SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

Year 10

Yearly Examination 2009

Mathematics

General Instructions

- Working time – 120 minutes
- Write using black or blue pen.
- *Approved* calculators may be used.
- All necessary working should be shown in every question if full marks are to be awarded.
- Marks may not be awarded for messy or badly arranged work.
- If more space is required, clearly write the number and the SECTION on the back page and answer it there. Indicate that you have done so.
- Write all answers in simplest exact form unless specified otherwise
- Clearly indicate your class by placing an X, next to your class

NAME:

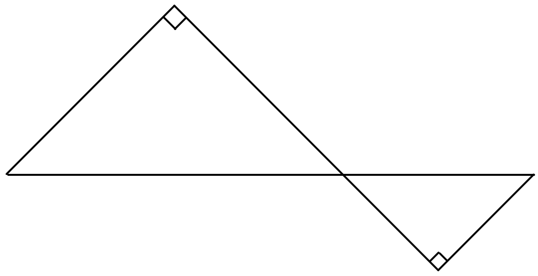
Examiner: *E. Choy*

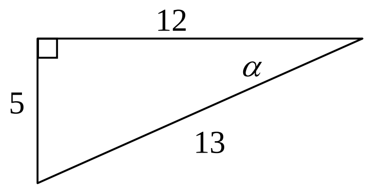
Class	Teacher	
10 A	Mr McQuillan	
10 B	Ms Roessler	
10 C	Ms Nesbitt	
10 D	Mr Fuller	
10 E	Mr Hespe	
10 F	Mr Gainford	
10 G	Ms Evans	

Section	Mark
1	/20
2	/20
3	/10
4	/20
5	/20
6	/20
7	/20
Total	**/130**

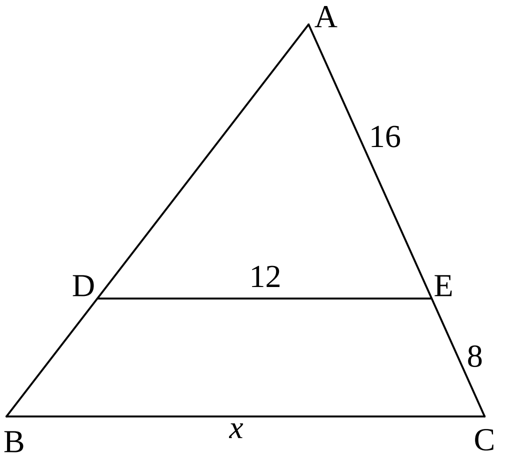
Question One (20 marks)		Answers	Marks
A	Write down the gradient of the line $y = 2x - 3$.		1
B	Simplify $(2m^3)^2$		1
C	Expand and simplify $(\sqrt{3}-1)(\sqrt{3}+1)$		1
D	Write down the exact value of $\sin 60°$.		1
E	Simplify $8^{\frac{1}{3}}$		1
F	Given that $\tan\alpha = 0.42$ and α is acute, use your calculator to find the angle α, correct to the nearest minute.		1
G	If $P(x) = 1 - 8x^2 + 14x^3 - 5x^4$, write down the degree of the polynomial $P(x)$.		1
H	Using the remainder theorem, find the remainder when the polynomial $P(x) = 2x^3 - x^2 + 3x - 1$ is divided by $(x-1)$.		1
I	Simplify $\frac{1}{a} + \frac{2}{a}$		1
J	Factorise $x^2 - 16$		1
K	Expand $(x-5)^2$		1
L	Write $\frac{1}{x}$ as a power of x.		1
M	Subtract $1 - x$ from $1 + x$.		1
N	Two similar statues have volumes in the ratio $1:64$. What is the ratio of their heights?		1

O	Sketch the graph of the line with equation $y = 5$.		1
P	Solve for x: $2x - 7 = 5 - x$.		1
Q	To what amount will $5000 grow over 6 years if it is invested at 8% p.a. compound interest compounded yearly. (Give your answer to the nearest cent.)		1
R	If $a = 2b\sqrt{\frac{c}{d}}$ express c in terms of a, b and d.		1
S	Express $\sqrt[3]{2.5 \times 10^6}$ in standard (scientific) notation.		1
T	Simplify $\sqrt{4 - 4x^2} - \sqrt{1 - x^2}$		1

Question Two (20 marks)		Answers	Marks
A	Factorise $a^2 + 2ab + b^2$		1
B	 What test can be used to show that these two triangles are similar?		1
C	Ron was asked to write down the value of $\sqrt{16}$. He remembered that every positive number has two square roots, so that he wrote $\sqrt{16} = \pm 4$. Is Ron's answer correct? Give a reason for your answer.		1
D	What is the value of $-x^2$ when $x = 5$?		1
E	Find the centre and exact radius of the circle with the equation $x^2 + y^2 + 2y - 10 = 0$, by first completing the square in y.		1
F	The midpoint of an interval is (2, 8). Find two distinct points that could be the end points of this interval.		1
G	The surface area of a closed hemisphere is 12π cm^2. Find its radius.		1
H	(i) Write down the minimum value of $(x-1)^2 + 4$. (ii) Without doing any further working, write down the number of solutions of $(x-1)^2 + 4 = 1$.		2

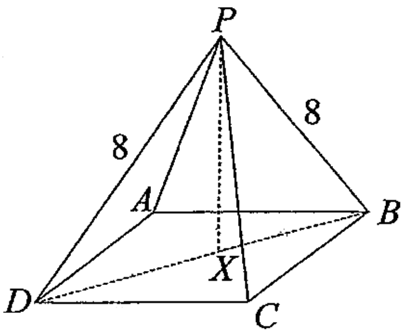
I	Solve the equation $2^x \times 4 = 32$ for *x*.		1
J	 Write down the exact value of : (i) $\tan\alpha$ (ii) $\tan(180° - \alpha)$		2
K	Given the formula $F = \frac{9}{5}C + 32°$, find the value of *C* if $F = 320°$.		1
L	A new car costs \$35 690. If it depreciates at a compound interest rate of 20% p.a., find its value, to the nearest dollar, at the end of four years.		1
M	Solve (algebraically) the pair of equations simultaneously. $y = 4x - 1$ $y = x + 2$		1
N	On separate diagrams sketch the graphs of (i) $y = -x^3$		3

	(ii) $y = 2x^2$ (iii) $y = -\sqrt{25 - x^2}$		
O	(i) Find the exact value of $\cos 150°$. (ii) Solve the equation $\sin\theta = \frac{1}{\sqrt{2}}$ for $0° \le \theta \le 180°$.		2

Question Three (10 marks)		Answers	Marks
A	 In the diagram above DE is parallel to BC, AE=16, CE=8 and DE=12. Let BC=*x*. (i) Show that ΔABC is similar to ΔADE . (ii) Find *x*.		3
B	A house has a hemispherical roof of diameter 15 metres. The roof is to be painted (on the outside only) with a special reflective coating that costs \$120 per litre. How much (correct to the nearest hundred dollars) will it cost to purchase enough of the coating to paint the roof if one litre of the coating will cover an area of 5m^2?		3
C	Two similar cones have volumes 27cm^3 and 64cm^3. (i) Write down the ratio of the surface area of the smaller cone to the larger cone. (ii) Find the radius of the smaller cone if its height is $\frac{9}{\pi}$cm.		4

Question Four (20 marks)		Answers	Marks
A	Let P be the parabola with equation $y = x^2 - 10x$. (i) Find the x-intercepts of P. (ii) Find the coordinates of the vertex of P. (iii) Sketch the graph of P. (iv) For what value of x does $x^2 - 10x$ take on its minimum possible value? (v) Show that the point A(2, –16) lies on P and write down the coordinates of the point B on P that is the reflection of A in the axis of symmetry of P.		9

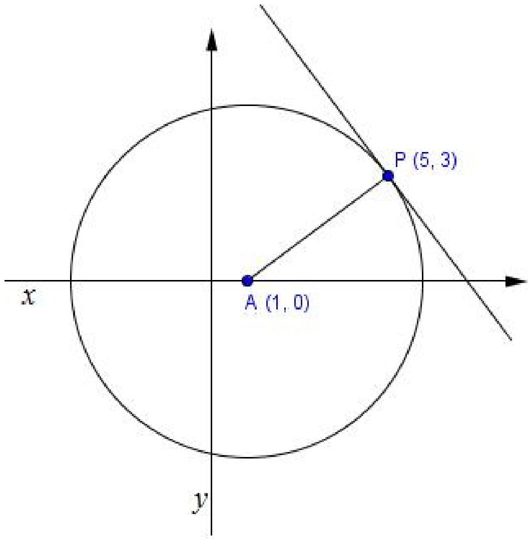
<table>
<tr><td>B</td><td>The line l with equation $4x + y = 7$ intersects the parabola P from part (A) in two distinct points.
(i) Use simultaneous equations to find the two points of intersection.

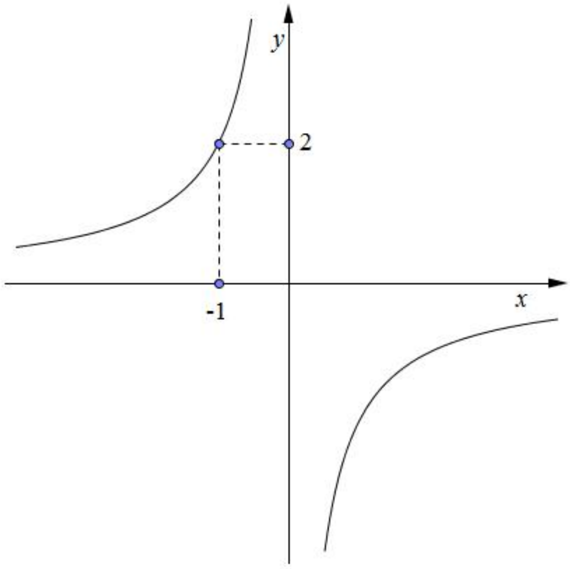
(ii) Go back to your sketch on part (A)(iii) and include the line l, showing clearly its points of intersection with the parabola P.</td><td></td><td>6</td></tr>
<tr><td>C</td><td>

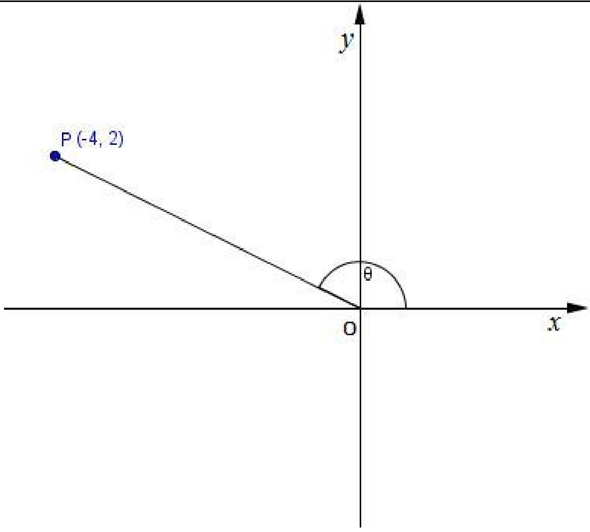
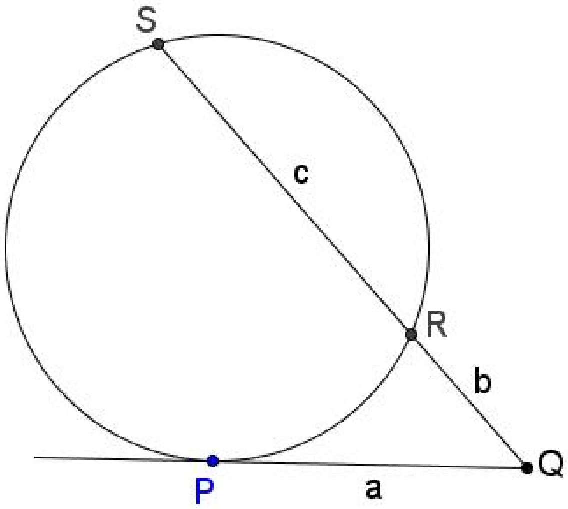
The diagram above shows a pyramid with square base ABCD. Point P is the apex of the pyramid. It is given that PD=PB=8 and $\angle PBD = 60°$. The point P lies vertically above the centre X of the square.

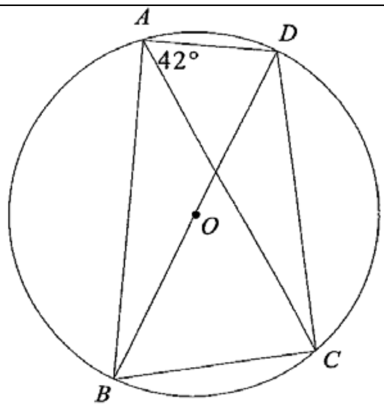
(i) Find length DB giving reasons.

(ii) Find the exact volume of the pyramid.</td><td></td><td>5</td></tr>
</table>

Question Five (20 marks)		Answers	Marks
A	 The diagram above shows the circle $(x-1)^2 + y^2 = 25$ with centre A(1, 0) and radius 5. The point P(5, 3) lies on the circumference of the circle. (i) Find the gradient of AP. (ii) Find, in general form, the equation of the tangent at P.		7

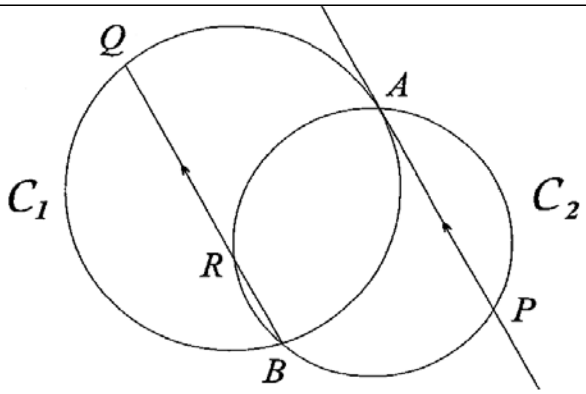
B	Consider the curve $y = ax^n$, where n is an integer and a is a constant. (i) If the curve passes through the point (–1, 2), find the possible values of a. (ii) It is known further that the graph has the form sketched below.  (α) Write down three possible values for n. (β) Given further that $\left(2, -\frac{1}{16}\right)$ lies on the curve, find the value of n.		7
C	There are 4 cards in a box. Each card has a letter on it. The letters on the cards are A, B, c and d. Two of the cards are chosen at random without replacement. (i) Draw a tree diagram to represent this experiment and list all the possible outcomes. (ii) Determine the probability that: (α) the two capital letters are chosen. (β) a capital letter and a lower case letter are chosen.		6

Question Six (20 marks)		Answers	Marks
A	 (i) In the diagram above, P has coordinates (–4, 2). Find length of OP. (ii) Write down the exact value of $\cos\theta$.		2
B	 In the diagram above, QP is a tangent to the circle, while QRS is a secant. If QP = a units, QR = b units and RS = c units. Write down an equation (do not prove this equation) showing the relationship between a, b and c.		2

<table>
<tr>
<td>C</td>
<td>

The diagram above shows a cyclic quadrilateral ABCD. The diagonal BD of the quadrilateral passes through the centre O of the circle and $\angle CAD = 42°$. Find, giving reasons:

(i) $\angle BAC$

(ii) $\angle BDC$</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>D</td>
<td>

In the diagram above, C1 and C2 are circles intersecting at A and B. The tangent to C1 at A meets C2 at P. Q is the point on C1 so that QB is parallel to AP. The chord QB intersects C2 at R.
(i) Draw in the intervals AQ, AB and PR.

(ii) Give a reason why $\angle PAB = \angle AQB$.

(iii) Give a reason why $\angle PAB = \angle PRB$

(iv) Explain why QA is parallel to RP.

(v) Are QA and RP equal? Explain your answer.</td>
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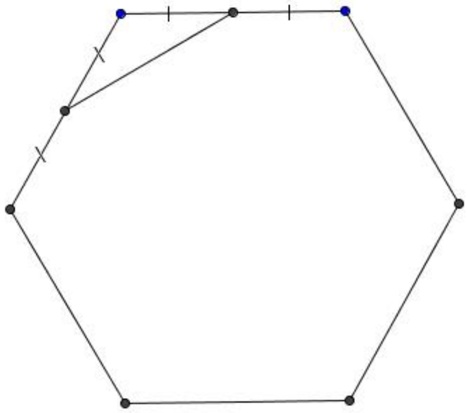
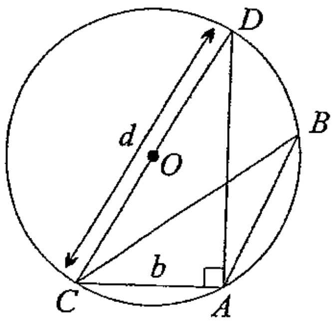
<td>10</td>
</tr>
</table>

Question Seven (20 marks)		Answers	Marks
A	 (i) Find the size of each interior angle in a regular hexagon. (ii) In the diagram above, the length of the straight line joining the midpoints of two adjacent sides of a regular hexagon is 12cm. (α) Calculate the exact length of one side of the regular hexagon. (β) Calculate the exact area of the regular hexagon.		7

<table>
<tr>
<td>B</td>
<td>


In the diagram above, the vertices of ΔABC and ΔADC are on the circumference of a circle with centre O, and $\angle CAD = 90°$.

Let the diameter CD = d and let AC = b.

(i) Explain why $\angle ADC = \angle ABC$.

(ii) Hence show that $\dfrac{b}{\sin B} = d$.
</td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>C</td>
<td>
The maximum daily temperatures $(°C)$ recorded in a city over a period of 20 days are given below.

<table>
<tr><td>25</td><td>24</td><td>27</td><td>28</td><td>26</td></tr>
<tr><td>30</td><td>28</td><td>25</td><td>29</td><td>30</td></tr>
<tr><td>31</td><td>34</td><td>32</td><td>30</td><td>26</td></tr>
<tr><td>23</td><td>24</td><td>26</td><td>30</td><td>27</td></tr>
</table>

(i) Find the range of the temperatures.

(ii) Find the interquartile range of the temperatures.

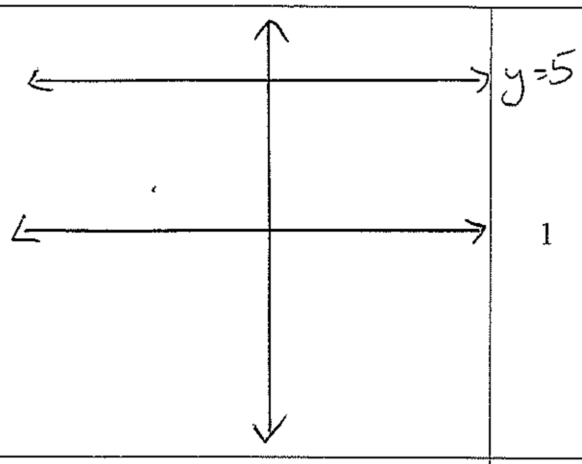
(iii) Find the standard deviation, correct to 1 decimal place.

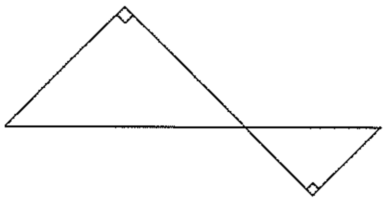
(iv) What would be the two most appropriate measures of spread for these temperatures? Why?
</td>
<td></td>
<td>4</td>
</tr>
</table>

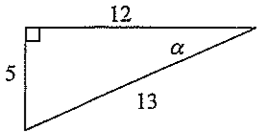
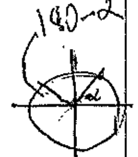
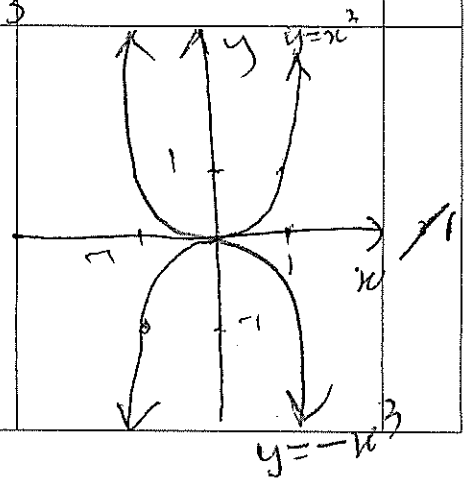
D	(i) Prove that $(a-b)^2+(b-c)^2+(c-a)^2=2(a^2+b^2+c^2-ab-bc-ca)$ (ii) Use part (i) to prove that $a^2+b^2+c^2 \geq ab+bc+ca$. (iii) Use part (ii) to prove that $(a+b+c)^2 \geq 3(ab+bc+ca)$.		3
E	(i) Without using a calculator, explain why $7^{\frac{1}{2}}$ is less than 3. (ii) Use methods similar to part (i) to explain why $7^{\frac{1}{2}}+7^{\frac{1}{3}}+7^{\frac{1}{4}}<7$. (Once again do not use a calculator.)		2

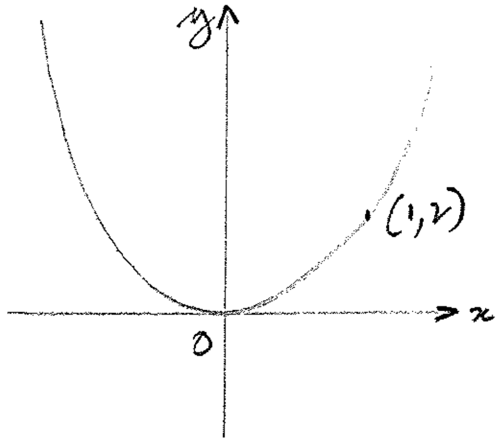
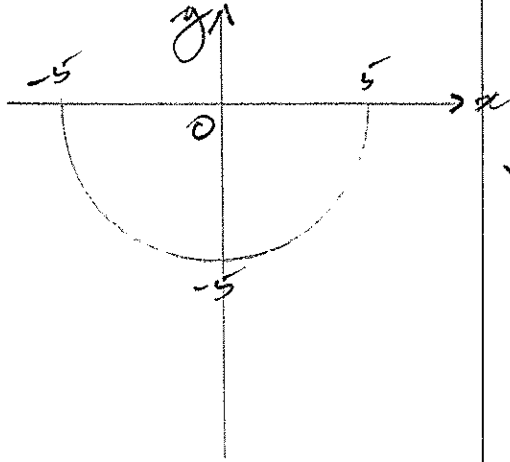
This is the end of the exam.

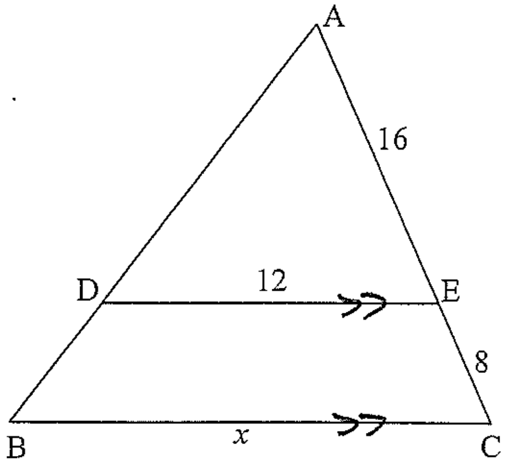
Question One (20 marks)		Answers	Marks
A	Write down the gradient of the line $y = 2x - 3$.	$m = 2$	1
B	Simplify $(2m^3)^2$	$4m^6$	1
C	Expand and simplify $(\sqrt{3}-1)(\sqrt{3}+1)$ $3 - 1$	$= 2$	1
D	Write down the exact value of $\sin 60°$.	$\sqrt{3}/2$	1
E	Simplify $8^{\frac{1}{3}}$	2	1
F	Given that $\tan\alpha = 0.42$ and α is acute, use your calculator to find the angle α, correct to the nearest minute.	$22° 47'$	1
G	If $P(x) = 1 - 8x^2 + 14x^3 - 5x^4$, write down the degree of the polynomial $P(x)$.	4	1
H	Using the remainder theorem, find the remainder when the polynomial $P(x) = 2x^3 - x^2 + 3x - 1$ is divided by $(x-1)$. $P(1) = 2 - 1 + 3 - 1$	$= 3$	1
I	Simplify $\frac{1}{a} + \frac{2}{a}$	$3/a$	1
J	Factorise $x^2 - 16$	$(x+4)(x-4)$	1
K	Expand $(x-5)^2$	$x^2 - 10x + 25$	1
L	Write $\frac{1}{x}$ as a power of x.	x^{-1}	1
M	Subtract $1 - x$ from $1 + x$. $1 + x - (1 - x)$	$2x$	1
N	Two similar statues have volumes in the ratio $1:64$. What is the ratio of their heights?	$1:4$	1

O	Sketch the graph of the line with equation $y = 5$.		1
P	Solve for x: $2x - 7 = 5 - x$. $2x - 7 = 5 - x$ $3x = 12$	$x = 4$.	1
Q	To what amount will \$5000 grow over 6 years if it is invested at 8% p.a. compound interest compounded yearly. (Give your answer to the nearest cent.) $A = 5000(1 + 0.08)^6$	\$7,934.37	1
R	If $a = 2b\sqrt{\frac{c}{d}}$ express c in terms of a, b and d. $\frac{a}{2b} = \sqrt{c/d}$ $\left(\frac{a}{2b}\right)^2 = c/d$	$c = d\left(\frac{a}{2b}\right)^2$ $= \frac{a^2d}{4b^2}$	1
S	Express $\sqrt[3]{2.5\times10^6}$ in standard (scientific) notation.	1.357208808×10^2	1
T	Simplify $\sqrt{4-4x^2} - \sqrt{1-x^2}$ $\sqrt{4(1-x^2} - \sqrt{1-x^2}$ $= 2\sqrt{1-x^2} - \sqrt{1-x^2}$	$\sqrt{1-x^2}$	1

Question Two (20 marks)		Answers	Marks
A	Factorise $a^2+2ab+b^2$	$(a+b)^2$	1
B	 What test can be used to show that these two triangles are similar?	equiangular	1
C	Ron was asked to write down the value of $\sqrt{16}$. He remembered that every positive number has two square roots, so that he wrote $\sqrt{16}=\pm 4$. Is Ron's answer correct? Give a reason for your answer.	No $\sqrt{16}$ is positive	1
D	What is the value of $-x^2$ when $x=5$?	-25	1
E	Find the centre and exact radius of the circle with the equation $x^2+y^2+2y-10=0$, by first completing the square in y.	$x^2+y^2+2y+1=10+1$ $x^2+(y+1)^2=11$ Centre $(0,-1)$ $r=\sqrt{11}$	1
F	The midpoint of an interval is (2, 8). Find two distinct points that could be the end points of this interval.	$(1,4)$, $(3,12)$ et al	1
G	The surface area of a closed hemisphere is 12π cm^2. Find its radius. $2\pi r^2+\pi r^2$ $=3\pi r^2=12\pi$ 	$\therefore r^2=4$ $r=2$	1
H	(i) Write down the minimum value of $(x-1)^2+4$. (ii) Without doing any further working, write down the number of solutions of $(x-1)^2+4=1$.	4 0	2

I	Solve the equation $2^x\times 4=32$ for x. $2^x=8$	$x=3$	1
J	 Write down the exact value of : (i) $\tan\alpha$ (ii) $\tan(180°-\alpha)$  $180-\alpha$	$\tan\alpha=\frac{5}{12}$ $\tan(180-\alpha)=-\frac{5}{12}$	2
K	Given the formula $F=\frac{9}{5}C+32°$, find the value of C if $F=320°$. $320=\frac{9}{5}C+32$ $1600=9C+160$	$9C=1440$ $C=160°$	1
L	A new car costs \$35 690. If it depreciates at a compound interest rate of 20% p.a., find its value, to the nearest dollar, at the end of four years. $A=P(1-\frac{r}{100})^n$ $A=35690(0.8)^4$	\$14619	1
M	Solve (algebraically) the pair of equations simultaneously. $y=4x-1$ $y=x+2$ $4x-1=x+2$ $3x=3$ $x=1$ $y=4\times 1-1=3$	$(1,3)$	1
N	On separate diagrams sketch the graphs of (i) $y=-x^3$		1

	(ii) $y = 2x^2$		✓
	(iii) $y = -\sqrt{25 - x^2}$		✓
O	(i) Find the exact value of $\cos 150°$. (ii) Solve the equation $\sin\theta = \frac{1}{\sqrt{2}}$ for $0° \le \theta \le 180°$.	$-\frac{\sqrt{3}}{2}$ $45°, 135°$ ✓	✓ 2

<table>
<tr><th colspan="2">Question Three (10 marks)</th><th>Answers</th><th>Marks</th></tr>
<tr><td>A</td><td>

In the diagram above DE is parallel to BC, AE=16, CE=8 and DE=12. Let BC=x.
(i) Show that ΔABC is similar to ΔADE.

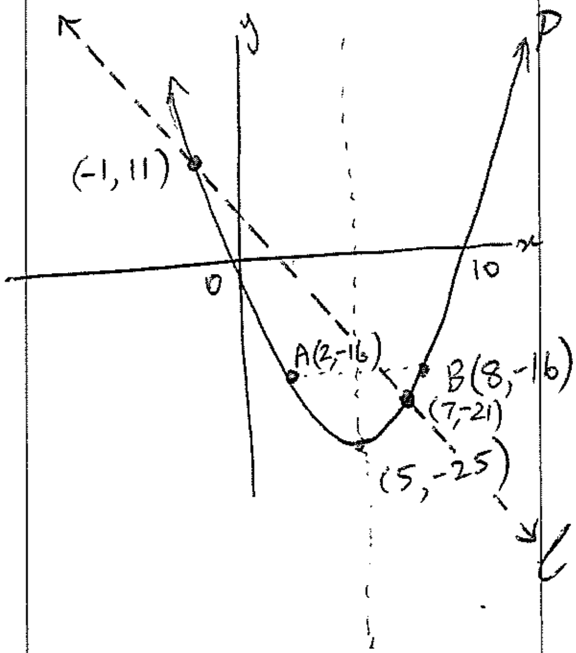
(ii) Find x.</td><td>✓ $D\hat{A}E = B\hat{A}C$ (common)
$A\hat{E}D = A\hat{C}B$ (corresp. ∠s, DE // BC) ✓
∴ $\triangle ABC \;|||\; \triangle ADE$ (equiangular)

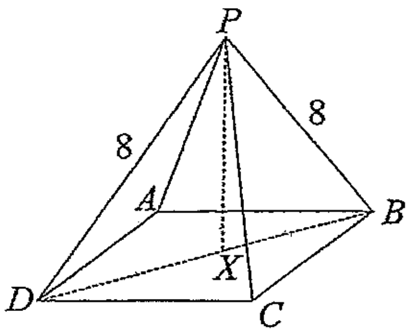
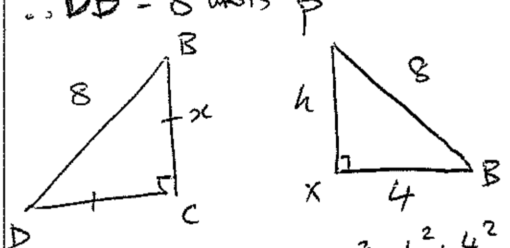
$\frac{x}{12} = \frac{16+8}{16}$
$x = 18$ ✓</td><td>3</td></tr>
<tr><td>B</td><td>A house has a hemispherical roof of diameter 15 metres. The roof is to be painted (on the outside only) with a special reflective coating that costs $120 per litre. How much (correct to the nearest hundred dollars) will it cost to purchase enough of the coating to paint the roof if one litre of the coating will cover an area of 5m^2?</td><td>Surf. Area $= 2\pi(7.5)^2$ ✓
Litres $= \frac{2\pi(7.5)^2}{5}$ ✓
Cost $= \frac{2\pi(7.5)^2}{5} \times 120$
$\doteqdot \$8500$ ✓</td><td>3</td></tr>
<tr><td>C</td><td>Two similar cones have volumes 27cm^3 and 64cm^3.

(i) Write down the ratio of the surface area of the smaller cone to the larger cone.

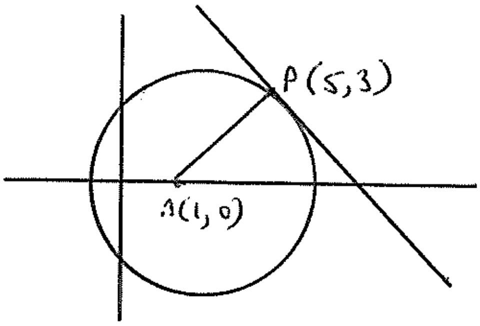
(ii) Find the radius of the smaller cone if its height is $\frac{9}{\pi}$ cm.</td><td>Side ratio = 3:4 ✓
∴ Area ratio = 9:16 ✓

$\frac{\pi}{3} \cdot r^2 \cdot \frac{9}{\pi} = 27$ ✓
$r^2 = 9$
$r = 3$ cm ✓</td><td>4</td></tr>
</table>

Question Four (20 marks)		Answers	Marks
	Let P be the parabola with equation $y = x^2 - 10x$. (i) Find the x-intercepts of P.	let $y=0$ $x^2-10x=0$ $x(x-10)=0$ $x=0,10$	
	(ii) Find the coordinates of the vertex of P.	when $x=5$ $y=(5)^2-10(5)$ $y=-25$ $\therefore$ P is $(5,-25)$	
A	(iii) Sketch the graph of P.	 	9
	(iv) For what value of x does $x^2 - 10x$ take on its minimum possible value?	$x=5$	
	(v) Show that the point A(2, −16) lies on P and write down the coordinates of the point B on P that is the reflection of A in the axis of symmetry of P.	$y=x^2-10x$ sub in $(2,-16)$ LHS $=-16$ RHS $=(2)^2-10(2)$ $=-16$ $\therefore$ A lies on P. B has coordinates $(8,-16)$	

B	The line l with equation $4x+y=7$ intersects the parabola P from part (A) in two distinct points. (i) Use simultaneous equations to find the two points of intersection. (ii) Go back to your sketch on part (A)(iii) and include the line l, showing clearly its points of intersection with the parabola P.	$y=x^2-10x$ ——— ① $y=7-4x$ ——— ② sub ① into ② $x^2-10x=7-4x$ $x^2-6x-7=0$ $(x-7)(x+1)=0$ $x=-1,\ 7$ sub into ② when $x=-1$: $y=7-4(-1)$, $y=11$ when $x=7$: $y=7-4(7)$, $y=-21$ $\therefore$ points of intersection are $(-1,11)$ & $(7,-21)$	6
C	 The diagram above shows a pyramid with square base ABCD. Point P is the apex of the pyramid. It is given that PD=PB=8 and $\angle PBD = 60°$. The point P lies vertically above the centre X of the square. (i) Find length DB giving reasons. (ii) Find the exact volume of the pyramid.	$PD=PB=8$ (given) $\therefore \triangle PBD$ is isosceles. $\angle PBD=60°$ (given) If an angle in an isosceles triangle is $60°$ the triangle is in fact <u>equilateral</u> $\therefore DB=8$ units  $8^2=x^2+x^2$ $2x^2=64$ $x^2=32$ $8^2=h^2+4^2$ $h^2=48$ $h=\sqrt{48}$ $h=4\sqrt{3}$ $V=\frac{1}{3}Ah$ $V=\frac{1}{3}x^2h$ $V=\frac{1}{3}(32)(4\sqrt{3})$ $V=\frac{128\sqrt{3}}{3}$ units^3	5

QUESTION FIVE (Y10)



$m_{AP} = \frac{3-0}{5-1}$

$= \frac{3}{4}$

Hence gradient of tangent is $\frac{-4}{3}$

Eqn of tangent is

$y - 3 = \frac{-4}{3}(x-5)$

$3y - 9 = -4x + 20$

$4x + 3y - 29 = 0$

B $\quad y = ax^n$

Since $(-1,2)$ lies on graph

$2 = a(-1)^n$

Since $(-1)^n = \pm 1$ $\quad n$ an integer

$a = \pm 2$

If $a = 2$

Then when $x > 0$, $y > 0$

But diagram shows

$x > 0$, $y < 0$

Hence $a = -2$

$y = -2x^n$

Considering all the possible integer values of n, $n \neq 0$ the only values that satisfy the diagram are $n = -1, -3, -5, \ldots$

Now if

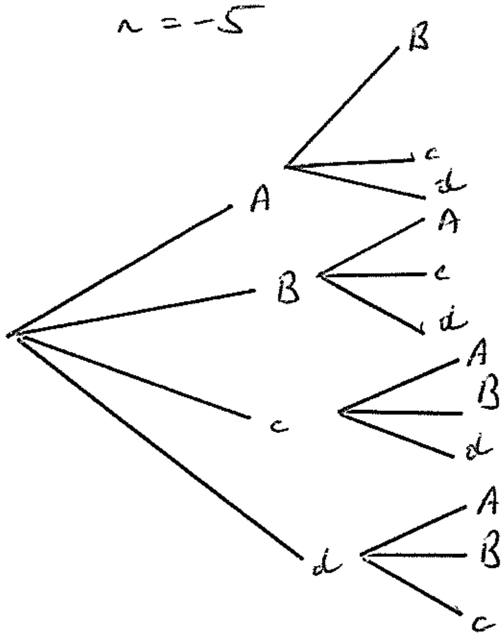
$y = -2x^n$

Since $(2, -\frac{1}{16})$ lies on curve

$-\frac{1}{16} = -2(2)^n$

$2^n = \frac{1}{32}$

$n = -5$

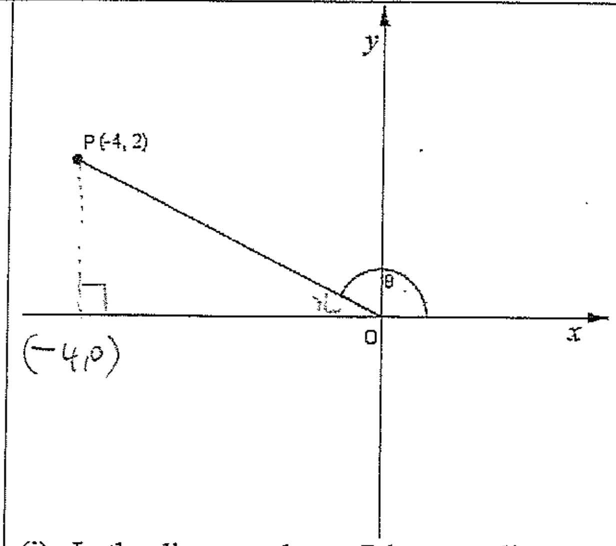
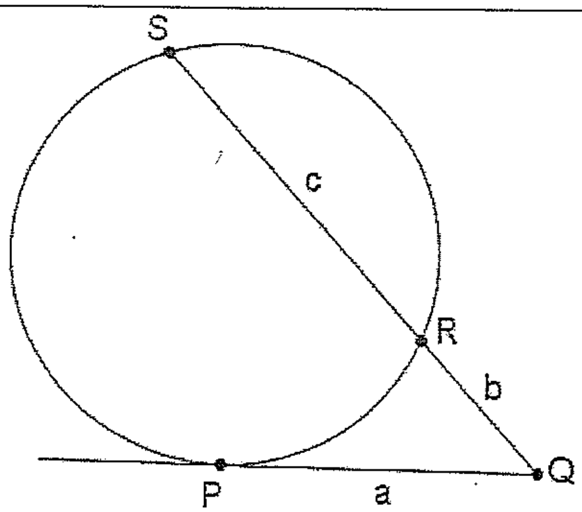


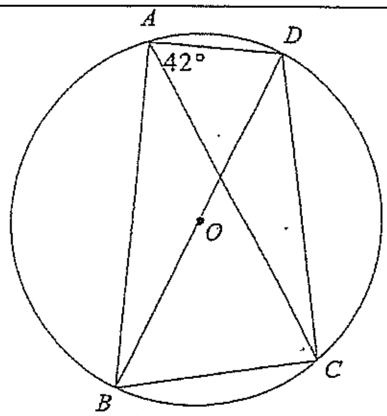
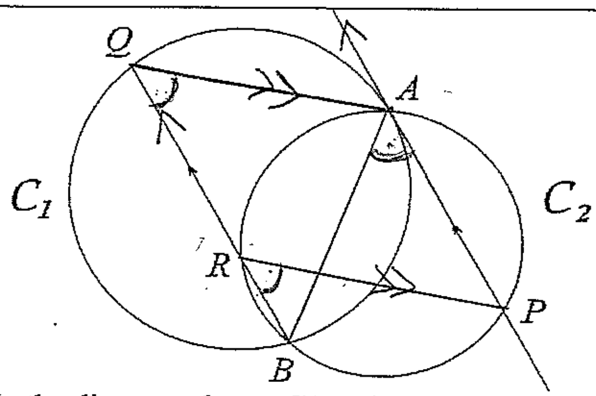
Sample Space is 12

$P(\text{2 capitals}) = \frac{2}{12} = \frac{1}{6}$

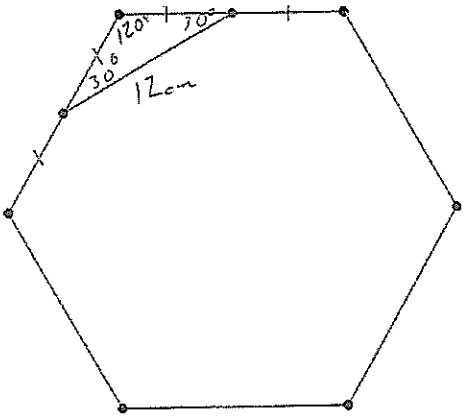
$P\left(\begin{array}{l}\text{Capital + lower case} \\ \text{in any order}\end{array}\right) = \frac{8}{12}$

$= \frac{2}{3}$

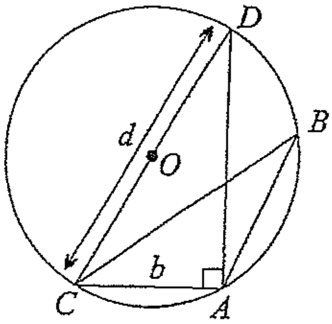
Question Six (20 marks)		Answers	Marks
A	 (i) In the diagram above, P has coordinates (–4, 2). Find length of OP. (ii) Write down the exact value of $\cos\theta$.	(i) $OP = \sqrt{2^2+4^2} = 2\sqrt{5}$ (ii) $\cos\theta = \frac{-2}{\sqrt{5}}$ or $\frac{-2\sqrt{5}}{5}$ $2\sqrt{5}$	1 1 2
B	 In the diagram above, QP is a tangent to the circle, while QRS is a secant. If QP = a units, QR = b units and RS = c units. Write down an equation (do not prove this equation) showing the relationship between a, b and c.	$a^2 = b(b+c)$ $= b^2+bc$	2

C	 The diagram above shows a cyclic quadrilateral ABCD. The diagonal BD of the quadrilateral passes through the centre O of the circle and $\angle CAD = 42°$. Find, giving reasons: (i) $\angle BAC$ (ii) $\angle BDC$	(i) $\angle BAC = 48°$ (angle in semi-circle is $90°$) (ii) $\angle BDC = 48°$ (angles at the circum. subtended by the same chord are equal)	6 3
D 1	 In the diagram above, C1 and C2 are circles intersecting at A and B. The tangent to C1 at A meets C2 at P. Q is the point on C1 so that QB is parallel to AP. The chord QB intersects C2 at R. (i) Draw in the intervals AQ, AB and PR. (ii) Give a reason why $\angle PAB = \angle AQB$. (iii) Give a reason why $\angle PAB = \angle PRB$ (iv) Explain why QA is parallel to RP. (v) Are QA and RP equal? Explain your answer.	(ii) alternate segment theorem (iii) $\angle PAB = \angle PRB$ (angles at the circum subtended by the same ~~chord~~ arc PB) (iv) $QA \parallel RP$ because $\angle AQR = \angle PRB$ (equal corres $\angle$s) (v) Yes $AP \parallel QB$ (given) $AQ \parallel PR$ (iv)	2 2 10 2 3

$\therefore QA = RP$ (parallelogram

	Question Seven (20 marks)	Answers	Marks
A	 (i) Find the size of each interior angle in a regular hexagon. (ii) In the diagram above, the length of the straight line joining the midpoints of two adjacent sides of a regular hexagon is 12cm. (α) Calculate the exact length of one side of the regular hexagon. (β) Calculate the exact area of the regular hexagon.	(i) 120° 1 (ii) $144 = 2a^2 - 2a^2\cos 120°$ (α) $144 = 3a^2$ 3 $a = 4\sqrt{3}$ 1 $8\sqrt{3}$ or $\frac{24}{\sqrt{3}}$  (β) $s = 12\sqrt{3}$ $\sqrt{12\sqrt{3}(12\sqrt{3}-8\sqrt{3})^3}$ $= \sqrt{12\sqrt{3}(4\sqrt{3})^2(4\sqrt{3})}$ $= \sqrt{144 \times 48}$ 2 3 $= 48\sqrt{3}$ Area $= 6 \times 48\sqrt{3}$ $= 288\sqrt{3}$ 1	7

B



In the diagram above, the vertices of ΔABC and ΔADC are on the circumference of a circle with centre O, and $\angle CAD = 90°$.

Let the diameter CD = d and let AC = b.

(i) Explain why $\angle ADC = \angle ABC$.

(ii) Hence show that $\dfrac{b}{\sin B} = d$.

(i) Angles in the same segment ①

(ii) $\sin D = \frac{b}{d}$. 1

$\frac{b}{\sin D} = d$. 1 ③

$\therefore \frac{b}{\sin B} = d$. 1

since $\angle D = \angle B$.

4

C

The maximum daily temperatures $(°C)$ recorded in a city over a period of 20 days are given below.

25	24	27	28	26
30	28	25	29	30
31	34	32	30	26
23	24	26	30	27

(i) Find the range of the temperatures.

(ii) Find the interquartile range of the temperatures.

(iii) Find the standard deviation, correct to 1 decimal place.

(iv) What would be the two most appropriate measures of spread for these temperatures? Why?

(i) 11°C.. 1

(ii) Q2 = 27.5

Q1 = 25.5

Q3 = 30

Q3 − Q1 = 4.5 1

(iii) 2.9 1

(iv) Range & Inter-Quartile Range. Two few for std dev to be useful. 1

4

D	(i) Prove that $(a-b)^2+(b-c)^2+(c-a)^2=2(a^2+b^2+c^2-ab-bc-ca)$ (ii) Use part (i) to prove that $a^2+b^2+c^2 \ge ab+bc+ca$. (iii) Use part (ii) to prove that $(a+b+c)^2 \ge 3(ab+bc+ca)$.	(i) $a^2-2ab+b^2+b^2-2bc+c^2+c^2-2ac+c^2$ $=2(a^2+b^2+c^2-ab-bc-ca)$ 1 (ii) $2(a^2+b^2+c^2)=2(ab+bc+ca)$ $+(a-b)^2+(b-c)^2+(c$ $\therefore a^2+b^2+c^2 \ge ab+bc+ca$ 1 since $(a-b)^2 \geqslant 0$ $(b-c)^2 \geqslant 0$ $(c-a)^2 \geqslant 0$. (iii) $a^2+b^2+c^2 \geqslant ab+bc+ca$. $(a+b+c)^2-2(ab+bc+ca) \geqslant ab+b$ $(a+b+c)^2 \geqslant 3(ab+bc+ca)$ 1	3
E	(i) Without using a calculator, explain why $7^{\frac{1}{2}}$ is less than 3. (ii) Use methods similar to part (i) to explain why $7^{\frac{1}{2}}+7^{\frac{1}{3}}+7^{\frac{1}{4}}<7$. (Once again do not use a calculator.)	$9^{\frac{1}{2}}=3 \Rightarrow 9=3^2$ $\Downarrow$ $\therefore 7^{\frac{1}{2}}<3 \Leftarrow 7<3^2$ 1 $7^{\frac{1}{2}}+7^{\frac{1}{3}}+7^{\frac{1}{4}}<9^{\frac{1}{2}}+8^{\frac{1}{3}}+16^{\frac{1}{4}}$ $=3+2+2$ $=7$.	2

This is the end of the exam.

Use this space if you wish to REWRITE any answers

Clearly indicate the QUESTION number

i)

f<a.

$9 = 3^2 \implies 7^{\frac{1}{2}} < 3$

$8 = 2^3 \implies 7^{\frac{1}{3}} < 2$

$16 = 2^4 \implies 7^{\frac{1}{4}} < 2.$