

# SYDNEY BOYS HIGH SCHOOL MoORE PARK, SURRY HILLS 

## Year 10

## Yearly Examination 2010

## Advanced Mathematics

## General Instructions

- Working time - 120 minutes
- Write using black or blue pen.
- Approved calculators may be used.
- All necessary working MUST be shown in every question if full marks are to be awarded.
- Marks may not be awarded for untidy or badly arranged work.
- If more space is required, clearly write the number of the QUESTION on one of the back pages and answer it there. Indicate that you have done so.
- All answers must be given in exact simplified form unless otherwise indicated.
- Clearly indicate your class by placing an X , next to your class

Examiner: B. Kilmore

NAME:

| Class | Teacher |  |
| :---: | :--- | :--- |
| 10 A | Mr Fuller |  |
| 10 B | Ms Nesbitt |  |
| 10 C | Ms Ward |  |
| 10 D | Ms Roessler |  |
| 10 E | Mr McQuillan |  |
| 10 F | Mr Boros |  |
| 10 G | Mr Hespe |  |


| Question | Mark |
| :---: | ---: |
| 1 | $/ 20$ |
| 2 | $/ 20$ |
| 3 | $/ 15$ |
| 4 | $/ 20$ |
| 5 | $/ 15$ |
| 6 | $/ 16$ |
| 7 | $/ 126$ |


| Question One (20 marks) |  | Answers | Marks |
| :---: | :---: | :---: | :---: |
| a | Write 23570000 in scientific notation. |  | 1 |
| b | Write the equation of the line shown below. |  | 2 |
| c | Write with a positive index: $\left(\frac{a}{b}\right)^{-2}$ |  | 1 |
| d | Write with a rational denominator in simplest form: $\frac{4}{3 \sqrt{2}}$ |  | 1 |
| e | Calculate the following, giving your answer correct to 2 decimal places. $\frac{4^{3}-5.14}{2+\sqrt{65-3.2^{2}}}$ |  | 1 |
| f | Given that $f(x)=x^{2}-3$, find the value of $f(-4)$ |  | 1 |
| g | Write the equation of a circle with centre the origin and a radius of 6 units. |  | 1 |
| h | Multiple Choice: <br> A distribution of 10 scores has a mean of 75. If the highest score is increased by 5 , the new mean will be: <br> A. 77.5 <br> B. 80 <br> C. 75.5 <br> D. cannot be determined |  | 1 |


|  | Question One (continued) | Answers | Marks |
| :---: | :---: | :---: | :---: |
| i | Solve : $3-2 x \leq 7$ |  | 2 |
| j | Find the least value of $3+(x-1)^{2}$ |  | 1 |
| k | Kelly runs the 14km City to Surf Race in 2 hours and 15 minutes. What is her speed in metres/second? Give your answer correct to two decimal places. |  | 2 |
| 1 | Sketch the region given by $\mathrm{x}^{2}+\mathrm{y}^{2}<9$ |  | 1 |
| m | Simplify $3 \sqrt{54}+\sqrt{24}$ |  | 1 |
| n | Expand and simplify: $(x-7)^{2}$ |  | 1 |
| 0 | Given that $\sin \theta=0.819$, find $\theta$ to the nearest degree if $\theta$ is an obtuse angle. |  | 1 |
| p | A letter is chosen at random from the word PARRAMATTA <br> i. What is the probability that it is a T? <br> ii. What is the probability that it is not a vowel? |  | 2 |





| Question Three (15 Marks) |  | Answers | Marks |
| :--- | :--- | :--- | :---: |
| aShade the region on the number <br> plane where <br> $x+y \leq 4$ and $2 y>3 x+6$ |  | 3 |  |
|  |  |  |  |
|  |  |  |  |


|  | Question Three (continued) <br> ii.$y=2^{x}$ <br> (Show the y-intercept as well <br> as one other point) <br>  <br> iii. <br>  <br> $y=(x-4)(x-1)(x+5)$ <br> (Indicate the intercepts) |  | Answers |
| :--- | :--- | :--- | :--- |


|  | Question Three (continued) |  | Answers |
| :--- | :--- | :--- | :---: |
| e | Find the value of the angle $a$, giving <br> reasons in full. (The centre of the <br> circle is labelled ' $\mathrm{O}^{\prime}$.) |  |  |


| Question Four (20 Marks) | Answers | Marks |
| :---: | :---: | :---: | :---: |
| aFor the parabola: <br> $y=x^{2}+2 x-8$ <br> find <br> i. <br> ine <br> ii. <br> The y-intercept <br> iii. <br> The vertex <br> iv. <br> Hence, sketch the graph |  | 4 |


|  | Question Four (continued) |  | Answers |
| :--- | :--- | :--- | :---: |
| cThe materials to make 25kg of an <br> alloy of copper and zinc cost $\$ 62$. If <br> the copper costs $\$ 3.20 / \mathrm{kg}$ and zinc <br> costs $\$ 1.40 / \mathrm{kg}$, find the composition <br> of the alloy. |  | 3 |  |


|  | Question Four (continued) |  | Answers |
| :--- | :--- | :--- | :---: |
| f | In the diagram below $\angle B O C=x^{\circ}$, <br> $\angle B D C=y^{\circ}$ and $\angle B A C=48^{\circ}$. <br> $O$ is the centre of the circle. Find the <br> values of $x^{\circ}$ and $y^{\circ}$ giving reasons <br> in full. |  | 3 |


| Question Five (20 Marks) | Answers | Marks |
| :--- | :--- | :--- | :---: |
| aSolve the following simultaneous <br> equations: <br> $y=x^{2}-5 x+8$ <br> $y=2 x-4$ |  | 3 |


|  | Question Five (continued) |  | Answers |
| :--- | :--- | :--- | :---: |
| d | Find the surface area of this object correct <br> to the nearest $c^{2}$ if it is to be coated <br> both inside and out with a rust protector. <br> The radius of the small cylinder is 5cm <br> and the radius of the large cylinder is <br> 15cm. |  |  |


|  | Ques | ion Five (continued) | Answers | Marks |
| :---: | :---: | :---: | :---: | :---: |
| f | $A B$ is an interval with $A(0,2)$ and $B(4,0)$ |  |  | 5 |
|  |  | Find the midpoint of AB |  |  |
|  |  | Find the gradient of the perpendicular bisector of $A B$ |  |  |
|  |  | Hence, or otherwise, find the equation of the perpendicular bisector of AB |  |  |



|  | Question Six (continued) |  | Answers |
| :--- | :--- | :--- | :---: |
| d | Given: $\mathrm{AB}=\mathrm{AC}$ and XAY is a tangent <br> to the circle at A. <br> Prove that BC // XY. |  |  |


|  | Question Six (continued) |  | Answers |
| :--- | :--- | :--- | :---: |
| f | A farmer has a triangular field ABC <br> which has side $\boldsymbol{a}=\mathbf{1 7 k m}$, side $\boldsymbol{b}=$ <br> $\mathbf{1 3 k m}$ and side $\boldsymbol{c}=\mathbf{1 1 \mathbf { k m } . \text { Calculate }}$ <br> the cost of fertilizer if the farmer needs <br> to use 1 tonne of fertilizer for every <br> square kilometre and fertilizer costs <br> \$155.50 per tonne (or part thereof). |  | 4 |
|  |  |  |  |


| Question Seven (16 Marks) | Answers | Marks |
| :--- | :--- | :--- | :---: |
| aTranspose the following formula to <br> make $b$ the subject. <br> $v=a\left(\frac{1}{b}-\frac{1}{c}\right)$ | 2 |  |


|  | Question Seven (continued) |  | Answers |
| :--- | :--- | :--- | :---: |
| c | The sum of the squares of two <br> consecutive positive odd integers <br> exceeds the product of the integers by <br> 147. Find them. |  | 3 |
| d | A plane leaves town $A$ and flies on a <br> bearing of $120^{\circ}$ for 600 km to point $P$. It <br> then changes direction to fly on a bearing of <br> $230^{\circ}$ until it reaches town $B$. The distance <br> between town $A$ and town $B$ is 1100km. |  |  |
| i.Draw a clear diagram showing the <br> plane's trip showing all salient <br> information.  <br> ii.  <br> Find the distance from town P to <br> town B <br> Show all angle calculations on <br> your diagram.  |  |  |  |


|  | Question Seven (continued) | Answers | Marks |
| :--- | :--- | :--- | :---: |
| e | By considering $x^{2}$ or otherwise, find the <br> value of $x$ as an integer: <br> $x=\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\ldots}}}}$ <br>  <br>  <br>  <br>  |  |  |


| Question One (20 marks) |  | Answers | Marks |
| :---: | :---: | :---: | :---: |
|  | Write 23570000 in scientific notation. | $2.357 \times 10^{7}$ | 1 |
| b | Write the equation of the line shown below. | $\begin{aligned} & y=-x+4 \\ & x+y-4=0 \end{aligned}$ | 2 |
| c | Write with a positive index: $\left(\frac{a}{b}\right)^{-2}$ | $\left(\frac{b}{a}\right)^{2}$ | 1 |
| d | Write with a rational denominator in simplest form: $\frac{4}{3 \sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}=\frac{4 \sqrt{2}}{3 \times 2}$ | $\frac{2 \sqrt{2}}{3}$ | 1 |
| e | Calculate the following, giving your answer correct to 2 decimal places. $\frac{4^{3}-5.14}{2+\sqrt{65-3.2^{2}}}$ | $6 \cdot 26$ | 1 |
| f | Given that $f(x)=x^{2}-3$, find the value of $f(-4)$ | $\begin{aligned} f(-4) & =(-4)^{2}-3 \\ & =16-3 \\ & =13 \end{aligned}$ | 1 |
| g | Write the equation of a circle with centre the origin and a radius of 6 units. | $x^{2}+y^{2}=36$ | 1 |
| h | Multiple Choice: <br> A distribution of 10 scores has a mean of 75. If the highest score is increased by 5 , the new mean will be: <br> A. 77.5 <br> B. 80 <br> C. 75.5 <br> D. cannot be determined | $\frac{10 \times 75+5}{10}=75.5$ | 1 |


|  | Question One (continued) | Answers | Marks |
| :---: | :---: | :---: | :---: |
| i | Solve: $3-2 x \leq 7$ | $\begin{aligned} -2 x & \leq 4 \\ x & \geqslant-2 \end{aligned}$ | 2 |
| j | Find the least value of $3+(x-1)^{2}$ | when $x=1$, ratue $\therefore 3$ | 1 |
| k | Kelly runs the 14 km City to Surf Race in 2 hours and 15 minutes. What is her speed in metres/second? Give your answer correct to two decimal places. | $\frac{14000 \mathrm{~m}}{2.25 \times 3600 \mathrm{~s}} \approx 1.73 \mathrm{~m} / \mathrm{s}$ | 2 |
| 1 | Sketch the region given by $\mathrm{x}^{2}+\mathrm{y}^{2}<9$ |  | 1 |
| m | Simplify $3 \sqrt{54}+\sqrt{24}=3 \sqrt{9 \times 6}+\sqrt{4 \times 6}$ | $\begin{aligned} & =9 \sqrt{6}+2 \sqrt{6} \\ & =11 \sqrt{6} \end{aligned}$ | 1 |
| n | Expand and simplify: $(x-7)^{2}$ | $x^{2}-7 x-7 x+49=x^{2}-14 x+49$ | 1 |
| o | Given that $\sin \theta=0.819$, find $\theta$ to the nearest degree if $\theta$ is an obtuse angle. $\sin ^{-1}(0.819) \div 54.98^{\circ}$ | $180^{\circ}-54.98^{\circ} \approx 125^{\circ}$ | 1 |
| p | A letter is chosen at random from the word PARRAMATTA <br> i. What is the probability that it is a T? <br> ii. What is the probability that it is not a vowel? | (i) $\frac{2}{10}=\frac{1}{5}$ <br> (ii) $\frac{6}{10}=\frac{3}{5}$ <br> or $1-\frac{4}{10}=\frac{3}{5}$ | 2 |





|  | estion Three (15 Marks) | Answers | Marks |
| :---: | :---: | :---: | :---: |
| a | Shade the region on the number plane where $x+y \leq 4$ and $2 y>3 x+6$ $\begin{aligned} & y>\frac{3}{2} x+3 \\ & y \leq-x+4 \end{aligned}$ |  | 3 |
| b | Find the value of $m$ such that $\frac{1-2 \sqrt{2}}{\sqrt{2}-1}=m-\sqrt{2}$ | $v$ $m=-3$ | 2 |
| c | Draw a sketch of the following functions: <br> i. $y=\frac{-1}{x}$ <br> (Indicate 2 points) |  | $\begin{array}{r} 4 \\ -1 \\ \hline \end{array}$ |


|  | Question Three (continued) | Answers | Marks |
| :---: | :---: | :---: | :---: |
|  | ii. $y=2^{x}$ <br> (Show the y-intercept as well as one other point) <br> iii. $y=(x-4)(x-1)(x+5)$ <br> (Indicate the intercepts) |  |  |
| d | Find $\theta$, correct to the nearest minute, in the triangle ABC drawn below. | $\begin{aligned} \frac{\sin \theta}{9} & =\frac{\sin 59}{11} \\ \sin \theta & =\frac{9 \sin \rho 4}{11} \\ \theta & =44^{\circ} 32^{1} \end{aligned}$ | 2 |



|  | uestion Four (20 Marks) | Answers | Marks |
| :---: | :---: | :---: | :---: |
| a | For the parabola: $y=x^{2}+2 x-8$ <br> find <br> i. The x -intercept <br> ii. The y-intercept <br> iii. The vertex <br> iv. Hence, sketch the graph | $\begin{aligned} & x=-4,2 \\ & y=-8 \\ & (-1,-9) \end{aligned}$  | 4 |
| b | Find the volume of this cone: | $\begin{aligned} V & =\frac{1}{3} \pi r^{2} h \\ & =\frac{1}{3} \times \pi \times 8^{2} \times 18 \\ \div & 1206.37 \mathrm{~cm}^{3} \\ & o r \\ & 384 \pi \mathrm{~cm}^{3} \end{aligned}$ | 2 |


|  | Question Four (continued) | Answers | Marks |
| :---: | :---: | :---: | :---: |
| c | The materials to make 25 kg of an alloy of copper and zinc cost $\$ 62$. If the copper costs $\$ 3.20 / \mathrm{kg}$ and zinc costs $\$ 1.40 / \mathrm{kg}$, find the composition of the alloy. | $\begin{gathered} x \text { copper }+y \text { zine }=25 \\ 3.20 x+1.40 y=62 \\ x+y=25-140 y=6200 \text { (0) } \end{gathered}$ <br> 15 lig copper <br> $10 \log$. $\operatorname{zinc}$ | 3 |
| d | Sketch this curve by first completing the square on $x$ : $\begin{aligned} & x^{2}-6 x+y^{2}=7 \\ & (x-3)^{2}+y^{2}=4^{2} \end{aligned}$ <br> centre $(3,0)$ <br> radius 4 |  | 2 |
| e | i. Prove that $\triangle L M P \\| \triangle P Q R$ <br> ii. Hence write an equation and solve it to find the value of $x$. | (i.) Equiangular $\text { (ii) } \begin{aligned} \frac{24}{40} & =\frac{x}{36} \\ x & =21.6 \end{aligned}$ | 3 |



|  | uestion Five (20 Marks) | Answers | Marks |
| :---: | :---: | :---: | :---: |
| a | Solve the following simultaneous equations: $\begin{align*} & y=x^{2}-5 x+8 \\ & y=2 x-4 \\ & 2 x-4=x^{2}-5 x+8  \tag{2}\\ & x^{2}-7 x+12=0  \tag{1}\\ & (x-3)(x-4)=0 \end{align*}$ | $\begin{aligned} & x=3,4,2 \\ & y=2 \text { or } 4 \end{aligned}$ <br> Qfor no 'y' values | 3 |
| b | Describe how the graph of $y=-(x-2)^{2}+1$ differs from the graph of $y=x^{2}$ | Moved up 1 unt MOVED RIGHT 2 UNTS REFLECTED in $x$ axi | $\begin{aligned} & D^{3} \\ & 1 \\ & 1 \end{aligned}$ |
| c | The chord of a circle to an external point $T$ cuts the circumference at Y and $Z$. A tangent from $T$ meets the circumference at $W$. <br> Given that $T Z=40 \mathrm{~cm}, Z Y=50 \mathrm{~cm}$, calculate the length of TW. <br> Give reasons for your answer. | $\begin{aligned} T W^{2} & =Y T \times z T \cdot(\text { Tand } \\ & =90 \times 40 \\ & =3600 \\ T W & =+\sqrt{3600} \\ T W & =60 \end{aligned}$ | 2 ent <br> Thn |




Find the mode of this data:
Lit =

$$
\begin{aligned}
& \frac{y-z-(x-z)+x-y}{(x-y x x-2)(y-z)} \\
& =0
\end{aligned}
$$

Cumulative Frequency Histogram


Score
c Use the remainder theorem to find the remainder for the following division:
$\left(2 x^{3}+7 x-13\right) \div(x-2)$
$P(2)=2(2)^{3}+2(2)-13$

$$
=17
$$

|  | Question Six (continued) | Answers | Marks |
| :---: | :---: | :---: | :---: |
| d | Given: $A B=A C$ and $X A Y$ is a tangent to the circle at A . <br> Prove that $\mathrm{BC} / / \mathrm{XY}$. | $\angle X A B=\angle A C B$ (HLT segment theown) <br> $\angle A C B=\angle A B C$ (base anjles of) soseles $\triangle A B C$ ) $\therefore \angle X A B=\angle A B C$ <br> $\therefore x y \\| B C$ (altemate is equai) | 3 |
| e | If $\theta$ is an acute angle and $\cos \theta=\frac{3}{7}$, Find $\sin \theta$. (Answer in simplest surd form. | $\begin{aligned} \operatorname{Sin} \theta & =\frac{\sqrt{40}}{7} \\ & =\frac{2 \sqrt{10}}{7} \end{aligned}$ | 3 |



|  | uestion Seven (16 Marks) | Answers | Marks |
| :---: | :---: | :---: | :---: |
| a | Transpose the following formula to make $b$ the subject. $v=a\left(\frac{1}{b}-\frac{1}{c}\right)$ | $\begin{aligned} & v=a\left(\frac{1}{b}-\frac{1}{c}\right) \\ & \frac{v}{a}=\frac{1}{b}-\frac{1}{c} \\ & \frac{v}{a}+\frac{1}{c}=\frac{1}{b} \\ & \frac{v c+a}{a c}=\frac{1}{b} \\ & b=\frac{a c}{v c+a} \end{aligned}$ | 2 |
| b | A lampshade is made by cutting off the top part of a cone. Find the area of material required to make this lampshade if the top opening has a radius of 7 cm and the bottom opening has a radius of 14 cm and the lampshade is 24 cm tall. | using similar triangles $\begin{aligned} \frac{x}{x+24} & =\frac{7}{14} \\ \frac{x}{x+24} & =\frac{1}{2} \\ 2 x & =x+24 \\ x & =24 \end{aligned}$ $\begin{aligned} & S_{1}^{2}=24^{2}+7^{2} \\ & S_{1}=625 \\ & S_{1}=25 \end{aligned}$ $S_{2}=50$ $\begin{aligned} \text { Area } & =\pi R S_{2}-\pi r S_{1} \\ & =\pi(14)(50)-\pi(7)(25) \\ & =525 \pi \mathrm{~cm}^{2} \\ & \approx 1649.34 \mathrm{~cm}^{2} \end{aligned}$ | 4 |


use sine rule twice: Find size of $\hat{A B P}$.
Find value of $x$.


