SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

Year 10

Yearly Examination 2015

Mathematics

General Instructions
- Working time – 120 minutes
- Reading time – 5 minutes
- Write using black or blue pen.
- Approved calculators may be used.
- All necessary working MUST be shown in every question if full marks are to be awarded.
- Marks may not be awarded for untidy or badly arranged work.
- All answers should be in simplest exact form unless specified otherwise.
- If more space is required, clearly write the number of the QUESTION on one of the back pages and answer it there. Indicate that you have done so.
- Clearly indicate your class by placing an X, next to your class

Examiner: *A. Fuller*

NAME:

Class	Teacher	
10 MaA	Mr Boros	
10 MaB	Mr Hespe	
10 MaC	Ms Ward	
10 MaD	Mr Parker	
10 MaE	Ms Millar	
10 MaF	Mr Elliott& Mr Choy	
10 MaG	Mr Gainford	

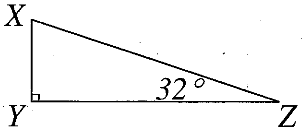
Section	Mark
A	/20
B	/15
C	/15
D	/15
E	/15
F	/14
G	/13
H	/13

/120

8) $\left(\sqrt{5}-1\right)^2 =$

(A) 4 (B) 6 (C) $6-2\sqrt{5}$ (D) $6-\sqrt{10}$

9)

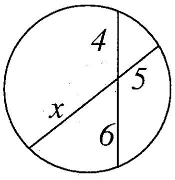


As a decimal correct to 3 decimal places, the ratio $\frac{YZ}{XZ}$ is

(A) $0{\cdot}530$ (B) $0{\cdot}625$

(C) $0{\cdot}848$ (D) unable to be calculated.

10)



The value of x is

(A) $3\frac{1}{3}$ (B) $4\frac{4}{5}$ (C) 7 (D) $7\frac{1}{2}$

11) Rationalize the denominator of $\frac{1}{\sqrt{7}-2}$.

(A) $\frac{\sqrt{7}-2}{3}$ (B) $\frac{\sqrt{7}+2}{3}$

(C) $\frac{\sqrt{7}-2}{5}$ (D) $\frac{\sqrt{7}+2}{5}$

12) The information below relates to a group's performance on two tests.

	Test I	Test II
Mean	60	68
Standard Deviation	6	10

What mark in Test II is equivalent to a mark of 72 in Test I?

(A) 72 (B) 80 (C) 84 (D) 88

13) If $\cos\theta < 0$ and $\sin\theta > 0$, then

(A) $0° < \theta < 90°$ (B) $90° < \theta < 180°$

(C) $180° < \theta < 270°$ (D) $270° < \theta < 360°$

14) Which does NOT have $m+1$ as a factor?

(A) m^2-1 (B) m^2+1

(C) m^2+m (D) m^2+2m+1

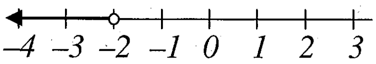
15) The centre and radius of the circle $(x - 1)^2 + (y + 2)^2 = 16$ are

(A) $(-1, 2)$ and 4 (B) $(1, -2)$ and 4

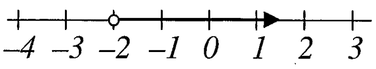
(C) $(-1, 2)$ and 16 (D) $(1, -2)$ and 16

16) Which graph illustrates the solution of $-4x > 8$?

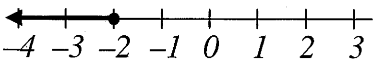
(A)



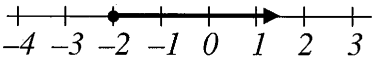
(B)



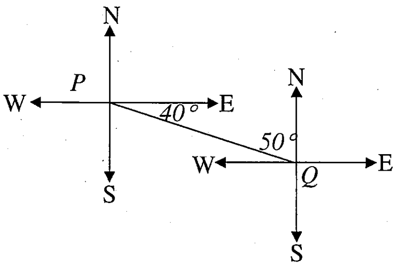
(C)



(D)



17) The bearing of P from Q is



(A) 040° (B) 050° (C) 130° (D) 310°

18) Which of the following is a polynomial?

(A) $x + \frac{2}{x}$ (B) $x^2 + 2^x$

(C) $(\sqrt{x} + 1)^2$ (D) $\sqrt{2}x^2 + \sqrt{3}x^3$

19) Three students are playing a game. They each toss a coin at the same time. A winner is declared if only one student tosses a head. What is the probability that a winner is declared?

(A) $\frac{1}{8}$ (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) $\frac{3}{8}$

20) For an item originally priced at \$$P$, its value \$$A$, after n years of depreciation at $r\%$ p.a. is given by

(A) $A = -P\left(1+\frac{r}{100}\right)^n$ (B) $A = -P\left(\frac{1+r}{100}\right)^n$

(C) $A = P\left(1-\frac{r}{100}\right)^n$ (D) $A = P\left(\frac{1-r}{100}\right)^n$

SECTION B (15 marks)

1) How many significant figures does 0.002030 have? [1]

2) $P(x) = x^3 + 2x^2 - 7x - 3$ and $Q(x) = x^2 - 3x + 7$ [4]

Find:

(a) The degree of $P(x) \times Q(x)$

(b) The constant term of $P(x) \times Q(x)$

(c) $P(x) + Q(x)$

(d) $P(x) - Q(x)$

3) Solve the following:

(a) $5 - \frac{x}{2} = 4$ [1]

(b) $x^2 - 11x + 24 = 0$ [2]

(c) $(3x-2)^2 = 49$ [2]

(d) $9x = 10x^2 + 2$ [2]

4) Let $f(x) = \frac{1}{2}x + 2$. [3]

(a) Evaluate $f(\text{-}3)$

(b) Simplify $f(3x-1)$

(c) Find the inverse function $f^{-1}(x)$

SECTION C (15 marks)

1) Factorise the following:

(a) $ap^2 - apq$ [1]

(b) $ap + aq - p - q$ [2]

2) Find the exact value of the following:

(a) $\tan 150°$ [1]

(b) $\cos\theta$, if θ is acute and $\tan\theta = \frac{\sqrt{2}}{2}$. [1]

3) Use the remainder theorem to find the remainder when $x^3 - 3x + 5$ is divided by $(x + 2)$. [1]

4) $1100 is invested for 6 years compounded yearly at 6% p.a. [3]

(a) How much is the investment worth after 6 years?

(b) How much interest is earned?

(c) What is the equivalent simple interest rate (to one decimal place)?

5) $P(x) = 5x^3 - 17x^2 - x + 11$ is divided by $A(x) = x - 2$. [3]

Find the quotient $Q(x)$ and the remainder $R(x)$.

6) A triangle has sides 4cm, 6cm and 7cm?

(a) What is the size of the smallest angle (to the nearest degree)? [2]

(b) Hence, what is the area of the triangle (to the nearest square centimetre)? [1]

SECTION D (15 marks)

1) Sketch the graph of the following (on the axes provided): [4]

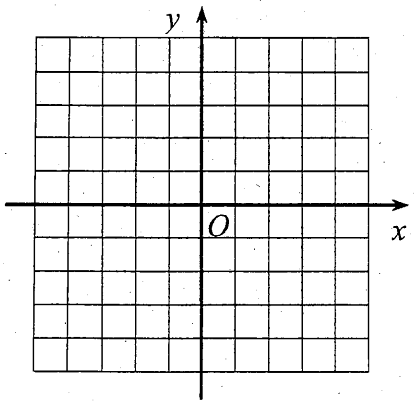
(a) $y = 4 - x^2$

(b) $y = \frac{2}{2+x}$

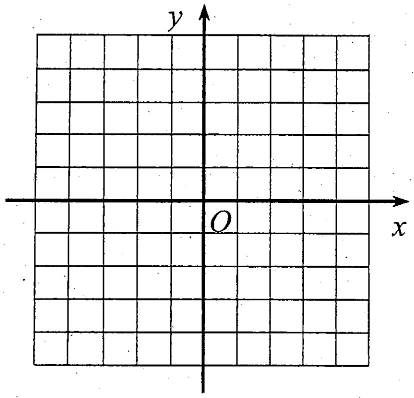
(c) $y = -4^x$

(d) $y = 2(2 + x)$

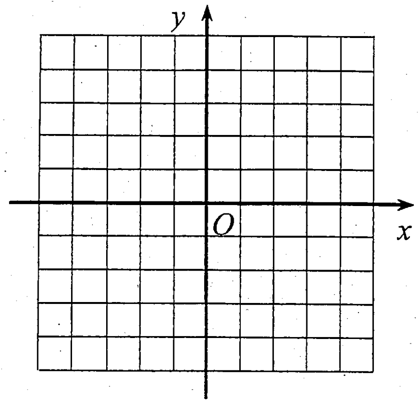
(a)



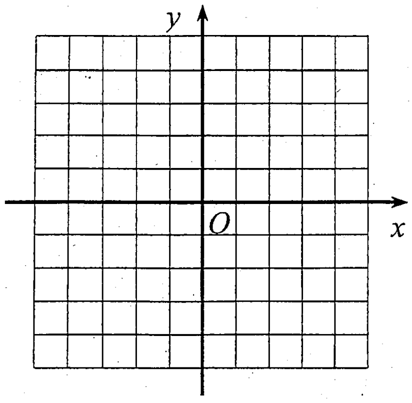
(b)



(c)



(d)



2) Give an example of a function f for which $f(a + b) = f(a) \times f(b)$ [1]

3) For the set of scores: 29, 31, 31, 34, 45, 39, 42, 45, 47, 53, 57, 61 [2]

Calculate:

(a) the range

(b) the inter-quartile range

4) Consider the parabola $y = x^2 - 6x + 7$.

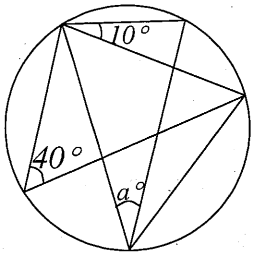
(a) Use the quadratic formula to find the x intercepts [2]

(b) Find the coordinates of the vertex. [2]

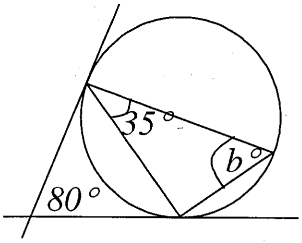
5) Find the value of the pronumerals in the following (no reasons required): [4]

Diagrams are NOT TO SCALE

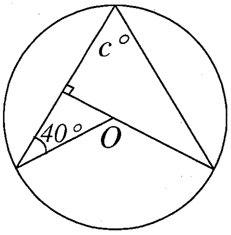
(a)



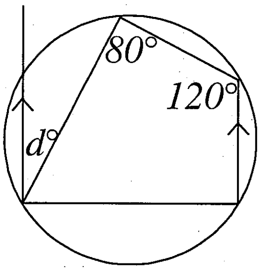
(b)



(c)

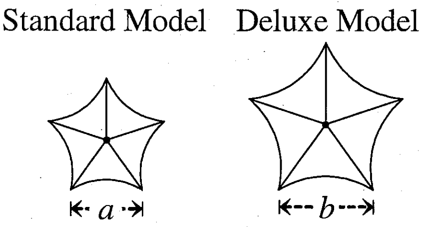


(d)



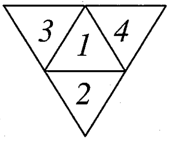
SECTION E (15 marks)

1) [2]



The two umbrellas above are similar shapes with $a : b = 4 : 5$. The standard model requires $1{\cdot}44$ m^2 of material. How much material is required for the deluxe model?

2) The net of the die is shown below.

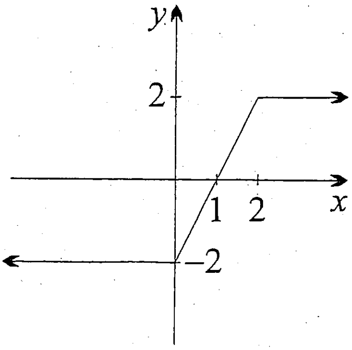


The faces are numbered *1, 2, 3* and *4.* The die is rolled twice. The number on the face that the die lands on is recorded each time.

(a) By considering a table, or otherwise, find the probability that the sum of the two recorded numbers is greater than 4. [2]

(b) Find the probability that the sum of the two recorded numbers is greater than 4 if it is known that a 3 appears on one of the dice. [1]

3) The graph of $y = f(x)$ is given below [3]



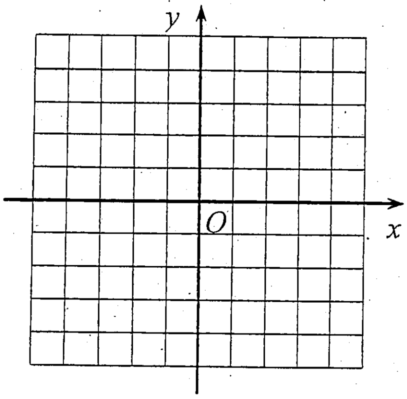
Sketch the following (on the axes provided)

(a) $y = f(x) - 1$

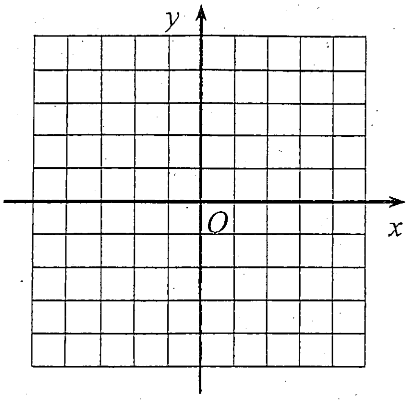
(b) $y = f\left(\frac{x}{2}\right)$

(c) $y = f(1 - x)$

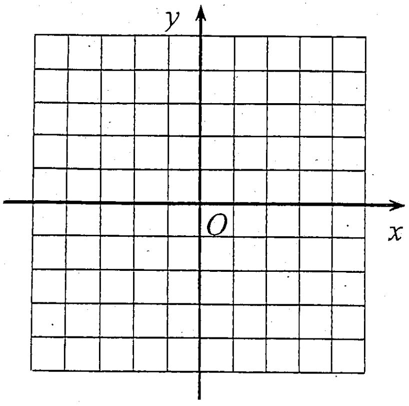
(a)



(b)



(c)

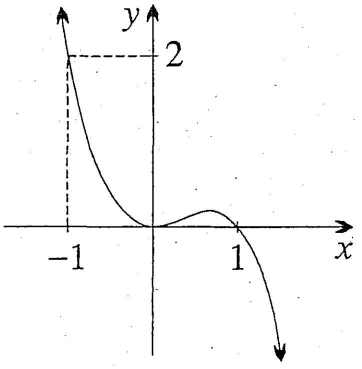


4) $y = -\sqrt{x + 4}$. What are the restrictions on x? [1]

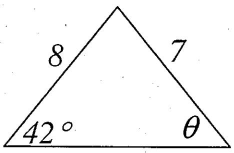
5) Write down the equation of the following cubic curves. [4]
(You may leave the equation in factored form)

(a) The x- intercepts are -2, -1 and 1. The y- intercept is -4.

(b)



6) [2]



Find the size of the angle θ to the nearest degree.

SECTION F (14 marks)

1) Write the following in the form a^b [2]

(a) $3^x \times 3^y$

(b) $3^x \times 5^x$

2) If the 17th day of a month is Thursday: [2]

(a) What is the first day of the month?

(b) What is the first day of the following month likely to be?

3) Make a the subject of $ab = ac + bd$ [2]

4) Andrew has a bucket of coins. Each coin has a zero on one side and an integer greater than zero on the other side. He randomly draws three coins, tosses them and calculates a score by adding the three numbers that appear.

(a) On Monday, Andrew draws coins with a 7, a 5 and a 10 for a score of 17. By considering a tree diagram, or otherwise, what other scores could he obtain by tossing the same three coins? [2]

(b) On Tuesday, Andrew draws three coins and tosses them three times, obtaining scores of 60, 110 and 130. On each of these tosses, exactly one of the coins shows a zero. Determine the maximum possible score that can be obtained by tossing these three coins. [2]

(c) On Wednesday, Andrew draws a coin with 25, one with a 50, and a third coin. He tosses these three coins and obtains a score of 170. Determine all possible numbers, other than zero, that could be on the third coin. [2]

5) Solve $2\sin x + 1 = 0$ for $0^\circ \leq x \leq 360^\circ$ [2]

SECTION G (13 marks)

1) (a) Expand $(x^2 + 2x)^2$ [1]

(b) Write $x^4 + 4x^3 - 5x^2 - 18x + 8$ in the form [1]

$(x^2 + 2x)^2 + A(x^2 + 2x) + B$

(c) Hence, by using the substitution $m = x^2 + 2x$, [3]

solve $x^4 + 4x^3 - 5x^2 - 18x + 8 = 0$

2) If $x^2 = 8x + y$ and $y^2 = x + 8y$ with $x \neq y$. What is the value of $x^2 + y^2$? [2]

3) $f(x) = 2x^2 - x^4$, $0 \leq x \leq 1$. Find the inverse function $f^{-1}(x)$. [3]

4) The lines $y = \frac{1}{2}x$ and $y = -\frac{1}{2}x$ are tangents to a circle at $(2,1)$ and $(2,-1)$. [3]

Find the equation of the circle.

SECTION H (13 marks)

1) A cone, a cylinder and a sphere all have radius r. The height of the cylinder is H and the height of the cone is h.

(a) If the cylinder and the sphere have the same volume, show that [1]

$H = \frac{4}{3}r.$

(b) If the cone and the cylinder have the same total surface area, show that [2]

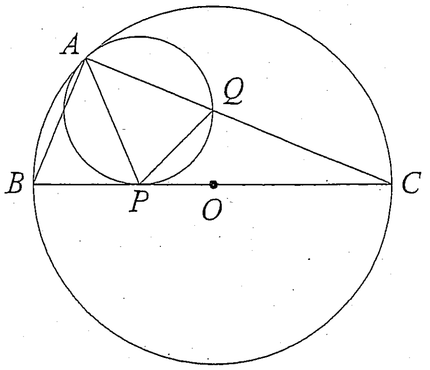
$r + 2H = \sqrt{r^2 + h^2}.$

(c) Hence, prove that h and H cannot both be integers. [3]

2) A monic cubic polynomial has a remainder of $(x + 8)$ when divided by $(x^2 + 4)$ and a remainder of -4 when divided by x. [3]

Find the polynomial in the form $ax^3 + bx^2 + cx + d$

3) [4]



Two circles touch internally at A where there is a common tangent. BC is a diameter of the larger circle, touching the smaller circle at P. AC cuts the smaller circle at Q. Prove that $\angle APQ + \angle ACP = 90°$.

End of Exam

Use this space if you wish to **rewrite** any answers.
Clearly *indicate* the **Question** number.

SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2015

Year 10 Yearly

Advanced Mathematics

Solutions

Sections	**Teacher**
A	-
B	AMG
C	EC
D	JM
E	PSP
F	AW
G	DH
H	RB

Multiple Choice Answers (Section A):

1.	B	**2.**	A	**3.**	D	**4.**	A	**5.**	B	**6.**	A	**7.**	D
8.	C	**9.**	C	**10.**	B	**11.**	B	**12.**	D	**13.**	B	**14.**	B
15.	B	**16.**	A	**17.**	D	**18.**	D	**19.**	D	**20.**	C		

SECTION A (20 marks) *Circle the correct answer*

$\frac{3(x-1)}{3} = x-1$

B √ 1) $\frac{3x-3}{3} =$

(A) x [✓ (B) $x-1$] (C) $x-3$ (D) $3x-1$

A √ 2) $10^{-2} =$

[✓ (A) $\frac{1}{100}$] (B) $\frac{1}{20}$ (C) -20 (D) -100

D √ 3)

1	1	1	2	2	3	3

These cards are shuffled and placed in a hat. One card is drawn from the hat at random.
What is the probability that it is a card with a 3 on it?

(A) $\frac{1}{3}$ (B) $\frac{1}{7}$ (C) $\frac{2}{5}$ [✓ (D) $\frac{2}{7}$]

A √ 4) $\sqrt{24} =$

[✓ (A) $2\sqrt{6}$] (B) $4\sqrt{3}$ (C) $4\sqrt{6}$ (D) $6\sqrt{2}$

√ 5)

B

Which of the following sets of scores has the greatest standard deviation?

I: 3, 4, 5, 6, 7 1.41
II: 3, 3, 5, 7, 7 1.7888
III: 3, 5, 5, 5, 7 01.26.

(A) I only [✓ (B) II only]
(C) III only (D) I, II and III have the same standard deviation.

A √ 6) Which point lies on both $y = x^3$ and $7x - 3y - 10 = 0$?

[✓ (A) $(-2, -8)$] √ (B) $(-1, -1)$ (C) $(1, 1)$ (D) $(2, 8)$

D 7) ?

Cf	Score	Frequency
3	5	3
4	6	1
6	7	2
13	8	7

$\Sigma f = 13$

with 13 scores the median is in position 7

For this set of scores, which of the following statements is correct?

~~(A)~~ There are 4 scores and their median is ~~6.5~~ 7.5
~~(B)~~ There are 4 scores and their median is ~~7~~ 8
(C) There are 13 scores and their median is ~~6.5~~ 7.5
[✓ (D) There are 13 scores and their median is ~~7~~ 8]

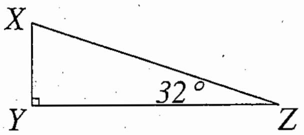
median 8

C ✓8) $(\sqrt{5}-1)^2=$ $5+1-2\sqrt{5}$

(A) 4 (B) 6 [✓ (C) $6-2\sqrt{5}$] (D) $6-\sqrt{10}$

✓9)

C



$\cos 32^\circ$

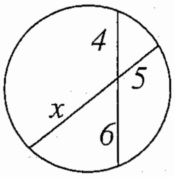
As a decimal correct to 3 decimal places, the ratio $\frac{YZ}{XZ}$ is

(A) 0·530 (B) 0·625

[✓ (C) 0·848] (D) unable to be calculated.

✓10)

B



$5x = 4\times6$

The value of x is

(A) $3\frac{1}{3}$ [✓ (B) $4\frac{4}{5}$] (C) 7 (D) $7\frac{1}{2}$

B ✓11) Rationalize the denominator of $\frac{1}{\sqrt{7}-2}$. $\times\frac{\sqrt{7}+2}{\sqrt{7}+2}$

(A) $\frac{\sqrt{7}-2}{3}$ [✓ (B) $\frac{\sqrt{7}+2}{3}$]

(C) $\frac{\sqrt{7}-2}{5}$ (D) $\frac{\sqrt{7}+2}{5}$

D 12) ✓ The information below relates to a group's performance on two tests.

	Test I	Test II
Mean	60	68
Standard Deviation	6	10

What mark in Test II is equivalent to a mark of 72 in Test I?

(A) 72 (B) 80 (C) 84 [✓ (D) 88]

72 in test 1 has z=2
∴ In test 2 68+2×10=88

B ✓13) If $\cos\theta < 0$ and $\sin\theta > 0$, then

(A) $0° < \theta < 90°$ [✓ (B) $90° < \theta < 180°$]

(C) $180° < \theta < 270°$ (D) $270° < \theta < 360°$

B 14) Which does NOT have $m + 1$ as a factor?

(A) m^2-1 [✓ (B) m^2+1]

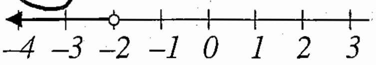
(C) m^2+m (D) m^2+2m+1

B. ✓15) The centre and radius of the circle $(x-1)^2 + (y+2)^2 = 16$ are

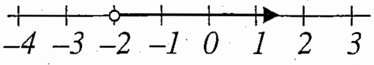
(A) $(-1, 2)$ and 4
[✓ (B)] $(1, -2)$ and 4
(C) $(-1, 2)$ and 16
(D) $(1, -2)$ and 16

A ✓16) Which graph illustrates the solution of $-4x > 8$?

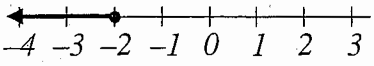
[✓ (A)]



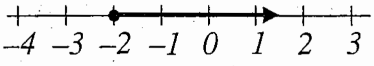
(B)



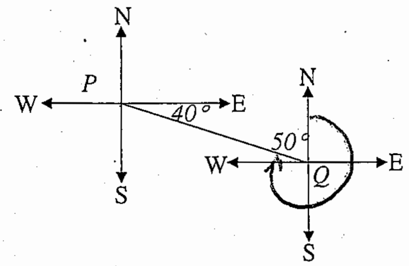
(C)



(D)



D. ✓17) The bearing of P from Q is



(A) 040°
(B) 050°
(C) 130°
[✓ (D)] 310°

D. ✓18) Which of the following is a polynomial?

~~(A)~~ $x + \frac{2}{x}$
~~(B)~~ $x^2 + 2^x$
~~(C)~~ $(\sqrt{x}+1)^2$
[✓ (D)] $\sqrt{2}x^2 + \sqrt{3}x^3$

D. ✓19) Three students are playing a game. They each toss a coin at the same time. A winner is declared if only one student tosses a head. What is the probability that a winner is declared?

(A) $\frac{1}{8}$
(B) $\frac{1}{6}$
(C) $\frac{1}{3}$
[✓ (D)] $\frac{3}{8}$

C ✓20) For an item originally priced at \$$P$, its value \$$A$, after n years of depreciation at r% p.a. is given by

(A) $A = -P\left(1+\frac{r}{100}\right)^n$
(B) $A = -P\left(\frac{1+r}{100}\right)^n$
[✓ (C)] $A = P\left(1-\frac{r}{100}\right)^n$
(D) $A = P\left(\frac{1-r}{100}\right)^n$

SECTION B (15 marks)

1) How many significant figures does 0.002030 have? [1]

[Very poorly understood. Many wrote '5' and many '3']

4

2) $P(x) = x^3 + 2x^2 - 7x - 3$ and $Q(x) = x^2 - 3x + 7$ [4]

Find:-

(a) The degree of $P(x) \times Q(x)$

[Many wrote x^5]

5

(b) The constant term of $P(x) \times Q(x)$

[Well answered.]

-21

(c) $P(x) + Q(x)$

[Well answered]

$x^3 + 3x^2 - 10x + 4$

(d) $P(x) - Q(x)$

$x^3 + x^2 - 4x - 10$

[Well answered.]

3) Solve the following:

(a) $5 - \frac{x}{2} = 4$ [1]

$10 - x = 8$

$-x = -2$

$x = 2$

[Well answered]

(b) $x^2 - 11x + 24 = 0$ [2]

$(x-3)(x-8) = 0$

$x = 3, 8$

[Well answered.]

[Here many failed to put ± 7, so got only one answer.]

(c) $(3x-2)^2 = 49$ [2]

$$3x-2 = \pm 7$$
$$3x = \pm 7 + 2$$
$$x = \frac{\pm 7 + 2}{3}$$

$x = 3, -\frac{5}{3}$

(d) $9x = 10x^2 + 2$ [2]

[Quite well answered]

$$10x^2 - 9x + 2 = 0$$
$$x = \frac{9 \pm \sqrt{81 - 80}}{20}$$
$$x = \frac{1}{2}, \frac{2}{5}$$

$x = \frac{1}{2}$ or $\frac{2}{5}$

4) Let $f(x) = \frac{1}{2}x + 2$. [3]

(a) Evaluate $f(-3)$

$f(-3) = -\frac{3}{2} + 2$ $f(-3) = \frac{1}{2}$

[Almost all got this right.]

(b) Simplify $f(3x-1)$

$$f(3x-1) = \frac{1}{2}(3x-1) + 2$$
$$= \frac{3}{2}x - \frac{1}{2} + 2$$

$f(3x-1) = \frac{3}{2}(x+1)$

[Many failed to do the last simplification step.]

(c) Find the inverse function $f^{-1}(x)$

$$y = \frac{1}{2}x + 2$$
$$\rightarrow x = \frac{1}{2}y + 2$$
$$2x = y + 4$$
$$\underline{y = 2x - 4}$$

[A sizeable proportion showed no understanding of the process.]

Section C Solutions

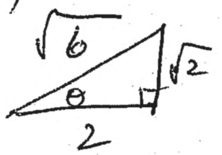
① (a) $ap(p-q)$ [1]

(b) $a(p+q)-(p+q)$
$= (p+q)(a-1)$ [2]

ⓘ This was ~~reasonably~~ well done, the most common wrong answers
(a)
(b) is $(p+q)(1-a)$ or $-(p+q)(a-1)$.

② (a) $\tan 150$
$= -\tan 30°$
$= -\frac{1}{\sqrt{3}}$ [1]

(b)



$\cos\theta$
$= \frac{2}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$

$\sqrt{\frac{2}{3}} = \frac{\sqrt{6}}{2}.$ [1]

② (a) Only a minority of student gave $\tan 150° = \frac{1}{\sqrt{3}}$
Answers ~~expressed~~ in non-rationalised form were common.

③ $P(-2) = -8+6+5$
$= 3.$
$\therefore$ remainder $= 3$ [1]

③ Well don

④

(a) $1100 \times (1.06)^6$

$= \$1560.37$ [1]

(b) 460.37

interest is earned

[1]

(c) $\frac{460.37}{11000} \div 6$

$\doteqdot 0.07$

approx 7%

$\doteqdot 0.00697$ [1]

$\doteqdot 7\%$

④

Those students who read the question carefully answered the question well.

(a) The main error was failure to compound the interest to 5 or 7 years instead of six.

(b) This was usually well done following on from the previous answer. A simple subtraction was required

(c) Well done

⑤

$$x-2 \overline{\smash{)}\, 5x^3 - 17x^2 - x + 11} \quad [3]$$

$$5x^2 - 7x - 15$$

$$5x^3 - 10x^2$$

$$-7x^2 - x$$

$$-7x^2 + 14x \quad +11$$

(23

$-7x - 15$

$-7x^2 - x + 11$

$-(-7x^2 + 14x)$

$-15x + 11$

$-(-15x + 30)$

$\boxed{-19}$

⑤ This was a introductory type of question, which was generally well done

⑥ (a)

$16 = 49 + 36 - 84\cos\theta$

$84\cos\theta = 69$

$\therefore \cos\theta = \frac{69}{84}$

$\theta = \cos^{-1}\left(\frac{69}{84}\right)$ [2]

$= 34^\circ 46'$

(b)

$A = \frac{1}{2} \times 42 \sin\theta$

$\approx 11{\cdot}976$ [1]

⑥

Answers here were generaly good, with some using the sine rule and others using right-angled triangle trigonometry

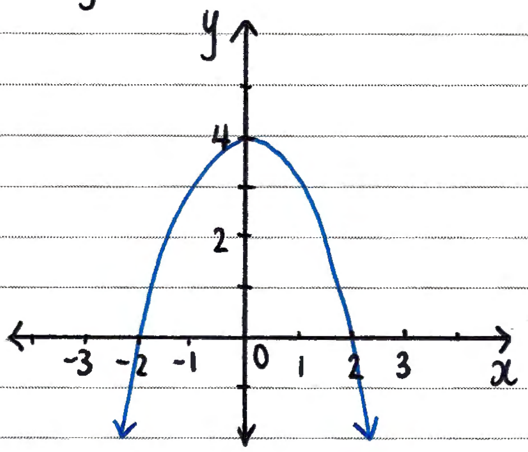
Rounding off to the nearest degree caused problem for students

With hindsight it is appropriate to emphasise two salient points

- the necessity for students not to round off until last step of a calculation and to show their calculator output before rounding off.
- the need to show all working; bald correct answers do not always score full marks

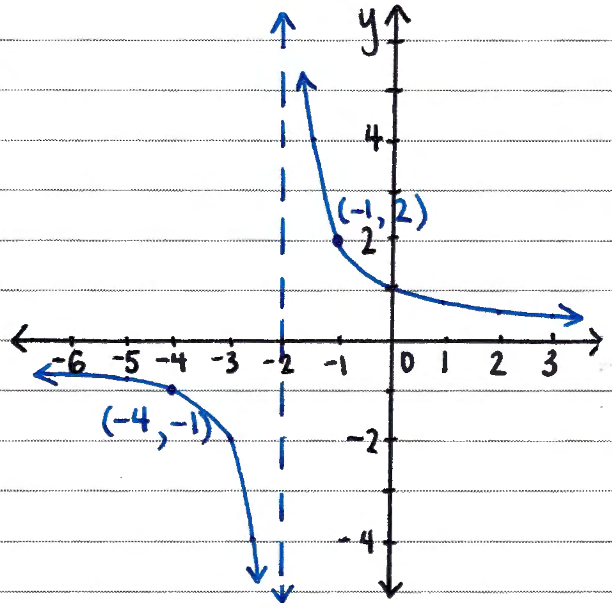
Section D

1\. (a) $y = 4 - x^2$

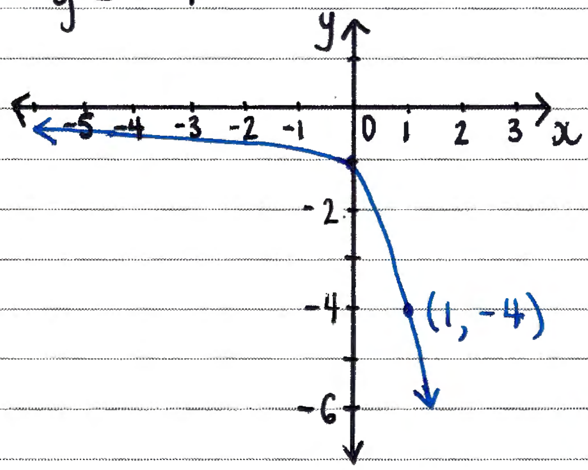


(b) $y = \dfrac{2}{2 + x}$

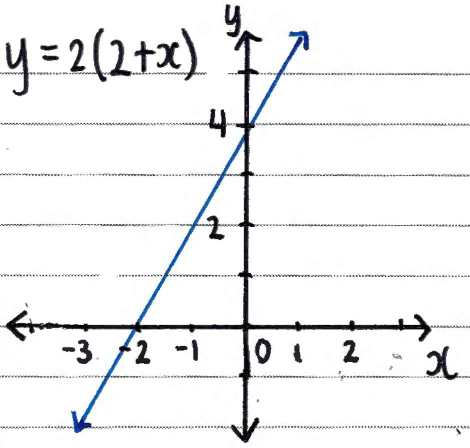
$2 + x \neq 0$
$\therefore x \neq -2$



(c) $y = -4^x$



(d) $y = 2(2+x)$



Comments

1.- Students need to make sure they label all key points, such as asymptotes, intercepts and at least two points.

2\. For $f(a+b) = f(a) \times f(b)$

An exponential as:

eg. $2^{a+b} = 2^a \times 2^b$

3\. (a) Range $= 61 - 29$
$= 32$

(b) Interquartile range
$= Q_3 - Q_1$
$= \left(\dfrac{53 + 47}{2}\right) - \left(\dfrac{34 + 31}{2}\right)$
$= 50 - 32.5$
$= 17.5$

Comments

2.- Not many students got this question.

3.- Some students incorrectly found the quartiles.

4. $y = x^2 - 6x + 7$

(a) $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(7)}}{2(1)}$$
$$= \frac{6 \pm \sqrt{8}}{2}$$
$$= \frac{6 \pm 2\sqrt{2}}{2}$$
$$= \frac{2(3 \pm \sqrt{2})}{2}$$
$$= 3 \pm \sqrt{2}$$

(b) $$x = \frac{-b}{2a}$$
(axis of symmetry)
$$x = \frac{-(-6)}{2(1)}$$
$$= 3$$

At $x = 3$: $y = (3)^2 - 6(3) + 7$
$= 9 - 18 + 7$
$= -2$

$\therefore$ vertex $= (3, -2)$

Comments

In (a), students need to make sure they factorise the numerator first before simplifying to minimise errors. Likewise in (b) students substituted x incorrectly.

5.

(a) $a = 30°$

(two lots of: angles in the same segment).

(b) $b = 50°$

(tangents from external point and angle in alternate segment)

(c) $c = 65°$

(angle sum of a triangle and angle at the centre and circumference.)

(d) $d = 20°$

(opposite angle in a cyclic quadrilateral and co-interior angles in parallel lines).

Comments

Very well answered

Note: no reasons needed

Section E Solutions

1. Sides $a : b$

$4 : 5$

SA $a^2 : b^2$

$4^2 : 5^2$

$1.44 : \Delta$

$\therefore$ Deluxe Model requires $\Delta = 1.44 \times \frac{5^2}{4^2} = 2.25 \text{ m}^2$

Comment: The main error was forgetting to square the ratio. This was a 1 mark penalty.

2. a) $\text{P}(> 4) = \frac{10}{16} = \frac{5}{8}$

+	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

$-\frac{1}{2}$ if fraction not reduced

b) $\text{P}(>4 \mid 3 \text{ on one face}) = \frac{5}{7}$ i.e. 5 of the shaded squares satisfy the condition.

+	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

$-\frac{1}{2}$ if fraction not reduced BUT not penalised twice

-1 for an answer that was impossible

Alternatively

$\text{P}(3 \text{ on one face}) = \frac{7}{16}$

$\text{P}(> 4 \text{ and at least one } 3) = \frac{5}{16}$

$\therefore \text{P}(>4 \mid 3 \text{ on one face}) = \frac{\frac{5}{16}}{\frac{7}{16}} = \frac{5}{7},$

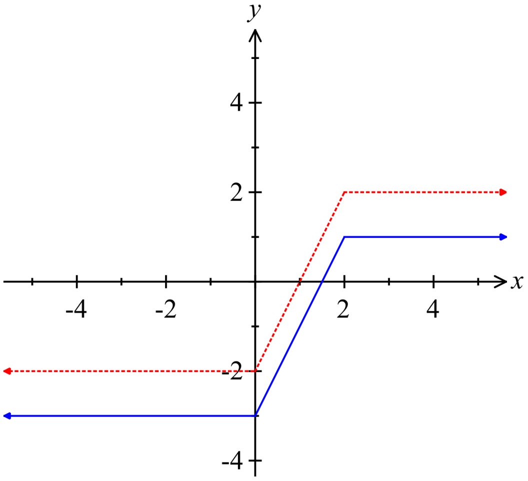
Comment: The advice given in the question was to use a table – this was good advice, especially for part b)

Part b) was not done very well. Practice with two-way tables is recommended.

3. The dotted graph is the original function and the solid is the answer.

(a) $y = f(x) - 1$

NB The answer is the original graph translated 1 unit downwards.



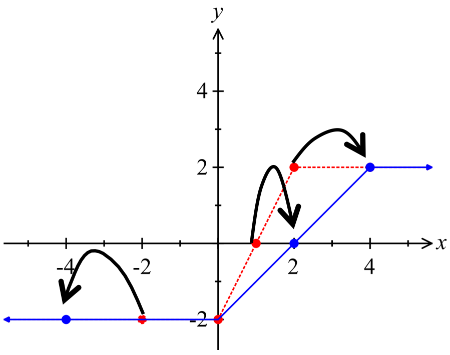
(b) $y = f\left(\dfrac{x}{2}\right)$

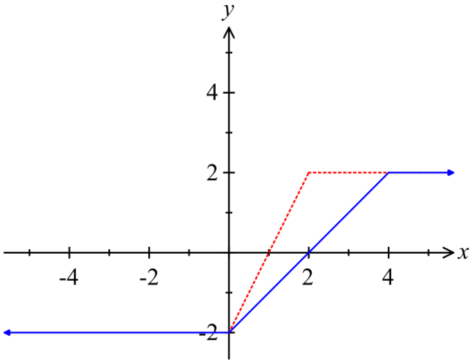
NB the answer is the original graph stretched horizontally by a factor of 2.

OR

By applying the function to some significant points (red dots), the transformation can be seen.

NB that (0, –2) doesn't "move" since $f\left(\dfrac{0}{2}\right) = f(0) = -2$



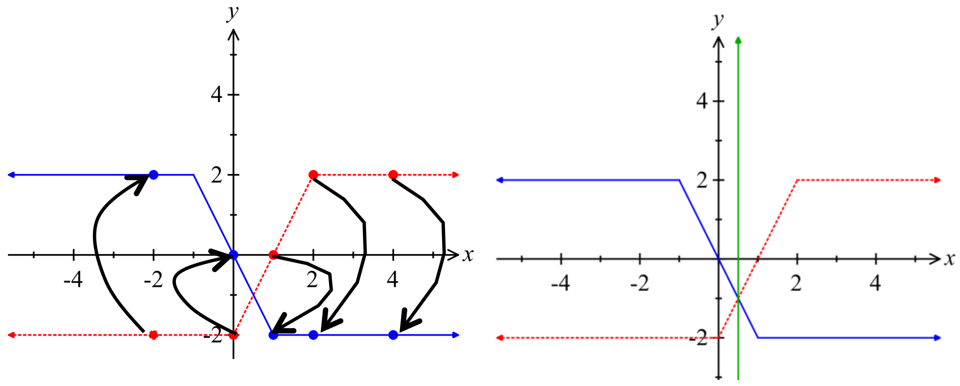


x	-2	0	2	4
$f(x)$	-2	-2	0	2
$f\left(\dfrac{x}{2}\right)$	$f\left(\dfrac{-2}{2}\right)$ $= f(-1)$ $= -2$	$f\left(\dfrac{0}{2}\right)$ $= f(0)$ $= -2$	$f\left(\dfrac{2}{2}\right)$ $= f(1)$ $= 0$	$f\left(\dfrac{4}{2}\right)$ $= f(2)$ $= 2$

3. (c) $y = f(1-x)$

NB the answer is the original graph reflected in the line $x = \frac{1}{2}$.

By applying the function to some significant points (red dots), the transformation can be seen.



x	-2	0	2	4
$f(x)$	-2	-2	0	2
$f(1-x)$	$f(1-(-2))$ $= f(3)$ $= 2$	$f(1-0)$ $= f(1)$ $= 0$	$f(1-2)$ $= f(-1)$ $= -2$	$f(1-4)$ $= f(-3)$ $= -2$

Comment: Parts (b) and (c) were not done well.
No half marks were awarded. The graph had to be "perfect".

4. The domain of $y = -\sqrt{x+4}$ is determined by $x + 4 \geq 0$

$\therefore x \geq -4$

Comment: A lack of understanding about square roots cost many students the mark.

5. (a) $y = k(x+2)(x+1)(x-1)$ [1 mark]

The y-intercept is –4 i.e. $-4 = k(-2)(-1)(1)$ [Substitute $x = 0$]

$\therefore k = 2$

$y = 2(x+2)(x+1)(x-1)$

(b) There is a double root at $x = 0$

$y = kx^2(x-1)$ [1 mark]

Substitute (–1, 2) $\Rightarrow 2 = k(-2)$

$\therefore k = -1$

$\therefore y = -x^2(x-1)$ or $x^2(1-x)$

Comment: Many students ignored the text about the two graphs being cubics.
Students with an incorrect answer, who showed no working, could only get a maximum of 1 mark if that.

6. Using the Sine Rule

$$\frac{\sin\theta}{8} = \frac{\sin 42^\circ}{7}$$

$$\therefore \sin\theta = \frac{8\sin 42^\circ}{7}$$

$$\therefore \theta \doteqdot 50^\circ, 180^\circ - 50^\circ$$

$$= 50^\circ, 130^\circ$$

Both angles fit the diagram.

Comment: Many students only scored 1.5 marks because they didn't consider the "ambiguous case".

Many students ignored the "nearest degree" which could have been costly.

SECTION F (14 marks)

1) Write the following in the form a^b [2]

(a) $3^x \times 3^y$

3^{x+y}. ①

DONE FAIRLY WELL.
$\frac{1}{2}$ for EACH COMPONENT.

(b) $3^x \times 5^x$

15^x. ①

2) If the 17th day of a month is Thursday: [2]

(a) What is the first day of the month?

17 = Thurs = 17 − 14 = 3rd Thurs.

1^{ST} = TUESDAY. ①

DONE FAIRLY WELL.

(b) What is the first day of the following month likely to be?

Thurs = 17 + 14 = 31

31 DAYS: Thurs = 31^{ST} ∴ 1^{ST} = FRIDAY.

30 DAYS: Wed = 30^{TH} ∴ 1^{ST} = THURSDAY. ①

DONE VERY WELL
THURSDAY AND/OR FRIDAY ACCEPTED

3) Make a the subject of $ab = ac + bd$ [2]

$ab - ac = bd$.

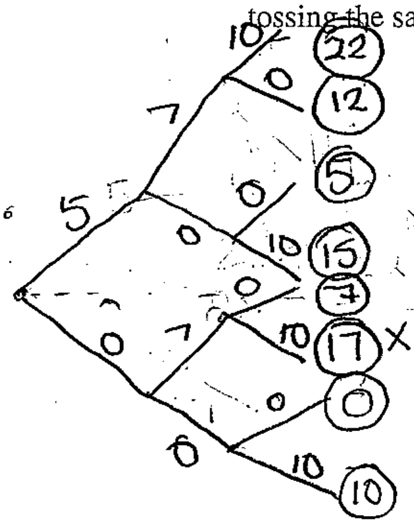
$a(b-c) = bd$.

$a = \frac{bd}{b-c}$. ②

DONE VERY WELL
① FOR UPTO $a(b-c) = bd$.

4) Andrew has a bucket of coins. Each coin has a zero on one side and an integer greater than zero on the other side. He randomly draws three coins, tosses them and calculates a score by adding the three numbers that appear.

(a) On Monday, Andrew draws coins with a 7, a 5 and a 10 for a score of 17. By considering a tree diagram, or otherwise, what other scores could he obtain by tossing the same three coins? [2]



Other scores =

0,5,7,10,12,15,22.

②

DONE FAIRLY WELL
- LAYOUT WAS POOR
① FOR CORRECT TREE DIAGRAM
(½) DEDUCTED FOR FIRST OMISSION.
① or (1½) DEDUCTED FOR MORE OMISSIONS.

(b) On Tuesday, Andrew draws three coins and tosses them three times, obtaining scores of 60, 110 and 130. On each of these tosses, exactly one of the coins shows a zero. Determine the maximum possible score that can be obtained by tossing these three coins. [2] ②

$a+b=60$ — ①
$b+c=110$ — ②
$a+c=130$ — ③

$$①+②+③ = 2a+2b+2c = 300$$

$$2(a+b+c) = 300$$

$$a+b+c = 150.$$

MAX POSSIBLE SCORE = 150

DONE FAIRLY WELL.
① FOR SET UP OF EQUATIONS
LAYOUT WAS EXTREMELY POOR.

(c) On Wednesday, Andrew draws a coin with 25, one with a 50, and a third coin. He tosses these three coins and obtains a score of 170. Determine all possible numbers, other than zero, that could be on the third coin. [2]

a+b

$a = 125$
$b = 50$
$c = 95$

$a = 0$
$b = 50$
$c = 120$

$a = 25$
$b = 0$
$c = 145$

$a = 0$
$b = 0$
$c = 170$

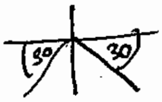
DONE VERY WELL
($\frac{1}{2}$) DEDUCTED FOR EACH OMISSION.

Third coin could be
95, 120, 145, 170. ②

5) Solve $2 \sin x + 1 = 0$ for $0° \leq x \leq 360°$ [2]

$\sin x = -\frac{1}{2}$.

$x = -30$.



②

$x = 210° \ \& \ 330°$

DONE POORLY
($\frac{1}{2}$) FOR $x = -30°$.
① FOR ONE SOLUTION.

MOST ONLY GAVE ONE SOLUTION

2015 Year 10 Mathematics Yearly: Section G Solutions

1. (a) Expand $\left(x^2+2x\right)^2$. $\boxed{1}$

Solution: $x^4+4x^3+4x^2$.
Comment: Generally well done.

(b) Write $x^4+4x^3-5x^2-18x+8$ in the form $\left(x^2+2x\right)^2+A\left(x^2+2x\right)+B$. $\boxed{1}$

Solution: $x^4+4x^3+4x^2-9x^2-18x+8=\left(x^2+2x\right)^2-9\left(x^2+2x\right)+8$.
Comment: Although most answered this part well, some did not seem to understand what was required and others, after finding A and B, did not explicitly answer the question.

(c) Hence, by using the substitution $m=x^2+2x$, solve $x^4+4x^3-5x^2-18x+8=0$. $\boxed{3}$

Solution:

$$\begin{aligned} m^2-9m+8 &= 0,\\ (m-8)(m-1) &= 0,\\ \therefore m &= 8,\ 1. \end{aligned}$$

$$\begin{array}{rlcrl} i.e.\ x^2+2x = & 8, & \text{or} & x^2+2x = & 1,\\ x^2+2x-8 = & 0, & & x^2+2x-1 = & 0,\\ (x+4)(x-2) = & 0, & & x = & \dfrac{-2\pm\sqrt{4+4}}{2},\\ x = & -4,\ 2. & & = & -1\pm\sqrt{2}. \end{array}$$

Comment: Well done by those who succeeded in (b).
Most subsequent errors were the result of a careless attempt to factorise x^2+2x-1 as $(x-1)^2$.

2. If $x^2=8x+y$ and $y^2=x+8y$ with $x\neq y$, what is the value of x^2+y^2? $\boxed{2}$

Solution: Method 1:—

$$\begin{aligned} x^2 &= 8x+y\ldots\ldots\ldots\boxed{1}\\ y^2 &= x+8y\ldots\ldots\ldots\boxed{2}\\ \boxed{1}-\boxed{2}:\quad x^2-y^2 &= 7x-7y,\\ (x+y)(x-y) &= 7(x-y),\\ x+y &= 7\ (\text{because } x\neq y).\\ \boxed{1}+\boxed{2}:\quad x^2+y^2 &= 9x+9y,\\ &= 9(x+y),\\ &= 9\times 7,\\ &= 63. \end{aligned}$$

Comment: Few got beyond simple addition, $9(x+y)$, which only garnered a half mark.

Solution: Method 2:—

$$\begin{aligned}
x+y &= 7 \text{ (as above).}\\
\text{Now } x^2 &= 7x+x+y,\\
\textit{i.e. } x^2 &= 7x+7, \text{ and similarly } y^2 = 7y+7,\\
x^2-7x-7 &= 0,\\
x &= \frac{7 \pm \sqrt{49+28}}{2},\\
&= \frac{7 \pm \sqrt{77}}{2}, \text{ and also } y = \frac{7 \pm \sqrt{77}}{2}.\\
\text{As } x \neq y, \text{ take } x &= \frac{7+\sqrt{77}}{2}, \; y = \frac{7-\sqrt{77}}{2},\\
\text{then } x^2+y^2 &= \frac{49+14\sqrt{77}+77}{4} + \frac{49-14\sqrt{77}+77}{4},\\
&= \frac{252}{4},\\
&= 63.
\end{aligned}$$

3. $f(x) = 2x^2 - x^4$, $0 \leqslant x \leqslant 1$. Find the inverse function $f^{-1}(x)$. $\boxed{3}$

Solution:

$$\begin{aligned}
\text{First put } y &= 2x^2 - x^4, \text{ with end-points } (0,\,0) \text{ and } (1,\,1),\\
\text{then for the inverse, } x &= 2y^2 - y^4,\\
\text{next put } k &= y^2,\\
x &= 2k - k^2,\\
&= -(k^2-2k+1)+1,\\
x-1 &= -(k-1)^2,\\
k-1 &= \pm\sqrt{1-x},\\
k &= 1 \pm \sqrt{1-x},\\
\text{but the inverse passes through } &(0,\,0) \text{ and } (1,\,1) \text{ like } f(x),\\
\text{so } k &= 1-\sqrt{1-x},\\
y^2 &= 1-\sqrt{1-x},\\
y &= \sqrt{1-\sqrt{1-x}}, \; \big(y \geqslant 0 \text{ as through } (0,\,0) \text{ and } (1,\,1)\big)\\
\textit{i.e. } f^{-1}(x) &= \sqrt{1-\sqrt{1-x}}.
\end{aligned}$$

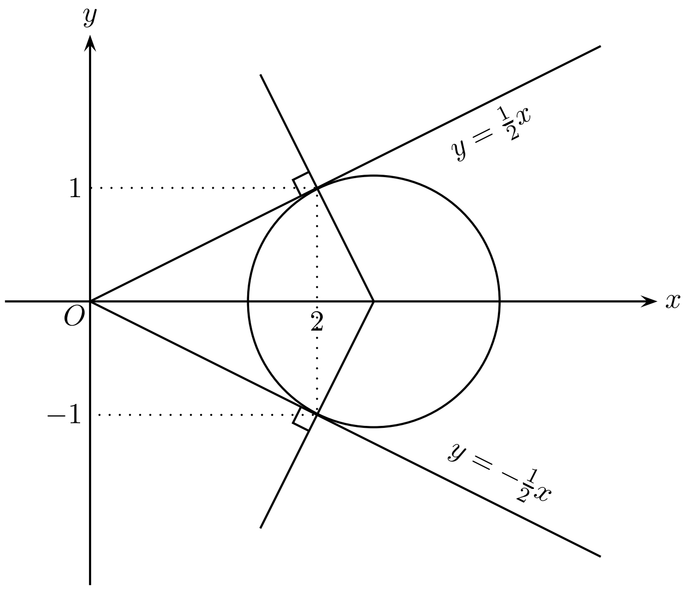
Comment: Many candidates failed to use completion of squares to extract y as the subject from $x = 2y^2 - y^4$.

Those who *did* then often failed to take account of whether the positive or negative root was appropriate to the given domain.

Sadly, quite a few students must have been away in Year 8 when $\sqrt{a^2+b^2} \neq a+b$ was discussed.

4. The lines $y = \frac{1}{2}x$ and $y = -\frac{1}{2}x$ are tangents to a circle at (2, 1) and (2, −1). Find the equation of the circle. $\boxed{3}$

Solution:



To find the centre, calculate where the perpendicular from the tangent at (2, 1) cuts the x-axis.

$$\begin{aligned} \text{Slope of normal} &= -2, \\ y - 1 &= -2(x-2). \\ \text{When } y &= 0,\ -1 = -2x + 4, \\ 2x &= 5, \\ x &= \frac{5}{2}. \\ \therefore \text{Centre } &\left(2\tfrac{1}{2},\ 0\right). \\ \text{Radius} &= \sqrt{\left(\frac{5}{2} - 2\right)^2 + 1^2}, \\ &= \frac{\sqrt{5}}{2}, \\ \therefore \text{Circle: } \left(x - \frac{5}{2}\right)^2 + y^2 &= \frac{5}{4}, \\ x^2 - 5x + \frac{25}{4} + y^2 &= \frac{5}{4}, \\ x^2 - 5x + y^2 + 5 &= 0. \end{aligned}$$

Comment: Many candidates failed to make a clear sketch which would have helped in planning the method to use. For too many candidates, this seemed to arise from an inconsistent mis-application of the order of ordered pairs.
An effective—but somewhat more cumbersome—method of finding the centre used by some candidates was to derive the equations of *both* normals and then to solve them simultaneously.

Comments about YR10 Yearly 2015 Section H.

1(a) students quoted $V=\pi r^2 h$, $V=\frac{4}{3}\pi r^3$ but did not show how $H=\frac{4}{3}r$ was arrived at.

(b) students quoted $SA=2\pi r^2+2\pi rH$ and $SA=\pi r^2+\pi rs$
Many did not show where $s=\sqrt{h^2+r^2}$ came from.

Most students getting this far were then successful.

(c) Very badly done. Only 5 students successful. See the solutions to understand the rigor involved.

2. A difficult monic cubic to find. Some marks given for parts of the answer. See the detailed solution(s) to understand.

3. Many students did not draw in the common tangent and then labelled it. See my diagram. In my opinion once this is done and a few alt. seg thm used the proof comes out easily.
I entertained many different solutions. But beware; chords may look equal but they are not; angles may look 90° but they are not; angles look bisected but they are not. All bad assumptions.
1 student drew in 4 construction lines. Overkill => yes.
Also logical; able to be followed lines of working are needed with reasons. There were 5 further pages students could use to set out a proof.
3 students scored 13/13 in Section H.

$\frac{}{13}$

SECTION H (13 marks)

1) A cone, a cylinder and a sphere all have radius *r*. The height of the cylinder is *H* and the height of the cone is *h*.

(a) If the cylinder and the sphere have the same volume, show that [1]

$$H = \frac{4}{3}r.$$

$V = \pi r^2 h$ } so $\cancel{\pi r^2} H = \frac{4}{3}\cancel{\pi} r^{\cancel{3}\,1}$

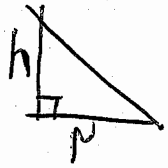
$V = \frac{4}{3}\pi r^3$ } $H = \frac{4}{3} r$ ①

(b) If the cone and the cylinder have the same total surface area, show that [2]

$$r + 2H = \sqrt{r^2 + h^2}.$$

$SA = 2\pi r^2 + 2\pi r H$

$SA = \pi r^2 + \pi r S \Rightarrow S = \sqrt{r^2 + h^2}$



So $2\pi r^2 + 2\pi r H = \pi r^2 + \pi r \sqrt{h^2 + r^2}$

$2\cancel{\pi r}(r + H) = \cancel{\pi r}(r + \sqrt{h^2 + r^2})$

(c) Hence, prove that *h* and *H* cannot both be integers. [3]

$2r + 2H = r + \sqrt{h^2 + r^2}$

$r + 2H = \sqrt{h^2 + r^2}$ ②

(c) $H = \frac{4}{3}r \Rightarrow r = \frac{3}{4}H$

Using (b) $\frac{3H}{4} + 2H = \sqrt{\frac{9H^2}{16} + h^2}$

$\frac{11H}{4} = \sqrt{\frac{9H^2 + 16h^2}{16}}$

so $\frac{11H}{4} = \frac{\sqrt{9H^2 + 16h^2}}{4}$

$11H = \sqrt{9H^2 + 16h^2}$

$121H^2 = 9H^2 + 16h^2$

$112H^2 = 16h^2$

$7H^2 = h^2$

thus taking positives.

$\sqrt{7}H = 1h$

$\frac{H}{h} = \frac{1}{\sqrt{7}}$

this ratio/fraction means that H and h cannot both be integers eg $H=1$, $h=\sqrt{7}$, or $H=\sqrt{7}$, $h=7$ etc.

③

2) A monic cubic polynomial has a remainder of $(x + 8)$ when divided by $(x^2 + 4)$ and a remainder of -4 when divided by x. [3]

Find the polynomial in the form $ax^3 + bx^2 + cx + d$

1st part of info : $P(x) = (x^2+4)\underbrace{(x+a)} + (x+8)$ must be a monic cubic $P(x)$

2nd part of info : $P(x) = x\underbrace{(x^2+b)} - 4$ must be a monic cubic $P(x)$

So $(x^2+4)(x+a) + (x+8) = x(x^2+b) - 4$

$x^3 + ax^2 + 4x + 4a + x + 8 = x^3 + bx - 4$

$x^3 + ax^2 + 5x + 4a + 8 = x^3 + bx - 4$

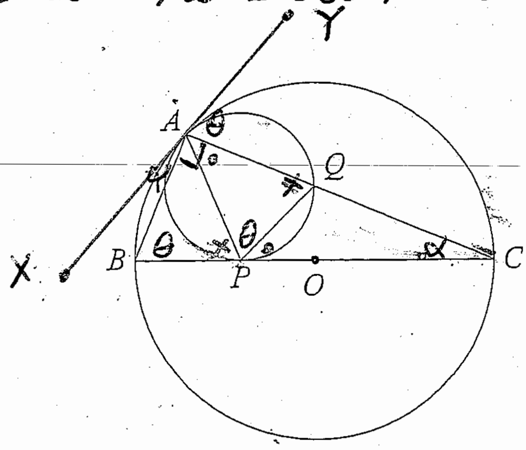
$a = 0$ is a possibility as $0x^2$ but $4a + 8 = -4$ and $b = 5$.

$4a = -12$

$a = -3$

So the monic $P(x)$ could be $1x^3 + 0x^2 + 5x - 4$ or $1x^3 - 3x^2 + 5x - 4$. By dividing by x and (x^2+4), to check, answer is $1x^3 - 3x^2 + 5x - 4$. ③

3) [4]



Two circles touch internally at A where there is a common tangent. BC is a diameter of the larger circle, touching the smaller circle at P. AC cuts the smaller circle at Q.

Prove that $\angle APQ + \angle ACP = 90°$.

In the large circle, $X\hat{A}B = B\hat{C}A = \alpha$ alt. seg. theorem

$B\hat{A}C = 90°$ angle in a semi circle

$Y\hat{A}Q = C\hat{B}A = \theta$ alt seg. theorem

In the small circle, ~~$C\hat{P}Q = P\hat{A}Q =$ alt seg theorem~~

$Y\hat{A}Q = Q\hat{P}A = \theta$ alt. seg. theorem

$A\hat{P}Q + A\hat{C}P = \theta + \alpha$

Now along the common tangent at A, $\alpha + 90° + \theta = 180°$ straight line angle

so $\alpha + \theta = 90°$ ④

and result is proved