**SYDNEY BOYS HIGH SCHOOL**
**MOORE PARK, SURRY HILLS**

# 2008

**Year 10**

**Yearly Examination**

# Mathematics

**General Instructions**

- Working time – 90 minutes
- Write using black or blue pen. Pencil maybe used for diagrams.
- Board approved calculators may be used.
- All *necessary* working should be shown in every question if full marks are to be awarded.
- Marks may **NOT** be awarded for messy or badly arranged work.
- If more space is required, clearly write the number of the QUESTION on one of the back pages and answer it there. Indicate that you have done so.
- Clearly indicate your class by placing an **X**, next to your class.
- Answer in simplest exact form unless otherwise instructed.

**NAME:**

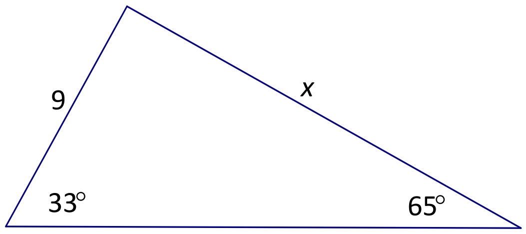
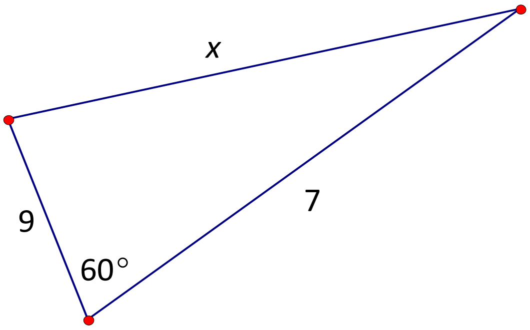
Examiner: *C.Kourtesis*

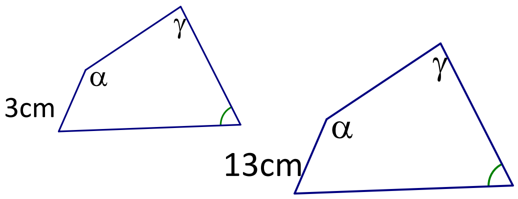
| Class | Teacher | |
|---|---|---|
| 10A | Mr. Fuller | |
| 10B | Mr. McQuillan | |
| 10C | Mr. Choy | |
| 10D | Ms. Ward | |
| 10E | Ms. Nesbitt | |
| 10F | Mr. Boros | |

| Question | Mark | |
|---|---|---|
| **1** | | **/20** |
| **2** | | **/16** |
| **3** | | **/15** |
| **4** | | **/16** |
| **5** | | **/15** |
| **6** | | **/18** |
| **Total** | | **/100** |

| Question One (20 Marks) | | Answers | Marks |
|---|---|---|---|
| A | Find 18% of \$640. | | |
| B | Simplify $\frac{a}{4}+\frac{2a}{3}$ | | |
| C | Simplify $\frac{12a-4}{4}$ | | |
| D | If $\sqrt{12}+\sqrt{3}=\sqrt{b}$ find the value of b. | | |
| E | Solve the inequality $5-3x<10$ | | |
| F | The volume of a cube is 64cm³. What is its surface area? | | |
| G | If $a=-3$ and $b=5$, evaluate $ba^2$. | | |
| H | Express $\sqrt{1.6\times10^9}$ in standard (scientific) notation. | | |

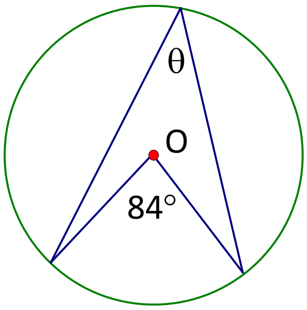
| | | | |
|---|---|---|---|
| I | Simplify $2(a+b)-(2a-b)$ | | |
| J | If $\sin\theta = \frac{\sqrt{3}}{2}$ where $0^\circ \le \theta \le 180^\circ$, find $\theta$. | | |
| K | Express with $h$ as the subject of the equation $d = 25\sqrt{\frac{h}{2}}$ | | |
| L | On separate diagrams sketch the graphs of:<br>(i) $y = x^2$<br>(ii) $xy = 1$<br>(iii) $x^2 + y^2 = 100$ | | |
| M | If $k = \frac{4}{a}$ find $k^{-3}$<br>(answer in positive index form) | | |

| Question Two (16 Marks) | | Answer | Marks |
|---|---|---|---|
| A |  <br>Find the value of *x*, correct to 2 decimal places | | |
| B |  <br>Use the Cosine Rule to find the value of x.<br>(Leave your answer in Surd form) | | |
| C | Ronald has a jar containing 120 jelly beans. Each jelly bean is either red, yellow or black. The ratio of red to yellow to black is 4 : 5 : 3.<br>Ronald chooses a jelly bean at random.<br>Find the probability it is:<br>(i) Black<br>(ii) Not Yellow | | |

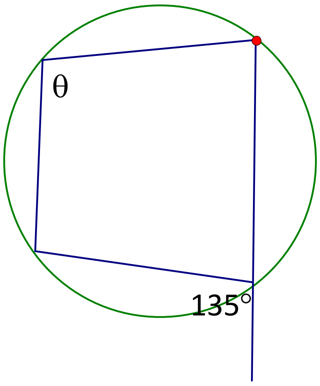
| | | | |
|---|---|---|---|
| D | The graph of $y = 4 - kx^2$ passes through the point (-5, 2). Find the value of k. | | |
| E | Consider the polygons:<br> <br>(i) Find the ratio of their areas.<br>(ii) If the area of the smaller polygon is 30cm², find the area of the larger. | | |
| F | The equation of a parabola is given by $y = x^2 - 4x + 3$<br>(i) Find the $x$ and $y$ intercepts.<br>(ii) Find the coordinates of the vertex.<br>(iii) Hence, sketch the graph of the parabola. | | |

<table>
<tr><th colspan="2">Question Three (15 Marks)</th><th>Answers</th><th>Marks</th></tr>
<tr><td>A</td><td>
Find the value of θ in each case. You are not required to give reasons. O is the centre of the circle:

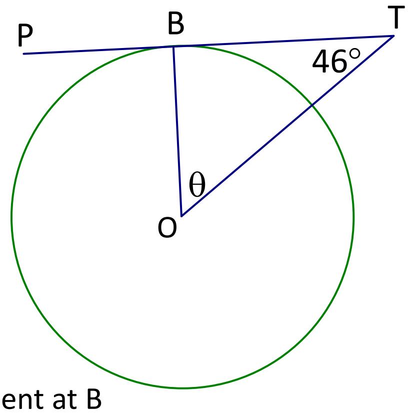
(i)



(ii)

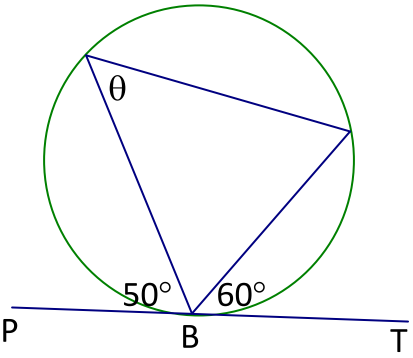


(iii)



PT is a tangent at B

(iv)



PT is a tangent at B
</td><td></td><td></td></tr>
</table>

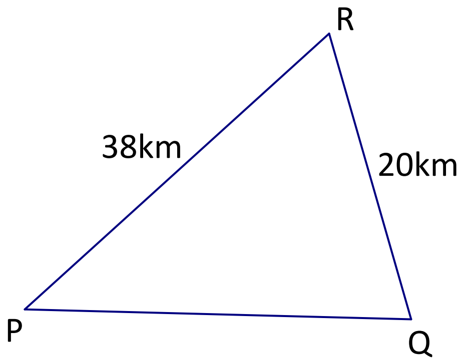
B

In 1954 a total of 6527mm of rain fell at Sprinkling Tarn and this set a UK record for annual rainfall. The tarn has a surface area of 23450m². How many litres of water fell on Sprinkling Tarn in 1954?

C

Factorise $A^2 - (B+C)^2$

D



In the diagram, the point Q is due east of P. The point R is 38km from P and 20km from Q. The bearing of R from Q is 325°.

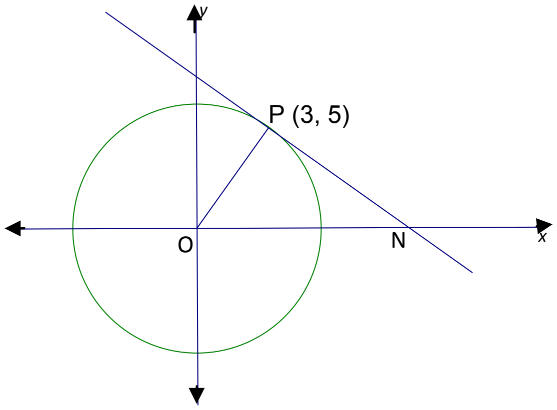
(i) What is the size of $\angle PQR$?

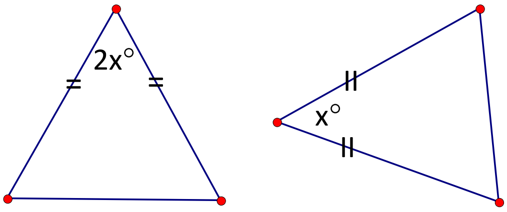
(ii) What is the bearing of R from P?

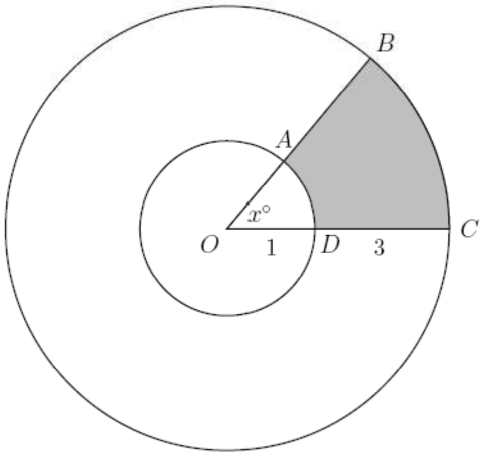
E

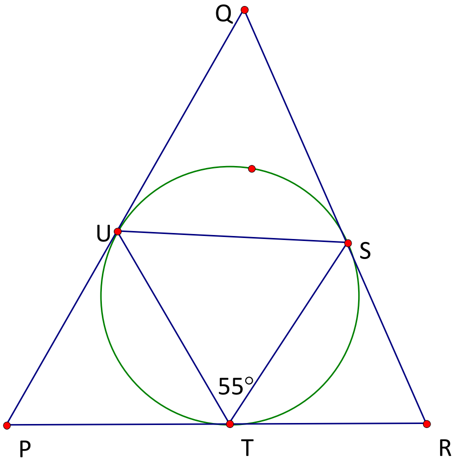
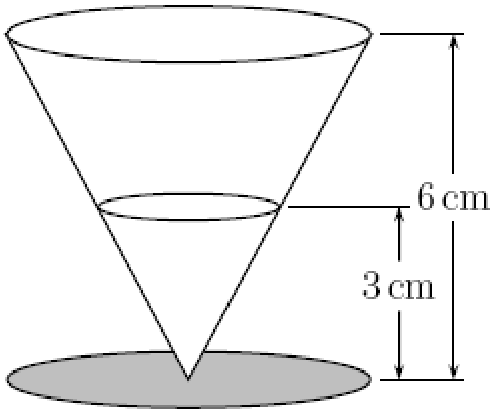
| Test | Kim's Mark | Class Mean | Class Standard Deviation |
|---|---|---|---|
| A | 79 | 60 | 20 |
| B | 70 | 60 | 10 |

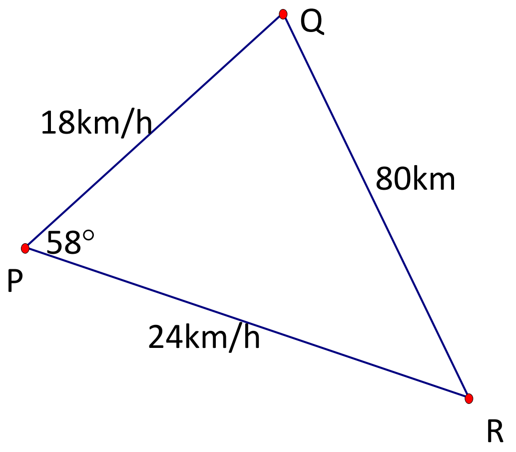
Indicate, giving reasons, in which test Kim performed better.

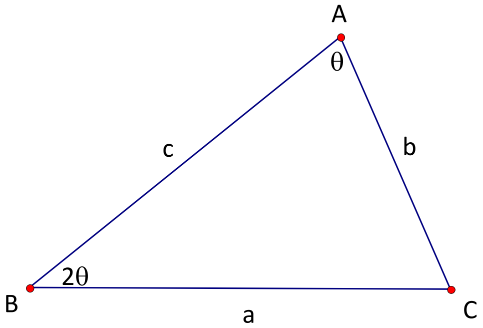
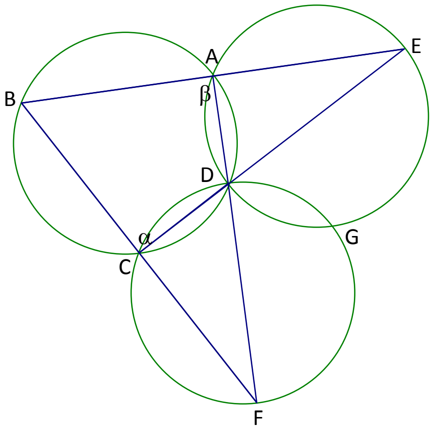
| Question Four (16 Marks) | | Answers | Marks |
|---|---|---|---|
| A |  <br>OP is a radius of the circle. PN is a tangent.<br>(i) Calculate the gradient of OP.<br>(ii) Show that the equation of PN is $3x + 5y - 34 = 0$.<br>(iii) Find the coordinates of N.<br>(iv) Write down the equation of the circle. | | |
| B | A 20cm by 5cm by 6cm block of lead is melted and cast into identical spherical fishing sinkers each of radius 1cm.<br>How many (whole) sinkers can be made? | | |

| | | | |
|---|---|---|---|
| C |  <br>The two triangles have equal areas and the four lengths are equal. What is the value of x? | | |
| D | The equation of a circle is<br>$x^2 + y^2 - 2x + y = 0$.<br>(i) Express this in the form:<br>$(x-a)^2 + (y-b)^2 = r^2$<br>(ii) Write down the coordinates of the centre and the length of the diameter. | | |

| Question Five (15 Marks) | | Answers | Marks |
|---|---|---|---|
| A | On separate diagrams sketch the graphs of the following, indicating the x and y intercepts in each case:<br>(i) $y=(x+3)(x-1)(x-4)$<br>(ii) $y=(x+1)^2(x-3)$<br>(iii) $y=1-(x-1)^4$ | | |
| B | O is the centre of both circles with radii 1cm and 4cm.<br> <br>(i) Show that the shaded are A is given by $A=\frac{\pi x}{24}$<br>(ii) If the shaded area is one sixth of the area of the outer circle find the value of x | | |

| | | | |
|---|---|---|---|
| C |  <br>The largest circle which it is possible to draw inside triangle PQR touches the triangle at S, T and U. If $\angle STU = 55°$, find the size of $\angle PQR$.<br>(Do Not Give Reasons). | | |
| D |  <br>A medicine glass in the shape of a cone has a height of 6cm. 3mL of liquid fills the cone to a height of 3cm. How many more mL of liquid is required to fill the cone to a height of 6cm? | | |

| Question Six (18 Marks) | | Answers | Marks |
|---|---|---|---|
| A |  <br>Two straight roads PQ and PR are inclined to each other at 58°. Two bike riders begin simultaneously from P and travel along the roads at 18km/h and 24km/h respectively. After t hours they are 80km apart in a direct line.<br>i) Show that $t=\frac{80}{\sqrt{(900-864\cos 58^{\circ})}}$<br>ii) Find the value of t (correct to 2 decimal places) | | |
| B | Two regular polygons have N and (N – 5) number of sides. The number of degrees of each of their angles differ by 1.<br>(i) Show that $N^{2}-5N-1800=0$<br>(ii) Find the possible value(s) of N. | | |

|  |  |  |  |
|---|---|---|---|
| C | <br> <br>Consider the triangle ABC.<br>i) Given the fact that Sin2θ = 2SinθCosθ, use the sine rule to show that $\cos\theta = \frac{b}{2a}$<br>ii) Hence prove that: $b^2 = a(a + c)$ where $a \neq c$. |  |  |
| D | <br> <br>Let ∠BCD = α and ∠BAD = β<br>ABCD is a cyclic quadrilateral. BA and CD are both produced and intersect at E. BC and AD produced intersect at F. The circles EAD, FCD intersect at G as well as at D. Prove that the points E, G and F are collinear. |  |  |

Use this space if you wish to **REWRITE** any answers.

Clearly *indicate* the **SECTION** and the **QUESTION** number.

| **Section** | **Question** | |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |

# Year 10 Yearly 2008

## Question 1

Ⓐ \$115.20 ①

Ⓑ $\frac{3a+8a}{12} = \frac{11a}{12}$ ①

Ⓒ $\frac{\cancel{4}(3a-1)}{\cancel{4}} = 3a-1$ ①

Ⓓ $\sqrt{12} = 2\sqrt{3}$

$2\sqrt{3} + \sqrt{3} = 3\sqrt{3}$

$= \sqrt{27}$

$b = 27$ ②

Ⓔ $5 - 3x < 10$

$-3x < 5$

$x > -\frac{5}{3}$ ①

Ⓕ $SA = 6x^2$

$V = x^3 = 64$

$x = 4$

$SA = 6 \times 4^2 = 96\ cm^2$ ②

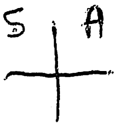
Ⓖ $5 \times (-3)^2 = 5 \times 9 = 45$ ①

Ⓗ $4.0 \times 10^4$ ① or $(16 \times 10^8)^{\frac{1}{2}}$

$= 4 \times 10^4$

etc

Ⓘ $\cancel{2a} + 2b - \cancel{2a} + b = 3b$ ①

Ⓙ 

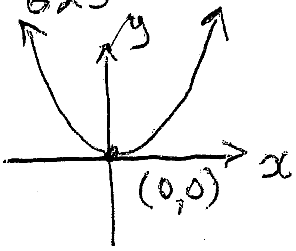
$\theta = 60°, 120°$ ②

Ⓚ $d = 25\sqrt{\frac{h}{2}}$

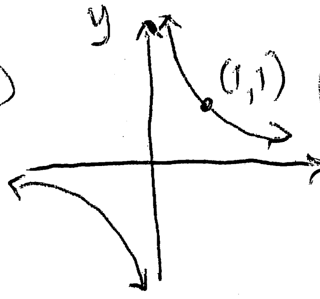
$\frac{d}{25} = \sqrt{\frac{h}{2}}$

$\frac{d^2}{625} = \frac{h}{2}$

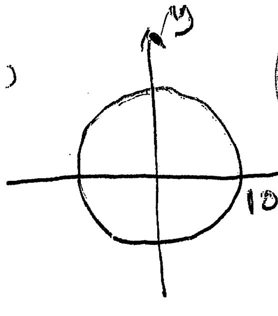
$h = \frac{2d^2}{625}$ ②

Ⓛ (i) 

$y = x^2$ ①

(ii) 

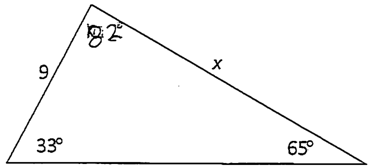
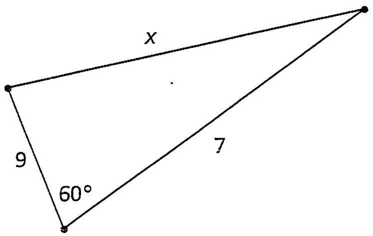
① $y = \frac{1}{x}$

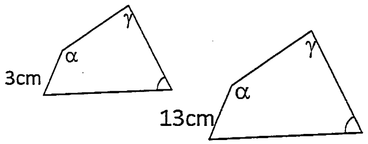
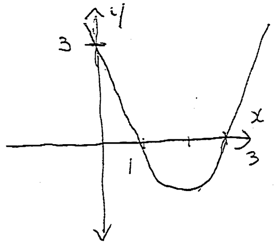
(iii) 

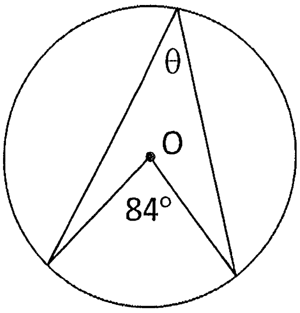
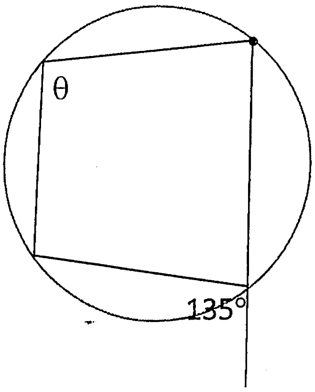
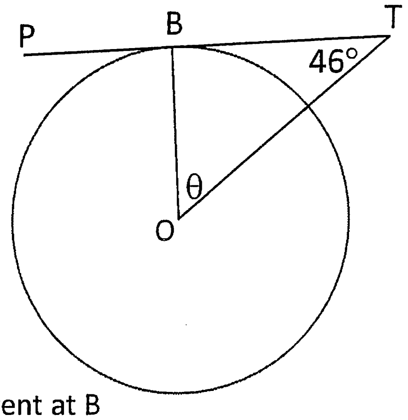
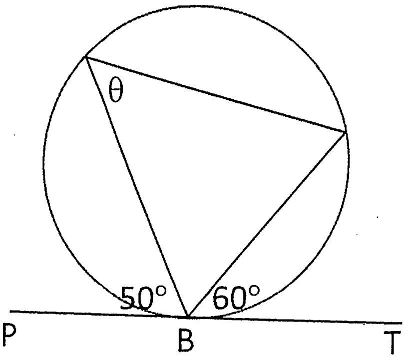
① $x^2 + y^2 = 100$

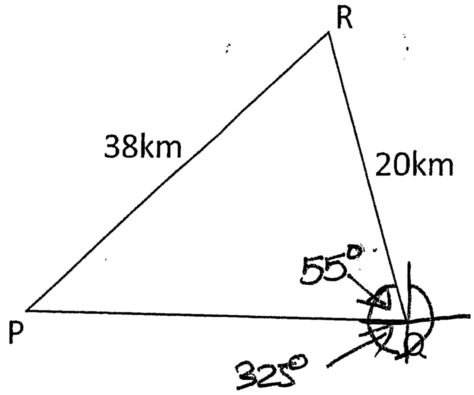
Ⓜ $k^{-3} = \frac{1}{k^3} = \frac{1}{\frac{4^3}{a^3}} = 1 \div \frac{64}{a^3}$

$= 1 \times \frac{a^3}{64} = \frac{a^3}{64}$ ②

| Question Two (16 Marks) | | Answer | Marks |
|---|---|---|---|
| A | <br> <br>Find the value of *x*, correct to 2 decimal places | $\frac{x}{\sin 33} = \frac{9}{\sin 65}$<br>$x = \frac{9 \sin 33}{\sin 65}$<br>$x = 5.4080$<br>$x = 5.41$ to 2dec pl | |
| B | <br> <br>Use the Cosine Rule to find the value of x. (Leave your answer in Surd form) | $x^2 = 7^2 + 9^2 - 2 \times 7 \times 9 \cos 60$<br>$x = \sqrt{67}$ | |
| C | Ronald has a jar containing 120 jelly beans. Each jelly bean is either red, yellow or black. The ratio of red to yellow to black is 4 : 5 : 3.<br>Ronald chooses a jelly bean at random.<br>Find the probability it is:<br>(i) Black<br>(ii) Not Yellow | 120 jelly beans<br>12 parts<br>Red $\frac{4}{12} \times 120 = 40$<br>yellow $\frac{5}{12} \times 120 = 50$<br>Black $3/12 \times 120 = 30$<br>$P(B) = \frac{30}{120} = 1/4$<br>$P(\tilde{y}) = \frac{70}{120} = 7/12$ | |

| | | | |
|---|---|---|---|
| D | The graph of $y = 4 - kx^2$ passes through the point (-5, 2). Find the value of k. | $2 = 4 - k \times 25$<br>$25k = 2$<br>$k = \frac{2}{25}$ | |
| E | Consider the polygons:<br><br> <br>(i) Find the ratio of their areas.<br>(ii) If the area of the smaller polygon is 30cm², find the area of the larger. | ratio of areas = $\frac{9}{169}$<br>9:169<br>Larger area = $\frac{30 \times 169}{9}$<br>$= 563\frac{1}{3}$ | |
| F | The equation of a parabola is given by $y = x^2 - 4x + 3$<br>(i) Find the *x* and *y* intercepts.<br>(ii) Find the coordinates of the vertex.<br>(iii) Hence, sketch the graph of the parabola. | $x = 0 \quad y = 3 \quad \underline{(0,3)}$ (y inte)<br>$y = 0 \quad (x-3)(x-1) = 0$<br>$x = 1, 3$<br>(1,0) (3,0) (x intercepts)<br>vertex $x = 2 \quad y = -1$<br><br> | |

| **Question Three (15 Marks)** | | **Answers** | **Marks** |
|---|---|---|---|
| A | Find the value of θ in each case. You are not required to give reasons. O is the centre of the circle: | | |
| | (i)  | $\theta = 42^{\circ}$ | |
| | (ii)  | $\theta = 135^{\circ}$. | |
| | (iii)  <br>PT is a tangent at B | $\theta = 44^{\circ}$. | |
| | (iv)  <br>PT is a tangent at B | $\theta = 60^{\circ}$. | |

<table>
<tr><td>B</td><td>In 1954 a total of 6527mm of rain fell at Sprinkling Tarn and this set a UK record for annual rainfall. The tarn has a surface area of 23450m$^2$. How many litres of water fell on Sprinkling Tarn in 1954?</td><td>153,058,150 L.</td><td></td></tr>
<tr><td>C</td><td>Factorise $A^2-(B+C)^2$</td><td>$(A-(B+C))(A+B+C)$<br>$=(A-B-C)(A+B+C)$</td><td></td></tr>
<tr><td>D</td><td>

In the diagram, the point Q is due east of P. The point R is 38km from P and 20km from Q. The bearing of R from Q is 325°.<br>(i) What is the size of $\angle PQR$?<br>(ii) What is the bearing of R from P?</td><td>(i) 55°.<br>(ii) $\sin P = \frac{20 \sin 55°}{38} = 25°32'$.<br>R from P = $90 - 25°32' = 64°28'$</td><td></td></tr>
<tr><td>E</td><td colspan="2">

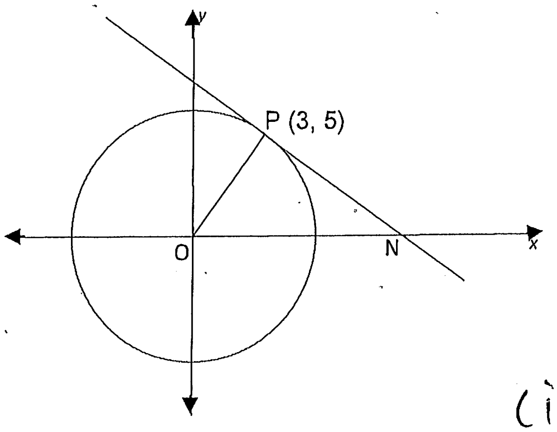
| Test | Kim's Mark | Class Mean | Class Standard Deviation |
|---|---|---|---|
| A | 79 | 60 | 20 |
| B | 70 | 60 | 10 |

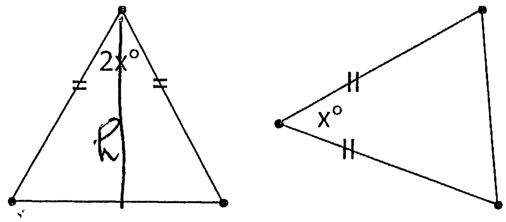
Indicate, giving reasons, in which test Kim performed better.

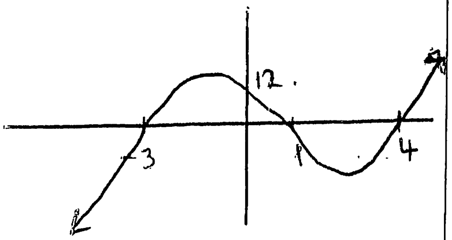
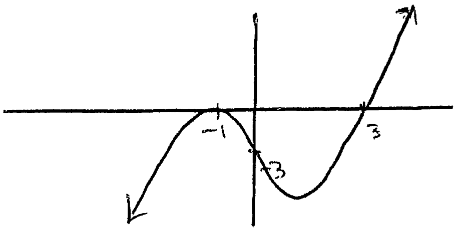
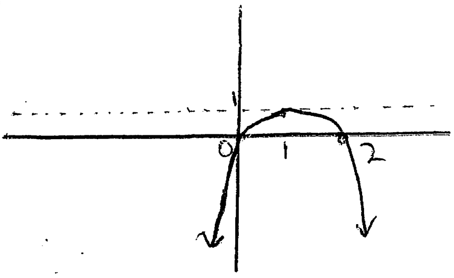
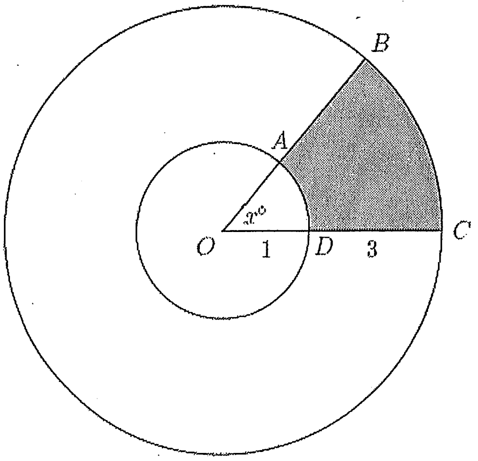
Test A, Kim is 19 away from mean and almost 1 standard deviation

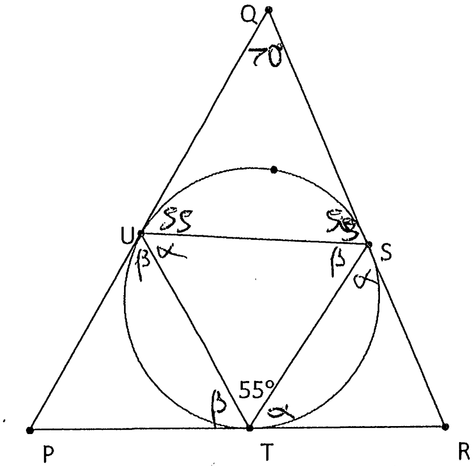
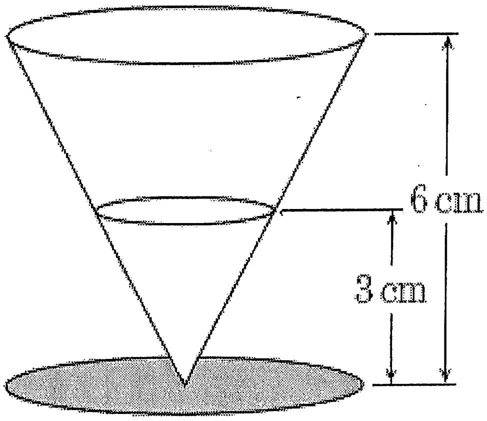
Test B, Kim is 10 away & exactly 1 standard deviation.

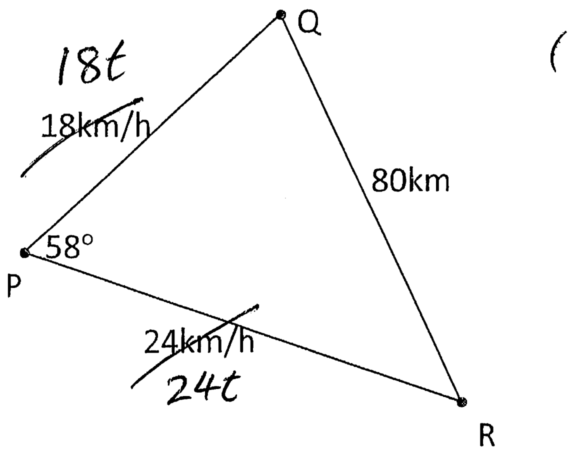
Did better in TEST B as his standard deviation is further away from mean</td><td></td></tr>
</table>

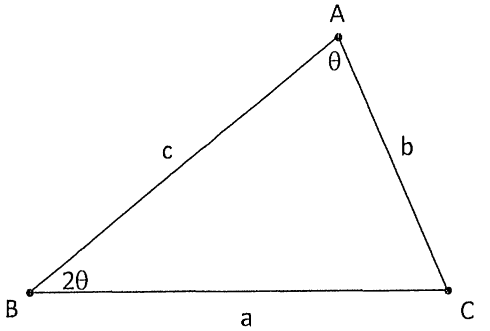
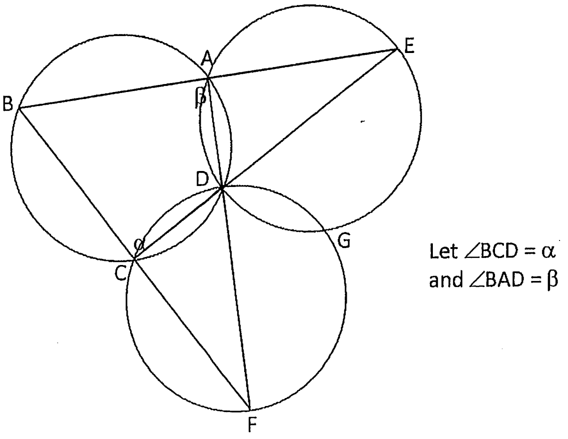
| Question Four (16 Marks) | | Answers | Marks |
|---|---|---|---|
| A |  <br>OP is a radius of the circle. PN is a tangent.<br>(i) Calculate the gradient of OP.<br>(ii) Show that the equation of PN is $3x+5y\cancel{+2}-34=0$.<br>(iii) Find the coordinates of N.<br>(iv) Write down the equation of the circle. | (i) $m_{OP} = 5/3$<br>$m_{PN} = -3/5$<br>$y-5 = -\frac{3}{5}(x-3)$<br>$5y - 25 = -3x + 9$<br>(ii) $\therefore 3x + 5y - 34 = 0$<br>(iii) When $y = 0$<br>$x = \frac{34}{3}$<br>$\therefore N\left(\frac{34}{3}, 0\right)$.<br>$\lvert OP \rvert = \sqrt{9+25}$<br>$= 3\cancel{4}$<br>(iv)<br>$\therefore x^2 + y^2 = 34$. | |
| B | A 20cm by 5cm by 6cm block of lead is melted and cast into identical spherical fishing sinkers each of radius 1cm.<br>How many (whole) sinkers can be made? | $V = 20 \times 30$<br>$= 600$<br>$V = \frac{4}{3}\pi r^3 = \frac{4\pi}{3}$<br>$= 143$ whole | |

| | | | |
|---|---|---|---|
| C | <br> <br>The two triangles have equal areas and the four lengths are equal. What is the value of x?<br>$\frac{1}{2}\cancel{x^2}\sin 2x = \cancel{\frac{1}{2}x^2}\sin x$<br>$\sin 2x = \sin x$<br>$\therefore\ x = 60^0$ | | |
| D | The equation of a circle is<br>$x^2+y^2-2x+y=0$.<br>(i) Express this in the form: $(x-a)^2+(y-b)^2=r^2$<br>(ii) Write down the coordinates of the centre and the length of the diameter.<br>$(x^2-2x+1)+(y^2+y+\frac{1}{4})=\frac{5}{4}$<br>$(x-2)^2+(y+\frac{1}{2})^2=\frac{5}{4}$<br>$\therefore\ C=(2,-\frac{1}{2})$<br>$r=\frac{\sqrt{5}}{2}\ \therefore$ | diameter $=\sqrt{5}$ | |

| Question Five (15 Marks) | | Answers | Marks |
|---|---|---|---|
| A | On separate diagrams sketch the graphs of the following, indicating the x and y intercepts in each case: | | |
| | (i) $y=(x+3)(x-1)(x-4)$ |  | 2 |
| | (ii) $y=(x+1)^2(x-3)$ |  | 2 |
| | (iii) $y=1-(x-1)^4$ |  | 2 |
| B | O is the centre of both circles with radii 1cm and 4cm.<br> | | |
| | (i) Show that the shaded are A is given by $A=\frac{\pi x}{24}$ | (i) $\frac{x}{360}(4^2\pi-1^2\pi)$<br>$=\frac{x}{360}(15\pi)$<br>$=\frac{\pi x}{24}.$ | 1 |
| | (ii) If the shaded area is one sixth of the area of the outer circle find the value of x | (ii) $6\times\frac{\pi x}{24}=16\pi$<br>$x=64°.$ | 2 |

| | | | |
|---|---|---|---|
| C |  <br>The largest circle which it is possible to draw inside triangle PQR touches the triangle at S, T and U. If $\angle STU = 55^\circ$, find the size of $\angle PQR$.<br>(Do Not Give Reasons). | $\angle PQR = 70^\circ$. | 3 |
| D |  <br>A medicine glass in the shape of a cone has a height of 6cm. 3mL of liquid fills the cone to a height of 3cm. How many more mL of liquid is required to fill the cone to a height of 6cm? | 1:2. lengths<br>1:8 volumes.<br>24mL.<br>i.e. 21mL more. | 3 |

| Question Six (18 Marks) | | Answers | Marks |
|---|---|---|---|
| A |  <br>Two straight roads PQ and PR are inclined to each other at 58°. Two bike riders begin simultaneously from P and travel along the roads at 18km/h and 24km/h respectively. After t hours they are 80km apart in a direct line.<br>i) Show that $t = \frac{80}{\sqrt{(900 - 864\cos 58^\circ)}}$<br>ii) Find the value of t (correct to 2 decimal places) | (i) $80^2 = (18t)^2 + (24t)^2 - 2(18t)(24t)\cos 58$<br>$80^2 = 324t^2 + 576t^2 - 864t^2\cos 58$<br>$t^2(900 - 864\cos 58) = 80^2$<br>$t^2 = \frac{80^2}{900 - 864\cos 58}$<br>$t = \pm\sqrt{\frac{80^2}{900 - 864\cos 58}}$<br>$= \frac{80}{\sqrt{(900 - 864\cos 58^\circ)}}$<br>since $t > 0$<br>(ii) $t = 3.80$ hours. | (3)<br>(1) |
| B | Two regular polygons have N and (N – 5) number of sides. The number of degrees of each of their angles differ by 1.<br>(i) Show that $N^2 - 5N - 1800 = 0$<br>(ii) Find the possible value(s) of N. | (i) $\frac{(N-2)180}{N} - \frac{(N-7)180}{N-5} = 1$<br>$(N-2)(N-5)180 - N(N-7)180 = N(N-5)$<br>$(N^2-7N+10)180 - (N^2-7N)180 = N^2-5N$<br>$\cancel{(N^2-7N)180} + 1800 - \cancel{(N^2-7N)180} = N^2-5N$<br>$N^2 - 5N - 1800 = 0$<br>(ii) $(N-45)(N+40) = 0$<br>$N = 45, -40$<br>only possible value of $N$ is 45 (since $N > 0$) | (3)<br>(2) |

| | | | |
|---|---|---|---|
| C |  <br>Consider the triangle ABC.<br>i) Given the fact that Sin2θ = 2SinθCosθ, use the sine rule to show that $\cos\theta = \frac{b}{2a}$<br>ii) Hence prove that: $b^2 = a(a + c)$ where $a \neq c$. | (i)<br>$\frac{\sin 2\theta}{b} = \frac{\sin\theta}{a}$<br>$\frac{2\cancel{\sin}\theta\cos\theta}{b} = \frac{\cancel{\sin}\theta}{a}$<br>$\cos\theta = \frac{b}{2a}$<br>(ii)<br>$\cos\theta = \frac{b^2+c^2-a^2}{2bc}$<br>$\frac{b}{2a} = \frac{b^2+c^2-a^2}{2bc}$<br>$\frac{\cancel{2}b^2c}{\cancel{2}a} = b^2+c^2-a^2$<br>$b^2c = b^2a + a(c^2-a^2)$<br>$b^2(c-a) = a(c-a)(c+a)$<br>$b^2 = \frac{a\cancel{(c-a)}(c+a)}{\cancel{(c-a)}}, a \neq c$<br>$b^2 = a(c+a)$ | (2)<br>(3) |
| D |  <br>Let ∠BCD = α and ∠BAD = β<br>ABCD is a cyclic quadrilateral. BA and CD are both produced and intersect at E. BC and AD produced intersect at F. The circles EAD, FCD intersect at G as well as at D. Prove that the points E, G and F are collinear. | Note $\alpha+\beta=180°$<br>(opp. ∠'s of cyclic quad. ABCD)<br>$\angle EGD = \beta$ (ext. ∠ of cyclic quad AEGD equal to opp. int. ∠)<br>$\angle DGF = \alpha$ (ext. ∠ of cyclic quad. DGFC equal to opp. int. ∠)<br>$\angle FGE = \alpha+\beta$<br>$= 180°$<br>∴ E, G and F are collinear | (4) |