

# NORTH SYDNEY GIRLS HIGH SCHOOL



## 2009 Year 10 Yearly Examination

# Mathematics

Name: \_\_\_\_\_ Class: \_\_\_\_\_

Teacher: \_\_\_\_\_

**Time Allowed:** 2 hours + 5 minutes reading time

### Directions to Candidates:

- Approved calculators may be used
- Answer Part A, the multiple choice questions, on the answer sheet provided.
- Answer Part B in the spaces provided
- For Part C, *each* question is to be started on a *new page*.
- Attempt every question.
- Show all necessary working. Do not use correction tape or fluid.
- Marks may be deducted for incomplete or poorly arranged work.

**At the end of the examination, staple this question paper to the front of your solutions and submit one bundle.**

	A	M	D	G	WM	Total
Part A						/20
Part B						/13
Part C Qu1	/7	/6				/13
Part C Qu2	/6		/7			/13
Part C Qu3	/13					/13
Part C Qu4			/6	/9		/15
Part C Qu5					/13	/13
<b>TOTAL</b>	/26	/6	/13	/9	/13	/100

**Part A: Multiple Choice (1 mark each)**

**Answer on the sheet provided by completely colouring the circle representing your answer. Use pencil only.**

1. Which of the following is greatest in value:

- (A)  $3^{-1}$                       (B)  $\sqrt{0.1}$                       (C) 33%                      (D)  $3.1 \times 10^{-1}$

2. If  $2x + 7$  is the largest of three consecutive numbers, the smallest is:

- (A)  $2x + 4$                       (B)  $2x + 5$                       (C)  $2x + 1$                       (D)  $2x$

3.  $-(2b^2)^4 =$

- (A)  $16b^8$                       (B)  $-8b^6$                       (C)  $8b^8$                       (D)  $-16b^8$

4. The surface area of a rectangular prism 12 cm by 10 cm by 8 cm is

- (A)  $960 \text{ cm}^2$                       (B)  $200 \text{ cm}^2$                       (C)  $592 \text{ cm}^2$                       (D)  $296 \text{ cm}^2$

5. The expression  $\frac{m}{c} + \frac{r}{c}$  is equivalent to:

- (A)  $m + r$                       (B)  $\frac{m+r}{2c}$                       (C)  $\frac{mc+rc}{c}$                       (D)  $\frac{m+r}{c}$

6. If  $x + 1$  is an even number then:

- (A)  $x + 2$  is even                      (B)  $x$  is even  
(C)  $\frac{x+1}{2}$  is an integer                      (D)  $x + 2$  is a multiple of 3

7. If 1 kilometre is approximately  $\frac{5}{8}$  mile, what fraction of a kilometre is half a mile?

- (A)  $\frac{4}{5}$                       (B)  $\frac{5}{16}$                       (C)  $\frac{1}{4}$                       (D)  $1\frac{1}{4}$

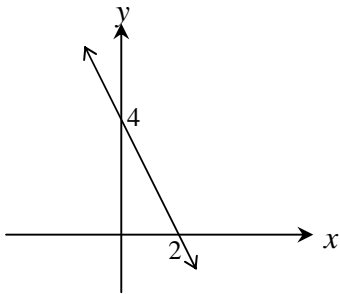
8. Which of the following is *not* linear?

- (A)  $y = 3$                       (B)  $y = \frac{1}{x} + 2$                       (C)  $x + 2y - 3 = 0$                       (D)  $y = \frac{x}{2} + 5$

9. Neema's coin collection appreciates at  $3\frac{1}{4}\%$  per year. If it is worth \$3600 now, what will it be worth in 2 years time?

- (A) \$3 369.80                      (B) \$3 624                      (C) \$3 837.80                      (D) \$23 400

10. The equation of the line illustrated is:



- (A)  $y = -2x$   
 (B)  $y = 4x - 2$   
 (C)  $y = 4 - 2x$   
 (D)  $y = 2x + 4$

11. Which line is parallel to  $y = 2x - 3$ ?

- (A)  $4x - 2y + 5 = 0$                       (B)  $y = x - 3$   
 (C)  $y + 2x - 3 = 0$                       (D)  $y = \frac{1}{2}x - 3$

12. Which of the following equations has the solutions 4 and  $-3$ ?

- (A)  $x^2 + x - 12 = 0$                       (B)  $x^2 - 7x - 12 = 0$   
 (C)  $x^2 - x - 12 = 0$                       (D)  $x^2 + 7x - 12 = 0$

13. Consider this solution of the equation  $2g^2 + 8g + 1 = 0$ . In which line does the first error occur?

$$g = -8 \pm \frac{\sqrt{8^2 - 4 \times 2 \times 1}}{2 \times 2} \quad \dots \text{Line 1}$$

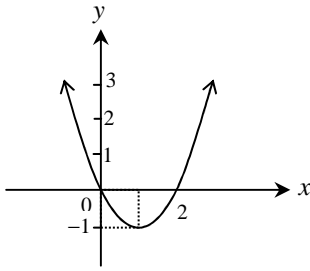
$$g = -8 \pm \frac{\sqrt{56}}{4} \quad \dots \text{Line 2}$$

$$g = -8 \pm \frac{4\sqrt{14}}{4} \quad \dots \text{Line 3}$$

$$g = -8 \pm \sqrt{14} \quad \dots \text{Line 4}$$

- (A) Line 1                      (B) Line 2                      (C) Line 3                      (D) Line 4

14.



The diagram shows the graph of the function

$y = (x-1)^2 + d$ . If  $d$  is an integer, then  $d$  is equal to:

- (A) -2
- (B) -1
- (C) 2
- (D) 1

15.

Two dice are rolled and the sum of the uppermost faces is calculated. What is the probability of obtaining a total of 2 or 3 is:

- (A)  $\frac{1}{12}$
- (B)  $\frac{1}{18}$
- (C)  $\frac{1}{6}$
- (D)  $\frac{1}{3}$

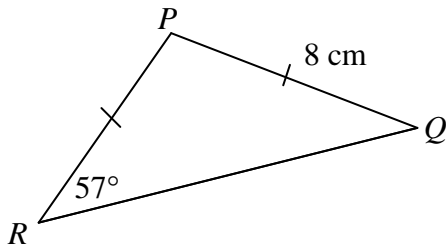
16.

Age	Frequency
14	5
15	4
16	1
17	6
18	4

The data in the table shows the ages of players in a cricket squad. If a new player aged 16 years joins the team, which of the following will change?

- (A) range
- (B) median
- (C) mean
- (D) mode

17.



The area of  $\triangle PQR$  in square centimetres is closest to:

- (A) 59
- (B) 29
- (C) 27
- (D) 13

18.

A car's fuel economy is stated as 13.5 L/100 km. If petrol costs 115.9 c/L, the fuel cost for a journey of 86 km is:

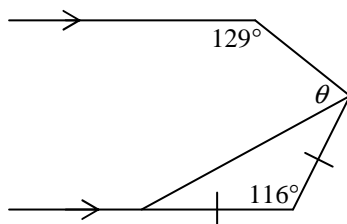
- (A) \$7.38
- (B) \$7.67
- (C) \$13.46
- (D) \$18.19

19.

0.000 38 in standard notation is:

- (A)  $3.8 \times 10^{-5}$
- (B)  $38 \times 10^{-5}$
- (C)  $3.8 \times 10^{-4}$
- (D)  $3.8 \times 10^4$

20.

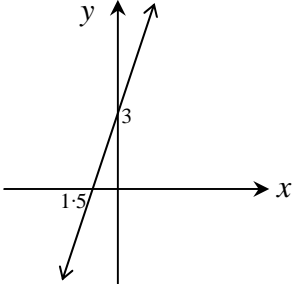
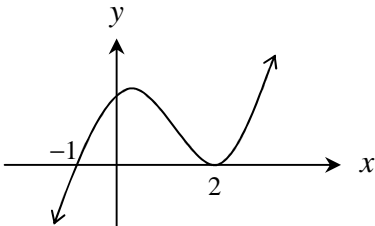
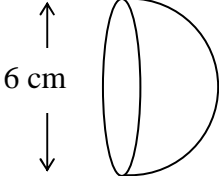
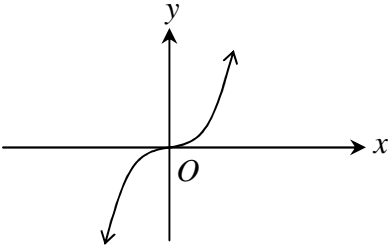


The value of  $\theta$  is:

- (A)  $33^\circ$
- (B)  $65^\circ$
- (C)  $83^\circ$
- (D)  $115^\circ$

**End of Part A**

**Part B: Write the answer only in the space provided. (1 mark each)**

(a)	Write in expanded form: $(5a + 3)^2$	
(b)	Solve $(x - 2)(2x + 5) = 0$	
(c)	Factorise: (i) $25x^2 - 9$ (ii) $2x^2 + 7x - 15$	
(d)	Simplify $\sqrt{12} + \sqrt{27}$	
(e)	Write $\frac{2}{\sqrt{3}}$ with a rational denominator	
(f)	Shade the region for which $y \geq 2x + 3$ .	
(g)	Write down in factored form, an equation which can be represented by this graph.	
(h)	Find the volume of this hemisphere, leaving your answer in terms of $\pi$ .	
(i)	The probability of getting the measles as a teenager is 0.018. How many of 700 000 teenagers will <i>not</i> be expected to contract measles?	
(j)	At a “25% off” sale, goods were sold for \$36. What was the price of the goods before the sale?	
(k)	The graph of $y = x^3$ is illustrated. On the same axes, draw the graph of $y = (x + 2)^3$ .	
(l)	Solve the equation $3^{2-x} = 9^x$ .	

**End of Part B**

**Part C:**

Use the examination pad provided.

Start each question on a new page.

Show all working.

**Question 1: (13 marks)**

**Marks**

- (a) Using  $u = \sqrt{x} - 1$ , solve

**3**

$$(\sqrt{x} - 1)^2 - 8(\sqrt{x} - 1) + 12 = 0$$

- (b) (i) If  $k$  is a positive integer, write down an expression in terms of  $k$  which will always generate odd numbers.

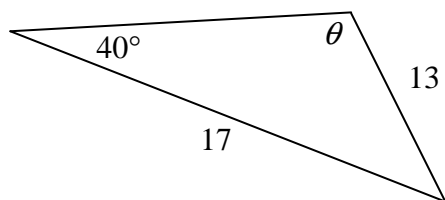
**1**

- (ii) Hence show that  $n^2 - 1$  is a multiple of 8 for all odd values numbers  $n$ .

**3**

- (c) Find the value of  $\theta$  correct to the nearest minute.

**3**



- (d) Sandra delivers mail to remote communities. She flies due east for 15 km and then turns on a bearing of  $312^\circ$  and flies a further 28 km. She then flies directly back to the starting point of the trip.

- (i) Draw a diagram illustrating the trip.

**1**

- (ii) What was the distance Sandra flew on the final leg of the journey? Give your answer correct to 2 significant figures.

**2**

**Question 2: (13 marks) Start a new page**

**Marks**

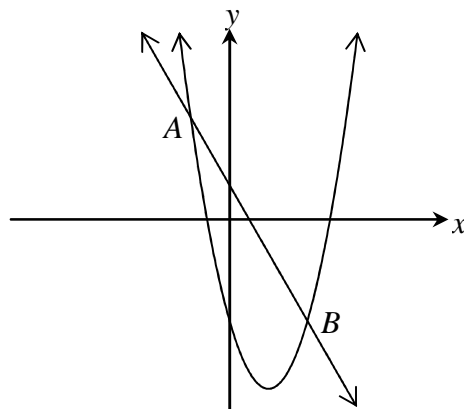
- (a) Draw a neat sketch of the following, showing the main features. Include any intercepts and asymptotes.

$$y = 3 + \frac{2}{x+1}$$

**3**

- (b) On the diagram below the graphs of  $y = 2x^2 - 4x - 3$  and  $y = 1 - 2x$  are drawn. Their points of intersection are labelled  $A$  and  $B$ . Solve a pair of simultaneous equations to find the coordinates of  $A$  and  $B$ .

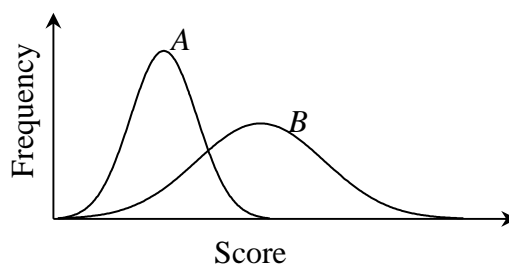
**3**



- (c) A child has pulled off the labels from 6 cans on a pantry shelf. Mum knew there were 2 cans of corn, 1 can of beans and 3 cans of soup. She now takes 2 cans from the shelf and opens them.

- (i) Draw a diagram for the two can selection sequence. Label each branch of the diagram with the appropriate probability. **1**
- (ii) What is the probability that Mum selects:
- ( $\alpha$ ) a can of soup and a can of corn? **2**
- ( $\beta$ ) exactly one can of soup? **2**

- (d) The graph shows the frequency curves for two sets of test results,  $A$  and  $B$ . Write a statement comparing the means and standard deviations of both sets of results. **2**



**Question 3: (13 marks) Start a new page****Marks**

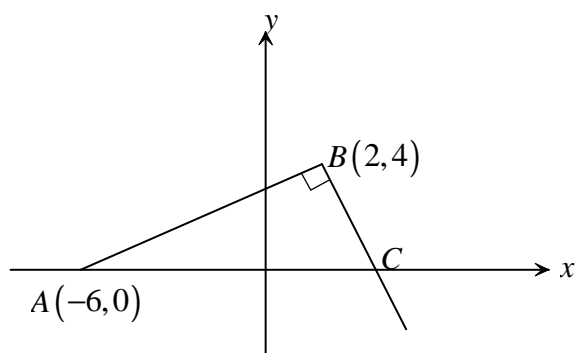
(a) Consider the formula  $\frac{n+t}{3} = \frac{n-x}{y}$ .

(i) Make  $n$  the subject of the formula. 2

(ii) State any restrictions which may apply to the variables. 2

(b) The diagram above illustrates the points  $A(-6,0)$  and  $B(2,4)$ .

The line  $BC$  is perpendicular to the line  $AB$  and the point  $C$  lies on the  $x$ -axis.



(i) Find the gradient of  $AB$ . 1

(ii) Write down the gradient of  $BC$ . 1

(iii) Show that the equation of  $BC$  is  $2x + y - 8 = 0$ . 2

(iv) Find the coordinates of  $C$ . 1

(v) Find the area of triangle  $ABC$ . 2

(vi) Find the equation of the circle with diameter  $AC$ . 2



**Question 4: (15 marks) Start a new page**

**Marks**

- (a) In a trial, 200 patients chosen at random, were given a blood test for liver disease. Some were suffering from the disease and some were not. The results of the test are shown in the two-way table below. A positive test result indicates that a person has liver disease even if they do not.

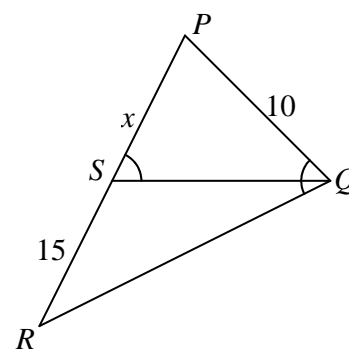
	Test Results		
	Accurate	Not Accurate	Total
Patients with the disease	13	3	16
Patients without the disease	144	40 (A)	184
Total	157	43	200

- (i) Explain the meaning of the value labelled **A**. **1**
- (ii) What percentage of test results are accurate? **1**
- (iii) How many patients had a negative test result? **1**
- (iv) What is the probability that a patient with a positive test result, selected at random, actually has liver disease? **1**
- (b) The following statistics were obtained from Year 10 Science and English tests. **2**

Subject	Mean	Standard deviation
English	60	6
Science	70	8

What mark in Science would be equivalent to a mark of 72 in English?

- (c) In the diagram  $\angle PQR = \angle PSQ$ ,  $PQ = 10$  units,  $RS = 15$  units and  $PS = x$  units.
- (i) Copy the diagram onto your examination pad.
- (ii) Prove that  $\triangle PQR$  is similar to  $\triangle PSQ$ . **3**
- (iii) Explain why  $\frac{x+15}{10} = \frac{10}{x}$ . **1**
- (iv) Hence find the value of  $x$ . **3**
- (v) If the area of  $\triangle PSQ$  is  $k$  square units, find the area of  $\triangle RSQ$  in terms of  $k$ . **2**



**Question 5: (13 marks) Start a new page**

**Marks**

(a) (i) Simplify  $\frac{1}{n} - \frac{1}{n+1}$ . **1**

(ii) Hence, using the above result, evaluate the following sum **2**

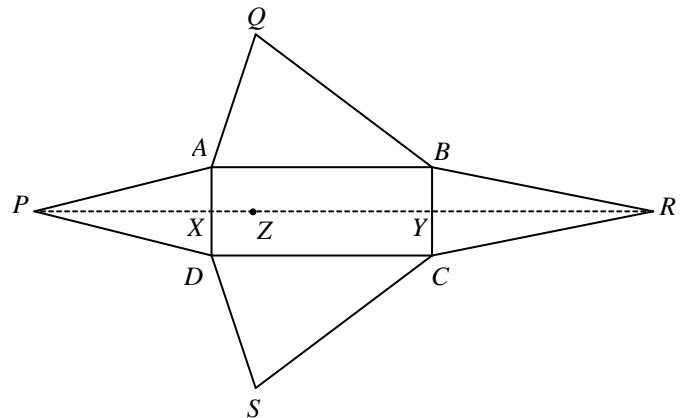
$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots + \frac{1}{100 \times 101}$$

(b) Evelyn has been investing her money for 5 years. Initially her investment was earning interest at 9% per annum compounded monthly. From the start of the global financial crisis 18 months ago, her investment began to lose value at 15% per annum each month.

(i) If Evelyn initially invested  $\$P$ , what was the value of her portfolio immediately before the global financial crisis? Give your answer in unsimplified form in terms of  $P$ . **1**

(ii) Evelyn's investment portfolio is now worth  $\$10\,000$ . What was the amount of her original investment? **2**

(c) The figure shows the net of a pyramid with a rectangular base. In this figure,  $PXZYR$  is a straight line,  $PX = 15$  cm,  $RY = 20$  cm,  $AB = 25$  cm and  $BC = 10$  cm. Further,  $AP = PD$  and  $BR = RC$ .



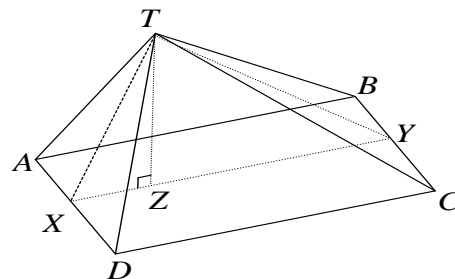
When the net is folded, points  $P, Q, R$  and  $S$  all meet at  $T$ , which lies vertically above the point  $Z$  in the horizontal base, as shown below.

(i) Show that  $\triangle TXY$  is right-angled. **1**

(ii) By considering the value of  $\sin \angle TXY$  or otherwise, show that  $T$  is 12 cm above the base. **2**

(iii) Find the volume of the pyramid. **2**

(iv) Find the angle that the face  $DCT$  makes with the base. **2**



**End of Paper**

Name: \_\_\_\_\_

Class: \_\_\_\_\_

**Part A: Multiple choice answer sheet.**

**Completely colour the circle representing your answer. Use pencil only.**

1.   Ⓐ Ⓑ Ⓒ Ⓓ

2.   Ⓐ Ⓑ Ⓒ Ⓓ

3.   Ⓐ Ⓑ Ⓒ Ⓓ

4.   Ⓐ Ⓑ Ⓒ Ⓓ

5.   Ⓐ Ⓑ Ⓒ Ⓓ

6.   Ⓐ Ⓑ Ⓒ Ⓓ

7.   Ⓐ Ⓑ Ⓒ Ⓓ

8.   Ⓐ Ⓑ Ⓒ Ⓓ

9.   Ⓐ Ⓑ Ⓒ Ⓓ

10.  Ⓐ Ⓑ Ⓒ Ⓓ

11.  Ⓐ Ⓑ Ⓒ Ⓓ

12.  Ⓐ Ⓑ Ⓒ Ⓓ

13.  Ⓐ Ⓑ Ⓒ Ⓓ

14.  Ⓐ Ⓑ Ⓒ Ⓓ

15.  Ⓐ Ⓑ Ⓒ Ⓓ

16.  Ⓐ Ⓑ Ⓒ Ⓓ

17.  Ⓐ Ⓑ Ⓒ Ⓓ

18.  Ⓐ Ⓑ Ⓒ Ⓓ

19.  Ⓐ Ⓑ Ⓒ Ⓓ

20.  Ⓐ Ⓑ Ⓒ Ⓓ

# 2009 Year 10 Yearly Examination

## Mathematics

Name: \_\_\_\_\_ **Solutions** \_\_\_\_\_

### Part A:

1.  A  B  C  D

2.  A  B  C  D

3.  A  B  C  D

4.  A  B  C  D

5.  A  B  C  D

6.  A  B  C  D

7.  A  B  C  D

8.  A  B  C  D

9.  A  B  C  D

10.  A  B  C  D

11.  A  B  C  D

12.  A  B  C  D

13.  A  B  C  D

14.  A  B  C  D

15.  A  B  C  D

16.  A  B  C  D

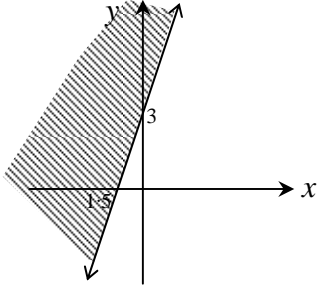
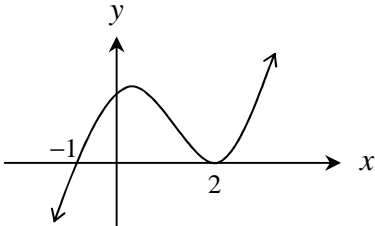
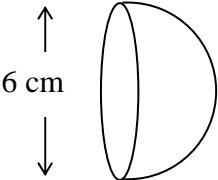
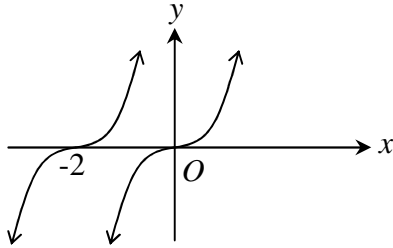
17.  A  B  C  D

18.  A  B  C  D

19.  A  B  C  D

20.  A  B  C  D

**Part B: Write the answer only in the space provided. (1 mark each)**

(a)	Write in expanded form: $(5a + 3)^2$	$25a^2 + 30a + 9$	
(b)	Solve $(x - 2)(2x + 5) = 0$	$x = 2, -2.5$	
(c)	Factorise: (i) $25x^2 - 9$	$(5x - 3)(5x + 3)$	
	(ii) $2x^2 + 7x - 15$	$(2x - 3)(x + 5)$	
(d)	Simplify $\sqrt{12} + \sqrt{27}$	$5\sqrt{3}$	
(e)	Write $\frac{2}{\sqrt{3}}$ with a rational denominator	$\frac{2\sqrt{3}}{3}$	
(f)	Shade the region for which $y \geq 2x + 3$ .		
(g)	Write down in factored form, an equation which can be represented by this graph.	$y = (x + 1)(x - 2)^2$	
			
(h)	Find the volume of this hemisphere, leaving your answer in terms of $\pi$ .		$18\pi \text{ cm}^3$
(i)	The probability of getting the measles as a teenager is 0.018. How many of 700 000 teenagers will <i>not</i> be expected to contract measles?	687 400	
(j)	At a “25% off” sale, goods were sold for \$36. What was the price of the goods before the sale?	\$48	
(k)	The graph of $y = x^3$ is illustrated. On the same axes, draw the graph of $y = (x + 2)^3$ .		
(l)	Solve the equation $3^{2-x} = 9^x$ .	$x = \frac{2}{3}$	

**End of Part B**

**Part C:**

**Question 1:**

(a)  $(\sqrt{x}-1)^2 - 8(\sqrt{x}-1) + 12 = 0$  using  $u = \sqrt{x}-1$

$$u^2 - 8u + 12 = 0$$

$$(u-2)(u-6) = 0$$

$$u = 2, 6$$

$$\therefore \sqrt{x}-1 = 2 \quad \text{or} \quad \sqrt{x}-1 = 6$$

$$\sqrt{x} = 3 \qquad \qquad \sqrt{x} = 7$$

$$\therefore x = 9 \text{ or } 49$$

(b) (i)  $n = 2k + 1$  or  $n = 2k - 1$

(ii)  $n^2 - 1 = (2k + 1)^2 - 1$  if  $n$  is odd

$$= 4k^2 + 4k + 1 - 1$$

$$= 4k^2 + 4k$$

$$= 4k(k + 1)$$

Now either  $k$  or  $k + 1$  will be even

$\therefore$  let  $k(k + 1) = 2m$  for some integer  $m$

$$\therefore n^2 - 1 = 4k(k + 1)$$

$$= 4(2m)$$

$$= 8m$$

Which is a multiple of 8.

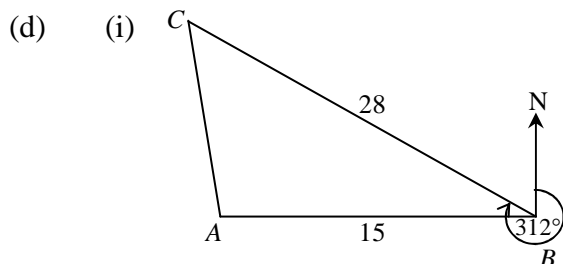
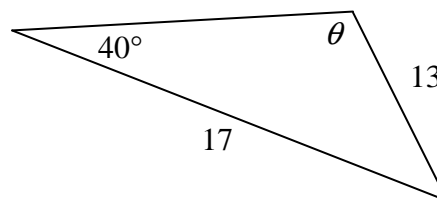
(c)  $\frac{\sin \theta}{17} = \frac{\sin 40^\circ}{13}$  by the sin rule

$$\sin \theta = \frac{17 \sin 40^\circ}{13}$$

$$= 0.8405\dots$$

$$\theta = 57.200\dots^\circ \text{ or } 122.7998\dots^\circ$$

$$= 57^\circ 12' \text{ or } 122^\circ 48' \quad \text{to the nearest minute}$$



(ii)  $\angle ABC = 312^\circ - 270^\circ = 42^\circ$

$$AC^2 = 15^2 + 28^2 - 2 \times 15 \times 28 \times \cos 42^\circ$$

$$= 384.758\dots$$

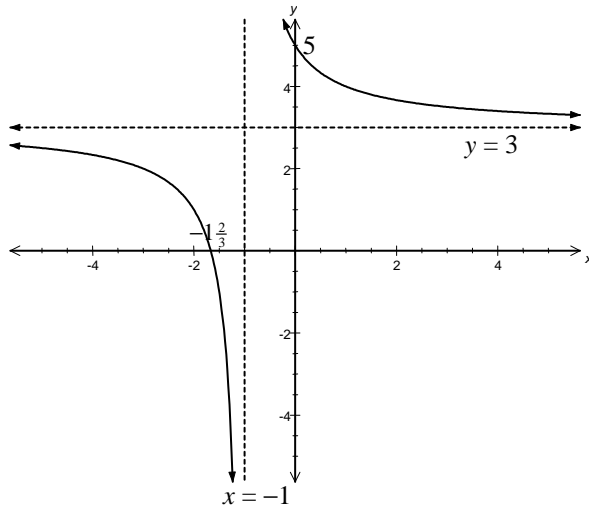
$$AC = \sqrt{384.758\dots}$$

$$= 19.615\dots$$

$\therefore$  she flew 20 km (to 2 sig fig)

**Question 2:**

(a)



$$y = 3 + \frac{2}{x+1}$$

(b)  $y = 2x^2 - 4x - 3$  .....①

$y = 1 - 2x$  .....②

Sub ① into ②:  $2x^2 - 4x - 3 = 1 - 2x$

$$2x^2 - 2x - 4 = 0$$

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

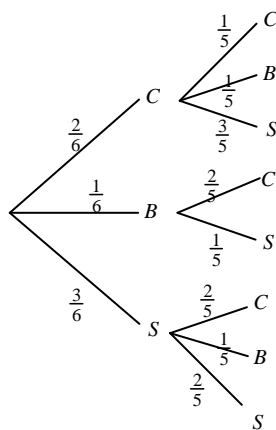
$$x = 2, -1$$

Sub into ②:  $x = 2: y = 1 - 2(2) = -3$

$x = -1: y = 1 - 2(-1) = 3$

$\therefore A = (-1, 3)$  and  $B = (2, -3)$

(c) (i)



(ii) (α)  $P(CS) = \frac{2}{6} \times \frac{3}{5} + \frac{3}{6} \times \frac{2}{5}$   
 $= \frac{2}{5}$

(β)  $P(\text{exactly 1 } S)$   
 $= \frac{2}{6} \times \frac{3}{5} + \frac{1}{6} \times \frac{3}{5} + \frac{3}{6} \times \frac{2}{5} + \frac{3}{6} \times \frac{1}{5}$   
 $= \frac{3}{5}$

(d)  $A$  has a lower mean than  $B$  and  $B$  has a greater standard deviation than  $A$ .

**Question 3:**

(a) (i) 
$$\frac{n+t}{3} = \frac{n-x}{y}$$

$$y(n+t) = 3(n-x)$$

$$yn + yt = 3n - 3x$$

$$yn - 3n = -3x - yt$$

$$n(y-3) = -3x - yt$$

$$n = \frac{-3x - yt}{y-3}$$

$$n = \frac{3x + yt}{3-y}$$

(ii)  $y \neq 0; y \neq 3$

(b) (i) 
$$m_{AB} = \frac{4-0}{2-(-6)}$$

$$= \frac{1}{2}$$

(ii)  $m_{BC} = -2$

(iii)  $y-4 = -2(x-2)$ 

$$y = -2x + 8$$

$$\therefore 2x + y - 8 = 0$$

(iv) At C,  $y = 0$ :  $2x - 8 = 0$ 

$$x = 4$$

$$\therefore C = (4, 0)$$

(v)  $AC = 10$  and is the base;  $B$  is 4 above the  $x$ -axis
$$A = \frac{1}{2}(10)(4)$$

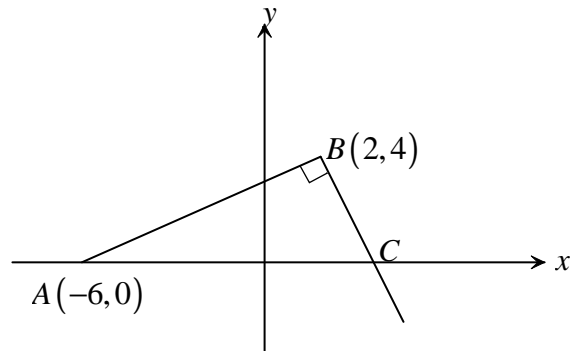
$$= 20$$

$$\therefore \text{Area } \triangle ABC \text{ is } 20 \text{ unit}^2$$

(vi)  $AC = 10 \therefore r = 5$ 

$$M_{AC} = (-1, 0)$$

$$\therefore \text{equation of the circle is } (x+1)^2 + y^2 = 25$$





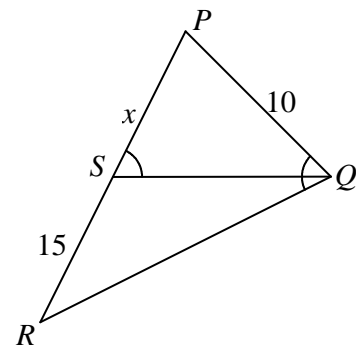
**Question 4: (15 marks) Start a new page**

**Marks**

- (a) (i) **A** is the number of patients for whom the test indicated that they had liver disease when they really did not.
- (ii) % accurate =  $\frac{157}{200} \times 100\%$   
 $= 78.5\%$
- (iii) Patients with a negative test result =  $3 + 144 = 147$
- (iv) Patients with a positive test result =  $13 + 40 = 53$
- $$P(\text{positive test result with the disease}) = \frac{13}{53}$$

- (b) In English,  $72 = 60 + 2 \times 6$   
 That is, the English mark is 2 standard deviations above the mean  
 $\therefore$  The equivalent mark in Science is  $70 + 2 \times 8 = 86$

- (c) (i)
- (ii) In  $\triangle PQR$  and  $\triangle PSQ$
1.  $\angle PQR = \angle PSQ$  (given)
  2.  $\angle P$  is common
- $\therefore \triangle PQR \parallel \triangle PSQ$  (equiangular)



- (iii)  $\frac{PQ}{PS} = \frac{QR}{SQ} = \frac{PR}{PQ}$  (corresponding sides of similar triangles)
- $$\therefore \frac{10}{x} = \frac{x+15}{10}$$
- i.e.  $\frac{x+15}{10} = \frac{10}{x}$  as required

- (iv)  $100 = x^2 + 15x$   
 $x^2 + 15x - 100 = 0$   
 $(x + 20)(x - 5) = 0$   
 $x = -20$  or  $5$   
 But  $x > 0 \therefore x = 5$

- (v) Area of  $\triangle PSQ = k$  square units  
 Ratio of sides of  $\triangle PQR$  : sides of  $\triangle PSQ = 10 : 5$   
 $= 2 : 1$   
 $\therefore$  Ratio of area of  $\triangle PQR$  : area of  $\triangle PSQ = 4 : 1$   
 $\therefore$  area of  $\triangle PQR = 4k$   
 By subtraction, area of  $\triangle PSQ = 3k$  square units

**Question 5:**

(a) (i) 
$$\frac{1}{n} - \frac{1}{n+1} = \frac{n+1-n}{n(n+1)}$$

$$= \frac{1}{n(n+1)}$$

(ii) 
$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots + \frac{1}{100 \times 101}$$

$$= \left( \frac{1}{1} - \frac{1}{2} \right) + \left( \frac{1}{2} - \frac{1}{3} \right) + \left( \frac{1}{3} - \frac{1}{4} \right) + \dots + \left( \frac{1}{99} - \frac{1}{100} \right) + \left( \frac{1}{100} - \frac{1}{101} \right)$$

$$= 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{99} - \frac{1}{100} + \frac{1}{100} - \frac{1}{101}$$

$$= 1 - \frac{1}{101}$$

$$= \frac{100}{101}$$

(b) (i) Months investing before the crisis =  $60 - 18 = 42$   
 9% pa = 0.75% per month  
 Value before crisis =  $P(1 + 0.75\%)^{42}$   
 $= P(1.0075)^{42}$

(ii) 15% pa = 1.25% per month  
 Value after crisis =  $P(1.0075)^{42} (1 - 1.25\%)^{18}$   
 $\therefore P(1.0075)^{42} (0.9875)^{18} = 10000$   

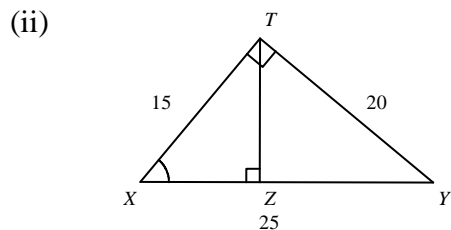
$$P = \frac{10000}{(1.0075)^{42} (0.9875)^{18}}$$

$$= 9163.045\dots$$

Evelyn's original investment was approximately \$9 163.

(c) (i)  $PX = 15, XY = AB = 25, YR = 20$   
 $15^2 + 20^2 = 625$   
 $= 25^2$

$\therefore \Delta TXY$  is right-angled by the converse of Pythagoras theorem



$$\sin \angle TXY = \frac{20}{25} = \frac{4}{5} \text{ in } \Delta XYT$$

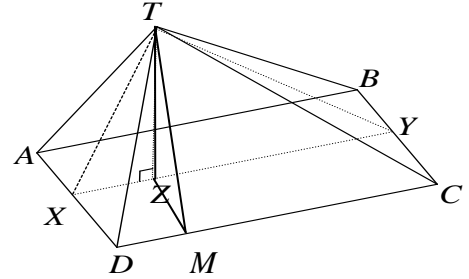
$$\begin{aligned} \text{In } \triangle XZT: \sin \angle TXZ &= \frac{TZ}{15} \\ TZ &= 15 \sin \angle TXZ \\ &= 15 \times \frac{4}{5} \\ &= 12 \end{aligned}$$

$\therefore T$  is 12 cm above the base.

(iii)  $V = \frac{1}{3} Ah$

$$\begin{aligned} V &= \frac{1}{3} \times 12 \times 10 \times 25 \\ &= 1000 \end{aligned}$$

$\therefore$  the volume of the pyramid is  $1000 \text{ cm}^3$



(iv) The angle that the face  $DCT$  makes with the base is  $\angle TMZ$   
 $\triangle TMZ$  is right-angled at  $Z$  and  $ZM = XD = 5$  also  $TZ = 12$

$$\therefore \tan \angle TMZ = \frac{12}{5}$$

$\therefore \angle TMZ = 67^\circ 23'$  correct to the nearest minute

**End of Solutions**