SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

Year 10

Yearly Examination 2013

Advanced Mathematics

General Instructions

- Working time – 120 minutes
- Write using black or blue pen.
- Approved calculators may be used.
- Marks may not be awarded for untidy or badly arranged work
- All answers should be given in simplest exact form unless specified otherwise
- If more space is required, clearly write the number of the QUESTION on the back page and answer it there. Indicate that you have done so
- Clearly indicate your class by placing an X, next to your class

Examiner: *A. Fuller*

Total Marks – 118

- Attempt questions 1 – 7
- Each question has 5 multiple choice, followed by extended response.
- CIRCLE the correct answer for the multiple choice. (A), (B), (C) or (D)
- Extended response should include relevant mathematical reasoning and/or calculations.

NAME: ______________________________

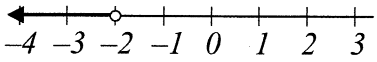
Class	Teacher	
10 A	Mr Boros	
10 B	Ms Ward	
10 C	Ms Millar	
10 D	Ms Nesbitt/Ms Likourezos	
10 E	Mr Hespe	
10 F	Mr Elliott/Ms Chen	
10 G	Mr Gainford	

Question	Mark
1	/18
2	/18
3	/18
4	/18
5	/16
6	/15
7	/15
Total	**/118**

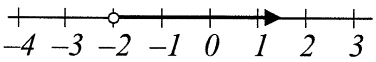
Question One (18 marks)

(a) Which graph illustrates the solution of $-3x > 6$?

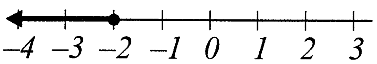
(A)



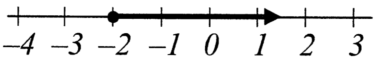
(B)



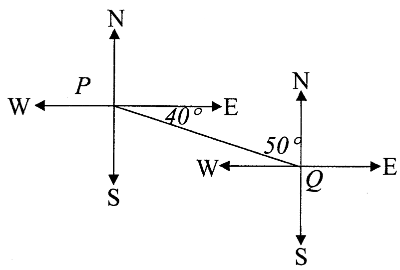
(C)



(D)



(b) The bearing of P from Q is

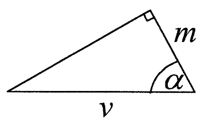


(A) 040° (B) 050° (C) 130° (D) 310°

(c) If $P(x)$ is of degree m and $Q(x)$ is of degree n, where $m > n$. $P(x) \times Q(x)$ is of degree:

(A) m (B) $m + n$ (C) $m \times n$ (D) n

(d)



(A) $m = v \sin \alpha$ (B) $v = m \sin \alpha$ (C) $m = v \cos \alpha$ (D) $v = m \cos \alpha$

(e) Which expression shows the product of p factors, each of which is m?

(A) pm (B) p^m (C) m^p (D) $p + m$

Question One (continued)

(f) $P(x) = 2x^2(2x-7)(2x+7)$. [2]

(I) What is the leading coefficient?

(II) What is the constant term?

(g) Circle the expressions which are polynomials: [1]

$3x + \frac{1}{x}$, $6x^5 + \sqrt{5}x$, 3, $2^x + 1$, $x^3 + x\sqrt{x}$

(h) Solve the following: [4]

(I) $x^2 - 9 = 0$

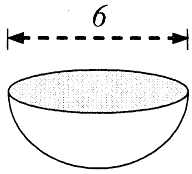
(II) $x^2 - 5x + 4 = 0$

(III) $3x^2 - 5x + 2 = 0$

(i) Find a quadratic equation in the form $x^2 + bx + c = 0$ which has [1]

solutions $x = -1$ and $x = 3$.

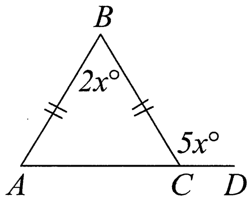
(j) [2]



(I) Calculate the volume of this solid hemisphere in cubic units.

(II) Calculate the surface area of this solid hemisphere in square units.

(k) $AB = BC$, $\angle ABC = 2x°$, $\angle BCD = 5x°$ [2]



(I) Find the size of $\angle ACB$ in terms of x (no reasons required)

(II) Hence, or otherwise, find the value of x.

Question Two (18 marks)

(a) What is the gradient of the line $2x - 3y + 7 = 0$?

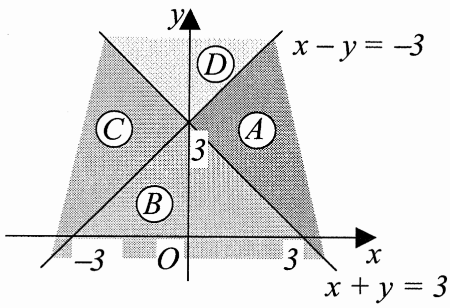
(A) $-\frac{3}{2}$ (B) $-\frac{2}{3}$ (C) $\frac{3}{2}$ (D) $\frac{2}{3}$

(b) $9x^2 - 4y^2 =$

(A) $(3x - 2y)^2$ (B) $(9x - 4y)^2$

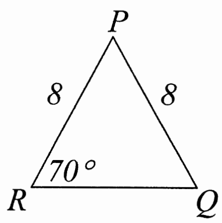
(C) $(3x - 2y)(3x + 2y)$ (D) $(9x - 4y)(9x + 4y)$

(c) Which region satisfies both $x - y \leq -3$ and $x + y \geq 3$?



(A) A (B) B (C) C (D) D

(d) Which expression gives the area of ΔPQR?



(A) $\frac{1}{2} \times 8 \times 8 \times \cos 70^\circ$ (B) $\frac{1}{2} \times 8 \times 8 \times \sin 70^\circ$

(C) $\frac{1}{2} \times 8 \times 8 \times \sin 40^\circ$ (D) $\frac{1}{2} \times 8 \times 8 \times \cos 40^\circ$

(e) Solve for x: $2x^2 - 5x - 1 = 0$

(A) $x = \frac{5 \pm \sqrt{17}}{4}$ (B) $x = \frac{-5 \pm \sqrt{17}}{4}$

(C) $x = \frac{5 \pm \sqrt{33}}{4}$ (D) $x = \frac{-5 \pm \sqrt{33}}{4}$

Question Two (continued)

(f) What is the remainder when $x^3 - 5x^2 + 2x + 5$ is divided by $x - 2$? [1]

(g) $1200 is invested for 10 years compounded annually at 4% p.a. [3]

(I) What is the final value of the investment?

(II) How much interest is earned in the 10 years?

(III) What is the equivalent simple interest rate?

(h) What is the equation of the axis of symmetry of the parabola [1]

$y = x^2 - 6x + 10$?

(i) $P(x) = 2x + 5$ and $Q(x) = x^3 - 7x + 4$. Find: [2]

(I) $P(x) - Q(x)$

(II) $P(x) \times Q(x)$

(j) Simplify $\frac{1}{x+2}+\frac{1}{x}$ [2]

(k) Sketch the following (on the axes provided): [4]

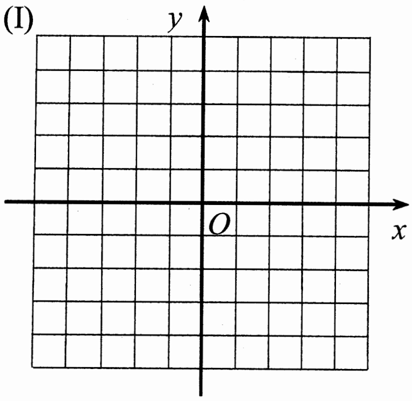
(I) $x+2y=2$

(II) $y=2x^2$

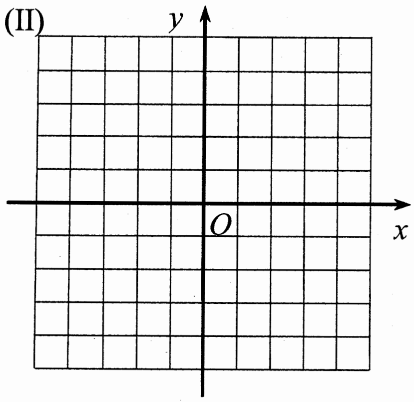
(III) $y=2^{-x}$

(IV) $xy=2$

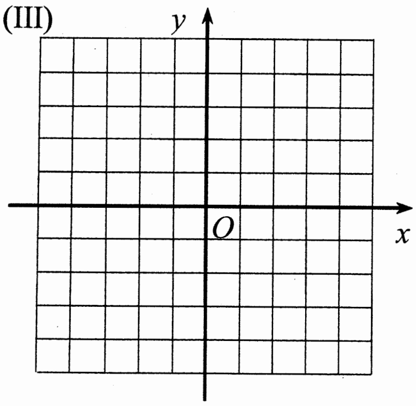
(I)



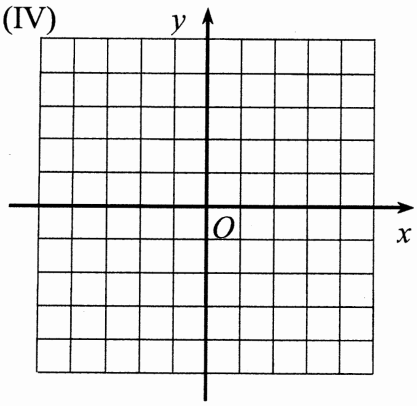
(II)



(III)



(IV)



Question Three (18 marks)

(a) Here are two statements:

I. $x^2 = 9x$ has 2 solutions

II. $x^2 = 9$ has 2 solutions

Which must be true?

(A) I only (B) II only

(C) I and II (D) Neither I nor II

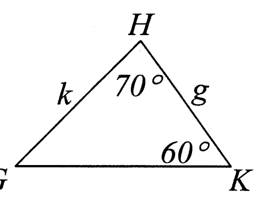
(b) $\left(\sqrt{5} - \sqrt{3}\right)^2 =$

(A) 2 (B) $2 - 2\sqrt{15}$ (C) 8 (D) $8 - 2\sqrt{15}$

(c) Convert 0.0035 cubic metres into cubic centimetres.

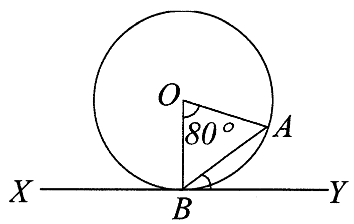
(A) 0.35 (B) 3.5 (C) 35 (D) 3500

(d) $\frac{k}{g} = ?$



(A) $\frac{\sin 50^\circ}{\sin 60^\circ}$ (B) $\frac{\sin 60^\circ}{\sin 50^\circ}$ (C) $\frac{\sin 50^\circ}{\sin 70^\circ}$ (D) $\frac{\sin 60^\circ}{\sin 70^\circ}$

(e)



The centre of the circle is O. XY is a tangent to the circle at B. $\angle BOA = 80°$.

The size of $\angle ABY$ in degrees is

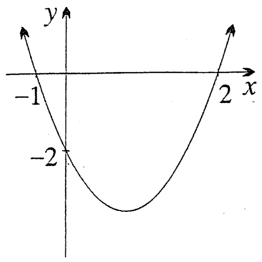
(A) 10 (B) 40 (C) 50 (D) 80

Question Three (continued)

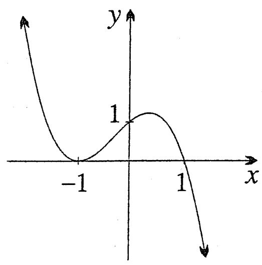
(f) Write down the equation of the following curves. [2]

(You may leave the equation in factored form)

(I)



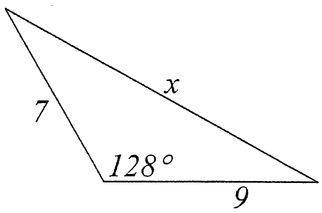
(II)



(g) What value of x would give a mean of 7 for the scores in this frequency distribution table? [1]

Score	Frequency
6	x
9	*7*

(h) Find the value of x correct to one decimal place. [2]



(i) What single percentage decrease has the same effect as three successive 10% reductions? [2]

(j) Solve $x^2 + 4x - 1 = 0$ by completing the square. [2]

(k) $P(x) = x^3 + 2x^2 - 11x + 25$ and $A(x) = x + 5$. [3]

Find $P(x) \div A(x)$, and hence, express $P(x)$ in the form $A(x) \times Q(x) + R$

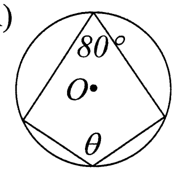
(l) Daniel's piano is currently valued at \$8800. Its value depreciates at the rate of 7.5% p.a. What will be the value of his piano in 2 years time? [1]

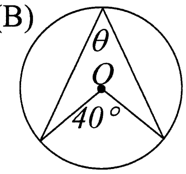
Question Four (18 marks)

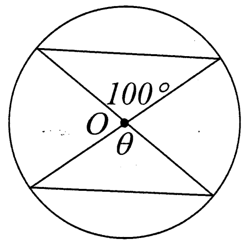
(a) What is 0.050143 correct to three significant figures?

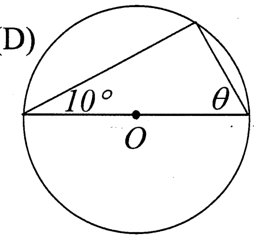
(A) 0.05 (B) 0.050 (C) 0.0501 (D) 0.05014

(b) O is the centre of each circle. In which diagram does θ equal $80°$?

(A)


(B)


(C)


(D)


(c) If $a < b < 1$ then which of the following statements is always true?

(A) $\frac{1}{a} > \frac{1}{b}$ (B) $-a > -b$ (C) $b^2 > a^2$ (D) $b^2 > ab$

(d) If $\sqrt{A} = n$, then $2A =$

(A) $2\sqrt{n}$ (B) $\sqrt{2n}$ (C) $2n^2$ (D) $4n^2$

(e) The same class sat for tests in English, Mathematics and Science. Eric's results are shown below:

TEST	CLASS MEAN	CLASS STANDARD DEVIATION	ERIC'S MARK
ENGLISH	75	5	80
MATHEMATICS	55	15	80
SCIENCE	60	10	80

In which test did Eric perform best, compared to the rest of his class?

(A) English (B) Mathematics

(C) Science (D) He performed as well in all three tests

Question Four (continued)

(f) Solve the following equations simultaneously: [2]

$3x + 2y = 7,\ x - y = 4$

(g) For the set of scores: 27, 28, 28, 33, 34, 38, 41, 43, 46, 52, 55, 56. [2]

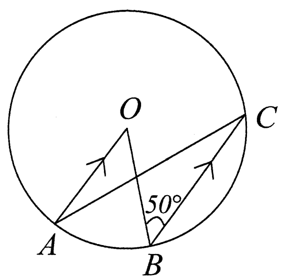
Calculate:

(I) the range

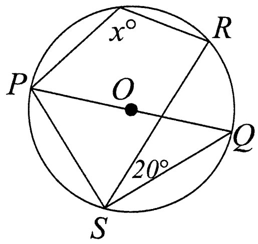
(II) the inter-quartile range

(h) *O* is the centre of the circle. *OA* is parallel to *CB*. [3]

Find the size of $\angle OAC$ giving reasons.



(i) [2]

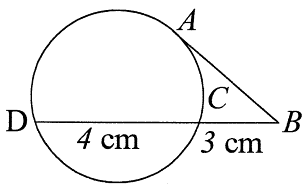


PQ and RS are straight lines. O is the centre of the circle.

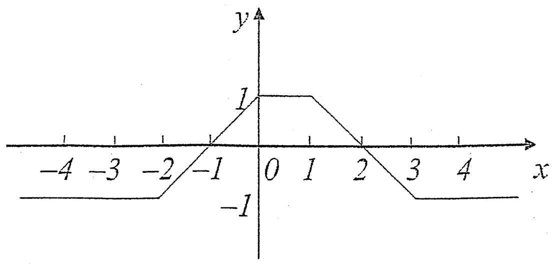
Find the value of x (giving reasons)

(j) AB is a tangent. $BC = 3$ cm, $CD = 4$ cm. [1]

Find the exact length of AB (no reasons required)



(k) The graph of $y = f(x)$ is given below. [3]



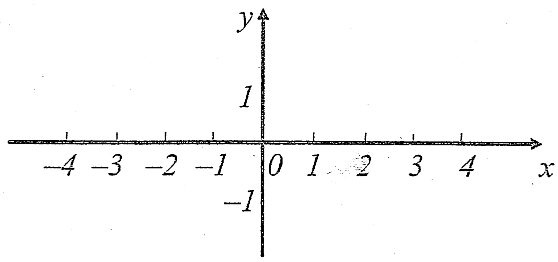
Sketch the following (on the axes provided):

(I) $y = -f(x)$

(II) $y = f(x) - 1$

(III) $y = f(x - 1)$

(I)



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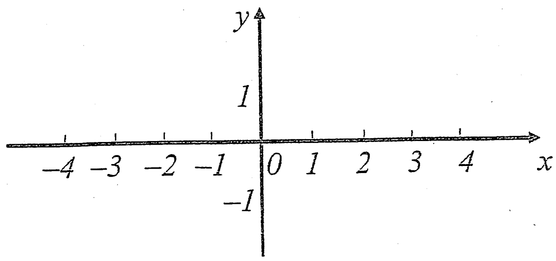
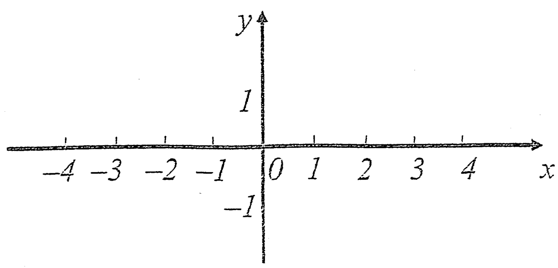
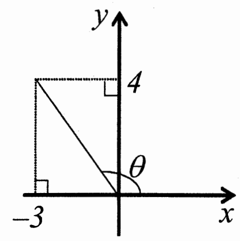


(III)



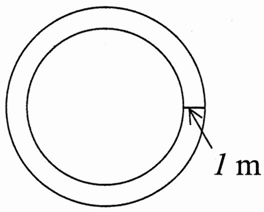
Question Five (16 marks)

(a) $\sin\theta = ?$



(A) $-\frac{4}{5}$ (B) $-\frac{3}{5}$ (C) $\frac{3}{5}$ (D) $\frac{4}{5}$

(b)



The diagram shows a circular running track, *1* metre wide. Two athletes circle the track once. One athlete runs on the inside line and the other on the outside line. What is the difference between the distances run by each athlete?

(A) It depends on the radius of the track (B) *1* m

(C) π m (D) 2π m

(c) Rationalize the denominator of $\dfrac{1}{\sqrt{7}-2}$.

(A) $\dfrac{\sqrt{7}-2}{3}$ (B) $\dfrac{\sqrt{7}+2}{3}$

(C) $\dfrac{\sqrt{7}-2}{5}$ (D) $\dfrac{\sqrt{7}+2}{5}$

(d) If $x:y = 1:2$ and $x:z = 3:5$ then $y:z =$?

(A) $2:5$ (B) $3:10$ (C) $5:6$ (D) $6:5$

(e) A steel cube with side length *3cm* has a mass of *210.6 g*.

What is the mass of $1cm^3$ of this steel?

(A) *7.8g* (B) *23.4g* (C) *35.1g* (D) *70.2g*

Question Five (continued)

(f) Simplify [3]

$$\frac{1-\frac{2}{x+1}}{x-\frac{2}{x+1}}$$

(g) The heights of two similar figures are $1.6m$ and $1.8m$. [3]

(I) If the volume of the smaller figure is $10.08m^3$.

Find the volume of the larger figure.

(II) If $800mL$ of paint is needed to give the smaller figure two coats of paint.

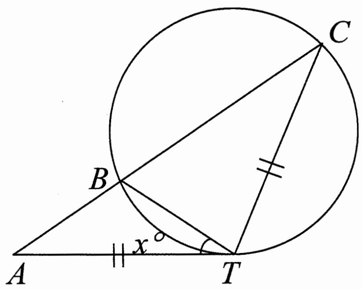
How much is required to give the larger figure two coats of paint?

(h) $P(x) = x^3 - x^2 - 16x - 20.$ [3]

(I) Show that $x + 2$ is a factor of $P(x)$.

(II) Hence, or otherwise, solve $P(x) = 0$.

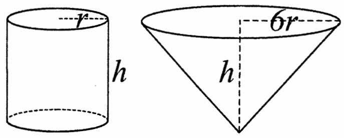
(i) AT is a tangent to the circle at T. AC cuts the circle at B. $TA = TC$. [2]



If $\angle ATB = x°$, prove that $\angle CBT = 2x°$.

Question Six (15 marks)

(a)



What is the ratio of the volume of the cylinder to the volume of the cone?

(A) $1:36$ (B) $1:12$ (C) $1:6$ (D) $1:2$

(b) $P(a,b)$ is in the first quadrant (ie. $a>0, b>0$).

$Q(c,d)$ is in the third quadrant (ie. $c<0, d<0$).

The midpoint of PQ is

(A) $\left(\frac{a+b}{2},\frac{c+d}{2}\right)$ (B) $\left(\frac{c-a}{2},\frac{d-b}{2}\right)$

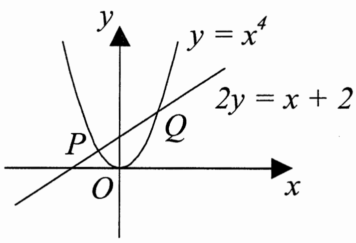
(C) $\left(\frac{a-c}{2},\frac{b-d}{2}\right)$ (D) $\left(\frac{a+c}{2},\frac{b+d}{2}\right)$

(c) Make G the subject of the formula $E=1-\sqrt{\frac{G}{R}}$.

(A) $G=R(1+E)^2$ (B) $G=R(1+E^2)$

(C) $G=R(1-E^2)$ (D) $G=R(1-E)^2$

(d)



The graph shows $y=x^4$ and $2y=x+2$ intersecting at P and Q.

The x values at P and Q are the solutions of

(A) $x^4-x-2=0$ (B) $x^4+x+2=0$

(C) $2x^4+x+2=0$ (D) $2x^4-x-2=0$

(e) Three students are playing a game. They each toss a coin at the same time. A winner is declared if only one student tosses a head.

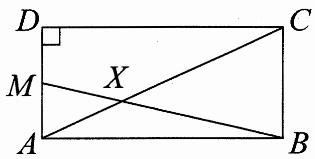
What is the probability that a winner is declared?

(A) $\frac{1}{8}$ (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) $\frac{3}{8}$

Question Six (continued)

(f) Factorise $x(x-1) - y(y-1)$ [2]

(g) In the diagram $ABCD$ is a rectangle and $AB = 2AD$. The point M is the midpoint of AD. The line BM meets AC at X. [5]



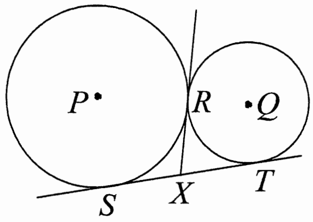
(I) Show that the triangles AXM and BXC are similar.

(II) Show that $3CX = 2AC$.

(III) Show that $9(CX)^2 = 5(AB)^2$.

(h) Two circles touch externally at R. ST is a common tangent. [3]

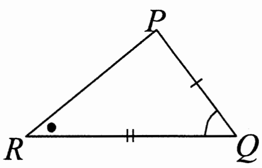
The common tangent at R meets ST in X.



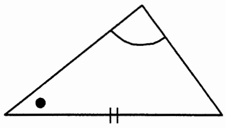
Prove that $\angle SRT$ is a right angle.

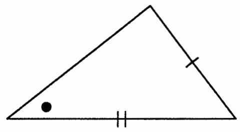
Question Seven (15 marks)

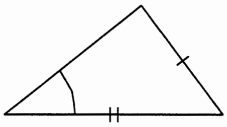
(a)

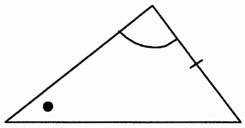


Which triangle below must be congruent to triangle PQR?

(A) 

(B) 

(C) 

(D) 

(b) The mean and standard deviation of a set of test scores are m and s respectively. If 4 marks are added to each score, what are the mean and standard deviation of the new set of scores?

(A) Mean = m, Standard Deviation = s

(B) Mean = $m + 4$, Standard Deviation = s

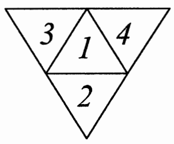
(C) Mean = m, Standard Deviation = $s + 4$

(D) Mean = $m + 4$, Standard Deviation = $s + 4$

(c) Which of the following triangles has the greatest area? With sides:

(A) 3, 4, 3 (B) 3, 4, 4 (C) 3, 4, 5 (D) 3, 4, 6

(d) The net of a die is shown below.



The faces are numbered $1, 2, 3$ and 4. The die is rolled twice. The number on the face that the die lands on is recorded each time. Find the probability that the sum of the two recorded numbers is 4.

(A) $\frac{1}{16}$ (B) $\frac{1}{8}$ (C) $\frac{3}{16}$ (D) $\frac{1}{4}$

(e) Each of the numbers $1, 2, 3, 4$ is assigned, in some order p, q, r, s.

What is the largest value of $p^q + r^s$

(A) 12 (B) 19 (C) 66 (D) 83

Question Seven (continued)

(f) The incircle of a triangle is the circle that is inscribed inside the triangle such that it touches each side of the triangle. [3]

Find the radius, r, of the incircle of a triangle with sides 3 cm, 4cm and 5 cm.

(g) 100 tickets are sold in a raffle. A ticket is drawn for first prize and then discarded. A second ticket is drawn for second prize, discarded and so on until all prizes have been given out.

If Andy has 2 tickets in the raffle:

(I) What is the probability of Andy winning 2^{nd} prize? [1]

(II) How many prizes are needed for Andy to have (at least) a 50% chance of winning a prize? [3]

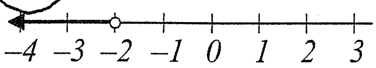
(h) What is the minimum value of 4^{x^2+x} in simplest exact form? [3]

END OF EXAM

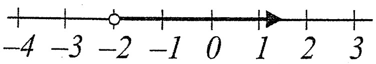
Question One (18 marks)

(a) Which graph illustrates the solution of $-3x > 6$?

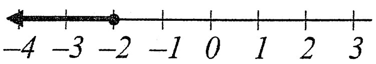
(A)



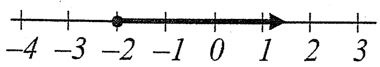
(B)



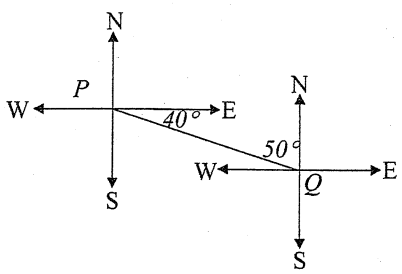
(C)



(D)



(b) The bearing of P from Q is

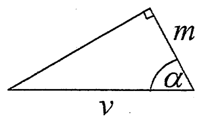


(A) 040° (B) 050° (C) 130° (D) 310°

(c) If $P(x)$ is of degree m and $Q(x)$ is of degree n, where $m > n$. $P(x) \times Q(x)$ is of degree:

(A) m (B) $m + n$ (C) $m \times n$ (D) n

(d)



(A) $m = v \sin \alpha$ (B) $v = m \sin \alpha$ (C) $m = v \cos \alpha$ (D) $v = m \cos \alpha$

(e) Which expression shows the product of p factors, each of which is m?

(A) pm (B) p^m (C) m^p (D) $p + m$

Question One (continued)

(f) $P(x) = 2x^2(2x-7)(2x+7)$. [2]

(I) What is the leading coefficient?

8

(II) What is the constant term?

0

(g) Circle the expressions which are polynomials: ~~[1]~~ [2]

$3x + \frac{1}{x}$, $6x^5 + \sqrt{5}x$, 3, $2^x + 1$, $x^3 + x\sqrt{x}$

(h) Solve the following: [4]

(I) $x^2 - 9 = 0$

$(x-3)(x+3) = 0$ $x = -3, 3$

(II) $x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$ $x = 1, 4$

(III) $3x^2 - 5x + 2 = 0$

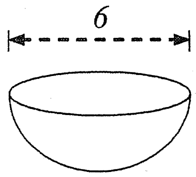
$(3x-2)(x-1) = 0$ $x = \frac{2}{3}, 1$

(i) Find a quadratic equation in the form $x^2 + bx + c = 0$ which has [1]

solutions $x = -1$ and $x = 3$.

$(x-3)(x+1) = 0$ $x^2 - 2x - 3 = 0$

(j) [2]



(I) Calculate the volume of this solid hemisphere in cubic units.

$r=3 \quad V=\frac{1}{2}\times\frac{4}{3}\times\pi r^3$

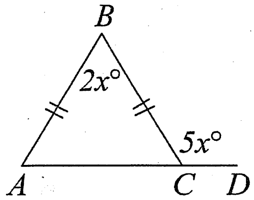
$=\frac{2}{3}\pi(3)^2 = 18\pi\ u^3 \doteqdot 56.55\ u^3$

(II) Calculate the surface area of this solid hemisphere in square units.

$SA=\frac{1}{2}\times 4\pi r^2+\pi r^2$

$=3\pi r^2 = 27\pi\ u^2 \doteqdot 84.82\ u^2$

(k) $AB = BC,\ \angle ABC = 2x°,\ \angle BCD = 5x°$ [2]



(I) Find the size of $\angle ACB$ in terms of x (no reasons required)

$8x = 180$

$\angle ACB = 180-5x$ OR $90-x$

OR $3x$

(II) Hence, or otherwise, find the value of x.

$x = 22°30'$

Question Two (18 marks)

(a) What is the gradient of the line $2x - 3y + 7 = 0$?

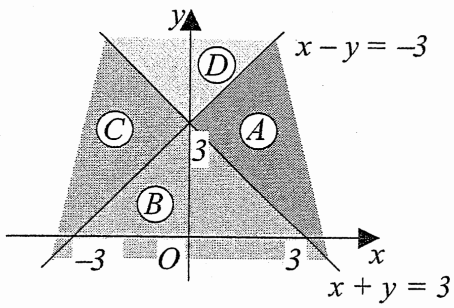
(A) $-\frac{3}{2}$ (B) $-\frac{2}{3}$ (C) $\frac{3}{2}$ (D) $\frac{2}{3}$

(b) $9x^2 - 4y^2 =$

(A) $(3x - 2y)^2$ (B) $(9x - 4y)^2$

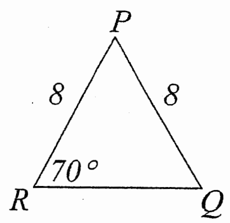
(C) $(3x - 2y)(3x + 2y)$ (D) $(9x - 4y)(9x + 4y)$

(c) Which region satisfies both $x - y \leq -3$ and $x + y \geq 3$?



(A) A (B) B (C) C (D) D

(d) Which expression gives the area of ΔPQR?



(A) $\frac{1}{2} \times 8 \times 8 \times \cos 70^\circ$ (B) $\frac{1}{2} \times 8 \times 8 \times \sin 70^\circ$

(C) $\frac{1}{2} \times 8 \times 8 \times \sin 40^\circ$ (D) $\frac{1}{2} \times 8 \times 8 \times \cos 40^\circ$

(e) Solve for x: $2x^2 - 5x - 1 = 0$

(A) $x = \frac{5 \pm \sqrt{17}}{4}$ (B) $x = \frac{-5 \pm \sqrt{17}}{4}$

(C) $x = \frac{5 \pm \sqrt{33}}{4}$ (D) $x = \frac{-5 \pm \sqrt{33}}{4}$

Question Two (continued)

(f) What is the remainder when $x^3 - 5x^2 + 2x + 5$ is divided by $x - 2$? [1]

$2^3 - 5(2)^2 + 2(2) + 5 = \boxed{-3}$

(g) $1200 is invested for 10 years compounded annually at 4% p.a. [3]

(I) What is the final value of the investment?

$1200(1 + 0.04)^{10}$

$= \boxed{\$1776.29}$

(II) How much interest is earned in the 10 years?

$\text{Interest} = 1776.29 - 1200 = \boxed{\$576.29}$

(III) What is the equivalent simple interest rate?

$576.29 = 1200(10)r$

$r = 4.8\%$

(h) What is the equation of the axis of symmetry of the parabola [1]

$y = x^2 - 6x + 10$?

$x = \frac{-b}{2a}$

$\boxed{x = 3}$

(i) $P(x) = 2x + 5$ and $Q(x) = x^3 - 7x + 4$. Find: [2]

(I) $P(x) - Q(x)$

$= \boxed{1 + 9x - x^3}$

(II) $P(x) \times Q(x) = (2x + 5)(x^3 - 7x + 4)$

$= 2x^4 - 14x^2 + 8x + 5x^3 - 35x + 20$

$= \boxed{2x^4 + 5x^3 - 14x^2 - 27x + 20}$

(j) Simplify $\frac{1}{x+2} + \frac{1}{x}$ [2]

$= \frac{x + x + 2}{x(x+2)}$

$= \frac{2x + 2}{x(x+2)}$

$= \frac{2(x+1)}{x(x+2)}$

(k) Sketch the following (on the axes provided): [4]

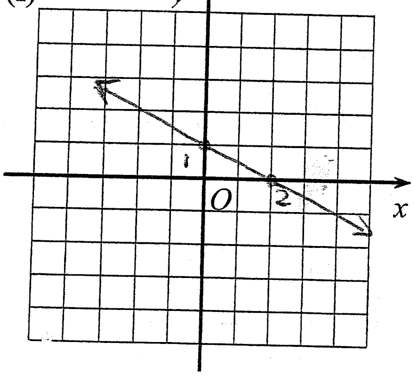
(I) $x + 2y = 2$

(II) $y = 2x^2$

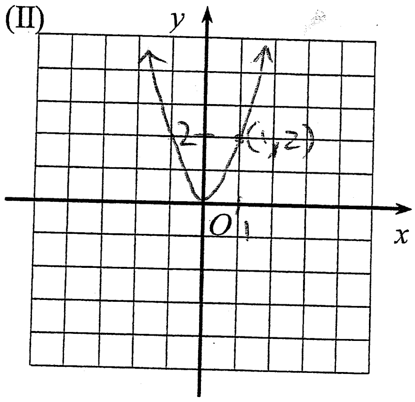
(III) $y = 2^{-x}$

(IV) $xy = 2$

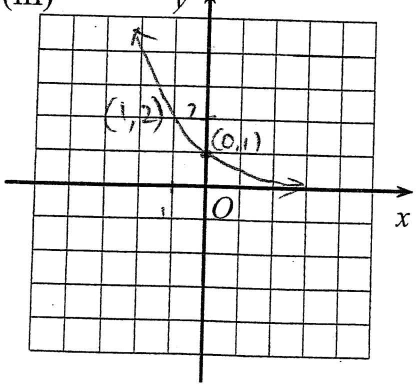
(I)



(II)



(III)



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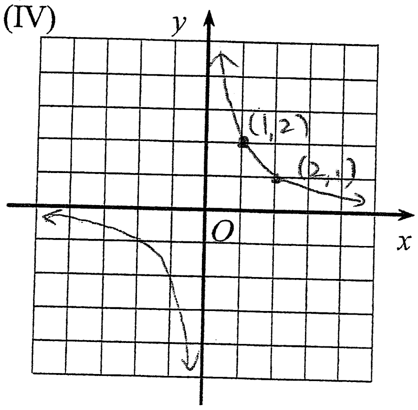


Question Three (18 marks)

(a) Here are two statements:

I. $x^2 = 9x$ has 2 solutions $x(x-9)=0$

II. $x^2 = 9$ has 2 solutions $x = \pm 3$

Which must be true?

(A) I only (B) II only

(C) I and II (D) Neither I nor II

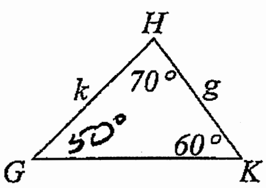
(b) $(\sqrt{5}-\sqrt{3})^2 =$ $5 - 2\sqrt{15} + 3$

(A) 2 (B) $2-2\sqrt{15}$ (C) 8 (D) $8-2\sqrt{15}$

(c) Convert 0.0035 cubic metres into cubic centimetres. 0.0035×100^3

(A) 0.35 (B) 3.5 (C) 35 (D) 3500

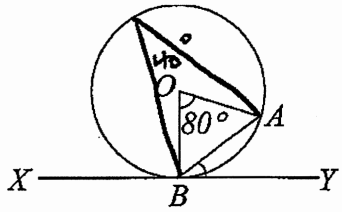
(d) $\frac{k}{g} = ?$



$\frac{k}{\sin 60°} = \frac{g}{\sin 50°}$

(A) $\frac{\sin 50^\circ}{\sin 60^\circ}$ (B) $\frac{\sin 60^\circ}{\sin 50^\circ}$ (C) $\frac{\sin 50^\circ}{\sin 70^\circ}$ (D) $\frac{\sin 60^\circ}{\sin 70^\circ}$

(e)



The centre of the circle is O. XY is a tangent to the circle at B. $\angle BOA = 80°$.

The size of $\angle ABY$ in degrees is

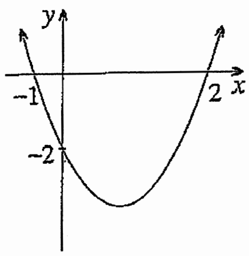
(A) 10 (B) 40 (C) 50 (D) 80

Question Three (continued)

(f) Write down the equation of the following curves. [2]

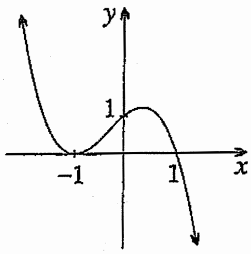
(You may leave the equation in factored form)

(I)



$y = (x+1)(x-2)$

(II)



$y = (x+1)^2(x-1)\times(-1)$

$= (x+1)^2(1-x)$

(g) What value of x would give a mean of 7 for the scores in this frequency distribution table? [1]

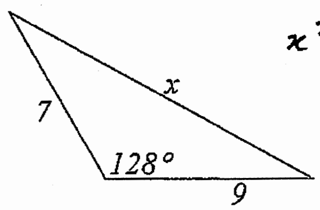
Score	Frequency
6	x
9	7

$\frac{6x+63}{x+7} = 7$

$6x+63 = 7x+49$

$x = 14$

(h) Find the value of x correct to one decimal place. [2]



$x^2 = 7^2 + 9^2 - 2\times7\times9\times\cos 128°$

≈ 207.573

$\therefore x \approx 14.4$

(i) What single percentage decrease has the same effect as three successive 10% reductions? [2]

$(0.9)^3 = 0.729$

$1 - 0.729 = 0.271$

$\therefore$ 27.1%

(j) Solve $x^2 + 4x - 1 = 0$ by completing the square. [2]

$x^2 + 4x + 4 = 1 + 4$

$(x+2)^2 = 5$

$x + 2 = \pm\sqrt{5}$

$x = -2 \pm \sqrt{5}$

(k) $P(x) = x^3 + 2x^2 - 11x + 25$ and $A(x) = x + 5$. [3]

Find $P(x) \div A(x)$, and hence, express $P(x)$ in the form $A(x) \times Q(x) + R$

$$\begin{array}{r|l} & x^2 - 3x + 4 \\ \hline x+5 & x^3 + 2x^2 - 11x + 25 \\ & x^3 + 5x^2 \\ \hline & -3x^2 - 11x \\ & -3x^2 - 15x \\ \hline & 4x + 25 \\ & 4x + 20 \\ \hline & 5 \end{array}$$

or

	1	2	-11	25
-5		-5	15	-20
	1	-3	4	(5)

$\therefore P(x) = (x+5)(x^2 - 3x + 4) + 5$

(l) Daniel's piano is currently valued at \$8800. Its value depreciates at the rate of 7.5% p.a. What will be the value of his piano in 2 years time? [1]

Value $= \$8800\left(1 - \frac{7.5}{100}\right)^2$

$= \$7529.50$

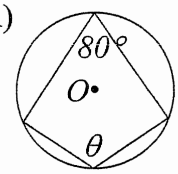
Question Four (18 marks)

(a) What is $0 \cdot 050143$ correct to three significant figures?

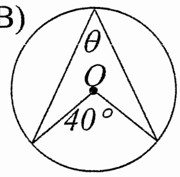
(A) $0 \cdot 05$ (B) $0 \cdot 050$ (C) $0 \cdot 0501$ (D) $0 \cdot 05014$

(b) O is the centre of each circle. In which diagram does θ equal $80°$?

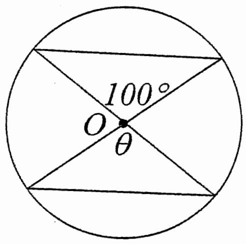
(A)



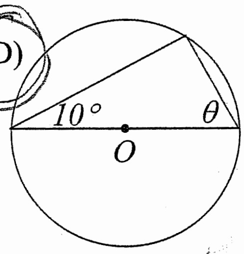
(B)



(C)



(D)



(c) If $a < b < 1$ then which of the following statements is always true?

(A) $\frac{1}{a} > \frac{1}{b}$ (B) $1 - a > 1 - b$ (C) $a + 1 > b - 1$ (D) $a - 1 > b + 1$

(d) If $\sqrt{A} = n$, then $2A =$

(A) $2\sqrt{n}$ (B) $\sqrt{2n}$ (C) $2n^2$ (D) $4n^2$

(e) The same class sat for tests in English, Mathematics and Science. Eric's results are shown below:

TEST	CLASS MEAN	CLASS STANDARD DEVIATION	ERIC'S MARK
ENGLISH	75	5	80
MATHEMATICS	55	15	80
SCIENCE	60	10	80

In which test did Eric perform best, compared to the rest of his class?

(A) English (B) Mathematics

(C) Science (D) She performed as well in all three tests

Question Four (continued)

(f) Solve the following equations simultaneously: [2]

$3x + 2y = 7, x - y = 4$

3x − 3y = 12   x + 1 = 4

5y = − 15

y = −1   x = 3

(g) For the set of scores: 27, 28, 28, | 33, 34, 38, | 41, 43, 46, | 52, 55, 56. [2]

Calculate:

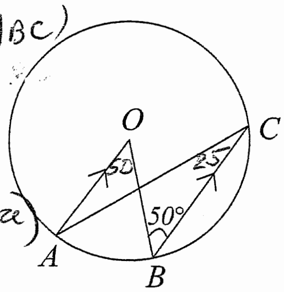
(I) the range   56 − 27   29

(II) the inter-quartile range   18½

(h) O is the centre of the circle. OA is parallel to CB. [3]

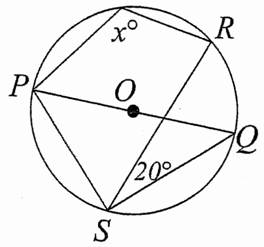
Find the size of $\angle OAC$ giving reasons.

∠AOB = 50 (ALT ∠s OA ∥ BC)
∠ACB = 25 (∠ at centre on same arc = 2× ∠ at circumference)



∠OAC = 25 (Alt. to ∠ACB OA ∥ BC)

(i) [2]



PQ and RS are straight lines. O is the centre of the circle.

Find the value of x (giving reasons)

PSR = 180 − x (cyclic quad PRQS)

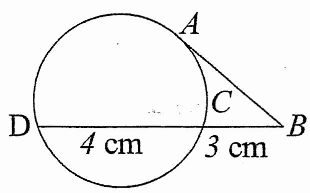
PSR = 70° (PSQ = 90° − ∠ in semicircle)

x = 180 − PSR

x = 110°

(j) AB is a tangent. $BC = 3$ cm, $CD = 4$ cm. [1]

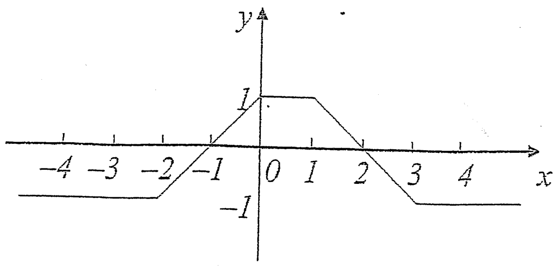
Find the exact length of AB (no reasons required)



$AB^2 = 7 \times 3$

$AB = \sqrt{21}$

(k) The graph of $y = f(x)$ is given below. [3]



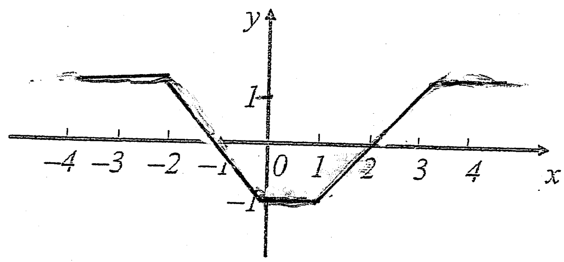
Sketch the following (on the axes provided):

(I) $y = -f(x)$

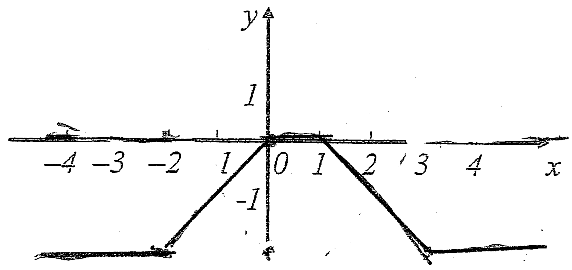
(II) $y = f(x) - 1$

(III) $y = f(x-1)$

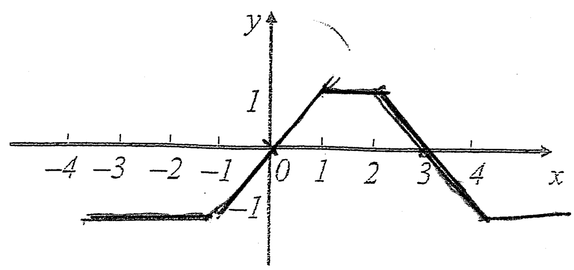
(I)



(II)



(III)



Y10 Yearly Question 5 - Solutions

a) $\sin\theta = \sin(180-\theta)$

$= \frac{4}{5}$ (D)

b) inner circle: $C = 2\pi r$

outer circle: $C = 2\pi(r+1)$

$= 2\pi r + 2\pi$

difference: outer - inner

$= 2\pi r + 2\pi - 2\pi r$

$= 2\pi$ m (D)

c) $\frac{1}{\sqrt{7}-2} \times \frac{\sqrt{7}+2}{\sqrt{7}+2}$

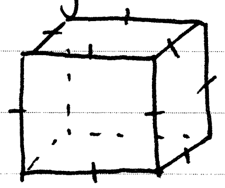
$= \frac{\sqrt{7}+2}{7-4}$

$= \frac{\sqrt{7}+2}{3}$ (B)

d) $x:y = 1:2$, $x:z = 3:5$

$\Rightarrow x:y = 3:6$

$\therefore y:z = 6:5$ (D)

e)  $= 210.6$ g

3 cm

$\therefore 27\text{ cm}^3 = 210.6$ g

$\Rightarrow 1\text{ cm}^3 = 7.8$ g (A)

f) $\dfrac{1 - \frac{2}{x+1}}{x - \frac{2}{x+1}}$

$= \frac{x+1-2}{x+1} \div \frac{x(x+1)-2}{x+1}$

$= \frac{x-1}{x+1} \times \frac{x+1}{x^2+x-2}$

$= \frac{x-1}{(x+2)(x-1)}$

$= \frac{1}{x+2}$

g) $h_s = 1.6\text{m}$, $h_l = 1.8\text{m}$

(I) $\frac{\text{Volume}_l}{\text{Volume}_s} = \frac{1.8^3}{1.6^3}$

$\frac{\text{volume}_l}{10.08\text{ m}^3} = \frac{1.8^3}{1.6^3}$

$\text{volume}_l = \frac{1.8^3}{1.6^3} \times 10.08$

$= 14.35\text{ m}^3$ (2.d.p)

(II) $\frac{\text{Area}_l}{\text{Area}_s} = \frac{1.8^2}{1.6^2}$

$\frac{\text{Area}_l}{800\text{mL}} = \frac{1.8^2}{1.6^2}$

$\text{Area}_l = \frac{1.8^2}{1.6^2} \times 800\text{mL}$

$= 1012.5$ mL

for 2 coats.

h) $P(x) = x^3 - x^2 - 16x - 20$

(I) $P(-2) = (-2)^3 - (-2)^2 - 16(-2) - 20$

$= -8 - 4 + 32 - 20$

$= 0$

$\therefore$ As $P(-2) = 0$ $x + 2$ is a factor.

(II)

$$\begin{array}{r}
x^2 - 3x - 10 \\
x + 2 \,\overline{)\, x^3 - x^2 - 16x - 20} \\
\underline{x^3 + 2x^2} \qquad\qquad \\
-3x^2 - 16x \qquad \\
\underline{-3x^2 - 6x} \qquad \\
-10x - 20 \\
\underline{-10x - 20} \\
0
\end{array}$$

$P(x) \Rightarrow (x+2)(x^2 - 3x - 10) = 0$

$(x+2)(x-5)(x+2) = 0$

$\therefore x = -2, 5$

i) $\angle TCB = x°$ (angle between a tangent and a chord drawn to the point of contact is equal to the angle in alternate segment).

$\angle CAT = x°$ (matching base angles in an isosceles triangles)

$\angle CBT = x° + x°$

$= 2x°$ (exterior angles of a Δ is equal to sum of the two opposite interior angles).

YEAR 10 YEARLY 2013.

Q6

(a) B. (1)

(b) D. (1)

(c) D. (1)

(d) D. (1)

(e) D. (1)

(f) $x(x-1)-y(y-1)$

$= x^2 - x - y^2 + y$ (2)

$= (x^2 - y^2) - (x-y)$

$= (x-y)(x+y-1)$

(g) In $\triangle AXM$ & $\triangle BXC$.

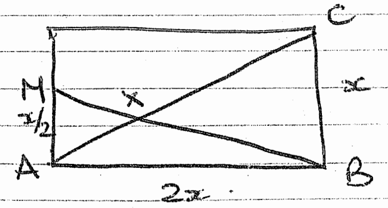
$\angle AXB = \angle BXC$ (vertically opposite) (1)

$\angle MAX = \angle BCX$ (Alternate $\angle$'s & $CB \parallel AD$) ABCD rectangle.

$\angle AMX = \angle CBX$ (" ").

$\therefore \triangle AXM \parallel\!| \triangle BXC$ (equiangular) 1

(ii)



$\frac{AM}{BC} = \frac{x/2}{x} = \frac{1}{2} = \frac{Ax}{Cx}$. Corresponding sid of similar $\triangle$s. $AxM \parallel\!| BxC$.

$AC = CX + AX$.

$= CX + \frac{1}{2}CX$. (2)

$AC = \frac{3}{2}CX$.

$2AC = 3CX$.

$3CX = 2AC$.

(iii) $9(CX)^2 = [3(CX)]^2$

$= (2AC)^2$

$= 4(AC)^2$. (2)

$= 4((2x)^2 + x^2)$

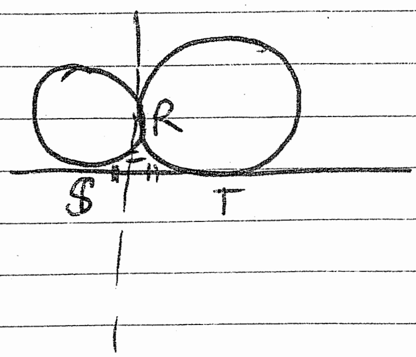
$= 20x^2$.

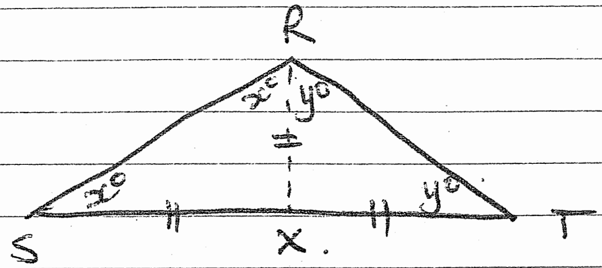
$5(AB)^2 = 5(2x)^2$.

$= 20x^2$.

$\therefore 9(CX)^2 = 5(AB)^2$.

(h)





SX = XR } equal tangents from an external p
TX = XR. }

Let. $\angle RSX = \angle XRS = x°$. (isosc. $\triangle$).
$\angle RTX = \angle XRT = y°$. (isosc $\triangle$).

$2x + 2y = 180°$ ($\angle$ sum $\triangle$).

$x + y = 90°$

$SRT = x + y = 90°$.

YR 10 Yearly Maths exam 2013.

$\frac{}{15}$.

Question 7

(a) D (1) (B was a chance but like the ambiguous case in the sine rule, an angle could have been obtuse).

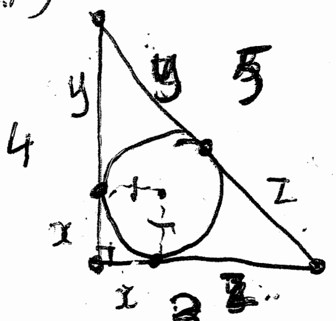
(b) B (1)

(c) C (1)

(d) C (1)

(e) D (1)

(f)



$x+y=4$
$x+z=3$
$y+z=5$

solving simultaneously
$y=2$
$x=1$
$z=2$

radius $r=1$cm. (3)

(g) (I) P(win first, win second) + P(not first, win second)

$= \frac{2}{100} \times \frac{1}{99} = \frac{1}{4950}$

$+ \frac{98}{100} \times \frac{2}{99} = \frac{49}{2475}$

sum: $\frac{1}{50}$ (1)

(II) $\frac{98}{100} \times \frac{97}{99} \times \frac{96}{98} \times \cdots \times \frac{(99-n)}{(101-n)}$

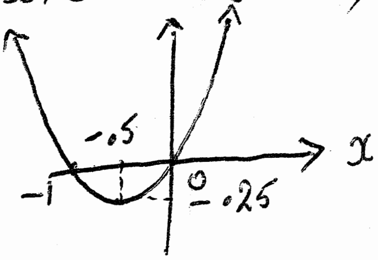
cancels to $\frac{(99-n)(100-n)}{9900}$

now we want $1 - \frac{(99-n)(100-n)}{9900} = 0\cdot$

expand and use quadratic formula

$n = 29$ (accept 30) (3)

(h) $x+x^2 = x(1+x)$



let $x=-0.5$, $4^{-0.25}$

$= 4^{-\frac{1}{4}}$

$= (2^2)^{-\frac{1}{4}}$

$= 2^{-\frac{1}{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{\sqrt{2}}{2}$ (3)