SYDNEY BOYS HIGH SCHOOL

MOORE PARK, SURRY HILLS

2014

Year 10
Yearly Examination

Mathematics

General Instructions

- Working time – 120 minutes.
- Write using black or blue pen.
- Pencil may be used for diagrams.
- Do not use Liquid paper or tape.
- Calculators may be used.
- All *necessary* working should be shown in every question if full marks are to be awarded.
- Marks may **NOT** be awarded for messy or badly arranged work.
- If more space is required, clearly write the number of the QUESTION on one of the back pages and answer it there. Indicate that you have done so.
- Clearly indicate your class by placing an **X**, next to your class.

NAME:

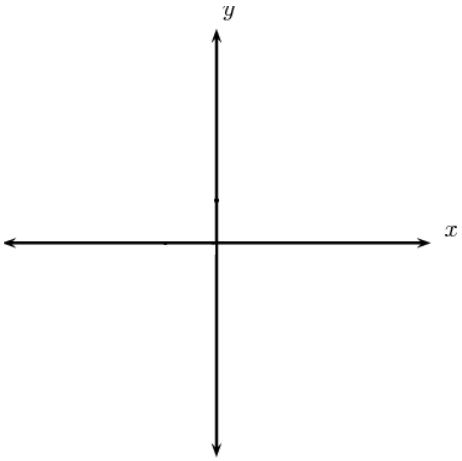
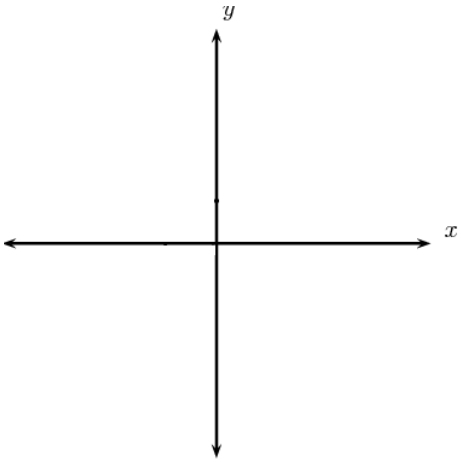
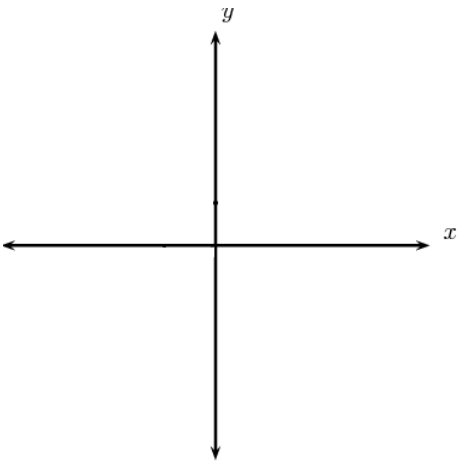
Examiner: *R. Boros*

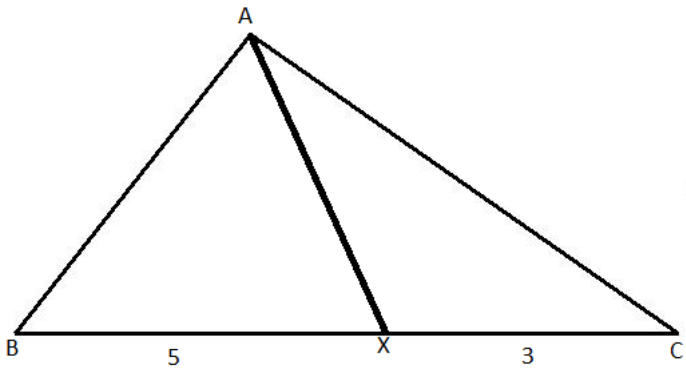
Class	Teacher	
10 A	Ms Kilmore	
10 B	Ms Chen / Mr Elliott	
10 C	Ms Millar	
10 D	Ms Nesbitt / Ms Likourezos	
10 E	Mr Hespe	
10 F	Mr Choy	
10 G	Mr Fuller	

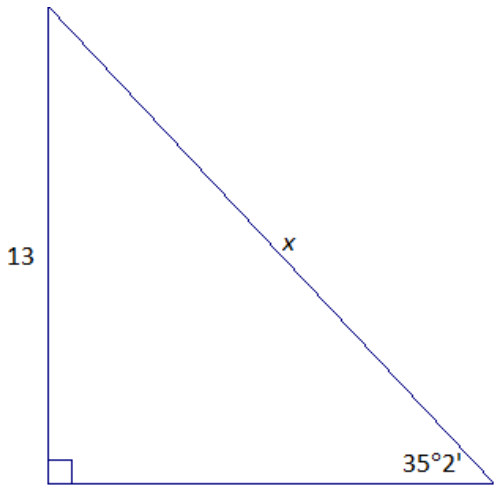
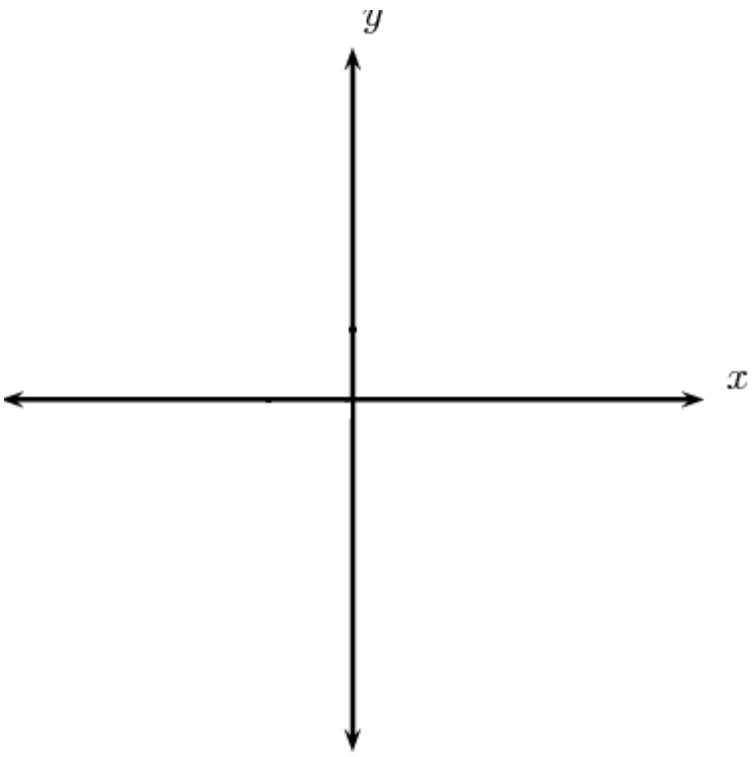
Question	Mark	
1		**/20**
2		**/20**
3		**/20**
4		**/20**
5		**/20**
6		**/15**
7		**/10**
Total		**/125**

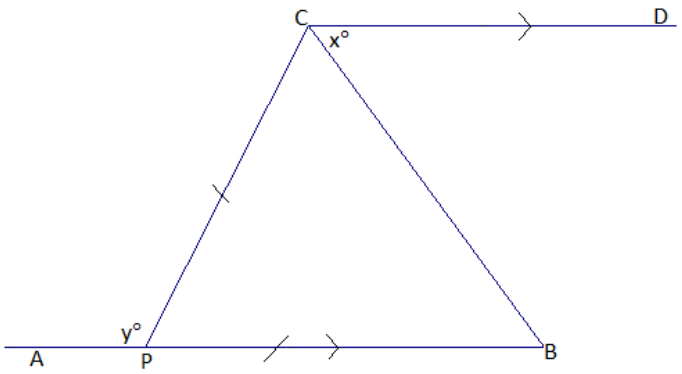
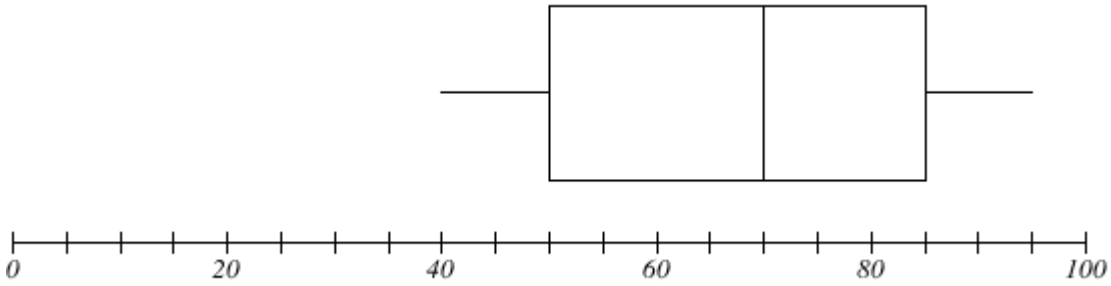
Question 1 (20 Marks)		**Marks**
a)	Write 5608200 in scientific notation.	1
b)	A bag contains 3 blue balls, 5 red balls and 4 yellow balls. If 1 ball is selected at random, find $P(\overline{\text{a yellow ball}})$	1
c)	When a pair of regular dice are thrown, what is the probability of a score of 9?	1
d)	The point $(2,k)$ lies on $x^2+y^2=8$. Find the value(s) of k.	1
e)	What is the minimum value of $4x^2+6$?	1
f)	Solve for x, $3(2x+1)(x+4)=0$	1
g)	If $4x+2y=8$ and $5x-2y=9$ are solved simultaneously, find the value of x.	1

h)	$5000 is invested at 4.00%p.a. compounded bi-annually. How much is the value of the investment (to the nearest cent) after 5 years?	2
i)	If the sides of a rectangle are increased by 50%. What is the area increased by?	1
j)	The ratio of the surface areas of 2 similar solids is $9:36$. Write down the ratio of the volumes of the solids.	1

k)	Draw neat sketches of (i) $y=-x^3$  (ii) $y=2^x$  (iii) $y=\frac{-1}{x-3}$  	1 1 1
l)	A 3 sided die labelled A,B,C is rolled and then an unbiased coin is tossed. Draw a tree diagram to illustrate the possible outcomes.	1

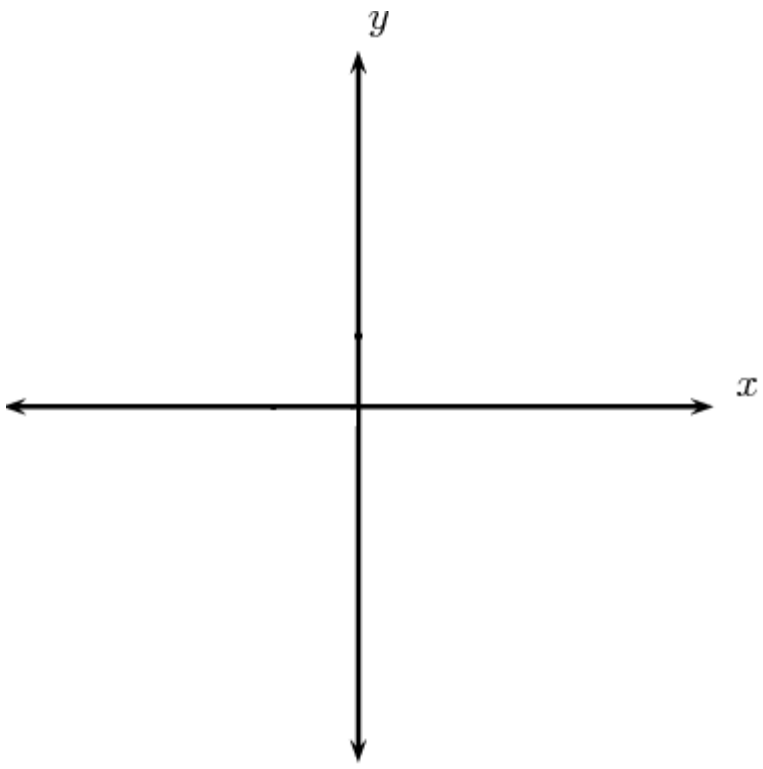
m)	What is the ratio of the area ΔABX to that of the area of ΔABC  	1
n)	Use the quadratic formula to solve $3x^2 - 5x + 1 = 0$. Leave your answer in surd form.	2
o)	Given $\cos\theta = -0.7$ and $0^\circ \le \theta \le 180^\circ$, Find θ in degrees and minutes.	1
p)	If $a * b = \frac{1}{a-b}$, Find 3*(3*5)	1

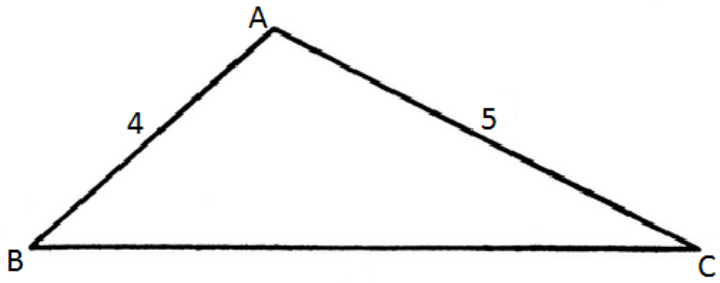
Question 2 (20 Marks)		Marks
a)	Find the value of x correct to 3 significant figures 	1
b)	Solve for k, $k-\frac{k+4}{8}=\frac{2(2k+1)}{3}$	2
c)	Draw neat sketches of $xy=1$ and $x^2+y^2=4$ on the same diagram. Using the graph, estimate the point(s) of intersection of these 2 equations to 1 decimal place. 	4

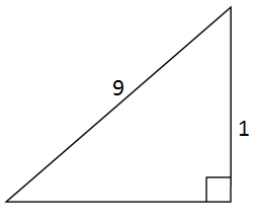
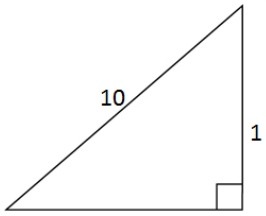
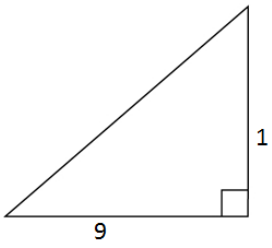
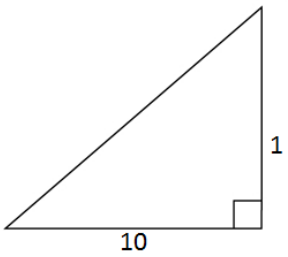
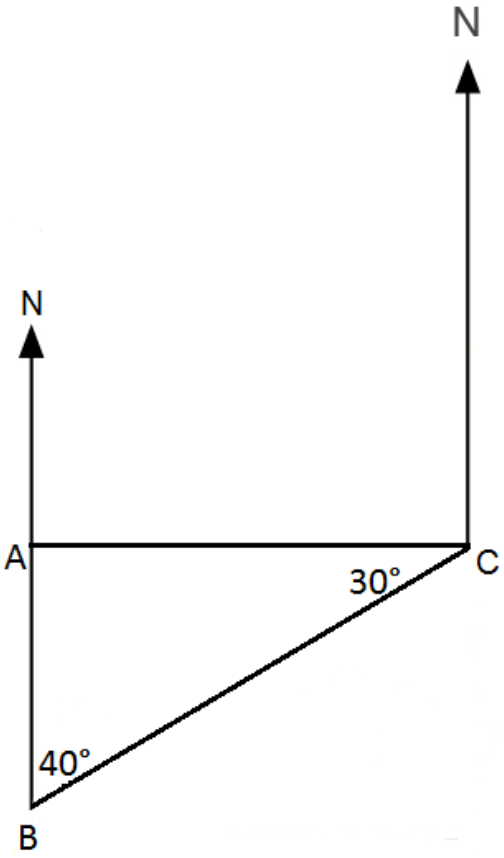
d)	Given $AB \| \| CD$ and $PC = PB$, find, giving reasons, a relationship connecting x and y.  	2
e)	The box and whisker plot represents the results of a yearly Latin test.  (i) What was the highest mark attained? (ii) Find the interquartile range. (iii) Find the median mark. (iv) Find the range.	1 1 1 1
f)	Make 'b' the subject of this formula: $$\frac{2}{a} = \frac{7}{b} - \frac{5}{c}$$	2

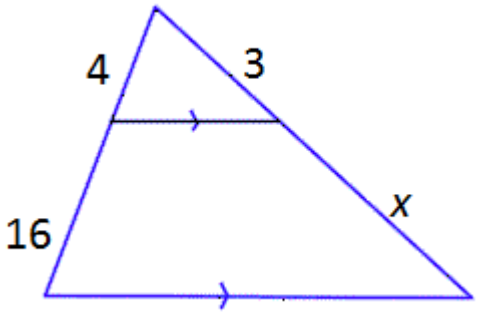
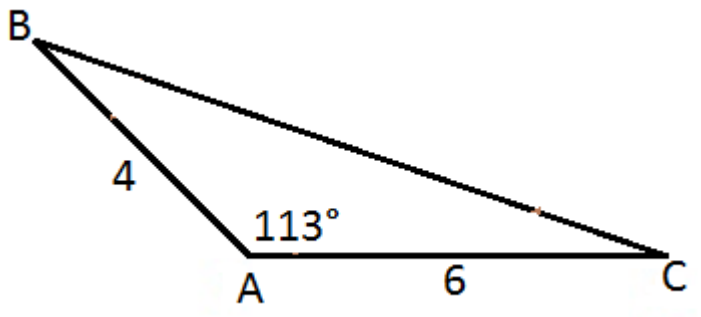
g)	Use the stem and leaf plot given to answer these questions. Stem | leaf 3 | 2 5 4 | 1 3 3 5 8 5 | 6 7 7 6 | 0 (i) Find the median. (ii) Find the mode(s). (iii) Find the mean.	1 1 1
h)	Find, by completion of the square, the minimum value of $3x^2 + 8x - 9$	2

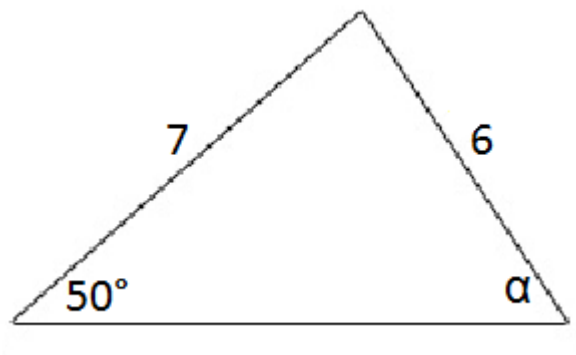
Question 3 (20 Marks)		Marks
a)	(i) Factorise $y = 3x^2 + 19x - 14$	1
	(ii) What is the y intercept?	1
	(iii) By making $3x^2 + 19x - 14 = 0$, solve for x.	1
	(iv) Find the equation of the axis of symmetry to the curve.	1
	(v) Find the coordinates of the vertex to this curve.	1
	(vi) Draw a neat sketch of all the above information.	1



b)	 Given $Cos\angle BAC = \frac{1}{8}$, (i) Calculate the exact value of $\sin\angle BAC$. (ii) Find the area of ΔABC as a surd. (iii) Find the length of side BC.	1 1 1
c)	A car purchased for \$8400 depreciates at 15%p.a. Find its value after 3 years. (to the nearest \$)	2

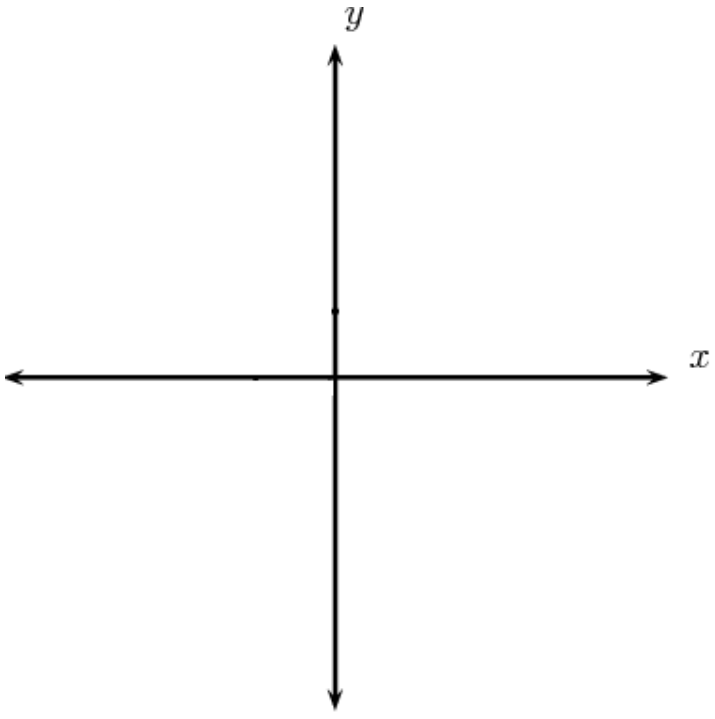
d)	Each of these diagrams represents the slope of a ramp. Which ramp is the steepest? A.  B.  C.  D.  	1
e)	Find the true or compass bearing of *A* from *C*.  (NOT TO SCALE)	1

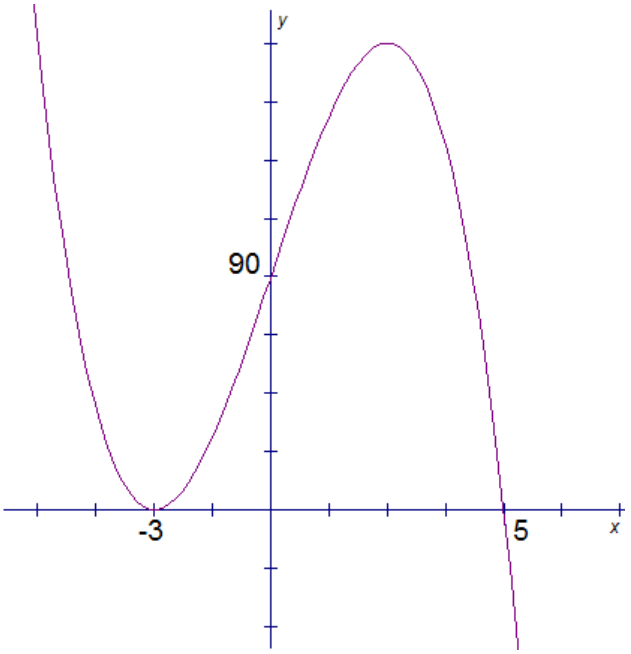
f)	Find the value of x. 	1
g)	Simplify $\frac{x^2-4x-5}{x^2+3x+2}$	2
h)	Find the total surface area of a hemisphere with a radius of 10cm.	2
i)	Find the length of BC correct to the nearest whole number. 	2

Question 4 (20 Marks)		Marks
a)	Express $\frac{2}{3-\sqrt{5}}$ with a rational denominator.	2
b)	Given $f(x)=3-2x$ (i) Find $f(-1)$	1
	(ii) Find a positive number Q such that $f(Q)=Q^2$	2
	(iii) Find R such that $f(2^R)<-5$	1
c)	Find the size of α correct to the nearest minute. 	2

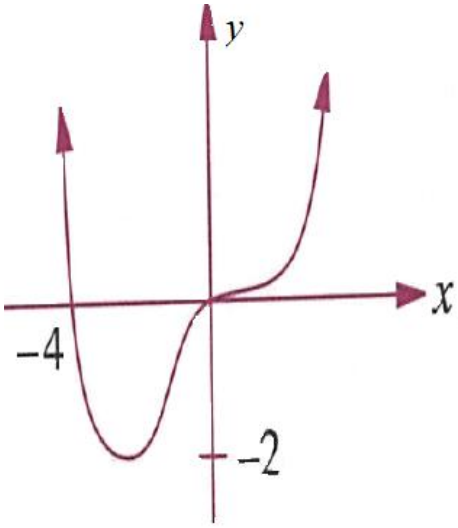
d)	What is the equation of the circle with radius 4 units and centre $(2,-1)$?	1
e)	P,Q,R are 3 towns. P and Q are 35km apart and Q is due east of P. The true bearing of R from P is 044^o and the true bearing of R from Q is 325^o. Find the distance of R from Q correct to 1 decimal place.	3
f)	The marks from 2 tests are shown below. Test A: 9, 11, 12, 12, 13, 13, 14, 15, 15, 16 Test B: 5, 7, 9, 10, 12, 13, 13, 15, 17, 19 Both tests have $\overline{x_A}=13$ and $\overline{x_B}=12$. $S.D_A=2.0$ (i) Find the standard deviation (to 1DP) of Test B. (ii) Give a convincing explanation why a mark of 15 in Test A is better than a mark of 15 in Test B	1 1

g)	State giving reason(s), whether or not the following is a polynomial, $10x^4 + 2x^3 - \frac{5}{x^2} + 10$	1
h)	If $P(x) = 5x + 2$ and $Q(x) = x^2 - 3x + 1$, find: (i) $P(x) - Q(x)$ (ii) $P(x)Q(x)$	1 2
i)	A polynomial $P(x)$ has degree 4, has leading coefficient 5 and a constant term of -3. Find a polynomial $P(x)$ which satisfies these properties.	2

Question 5 (20 Marks)		Marks
a)	Find the coordinates of the points of intersection of $y = 2x^3 - x^2$ and $y - 8x + 4 = 0$.	4
b)	Provide a neat sketch of $y = 5x(x-2)^2$ 	2

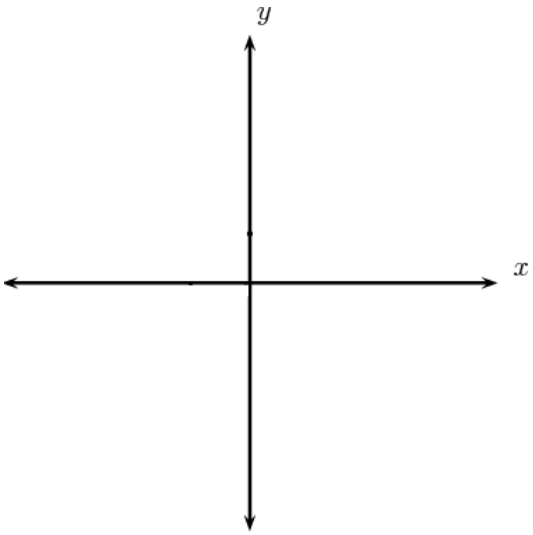
c)	Given $x^3-5x^2+7x-2=(x-2)(ax^2+bx+c)$ By long division or otherwise, find the values of a,b and c	3
d)	Find the equation of the cubic polynomial in this diagram. Leave your answer in factorised form. 	3

e)	Given the graph of $y = P(x)$ in the diagram below	8

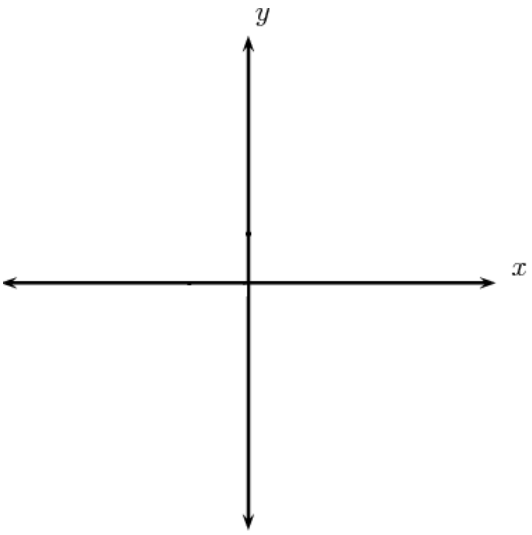


Sketch the graphs of the following on separate diagrams.

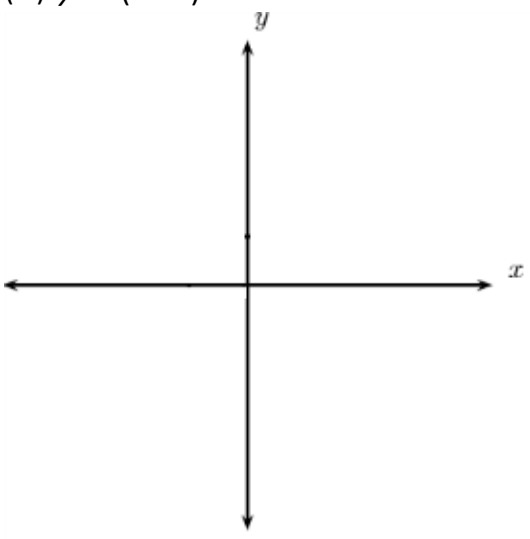
(i) $y = P(-x)$



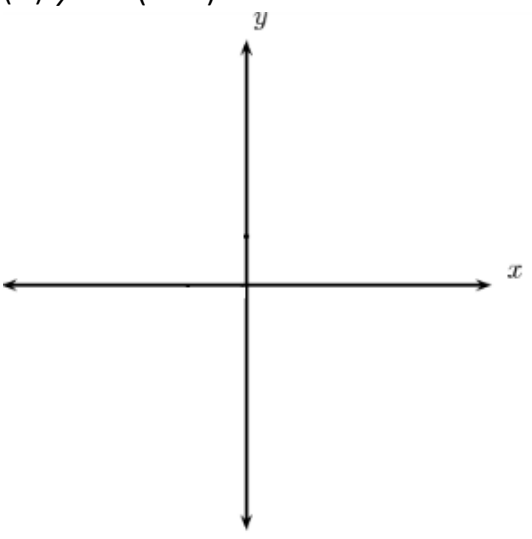
(ii) $y = P(x) + 2$

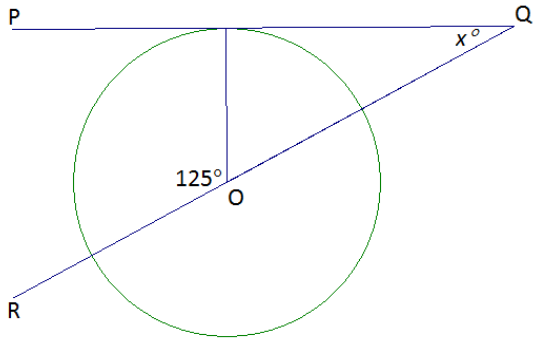
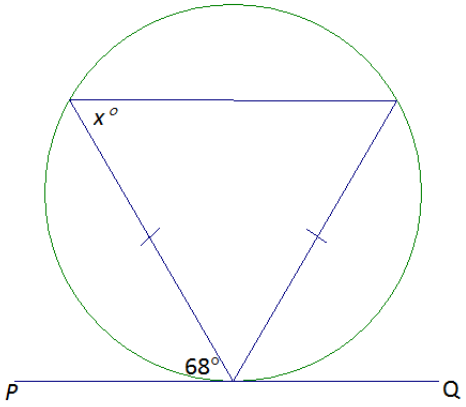
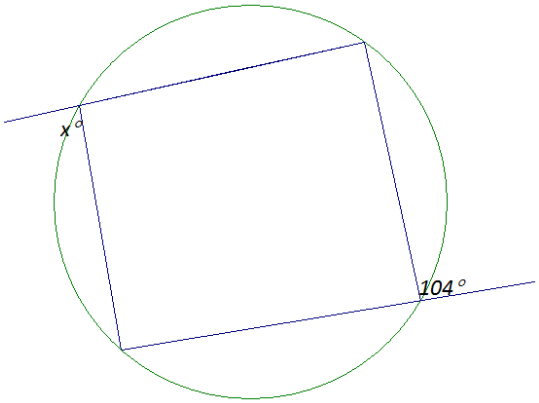
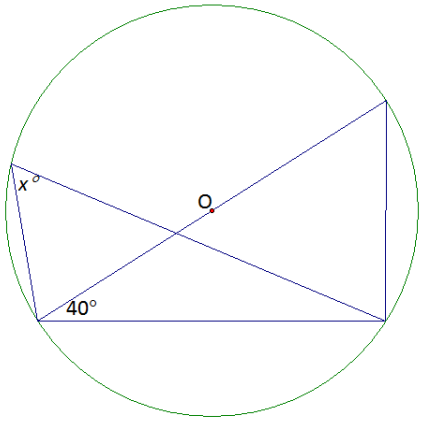


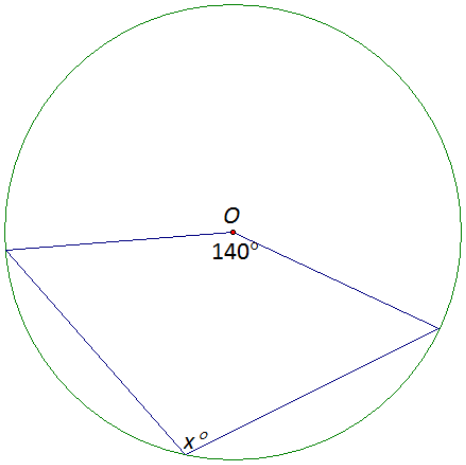
(iii) $y = P(x + 4)$

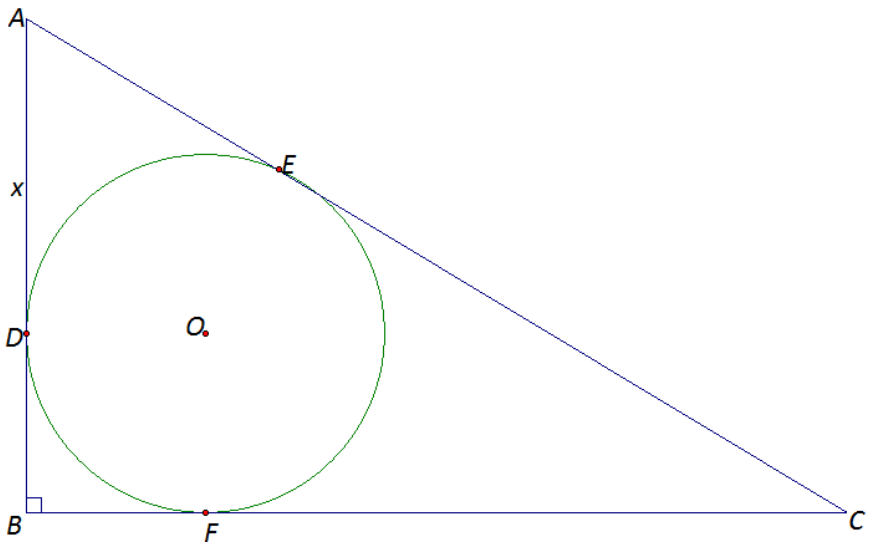
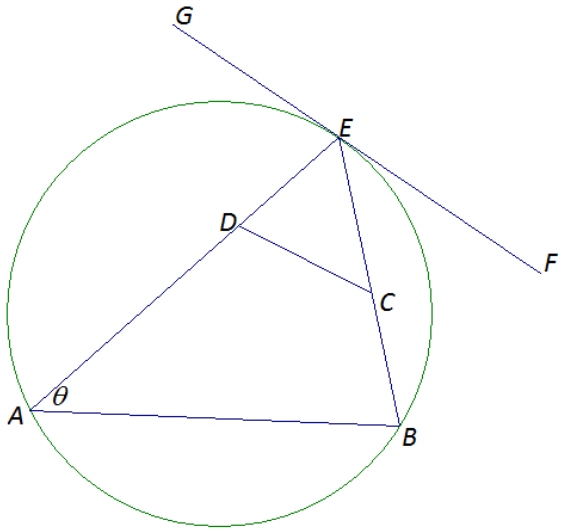


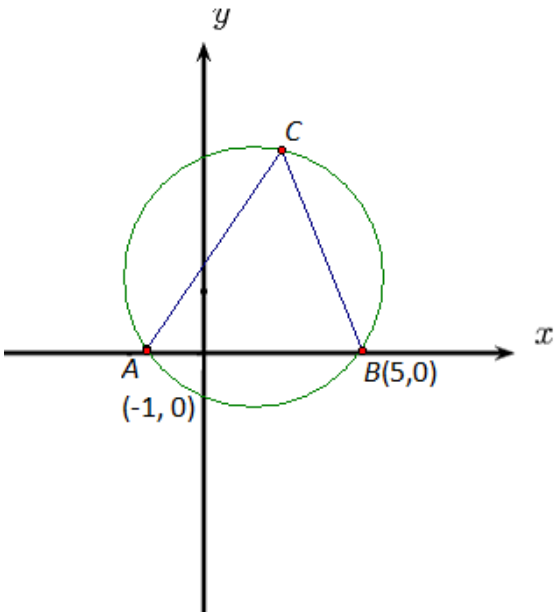
(iv) $y = -P(x - 4)$



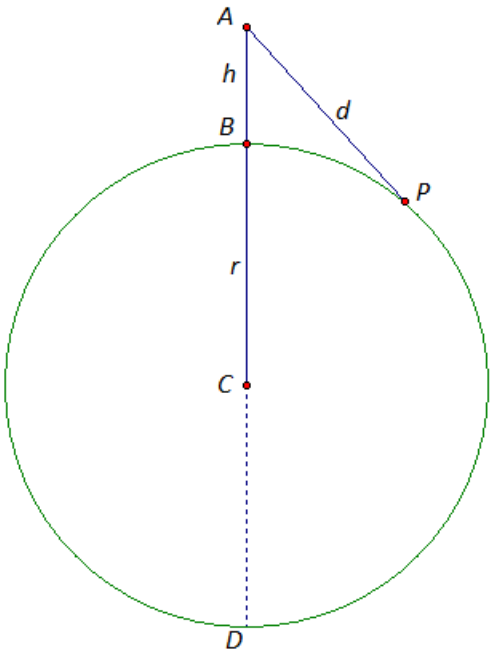
Question 6 (15 Marks)		Marks
a)	Find x in each case. The centre of the circle is denoted by O. Do <u>not</u> give reasons.	
	(i)  PQ is a tangent.	1
	(ii)  PQ is a tangent.	1
	(iii) 	1
	(iv) 	

	(v)  	1 1
b)	Show that $(x-2)$ is a factor of $P(x)=x^3+2x^2-5x-6$ using the remainder-factor theorem.	2
c)	Given x^2-x-6 is a factor of $2x^3+mx^2-13x+n$, find the values of m and n.	2

d)	 $BC = 24cm,\ DB = 3cm,\ AD = x$ Find the value of x.	2
e)	 GF is a tangent to the circle at E. Quadrilateral $ABCD$ is cyclic. Prove $DC \parallel GF$.	4

Question 7 (10 Marks)		Marks
a)	If the equation $x^2+y^2+ax+by+c=0$ always represents a circle, prove that $a^2>4c-b^2$.	3
b)	 Triangle ABC is equilateral. The vertices A, B and C lie on the circumference of the circle. Find the exact coordinates of the centre of the circle.	3

c)



C is the centre of this circle.

The distance d of the visible horizon from an observer at height h above the earth's surface is given by the formula $d^2 = h^2 + 2hr$ where r is the radius of the earth.

(i) Prove the tangent/secant formula. 2

(ii) Hence or otherwise prove $d^2 = h^2 + 2hr$ 2

END OF TEST

2014 Year 10 Yearly - Solutions

Question 1 (20 Marks)

		Marks
a)	Write 5608200 in scientific notation. 5.6082×10^6	1
b)	A bag contains 3 blue balls, 5 red balls and 4 yellow balls. If 1 ball is selected at random, find $P(\text{a yellow ball})$ $P = 1 - \frac{4}{12} = \frac{2}{3}$	1
c)	When a pair of regular dice are thrown, what is the probability of a score of 9? 3 6, 6 3, 4 5, 5 4 $= 4/36 = 1/9$	1
d)	The point $(2,k)$ lies on $x^2+y^2=8$. Find the value(s) of k. $4 + y^2 = 8$, $y^2 = 4$, $y = \pm 2$, $\therefore k = \pm 2$	1
e)	What is the minimum value of $4x^2+6$? 6	1
f)	Solve for x, $3(2x+1)(x+4)=0$ $2x+1=0$, $2x=-1$, $x=-1/2$; $x+4=0$, $x=-4$; $x = -1/2, -4$	1
g)	If $4x+2y=8$ and $5x-2y=9$ are solved simultaneously, find the value of x. $5x-2y=9$, $9x=17$, $x = 17/9$ $17/9$	1

h) $5000 is invested at 4.00%p.a. compounded bi-annually. How much is the value of the investment (to the nearest cent) after 5 years? 2

$A = 5000(1 + 0.02)^{10}$

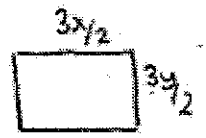
$= 6094.97$

If they've shown some sort of working in compound interest form = 1 mark

i) If the sides of a rectangle are increased by 50%. What is the area increased by? 1

$A_1 = xy$

½ only for 225%.



$A_2 = \frac{9xy}{4}$

$A_2 - A_1 = \frac{5}{4}$ ∴ inc 125%

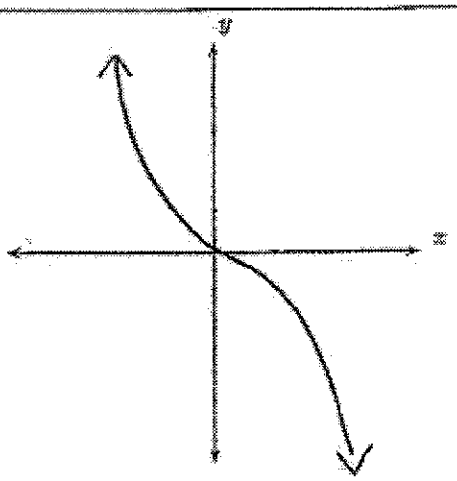
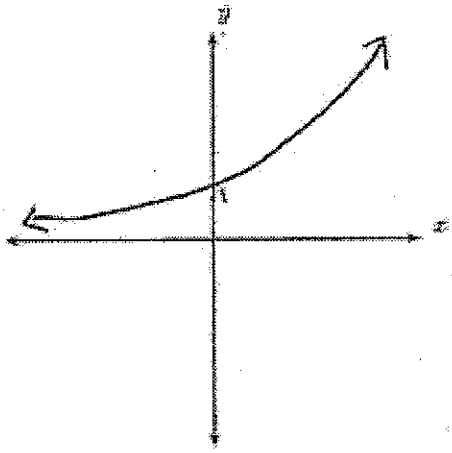
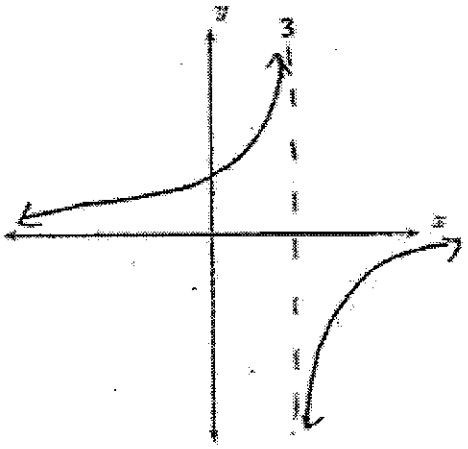
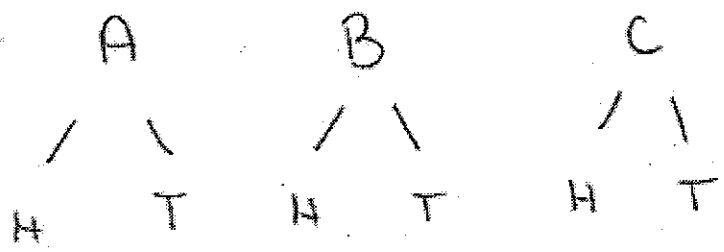
j) The ratio of the surface areas of 2 similar solids is 9:36. Write down the ratio of the volumes of the solids. 1

SA 9 : 36

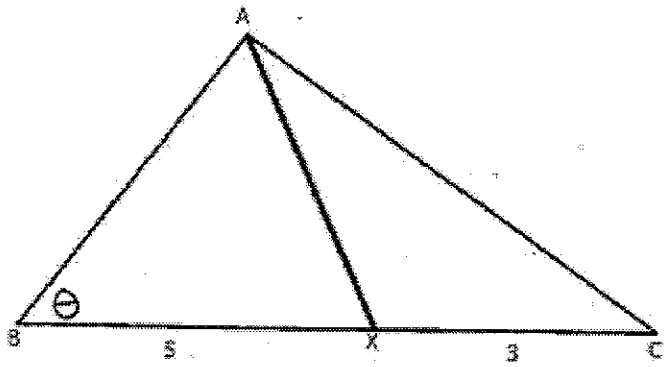
↓

$V = (\sqrt{9})^3 : (\sqrt{36})^3$

$= 27 : 216$ ⇒ 1 : 8

k)	Draw neat sketches of (i) $y=-x^3$ 	1
	(ii) $y=2^x$ 	1
	(iii) $y=\frac{-1}{x-3}$ 	1
l)	A 3 sided die labelled A,B,C is rolled and then an unbiased coin is tossed. Draw a tree diagram to illustrate the possible outcomes. 	1

m) What is the ratio of the area ΔABX to that of the area of ΔABC — 1



method 1:

5 : 8

A 25 : 64

method 2

$\frac{1}{2} \times AB \times 5 \times \sin\theta : \frac{1}{2} \times AB \times 8 \times \sin\theta$

5 : 8

n) Use the quadratic formula to solve $3x^2 - 5x + 1 = 0$. Leave your answer in surd form. — 2

$$x = \frac{+5 \pm \sqrt{(-5)^2 - 4 \times 3 \times 1}}{2 \times 3}$$

$$= \frac{5 \pm \sqrt{25 - 12}}{6} \qquad = \frac{5 \pm \sqrt{13}}{6}$$

$=$

o) Given $\cos\theta = -0.7$ and $0° \leq \theta \leq 180°$, Find θ in degrees and minutes. — 1

$134° 26'$

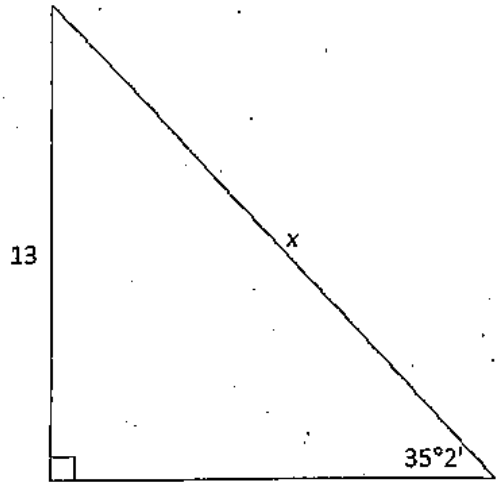
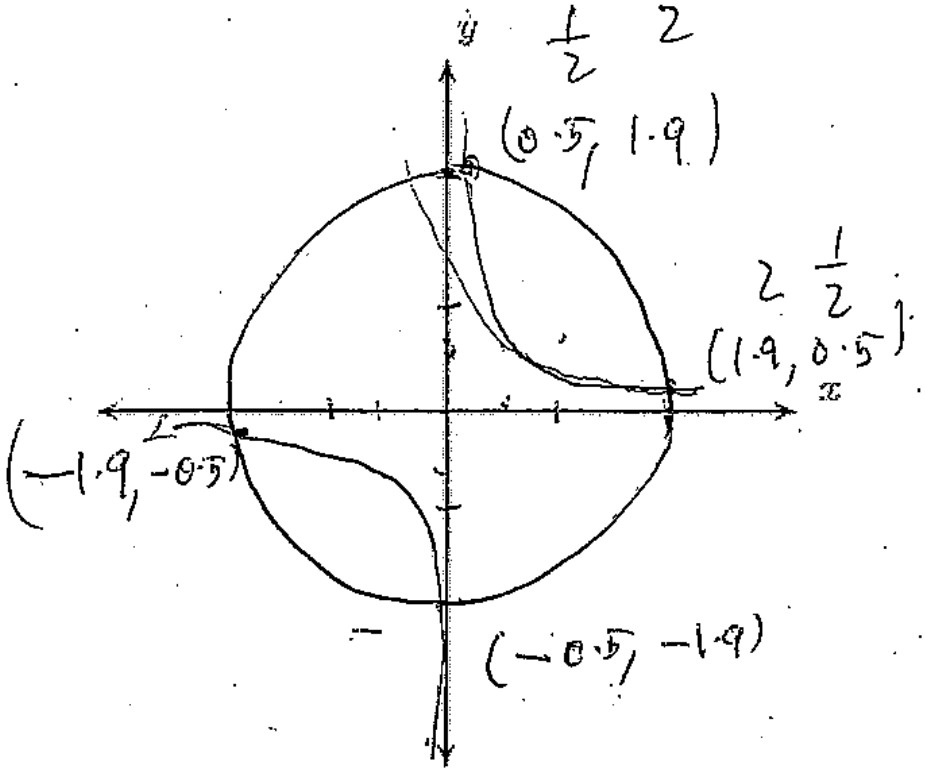
p) If $a * b = \frac{1}{a-b}$, Find 3*(3*5) — 1

$3 * 5 = \frac{1}{3-5}$

$= -\frac{1}{2}$

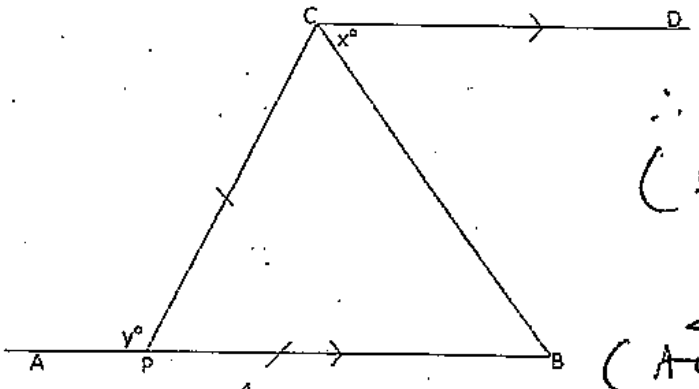
$3 * -\frac{1}{2} = \frac{1}{3+\frac{1}{2}}$

$= 2/7$

Question 2 (20 Marks)		Marks
a)	Find the value of x correct to 3 significant figures  $\frac{13}{x} = \sin 35^\circ 2'$ $\therefore x = \frac{13}{\sin 35^\circ 2'}$ $= 22.6$	1
b)	Solve for k, $k - \frac{k+4}{8} = \frac{2(2k+1)}{3}$ $24k - 3(k+4) = 16(2k+1)$ $24k - 3k - 12 = 32k + 16$ $21k - 12 = 32k + 16$ $-11k = 28$ $\therefore k = -\frac{28}{11}$	2
c)	Draw neat sketches of $xy = 1$ and $x^2 + y^2 = 4$ on the same diagram. Using the graph, estimate the point(s) of intersection of these 2 equations to 1 decimal place.  	4

$x - y/2 = 0$ $2x = y$

d) Given $AB \parallel CD$ and $PC = PB$, find, giving reasons, a relationship connecting x and y. 2



$\triangle PCB$ is isos.

$\therefore \angle PCB = \angle PBC = y/2$

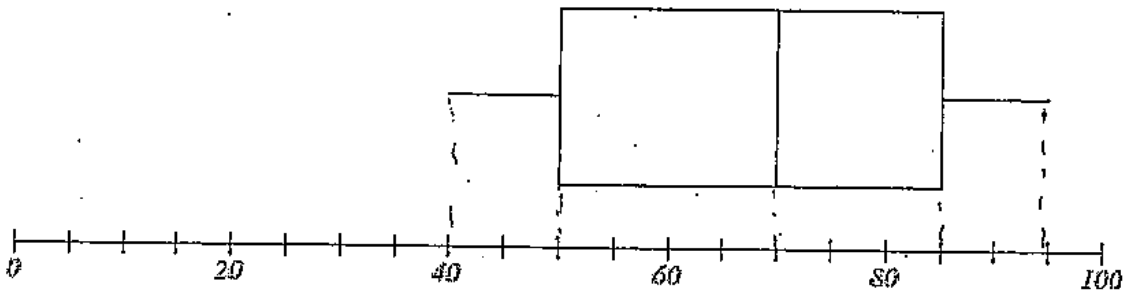
(ext. $\angle$ = Sum of interior opp. angles)

$\angle CPB = 180 - y$.

(Angle sum of st. line)

$180 - y + y/2 + x = 180$ (Corat $\angle$s).

e) The box and whisker plot represents the results of a yearly Latin test.



(i) What was the highest mark attained? 95 1

(ii) Find the interquartile range. $85 - 50 = 35$ 1

(iii) Find the median mark. 70 1

(iv) Find the range. $95 - 40 = 55$ 1

f) Make 'b' the subject of this formula: 2

$$\frac{2}{a} = \frac{7}{b} - \frac{5}{c}$$

$$\frac{2}{a} + \frac{5}{c} = \frac{7}{b}$$

$$\frac{2c + 5a}{ac} = \frac{7}{b}$$

$$\therefore b = \frac{7ac}{5a + 2c}.$$

g) Use the stem and leaf plot given to answer these questions.

Stem	leaf
3	2 5
4	1 3 3 5 8
5	6 7 7
6	0

(i) Find the median. 45. 1

(ii) Find the mode(s). 43, 57. 1

(iii) Find the mean. 47 1

h) Find, by completion of the square, the minimum value of $3x^2+8x-9$ 2

$$3\left(x^2+\frac{8}{3}x+\frac{16}{9}-\frac{16}{9}-\frac{27}{9}\right)$$

$$=3\left[\left(x+\frac{4}{3}\right)^2-\frac{43}{9}\right]$$

$$\therefore \text{Min value} = \frac{-43}{3}$$

3. (a) (i) Factorise $y = 3x^2 + 19x - 14$. [1]

Solution:

P	-42	$y = 3x^2 + 21x - 2x - 14,$
S	19	$= 3x(x+7) - 2(x+7),$
F	$21, -2$	$= (3x-2)(x+7)$

(ii) What is the y intercept? [1]

Solution: $y = -14$.

(iii) By making $3x^2 + 19x - 14 = 0$, solve for x. [1]

Solution: $(3x-2)(x+7) = 0,$
$\therefore x = \dfrac{2}{3}, -7.$

(iv) Find the equation of the axis of symmetry of the curve. [1]

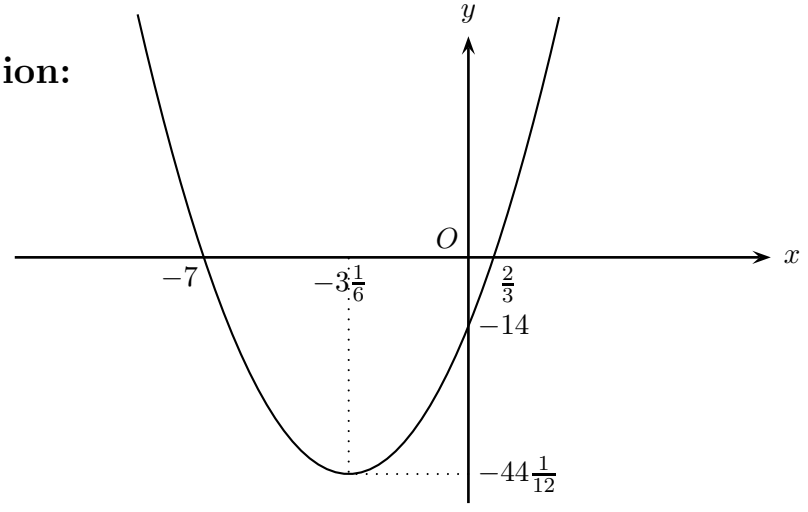
Solution: $x = -\dfrac{19}{6}$.

(v) Find the coordinates of the curve's vertex. [1]

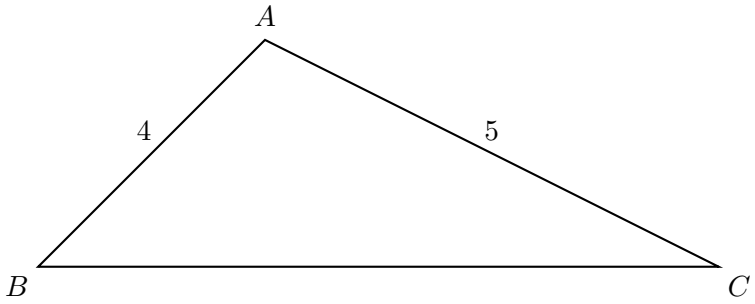
Solution: $3\left(-\frac{19}{6}\right)^2 + 19\left(-\frac{19}{6}\right) - 14 = -\dfrac{529}{12}$.
$\therefore$ Vertex is $\left(-3\frac{1}{6}, -44\frac{1}{12}\right)$.

(vi) Draw a neat sketch of all the above information. [1]

Solution:



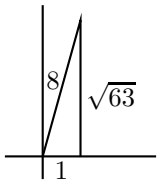
(b)



Given $\cos B\widehat{A}C = \dfrac{1}{8}$:

(i) Calculate the exact value of $\sin B\widehat{A}C$. 1

Solution: $\dfrac{\sqrt{63}}{8} = \dfrac{3\sqrt{7}}{8}$.



$8^2 - 1 = 63$

(ii) Find the area of $\triangle ABC$ as a surd. 1

Solution: $\dfrac{1}{2} \times 4 \times 5 \times \dfrac{\sqrt{63}}{8} = \dfrac{15\sqrt{7}}{4}$.

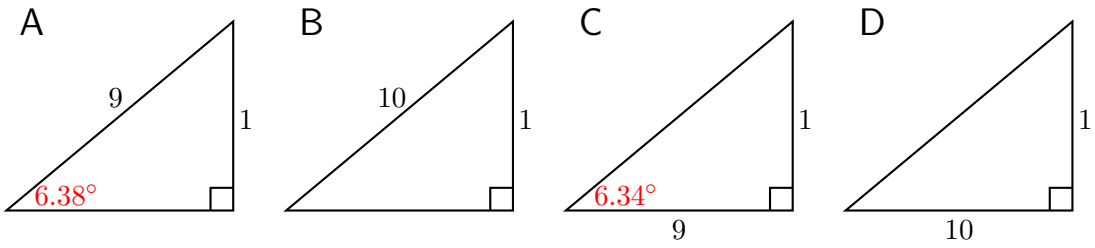
(iii) Find the length of side BC. 1

Solution:
$$\begin{aligned} BC^2 &= 4^2 + 5^2 - 2 \times 4 \times 5 \times \frac{1}{8}, \\ &= 36. \\ \therefore BC &= 6. \end{aligned}$$

(c) A car purchased for \$8400 depreciates at 15% p.a. 2
Find its value after 3 years (to the nearest \$).

Solution: $\$8400\left(1 - \frac{15}{100}\right)^3 = \$5158.65$.
$\therefore$ Value is \$5159.

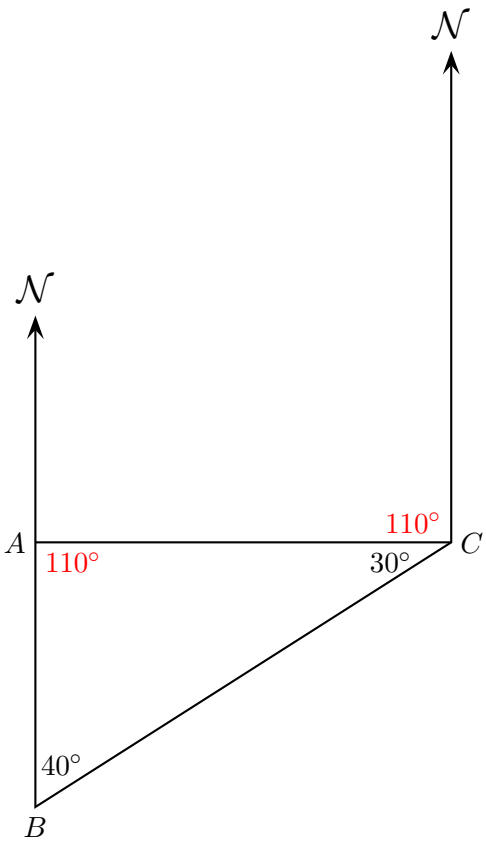
(d) Each of these diagrams represents the slope of a ramp. 1
Which ramp is the steepest?



Solution: A is the steepest.

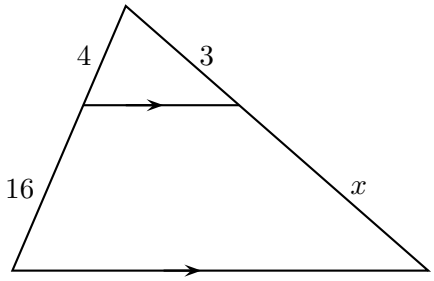
(e) Find the true or compass bearing of A from C. 1

Solution: $360 - 110 = 250$.
$\therefore$ 250°T or S70°W.



(NOT TO SCALE)

(f)



Find the value of x. 1

Solution:
$$\begin{aligned} \frac{x+3}{3} &= \frac{20}{4}, \\ 4x+12 &= 60, \\ 4x &= 48, \\ x &= 12. \end{aligned}$$

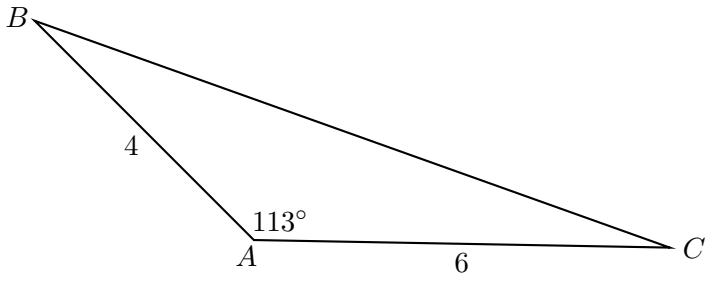
(g) Simplify $\dfrac{x^2-4x-5}{x^2+3x+2}$. 2

Solution: $\dfrac{(x-5)(x+1)}{(x+2)(x+1)} = \dfrac{x-5}{x+2},\ x \neq -1.$

(h) Find the total surface area of a hemisphere with a radius of 10 cm. 2

Solution: $\frac{1}{2} \times 4\pi \times 10^2 + \pi \times 10^2 = 300\pi\,\text{cm}^2.$

(i) Find the length of BC correct to the nearest whole number. 2



Solution:
$$\begin{aligned} BC^2 &= 4^2 + 6^2 - 2 \times 4 \times \cos 113^\circ, \\ &\approx 70.7551. \\ \therefore BC &\approx 8. \end{aligned}$$

Question 4 (20 Marks)		Marks
a)	Express $\frac{2}{3-\sqrt{5}}$ with a rational denominator.	2

$$\frac{2}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}}$$

$$= \frac{6+2\sqrt{5}}{4}$$

$$= \frac{3+\sqrt{5}}{2}$$

b) Given $f(x)=3-2x$

(i) Find $f(-1)$ — 1

$$3-2(-1) = 5$$

(ii) Find a positive number Q such that $f(Q)=Q^2$ — 2

$3-2(-3) = 9 = (-3)^2$

$3-2(1) = 1 = 1^2$

$Q = 1 \ (>0)$

(iii) Find R such that $f(2^R)<-5$ — 1

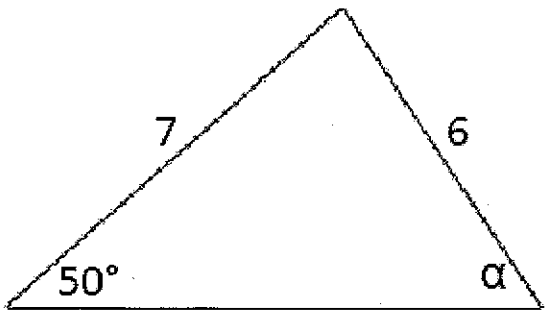
$3-2.2^r < -5$

$-2.2^r < -8$

$2^r > 4$

$r > 2$

c) Find the size of α correct to the nearest minute. — 2



$$\frac{6}{\sin 50} = \frac{7}{\sin \alpha}$$

$$\sin \alpha = \frac{7 \sin 50}{6}$$

$$\alpha = 63.20'39''$$

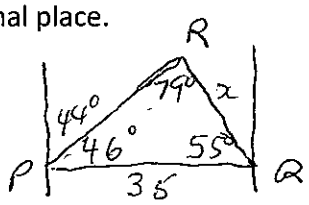
$$= 63°21'$$

or $116°39'$

d) What is the equation of the circle with radius 4 units and centre (2,−1)? 1

$(x-2)^2 + (y+1)^2 = 16$

e) P,Q,R are 3 towns. P and Q are 35km apart and Q is due east of P. The true bearing of R from P is 044° and the true bearing of R from Q is 325°. Find the distance of R from Q correct to 1 decimal place. 3



$$\frac{x}{\sin 46} = \frac{35}{\sin 79}$$

$x = 25.64$

$= 25.6$ (1 d.p.)

f) The marks from 2 tests are shown below.

Test A	9	11	12	12	13	13	14	15	15	16
Test B	5	7	9	10	12	13	13	15	17	19

Both tests have $\overline{x_A} = 13$ and $\overline{x_B} = 12$.

$S.D_A = 2.0$

(i) Find the standard deviation (to 1DP) of Test B. 1

4.1 (1 d.p.)

(ii) Give a convincing explanation why a mark of 15 in Test A is better than a mark of 15 in Test B 1

In test A 15 is more than one SD above the mean
In test B 15 is less than one SD above the mean.

g)	State giving reason(s), whether or not the following is a polynomial, $10x^4+2x^3-\frac{5}{x^2}+10$ not a polynomial $\frac{1}{x^2}$ has a negative index (-2)	1
h)	If $P(x)=5x+2$ and $Q(x)=x^2-3x+1$, find: (i) $P(x)-Q(x)$ $-x^2+8x+1$	1
	(ii) $P(x)Q(x)$ $5x^3-13x^2-x+2$	2
i)	A polynomial $P(x)$ has degree 4, has leading coefficient 5 and a constant term of -3. Find a polynomial $P(x)$ which satisfies these properties. $5x^4-3$ (may have additional terms with indices of 3, 2 or 1)	2

Question 5 (20 Marks)		Marks
a)	Find the coordinates of the points of intersection of $y=2x^3-x^2$ and $y-8x+4=0$.	4

$y = 2x^3 - x^2 \quad (1)$

$y = 8x - 4 \quad (2)$

$(1) = (2)$

$2x^3 - x^2 = 8x - 4$

$2x^3 - x^2 - 8x + 4 = 0$

$x^2(2x-1) - 4(2x-1) = 0$

$(2x-1)(x^2-4) = 0$

$\therefore (2x-1)(x-2)(x+2) = 0$

$\therefore x = \frac{1}{2}, 2, -2$

* when $x = \frac{1}{2}$, $y = 8(\frac{1}{2}) - 4$
$= 0$

$(\frac{1}{2}, 0)$

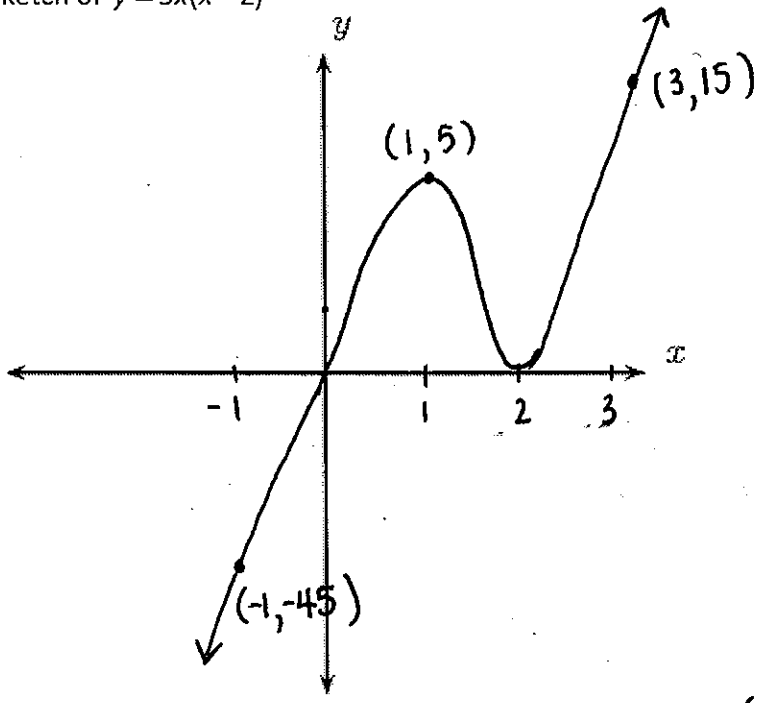
* when $x = 2$, $y = 8(2) - 4$
$= 12$

$(2, 12)$

* when $x = -2$, $y = 8(-2) - 4$
$= -20$

$(-2, -20)$

b)	Provide a neat sketch of $y=5x(x-2)^2$	2



(N.T.S)

c) Given $x^3-5x^2+7x-2=(x-2)(ax^2+bx+c)$ — 3

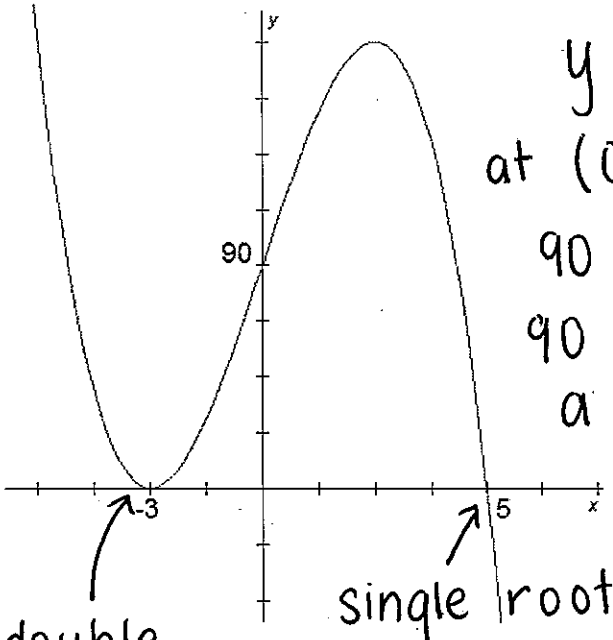
By long division or otherwise, find the values of a, b and c

$$
\begin{array}{r}
x^2-3x+1 \\
(x-2)\overline{)\,x^3-5x^2+7x-2} \\
x^3-2x^2 \\
\hline
-3x^2+7x \\
-3x^2+6x \\
\hline
x-2 \\
x-2 \\
\hline
0
\end{array}
$$

$\therefore x^3-5x^2+7x-2=(x-2)(x^2-3x+1)$

$\therefore$ $\boxed{a=1,\ b=-3,\ c=1}$

d) Find the equation of the cubic polynomial in this diagram. Leave your answer in factorised form. — 3



double root

single root

$y=a(x+3)^2(x-5)$

at $(0,90)$

$90=a(0+3)^2(0-5)$

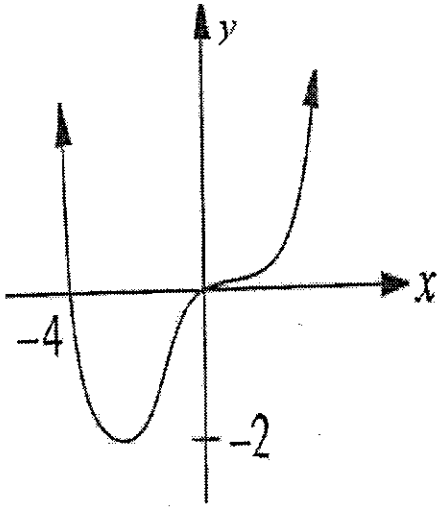
$90=-45a$

$a=-2$

$\therefore$ $\boxed{y=-2(x-5)(x+3)^2}$

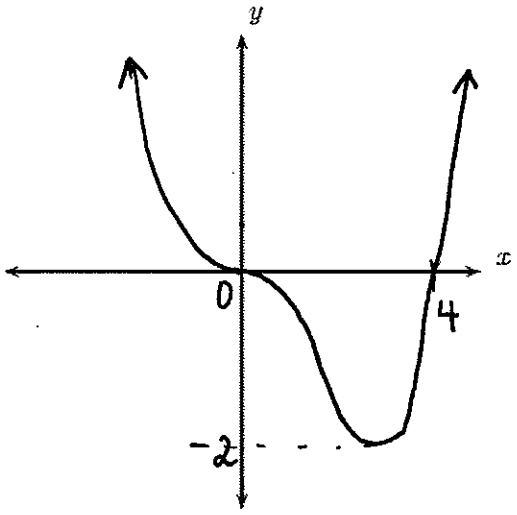
or $\boxed{y=2(5-x)(x+3)^2}$

e)	Given the graph of $y=P(x)$ in the diagram below	8

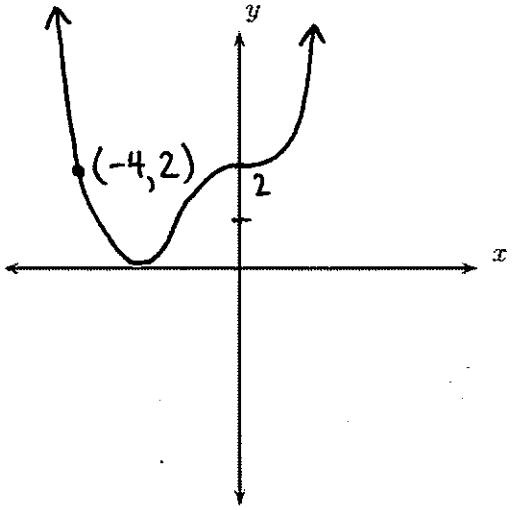


Sketch the graphs of the following on separate diagrams.

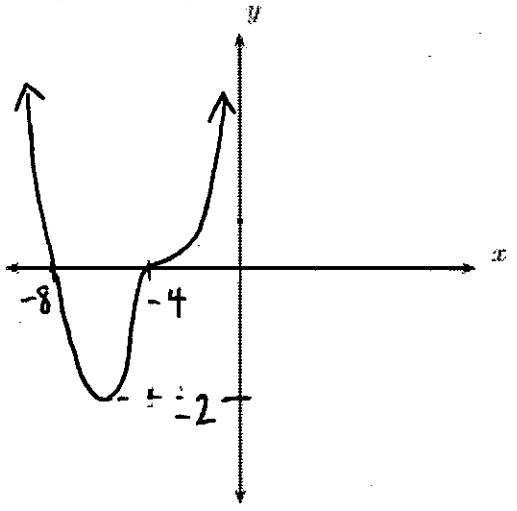
(i) $y=P(-x)$



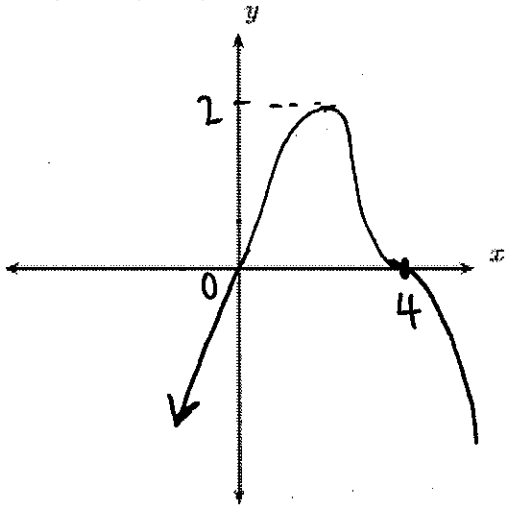
(ii) $y=P(x)+2$



(iii) $y=P(x+4)$



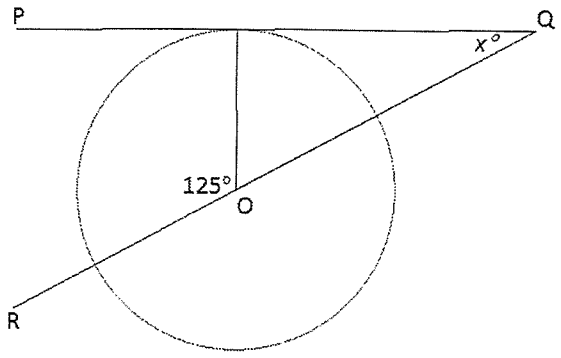
(iv) $y=-P(x-4)$



Question 6 (15 Marks) [all sections into Yearly 2014] **Marks**

a) Find x in each case.
The centre of the circle is denoted by O.
Do not give reasons.

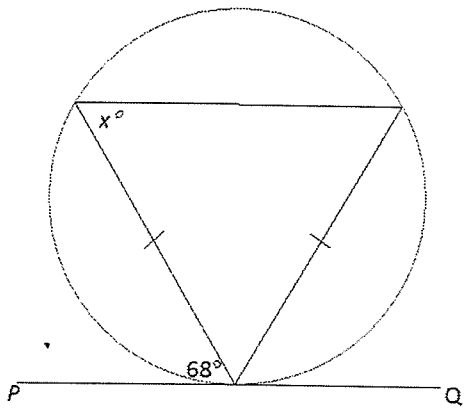
(i)



PQ is a tangent. 1

$x = 35°$

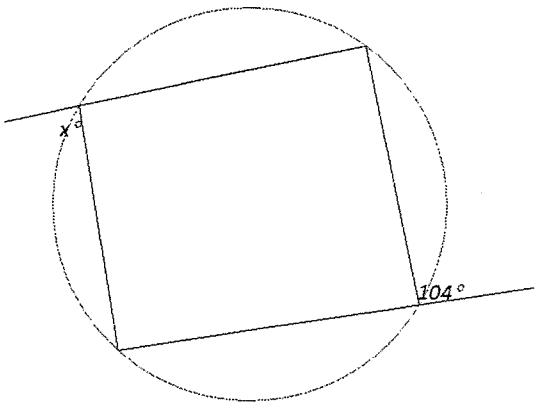
(ii)



PQ is a tangent. 1

$x = 68°$

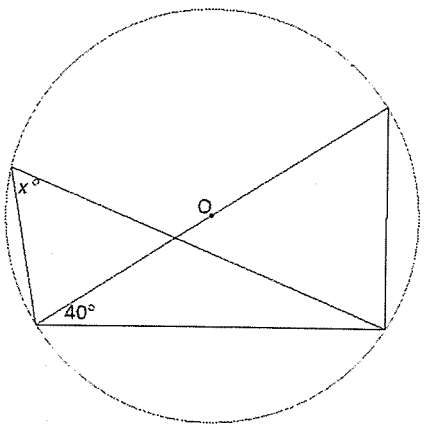
(iii)



1

$x = 76°$

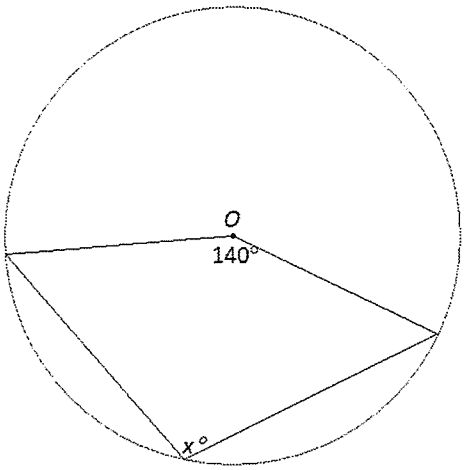
(iv)



$x = 50°$

1

(v)



$x = 110°$ 1

b) Show that $(x-2)$ is a factor of $P(x)=x^3+2x^2-5x-6$ using the remainder-factor theorem. 2

$P(2) = 2^3+2(2)^2-5(2)-6$

$= 8+8-10-6$

$= 0$

c) Given x^2-x-6 is a factor of $2x^3+mx^2-13x+n$, find the values of m and n. 2

$(x-3)(x+2)$

$2x^3+mx^2-13x+n = (x^2-x-6)Q(x)$

$2(3)^3+9m-39+n = 0$

$9m+n=-15$ — (1) (½)

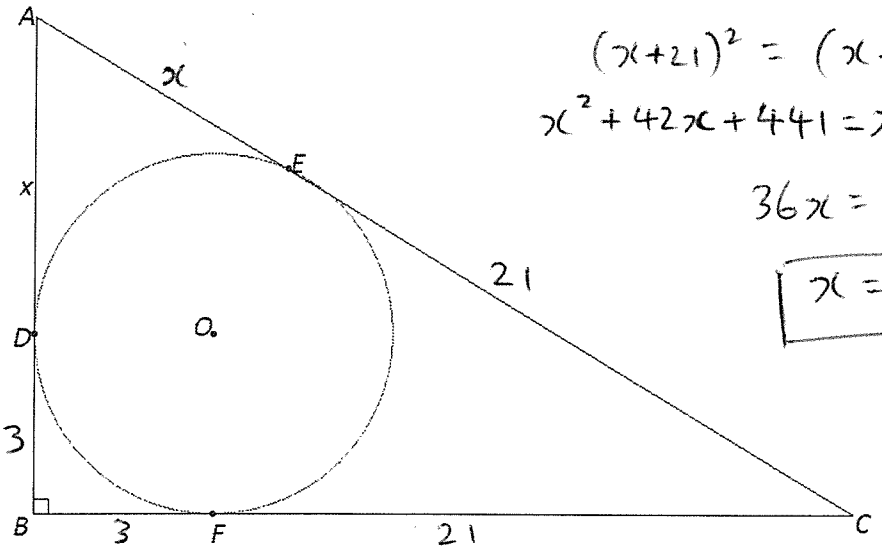
$2(-2)^3+m(-2)^2-13(-2)+n=0$

$4m+n=-10$ — (2) (½)

(1) − (2) $5m=-5$

$m=-1$ (½) $n=-6$ (½)

d)



$(x+21)^2 = (x+3)^2 + 24^2$

$x^2 + 42x + 441 = x^2 + 6x + 9 + 576$

$36x = 144$

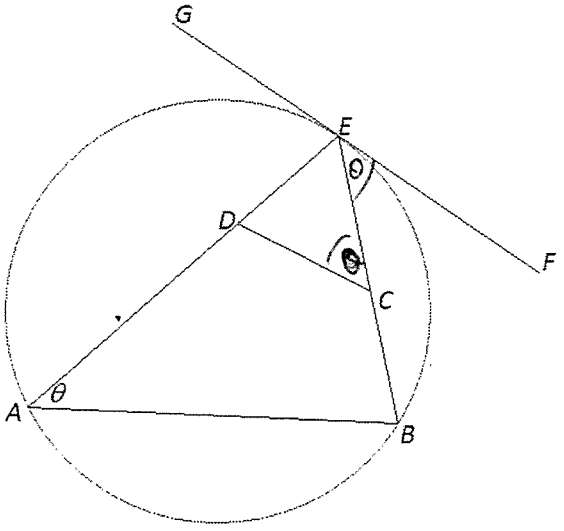
$x = 4$

2

$BC = 24cm$, $DB = 3cm$, $AD = x$

Find the value of x.

e)



4

GF is a tangent to the circle at E.

Quadrilateral $ABCD$ is cyclic.

Prove $DC \parallel GF$.

$\angle ECD = \angle DAC = \theta$ (ext. $\angle$ of a cyclic quad is equal to the opp. int. $\angle$)

$\angle FEB = \angle DAC = \theta$ (The ~~tangent~~ angle between a ~~tangent~~ and a chord is equal to the angle in the alternate segment)

$\therefore \angle ECD = \angle FEC$ but they are also alternate angles.

$\therefore DC \parallel GF$

Question 7 (10 Marks)		Marks
a)	If the equation $x^2+y^2+ax+by+c=0$ always represents a circle, prove that $a^2>4c-b^2$.	3

$x^2+ax+y^2+by=-c$

$x^2+ax+\frac{a^2}{4}+y^2+by+\frac{b^2}{4}=-c$ ✓

$\left(x+\frac{a}{2}\right)^2+\left(y+\frac{b}{2}\right)^2=\frac{a^2+b^2}{4}-c$

$=\frac{a^2+b^2-4c}{4}$ ✓

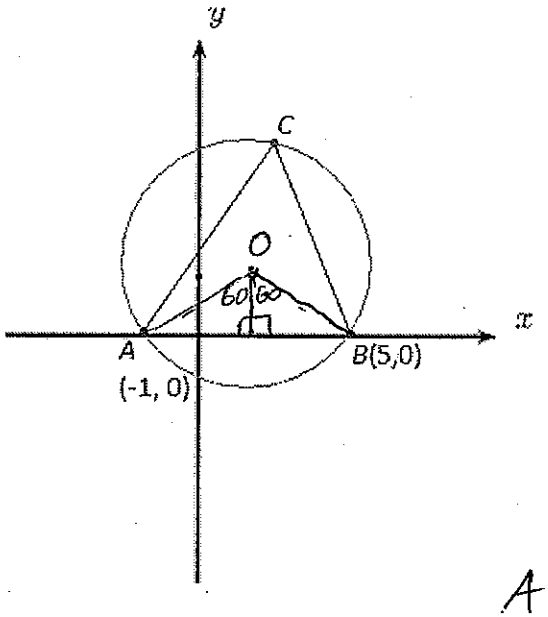
Now $\left(x+\frac{a}{2}\right)^2\geq 0$ and $\left(y+\frac{b}{2}\right)^2\geq 0$ and radius >0

$\therefore \frac{a^2+b^2-4c}{4}>0$

$a^2+b^2>4c$

$a^2<4c-b^2$ ✓

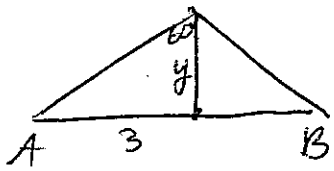
b) 3



Triangle ABC is equilateral.
The vertices A, B and C lie on the circumference of the circle.
Find the exact coordinates of the centre of the circle.

$AB=6$ units $\frac{1}{2}$

By symmetry, $x_0=2$



$\tan 60=\frac{3}{y}$

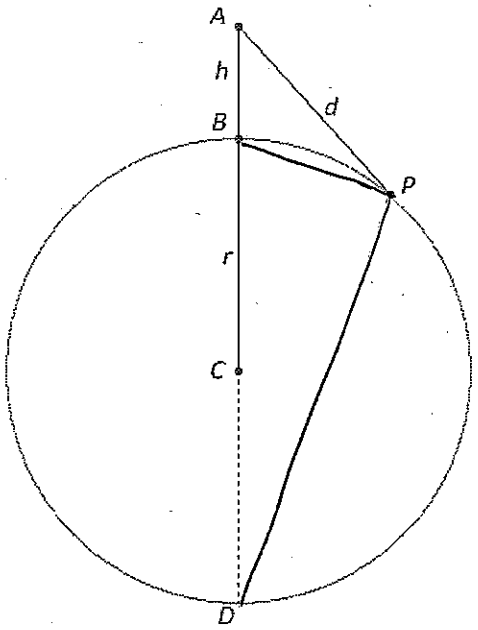
$y_0=\frac{3}{\tan 60}$

$=\frac{3}{\sqrt{3}}$

$y_0=\sqrt{3}$

Centre, $O=(2,\sqrt{3})$ ✓ (1½)

c)



C is the centre of this circle.

The distance d of the visible horizon from an observer at height h above the earth's surface is given by the formula $d^2 = h^2 + 2hr$ where r is the radius of the earth.

(i) Prove the tangent/secant formula. 2

Construct PB, PD

In $\triangle ABP$ and $\triangle APD$,

$\angle APB = \angle BDP$ (Alternate Segment Theorem) ✓

$\angle A$ in common

$\therefore \triangle ABP \,|||\, \triangle APD$ (Equiangular)

Then $\frac{AB}{AP} = \frac{AP}{AD}$ (Matching sides in proportion) $\Rightarrow \therefore AP^2 = (AB)(AD)$ ✓

(ii) Hence or otherwise prove $d^2 = h^2 + 2hr$ 2

$AP^2 = AB \cdot AD$

$d^2 = h(2r + h)$ ✓

$\Rightarrow d^2 = h^2 + 2hr$ ✓

OR ~~From Pyth~~ $\angle APD = 90°$ (tangent meets radius at 90° at point of contact.) ✓

From Pyth. Theorem, $r^2 + d^2 = (h+r)^2$

$r^2 + d^2 = h^2 + 2hr + r^2$ ✓

$d^2 = h^2 + 2hr$ #

END OF TEST