x$

$= 121 \times 2 \times 10^7 \div 2.6$

$= 930\,769\,230.8$

$\therefore$ population is $\doteq$ 931 million people

e)

cone h = 16cm

h : 2r is 4 : 1

$\therefore$ cone r = 2cm

cone : sphere is 1 : 2

$\therefore$ sphere r = 4cm.

$V_{cone} = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(2)^2(16)$

$= \frac{64\pi}{3}$ cm^3

$V_{sphere} = \frac{4}{3}\pi r^3 \times \frac{1}{2}$

$= \frac{4}{3}\pi(4)^3 \times \frac{1}{2}$

$= \frac{256\pi}{3}$ cm^3

But top 4cm of cone is "embedded" in sphere

A similar cone $\frac{1}{4}$ height has $(\frac{1}{4})^3$ the volume of the original.

$\therefore$ Total volume $= \frac{128\pi}{3} + \frac{64\pi}{3} - \frac{1}{64} \times \frac{64\pi}{3}$

$= \frac{191\pi}{3}$ cm$^3 \doteq 200$cm^3

(3 sig fig)