NORTH SYDNEY GIRLS HIGH SCHOOL

2010 Year 10 Yearly Examination

Mathematics

Name:_____ Class: _____

Teacher:_____

Time Allowed: 2 hours + 5 minutes reading time

Directions to Candidates:

- Approved calculators may be used
- Answer Part A, the multiple choice questions, on the answer sheet provided.
- Answer Part B in the spaces provided
- For Part C, ***each*** question is to be started on a ***new page.***
- Attempt every question.
- Show all necessary working. Do not use correction tape or fluid.
- Marks may be deducted for incomplete or poorly arranged work.

At the end of the examination, staple this question paper to the front of your solutions and submit one bundle.

	A	N	M	D	G	WM	Total
Part A							/20
Part B	/7	/4		/11			/22
Part C Q1	/7		/9				/16
Part C Q2	/8			/9			/17
Part C Q3	/10				/3	/4	/17
Part C Q4	/8				/5	/5	/18
Part C Q5						/14	/14
TOTAL	/40	/4	/9	/20	/8	/23	/124

Part A: Multiple Choice (1 mark each)

Answer on the sheet provided by completely colouring the circle representing your answer. Use pencil only.

1. If n is any integer then a number which must be divisible by 3 would be:

   (A) $n + 3$ (B) $n - 3$ (C) $n + n$ (D) $6n$

2. $\tan 60^o =$

   (A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{3}}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\sqrt{3}$

3. If $x = 2at$, then $x^2 - tx =$

   (A) 0 (B) $2at^2$ (C) $2a^2t^2 - 2at^2$ (D) $4a^2t^2 - 2at^2$

4. Consider this solution of the equation $2g^2 + 8g + 1 = 0$. In which line does the first error occur?

   (A) $g = -8 \pm \frac{\sqrt{8^2 - 4 \times 2 \times 1}}{2 \times 2}$

   (B) $g = -8 \pm \frac{\sqrt{56}}{4}$

   (C) $g = -8 \pm \frac{4\sqrt{14}}{4}$

   (D) $g = -8 \pm \sqrt{14}$

5. $16m^{-8} \div 4m^2 =$

   (A) $4m^{-10}$ (B) $4m^{-6}$ (C) $4m^{-4}$ (D) $4m^6$

6. A line perpendicular to $y = 4x - 6$ is:

   (A) $y = -\frac{1}{4}x$ (B) $y = 4x + 8$ (C) $y = \frac{1}{4}x + 2$ (D) $y = -\frac{1}{4}$

7. Which statement is true:

   (A) $\sin 40° = 2\sin 20°$

   (B) $\tan 60° = \frac{\cos 60^\circ}{\sin 60^\circ}$

   (C) $\sin 40° = \cos 50°$

   (D) $\frac{\sin 60°}{2} = \sin 30°$

8. This set of data is arranged in order from smallest to largest.

5, 6, 11, x, 13, 18, 25

The range is six less than twice the value of x.

Which one of the following is true?

(A) The median is 12 and the interquartile range is 7.

(B) The median is 12 and the interquartile range is 12.

(C) The median is 13 and the interquartile range is 7.

(D) The median is 13 and the interquartile range is 12.

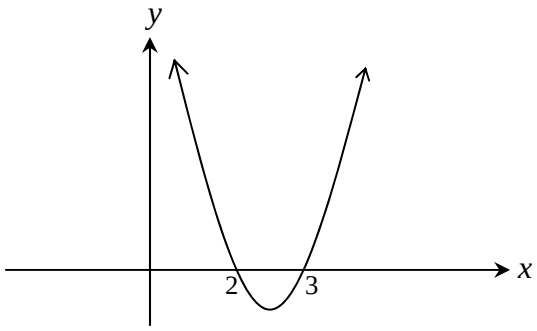
9. If $K = Ft^3$ and $F = 5$ and $t = 0.715$, find the value of K correct to three significant figures?

(A) 1.82 (B) 1.827 (C) 1.828 (D) 1.83

10. The parabola $y = -2x^2 + 8x - 1$ has a maximum value of:

(A) 7 (B) 2 (C) −1 (D) −25

11. The equation of the parabola illustrated could be:



(A) $y = (x + 2)(x + 3)$

(B) $y = (x - 2)(x - 3)$

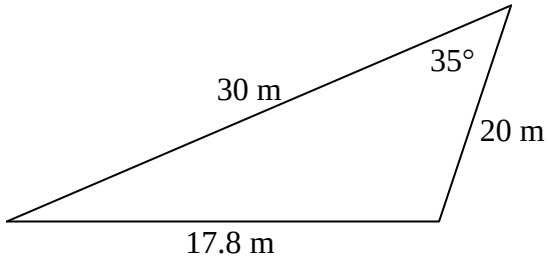
(C) $y = (x + 2)(x - 3)$

(D) $y = x^2 + 5x - 6$

12. If $A = 6x + 10$, and x is increased by 2, what will be the corresponding increase in A?

(A) 12 (B) 2 (C) $6x + 12$ (D) $6x + 22$

13. What is the area of the triangle to the nearest square metre?



NOT TO SCALE

(A) 102 m^2 (B) 153 m^2 (C) 172 m^2 (D) 178 m^2

14. A rectangular prism has length $4k$, width $2k$ and height k. It's surface area is:

(A) $14k^2$ (B) $28 k^3$ (C) $8k^3$ (D) $28k^2$

15. Dora works for $9.60 per hour for eight hours each day on Thursday and Friday.
On Saturday she works for six hours at time-and-a-half.
How much does Dora earn in total for Thursday, Friday and Saturday?

(A) $192.00 (B) $211.20 (C) $240.00 (D) $316.80

16. Using the tax table below, determine the tax payable on a taxable income of $47 000.

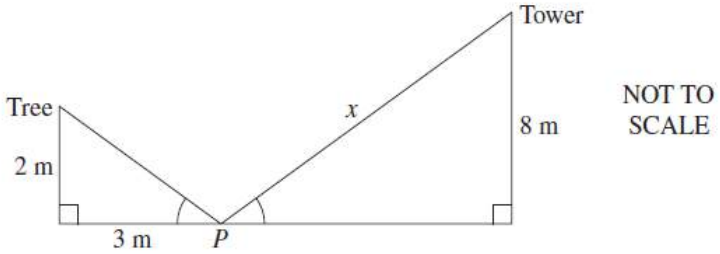
Taxable Income	*Tax on this income*
$0 – $6 000	NIL
$6 001 – $22 000	16 cents for each $1 over $6 000
$22 001 – $45 000	$2 560 plus 25 cents for each $1 over $22 000
$45 001 – $60 000	$8 310 plus 40 cents for each $1 over $45 000
$60 001 and over	$14 310 plus 48 cents for each $1 over $60 000

(A) $8 310.40 (B) $9 109.60 (C) $9 110.00 (D) $10 310.40

17. Kerry has a credit card. She is charged 0.05% compound interest per day on outstanding balances. How much interest is Kerry charged on an amount of $250, which is outstanding on her credit card for 30 days?

(A) $3.75 (B) $3.78 (C) $253.75 (D) $253.78

18. A point P lies between a tree, 2 metres high, and a tower 8 metres high. P is 3 metres away from the base of the tree.
From P, the angles of elevation to the top of the tree and to the top of the tower are equal.



What is the distance, x, from P to the top of the tower?

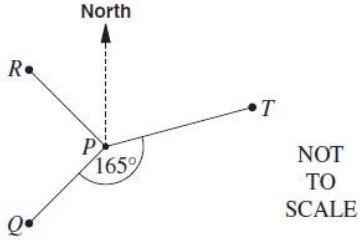
(A) 9 m (B) 9.61 m (C) 12.04 m (D) 14.42 m

19. A sphere and a closed cylinder have the same radius. The height of the cylinder is four times the radius. What is the ratio of the volume of the cylinder to the volume of the sphere?

(A) 2:1 (B) 3:1 (C) 4:1 (D) 8:1

20. The diagram shows the position of Q, R and T relative to P.

In the diagram, Q is SW of P.
R is NW of P.
$\angle QPT = 165°$



What is the bearing of T from P?

(A) 060° (B) 075° (C) 105° (D) 120°

End of Part A

Part B: Write the answer only in the space provided. (1 mark each unless otherwise indicated)

(a) Write in expanded form: $\left(t+\frac{1}{t}\right)^2$

(b) Solve $(x-2)(2x+5)=0$.

(c) Factorise: (i) $x^2-9x-90$

(ii) $25y-y^3$

(d) Simplify $\frac{a}{ax+ay}$.

(e) Solve the inequality: $11-3x\le 20$.

(f) If $w=3x+2$, find w if $x=2t+1$.

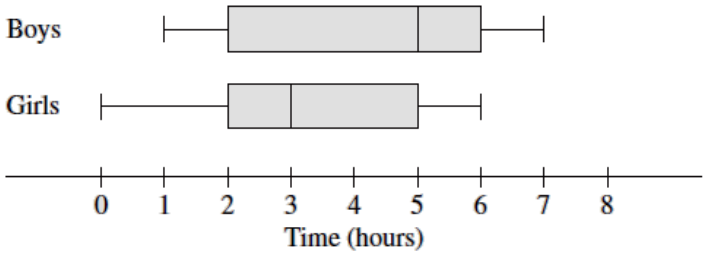
(g) Write $\frac{7}{\sqrt{7}}$ with a rational denominator.

(h) Simplify $\frac{\sqrt{8}}{\sqrt{50}}$.

(i) Chris buys a new telescope for \$4200. It depreciates in value by 10% in the 1st year then another 20% in the 2nd year.
What is the telescope's value after 2 years?

(j) The equal sides of an isosceles triangle are each 2 cm longer than the third side. The third side has length $(x+2)$ cm.
What is the perimeter of the triangle?

(k) In a school, boys and girls were surveyed about the time they usually spend on the internet over a weekend.
These results were displayed in box-and-whisker plots, as shown below.



(1) Find the median for boys.

(2) What percentage of girls usually spend 5 or less hours on the internet over a weekend?

(3) Jenny said that the graph shows that the same number of boys as girls usually spend between 5 and 6 hours on the internet over a weekend. Under what circumstances would this statement be true?

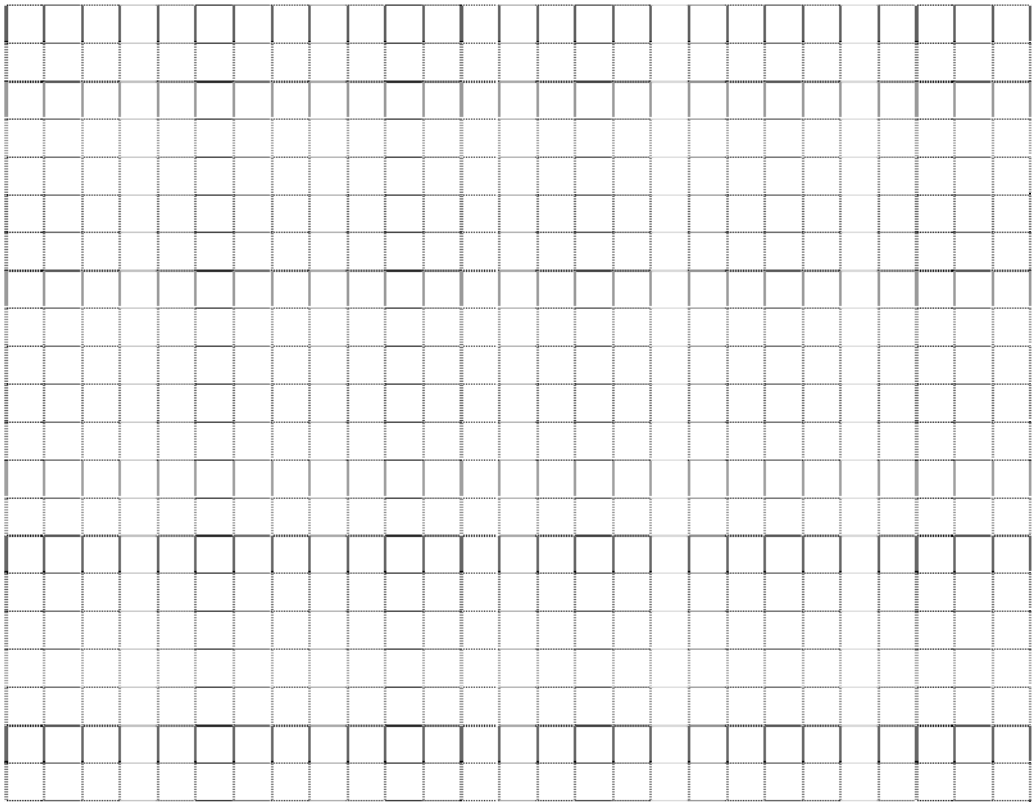
(l) A bag contains 3 red lollies and 2 green lollies. Jane randomly chooses one, eats it, then selects and eats another one.
Find the probability of choosing two green lollies.

(m) The marks of 42 mathematics students are shown below in the table.

Class	Class Centre	Frequency	Cumulative Frequency
21 - 25		1	
26 – 30		2	
31 – 35		4	
36 – 40		7	
41 – 45		15	
46 - 50		10	
51 – 55		2	
56 – 60		1	

(1) Complete the table. **2**

(2) Draw a cumulative frequency histogram and polygon.



3

(3) Find the interquartile range. Show working.

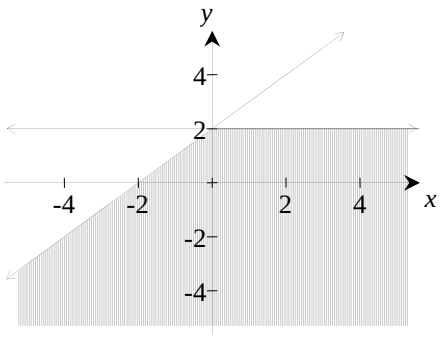
2

End of Part B

Part C: **Use the examination pad provided.**
Start each question on a new page.
Show all working.

Question 1: (16 marks) **Marks**

(a) The shaded region is bound by the lines $y = x + 2$ and $y = 2$. Write down the two inequalities that fully describe the shaded region. **2**



(b) Given that k is a positive number specify the largest and the smallest of the following numbers: **2**

$2^{-\frac{1}{2}k}$, $2^{\frac{1}{2}k}$, 2^{k}, 2^{-k}

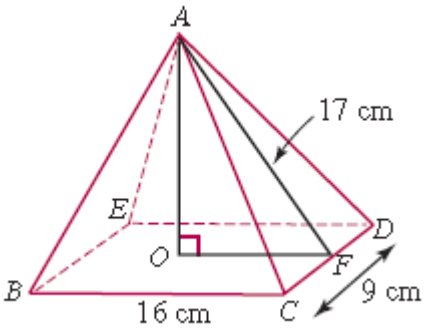
(c) Line k has equation: $9x - 2y + 20 = 0$ and line m has equation: $3x + y - 10 = 0$. Show that k and m intersect at a point P on the y-axis. **3**

(d) In the rectangular pyramid shown, BC = 16 cm, CD = 9 cm and AF = 17 cm.

(i) Write down the length of OF. **1**

(ii) Find AO, the height of the pyramid. **2**

(iii) Hence find the volume of the pyramid. **2**



(e) Consider the quadrilateral $ABCD$.

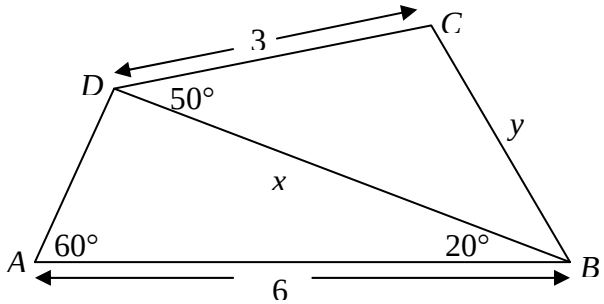


Figure not to scale
Lengths are in metres

Find correct to 2 significant figures:

(i) x **2**

(ii) y **2**

Question 2: (17 marks) Start a new page **Marks**

(a) Solve the equation $(x-4)^2 = 6(x-4)+27$ using the substitution $m = x-4$. **3**

(b) Solve the equation $9\left(3^{2x}\right)-10\left(3^{x}\right)+1=0$. **3**

(c) Find all values of x with $0° \le x \le 360°$ for which $1+\sqrt{3}\tan x = 0$. **2**

(d) The mean of a set of ten scores is 14. Another two scores are included and the new mean is 16. What is the mean of the two additional scores? **2**

(e) The mean of the heights of a large number of people is 155 cm and the standard deviation is 11.2 cm. A person is added to this group with a height of 170 cm. Explain what effect this will have on the standard deviation. **1**

(f) There are two prizes in a raffle in which 53 tickets are sold.
The first prize is obtained by drawing a ticket at random, and this ticket is not replaced for the draw for second prize. A man buys two tickets in the raffle.

(i) Draw a tree diagram to illustrate the situation. **1**

Find the probability that the man:

(ii) wins the first prize; **1**

(iii) wins both prizes; **2**

(iv) wins at least one prize? **2**

Question 3: (17 marks) Start a new page **Marks**

(a) Determine whether the point $(2,3)$ lies inside, outside or on the circle. **2**
$(x-1)^2+(y+1)^2=8$. Show working.

(b) On separate diagrams, draw neat sketches of each of the following, showing intercepts, asymptotes and any other important features.

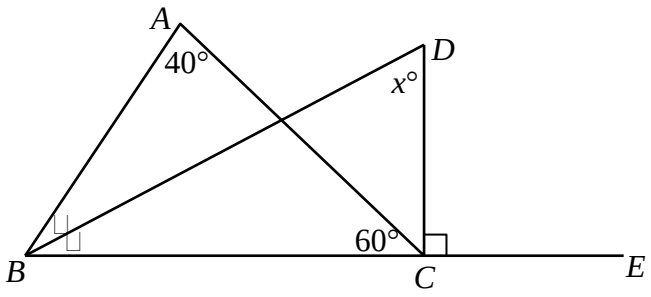
(i) $xy=-2$ **2**

(ii) $y=(x+2)^2+5$ **2**

(iii) $x^2+2x+y^2=3$ **2**

(c) If $3p-2=10$, find the value of p^2. **2**

(d) In the figure, *BD* bisects $\angle ABC$. *DC* is perpendicular to *BC*.
$\angle BAC=40°$, $\angle ACB=60°$ and $\angle BDC=x°$.



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(i) Copy the diagram on your answer sheet.

(ii) Find the value of *x*, giving reasons. **3**

(e) Prove that the sum of 3 odd numbers is odd. **2**

(f) For what values of *x* is the expression $\sqrt{x+3}+\sqrt{2-x}$ possible to evaluate? **2**

Question 4: (18 marks) Start a new page **Marks**

(a) Consider the formula: $t = \dfrac{6-r}{rx-x}$

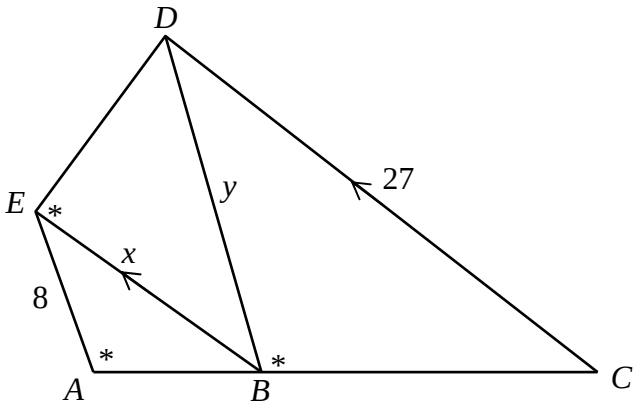
(i) Make x the subject of the formula. **2**

(ii) State any restrictions which may apply to the variables. **3**

(b) (i) Factorise $x^2 + 4x + 4$. **1**

(ii) Hence factorise $x^4 - x^2 - 4x - 4$. **2**

(c) In the diagram, BE is parallel to CD.
$CD = 27$, $AE = 8$, $BE = x$ and $BD = y$. Also $\angle BAE = \angle CBD = \angle BED$.



(i) Copy the diagram at the beginning of a new page.

(ii) Prove that $\Delta ABE \,|||\, \Delta BCD$. **2**

(iii) Explain why $\angle AEB = \angle BDC$. **1**

(iv) Prove that $\Delta ABE \,|||\, \Delta EDB$. **2**

(v) Hence prove that $x^2 = 8y$ **2**

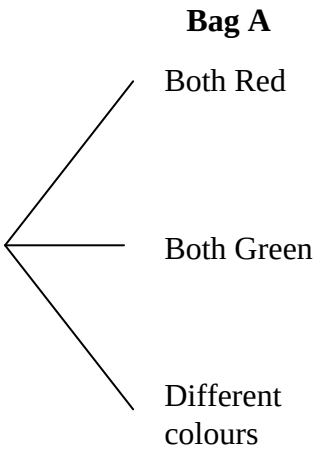
(vi) Using parts (ii), and (v), find the value of x and y. **3**

Question 5: (14 marks) Start a new page

(a) To estimate the number of fish in a pond in a fish farm, 75 similar fish were tagged and released into the pond. Later a sample of 42 fish were netted from the pond. It was noted that 5 of these were tagged. Estimate the total number of fish in the dam. **2**

(b) Bag A contains 4 green marbles and 4 red marbles. Bag B contains 3 green marbles and 1 red marble. Two marbles are drawn at random from Bag A and placed in Bag B. Two marbles are then drawn from Bag B.

(i) Copy and complete the probability tree diagram started below to illustrate the situation. **2**



(ii) Calculate the probability that the two marbles drawn from Bag B are of different colours. **3**

(c) A picture on a wall measures 22cm by 18cm. It is surrounded by a frame of uniform width.

(i) If the width of the frame is x cm, show that the area of the frame is given by **2**

$$A = 4x^2 + 80x \text{ cm}^2$$

(ii) The area of the frame is 384 cm^2. What is the width of the frame? **2**

(d) If $a = 3\left(x - x^{-1}\right)$ and $b = 6\left(x + x^{-1}\right)$ find an equation connecting a and b which is independent of x. (i.e. without any terms in x) **3**

End of Paper

Name:_____ Class: _____

<u>Part A:</u> Multiple choice answer sheet.

Completely colour the circle representing your answer. Use pencil only.

1. Ⓐ Ⓑ Ⓒ Ⓓ
2. Ⓐ Ⓑ Ⓒ Ⓓ
3. Ⓐ Ⓑ Ⓒ Ⓓ
4. Ⓐ Ⓑ Ⓒ Ⓓ
5. Ⓐ Ⓑ Ⓒ Ⓓ
6. Ⓐ Ⓑ Ⓒ Ⓓ
7. Ⓐ Ⓑ Ⓒ Ⓓ
8. Ⓐ Ⓑ Ⓒ Ⓓ
9. Ⓐ Ⓑ Ⓒ Ⓓ
10 Ⓐ Ⓑ Ⓒ Ⓓ
11. Ⓐ Ⓑ Ⓒ Ⓓ
12. Ⓐ Ⓑ Ⓒ Ⓓ
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14. Ⓐ Ⓑ Ⓒ Ⓓ
15. Ⓐ Ⓑ Ⓒ Ⓓ
16. Ⓐ Ⓑ Ⓒ Ⓓ
17. Ⓐ Ⓑ Ⓒ Ⓓ
18. Ⓐ Ⓑ Ⓒ Ⓓ
19. Ⓐ Ⓑ Ⓒ Ⓓ
20. Ⓐ Ⓑ Ⓒ Ⓓ

2010 YR 10 Yearly Solutions

PART A

1 D	5 A	9 D	13 C	17 B
2 D	6 A	10 A	14 D	18 D
3 D	7 C	11 B	15 C	19 B
4 A	8 D	12 A	16 C	20 A

Part B: Write the answer only in the space provided. (1 mark each unless otherwise indicated)

(a) Write in expanded form: $\left(t+\frac{1}{t}\right)^2$

$t^2+\frac{1}{t^2}+2$

(b) Solve $(x-2)(2x+5)=0$.

$x=2$ and $x=-\frac{5}{2}$

(c) Factorise: (i) $x^2-9x-90$

$(x-15)(x+6)$

(ii) $25y-y^3$

$y(5+y)(5-y)$

(d) Simplify $\frac{a}{ax+ay}$.

$\frac{1}{x+y}$

(e) Solve the inequality: $11-3x\leq 20$.

$x\geq -3$

(f) If $w=3x+2$, find w if $x=2t+1$.

$w=6t+5$

(g) Write $\frac{7}{\sqrt{7}}$ with a rational denominator.

$\sqrt{7}$

(h) Simplify $\frac{\sqrt{8}}{\sqrt{50}}$.

$\frac{2}{5}$

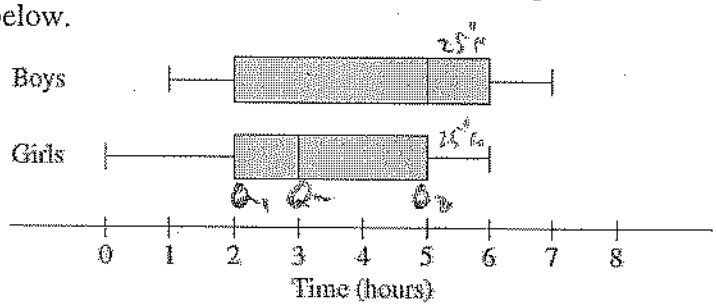
(i) Chris buys a new telescope for \$4200. It depreciates in value by 10% in the 1st year then another 20% in the 2nd year.
What is the teles cope's value after 2 years?

\$3024

(j) The equal sides of an isosceles triangle are each 2 cm longer than the third side. The third side has length $(x+2)$ cm.
What is the perimeter of the triangle?

$P=3x+10$

(k) In a school, boys and girls were surveyed about the time they usually spend on the internet over a weekend.
These results were displayed in box-and-whisker plots, as shown below.



(1) Find the median for boys.

5

(2) What percentage of girls usually spend 5 or less hours on the internet over a weekend?

75%

(3) Jenny said that the graph shows that the same number of boys as girls usually spend between 5 and 6 hours on the internet over a weekend. Under what circumstances would this statement be true?

only true if the total no. of boys = total no. of girls

(l) A bag contains 3 red lollies and 2 green lollies. Jane randomly chooses one, eats it, then selects and eats another one.
Find the probability of choosing two green lollies.

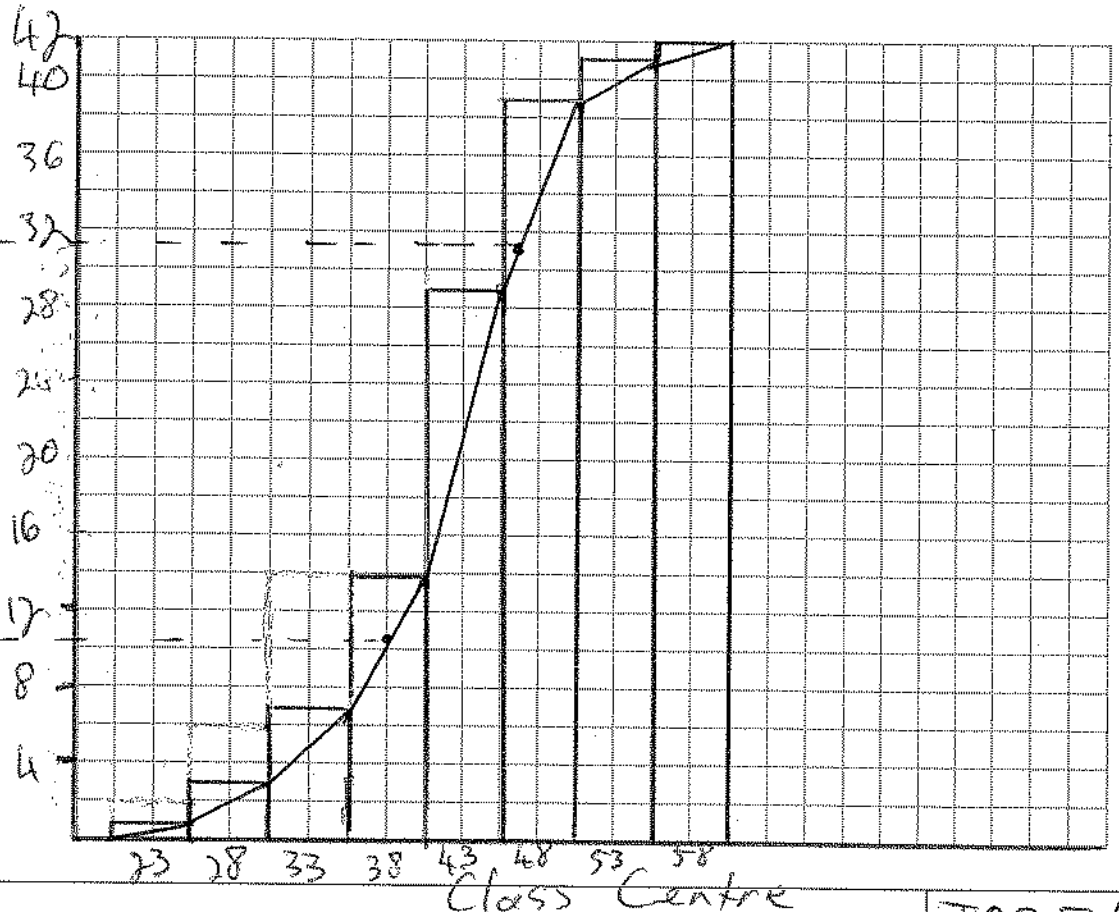
$P(GG)=\frac{2}{5}\times\frac{1}{4}=\frac{1}{10}$

(m) The marks of 42 mathematics students are shown below in the table.

Class	Class Centre	Frequency	Cumulative Frequency
21 - 25	23	1	1
26 – 30	28	2	3
31 – 35	33	4	7
36 – 40	38	7	14
41 – 45	43	15	29
46 - 50	48	10	39
51 – 55	53	2	41
56 – 60	58	1	42

(1) Complete the table. **2**

(2) Draw a cumulative frequency histogram and polygon.



3

(3) Find the interquartile range. Show working.

IQR = $Q_3 - Q_1$
= 46 − 38
= 8

2

End of Part B

Part C

Q1

a) $y \leq x+2$ and $y \leq 2$

b) $\frac{1}{2^{\frac{1}{2}k}}, 2^{\frac{1}{2}k}, 2^k, \frac{1}{2^k}$

In order $\frac{1}{2^k}, \frac{1}{2^{\frac{1}{2}k}}, 2^{\frac{1}{2}k}, 2^k$

Largest $= 2^k$, smallest $= 2^{-k}$

c) $k: 9x - 2y + 20 = 0$

$m: 3x + y - 10 = 0$

$2m + k \quad 15x = 0$

$\therefore x = 0$ (on y-axis)

ii intersect at a point P on the y-axis

d) i $OF = 8\text{cm}$

ii Pyth. T. $(AO)^2 + 8^2 = 17^2$

$AO = 15\text{cm}$ (Height)

iii $V = \frac{1}{3}AH$

$= \frac{1}{3} \times (16 \times 9) \times 15$

Volume $= 720\text{ cm}^3$

e)

i $\frac{6}{\sin 100^\circ} = \frac{x}{\sin 60^\circ}$ (Sin rule)

$x = \frac{6 \sin 60^\circ}{\sin 100^\circ}$

$x = 5.3\text{ m}$ (2 s.f)

ii $y^2 = 3^2 + (5.27\ldots)^2 - 2 \times 3 \times 5.27\ldots \cos 50^\circ$

$y^2 = 36.839\ldots - 20.349\ldots$

$y = 4.1\text{ m}$ (2 s.f)

Q2

a) $(x-4)^2 = 6(x-4) + 27$

$m^2 = 6m + 27$

$m^2 - 6m - 27 = 0$

$(m-9)(m+3) = 0$

$m = 9$ and $m = -3$

ie $x-4 = 9$ and $x-4 = -3$

$x = 13$ $\quad$ $x = 1$

b) $9(3^{2x}) - 10(3^x) + 1 = 0$

$9(3^x 3^x) - 10(3^x) + 1 = 0$

Let $m = 3^x$

$9m^2 - 10m + 1 = 0$

$(9m-1)(m-1) = 0$

$m = \frac{1}{9}$ and $m = 1$

$\frac{1}{3^2} = 3^x$ and $3^x = 3^0$

$x = -2$ and $x = 0$

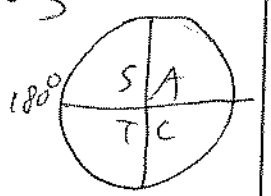
c) $1 + \sqrt{3}\tan x = 0 \quad 0° \leq x \leq 360°$

ie $\tan x = \frac{-1}{\sqrt{3}}$

But $\tan 30° = \frac{1}{\sqrt{3}}$

So

$x = 150°, 330°$



d) If x = sum of the scores

then $\frac{x}{10} = 14$ ie $x = 140$

So $\frac{140 + (Y+Z)}{12} = 16$ $\quad$ Y & Z are extra scores

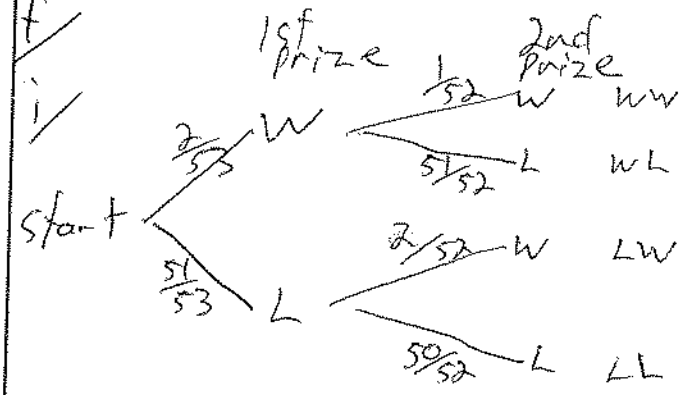
$140 + (Y+Z) = 192$

$Y + Z = 52$

e) It will increase the standard deviation since 11.2 is outside the range of 1 σ which includes most heigh

f)

i)



ii) $P(\text{1st prize}) = \frac{2}{53}$

iii) $P(\text{Both prizes}) = \frac{2}{53} \times \frac{1}{52} = \frac{1}{137}$

iv) $P(\text{at least one prize}) = 1 - P(\text{no prizes})$

$= 1 - \left(\frac{51}{53} \times \frac{50}{52}\right)$

$= \frac{103}{1378}$

Q3

a/ $(x-1)^2+(y+1)^2=8$

Has centre $(1,-1)$

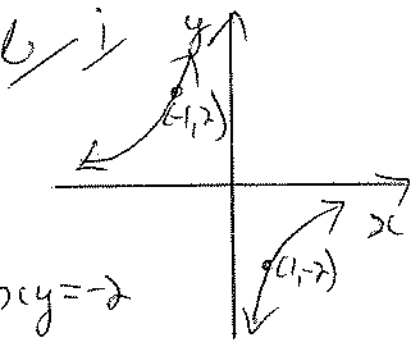
Radius $=\sqrt{8}=2\sqrt{2}\sim 2.8$

Distance from $(1,-1)$ to $(2,3)$

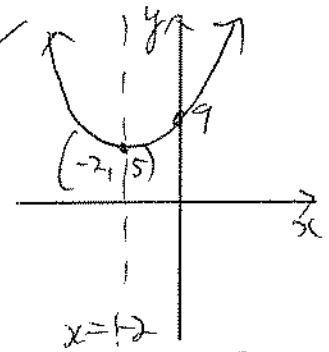
$=\sqrt{(2-1)^2+(3+1)^2}=\sqrt{17}\sim 4.1$

$\therefore$ $(2,3)$ lies outside the circle

b/ i



ii/



$y=(x+2)^2+5$

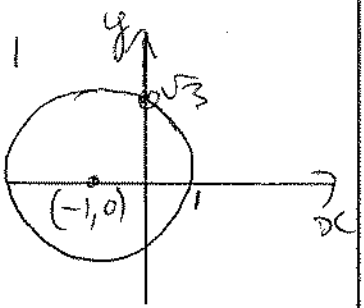
iii/ $x^2+2x+y^2=3$

$(x+1)^2+y^2=3+1$

$(x+1)^2+y^2=4$

Centre $(-1,0)$

radius $=2$



c/ $3p-2=10$

$3p=12$

$p=4$

$p^2=16$

d/ Let p, q and r be any integers.

Then $(2p+1)$, $(2q+1)$ and $(2r+1)$ are odd numbers.

Their sum is:

$2p+2q+2r+3$

$=2(p+q+r+1)+1$

This is not divisible by 2,

$\therefore$ It is odd.

e/ $x+3\geq 0$ | $2-x\geq 0$

$x\geq -3$ | $-x\geq -2$

$x\leq 2$

So

$-3\leq x\leq 2$

f/ Let $y=\angle DBC$

$\triangle BAC$ $2y+40+60^\circ=180^\circ$ (Angle sum of $\triangle$)

$y=40^\circ$

$\triangle BDC$

$40^\circ+x+90^\circ=180^\circ$ (Angle sum of $\triangle$)

$x=50^\circ$

Q4

(a) (i) $t = \dfrac{6-r}{rx-x}$

$= \dfrac{6-r}{x(r-1)}$

$\therefore xt = \dfrac{6-r}{(r-1)}$

$\therefore x = \dfrac{6-r}{t(r-1)}$

(ii) $x \neq 0, t \neq 0, r \neq 1, r \neq 6$

(b) (i) $x^2+4x+4 = (x+2)^2$

(ii) $x^4 - x^2 - 4x - 4 = x^4 - (x^2+4x+4)$

$= x^4 - (x+2)^2$

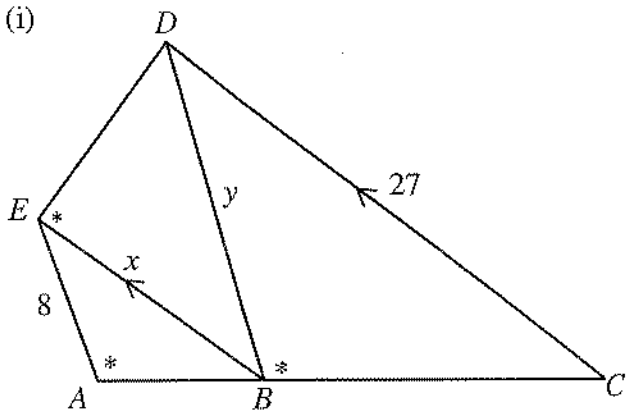
$= (x^2)^2 - (x+2)^2$

$= [x^2 - (x+2)][x^2 + (x+2)]$

$= (x^2 - x - 2)(x^2 + x + 2)$

$= (x-2)(x+1)(x^2+x+2)$

(c) (i)



(ii) In ΔABE and ΔBCD

$\angle EAB = \angle DBC$ (given)

$\angle EBA = \angle DCB$ (corresp. $\angle$s $EB \parallel DC$)

$\therefore \Delta ABE \,|||\, \Delta BCD$ (equiangular)

(iii) $\angle AEB = \angle BDC$ (corresp. $\angle$s of sim Δs)

(iv) In ΔABE and ΔEDB

$\angle EAB = \angle DEB$ (given)

$\angle AEB = \angle BDC$ (proven above)

$\angle BDC = \angle EBD$ (alt. $\angle$s, $EB \parallel CD$)

$\therefore \angle AEB = \angle EBD$

$\therefore \Delta ABE \,|||\, \Delta EDB$ (equiangular)

(v) $\dfrac{AB}{ED} = \dfrac{BE}{DB} = \dfrac{AE}{EB}$ (ratio of sides of sim Δs)

$\therefore \dfrac{x}{y} = \dfrac{8}{x}$

$\therefore x^2 = 8y \quad -(1)$

(vi) $\dfrac{AB}{BC} = \dfrac{BE}{CD} = \dfrac{AE}{BD}$ (From (ii), ratio of sides of sim Δs)

$\therefore \dfrac{x}{27} = \dfrac{8}{y}$

$\therefore xy = 216 \quad -(2)$

From (1) $y = \frac{1}{8}x^2$

Sub into (2)

$\therefore x\left(\dfrac{x^2}{8}\right) = 216$

$\therefore x^3 = 1728$

$\therefore x = 12$

Sub into (2), $y = \frac{216}{12} = 18$

$\therefore x = 12, y = 18$

Q5

a/ Sample : tagged

42 : 5

8.4 : 1

$\therefore 8.4 \times 75 = 630$

b/ $a = 3\left(x - \frac{1}{5x}\right)$, $b = 6\left(x + \frac{1}{5x}\right)$

$\left(\frac{a}{3}\right)^2 = \left(x - \frac{1}{5x}\right)^2$ and $\left(\frac{b}{6}\right)^2 = \left(x + \frac{1}{5x}\right)^2$

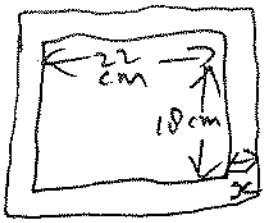
$\frac{a^2}{9} = x^2 - 2 + \frac{1}{25x^2}$ and $\frac{b^2}{36} = x^2 + 2 + \frac{1}{25x^2}$

or $x^2 + \frac{1}{25x^2} = \frac{a^2}{9} + 2$ and $x^2 + \frac{1}{25x^2} = \frac{b^2}{36} - 2$

$\therefore \frac{a^2}{9} + 2 = \frac{b^2}{36} - 2$

or $a^2 = \frac{b^2}{4} - 36$

c/ (i)



Total Area = Area of frame + Area of picture

$(22 + 2x)(18 + 2x) = A_f + 22 \times 18$

$396 + 44x + 36x + 4x^2 = A_f + 396$

$A = 4x^2 + 80x \text{ cm}^2$

(ii) $384 = 4x^2 + 80x$

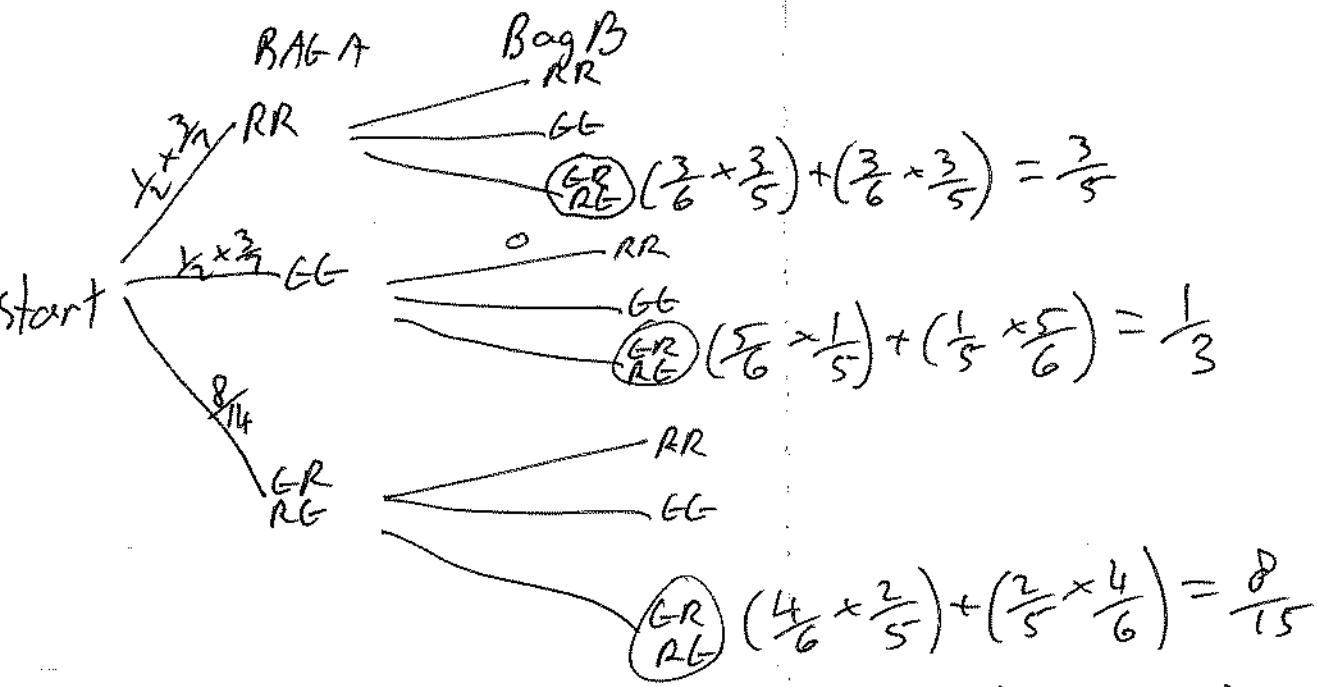
$0 = x^2 + 20x - 96$

$0 = (x - 4)(x + 24)$

$x = 4 \text{ cm}$ ($x = -24$ not realistic)

Width of frame = 4 cm

b/ A (4G 4R) B (3G 1R)



$\therefore P(\text{Two marbles drawn from B of different colour}) = \left(\frac{3}{14} \times \frac{3}{5}\right) + \left(\frac{3}{14} \times \frac{1}{3}\right) + \left(\frac{0}{14} \times \frac{8}{15}\right)$

$= \frac{53}{105} \sim 0.505$