Sydney Boys High School

MOORE PARK, SURRY HILLS

YEAR 10 ADVANCED MATHEMATICS

Yearly Examination 2016

General Instructions:

- All questions may be attempted.
- **Marks may be deducted for careless or badly arranged work.**
- All working and answers are to be written in this test booklet.
- If you wish to rewrite an answer, draw a line through your faulty answer and rewrite your answer on the back pages of this booklet. **Show the number and part of the answer being rewritten.**
- Leave your answers in the simplest exact form, unless otherwise stated.
- Board approved calculators may be used.
- Clearly indicate your class by placing an ✕ next to your class.

Time Allowed: 120 minutes
Reading Time: 5 minutes
Write using black or blue pen.

Examiner: RB

Name: _____

Class	Teacher	
10**A**	Ms Kilmore	
10**B**	Mr Choy & Mr Elliott	
10**C**	Ms Millar & Ms Evans	
10**D**	Mr Wang	
10**E**	Mr Fuller	
10**F**	Ms Ward	
10**G**	Mr Parker & Mr Elliott	

Section	Marks
A	/ 10
B	/ 18
C	/ 17
D	/ 22
E	/ 18
F	/ 16
G	/ 13
H	/ 11
Total	/ 125

SECTION A: MULTIPLE CHOICE (10 MARKS)

Circle the correct letter *A, B, C, D* in these questions

1 A customer pays \$714 in cash for an article on which he has been given 16% discount. What is the selling price of the article?

A. \$599.76	B. \$615.50	C. \$850	D. \$865

2 The balance of an investment at compound interest on \$$P$ for 20 years at r% per annum payable half yearly is:

A. $\$P\left(1+\frac{r}{100}\right)^{20}$	B. $\$P\left(1+\frac{r}{200}\right)^{40}$	C. $\$P\left(1+\frac{r}{200}\right)^{20}$	D. $\$P\left(1+\frac{40r}{100}\right)$

3 A cylinder and a cone have the same height. If the ratio of their base radii is 1 : 2, find the ratio of their volumes.

A. 3 : 4	B. 4 : 5	C. 1 : 2	D. 1 : 4

4 If $4P = 9Q$ then $\frac{4P^2}{9Q^2}$ is equal to

A. $\frac{4}{9}$	B. $\frac{81}{16}$	C. 1	D. $\frac{9}{4}$

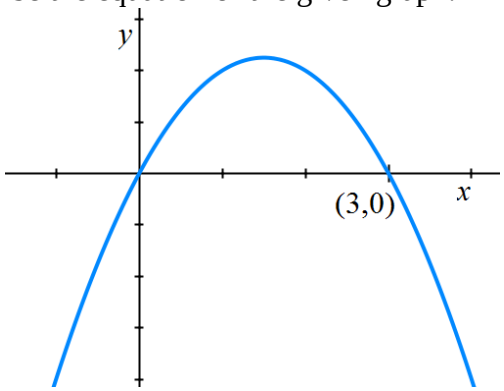
5 The square of $-2x + x^2$ is

A. $4x^2 + x^4$	B. $4x^2 - 4x^3 + x^4$	C. $4x^2 + 4x^3 + x^4$	D. $4x - 4x^2 + x^3$

6 The solution to the equation $2x^2 = 7x$ is $x =$

A. $3\frac{1}{2}$ only	B. 0 or $-3\frac{1}{2}$	C. $3\frac{1}{2}$ or $-3\frac{1}{2}$	D. 0 or $3\frac{1}{2}$

7 Which of the following could be the equation of the given graph.



A. $y = 3x - x^2$	B. $y = 3x^2 - x$	C. $y = x - 3x^2$	D. $y = x^2 - 3x$

8 The solution to $4 - x \le \frac{2}{3}x - 3$ is:

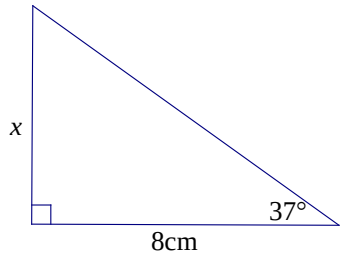
A. $x \le 4\frac{1}{5}$	B. $x \le \frac{5}{21}$	C. $x \ge 4\frac{1}{5}$	D. $x \ge \frac{5}{21}$

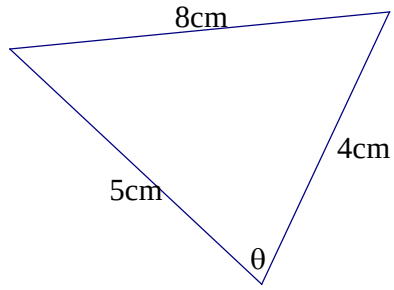
9 $\log_3 15 + \log_3 18 - \log_3 10 =$

A. 1	B. 2	C. -2	D. 3

10 A fair coin is tossed 3 times. Find the probability that there is at least 1 tail.

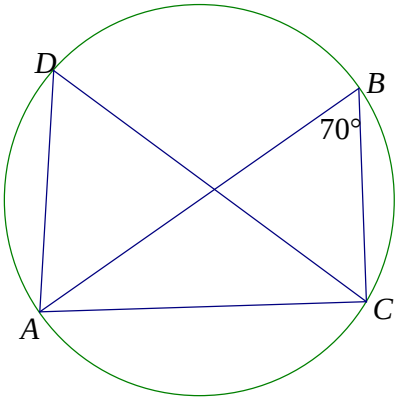
A. $\frac{7}{8}$	B. $\frac{1}{2}$	C. $\frac{3}{8}$	D. $\frac{5}{8}$

SECTION B (18 Marks)		Marks
1	Find the cube root of $-64x^9$	**1**
2	Find the square root of $1\frac{7}{9}$ in exact form	**1**
3	Find the exact value of $\cos 150^\circ$	**1**
4	Find the circumference of a circle with diameter 13.8cm. Answer correct to 3 S.F.	**2**
5	Find x, correct to 4 D.P.  	**2**
6	What is the supplement of 83º	**1**
7	What is 6308992 written in scientific notation correct to 2 S.F.	**1**

8	Given $\tan\theta = 0.8$ and $\cos\theta < 0$, find θ correct to the nearest degree. $0^\circ \le \theta \le 360^\circ$	**2**
9	A number is picked at random from the set {1, 2, 3, 4, ……, 11}. Find the probability that the number picked is not prime.	**1**
10	Make A the subject in $T = \sqrt{\frac{B}{A}}$	**1**
11	Find θ, correct to the nearest minute.  	**2**
12	A student invests \$200 into a savings account earning interest at 7% compounded annually. (a) How much does he have in the account (to the nearest cent), after 2 years? (b) After how many complete years will he first have more than \$3000?	**1** **2**

SECTION C (17 Marks)		Marks
1	In a game, 1 red die and 1 blue die are used. Both dice are unbiased but the faces of the red die are numbered 1, 1, 2, 3, 4, 5 and the faces of the blue die are numbered 1, 1, 2, 2, 4, 4. The 2 dice are thrown together. Find the probability that: (a) the number on the red die is odd.	**1**
	(b) the number on the blue die is greater than the number on the red die	**2**
	(c) the numbers on the dice are identical.	**1**
2	Given the points $A(-2,1)$, $B(5,-1)$ and $C(3,3)$. Find (a) the equation of the line through A, parallel to BC. (Answer in general form)	**2**
	(b) the equation of the perpendicular bisector of BC. (written in gradient/intercept form)	**2**

3	A glass sphere has a radius of 5.75cm. (a) Calculate its volume correct to 2 D.P. (b) The glass sphere is tightly packed into a cylindrical gift box such that it just touches the curved surface of the box and the top and bottom of the box. Find the total surface area of the interior of the box correct to 2 D.P.	**2** **2**
4	Find the exact value of $(10\,000\,000\,001)^2 - (9\,999\,999\,999)^2$	**2**
5	Given $\log_{10} Z = \log_{10} A + \log_{10} B - \log_{10} C$, express Z in terms of A, B and C.	**1**
6	Solve for x, $\log_{10}(10x^2 + 12x - 3) = 1 + 2\log_{10} x$	**2**

SECTION D (22 Marks)		Marks
1	Given $f(x) = 3 - 2x,$ find	
	(a) $f(-1)$	**1**
	(b) $f^{-1}(x)$	**1**
	(c) $f^{-1}(2)$	**1**
	(d) a positive number q such that $f(q) = q^2$	**2**
	(e) find r such that $f(2^r) < -5$	**2**
2	We are told that AB is a diameter and $\angle ABC = 70^\circ$  Find:	
	(a) $\angle BCA$	**1**
	(b) $\angle ADC$	**1**

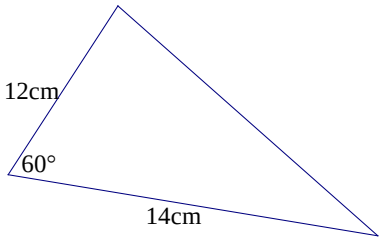
3 For the equation $y = x^2 - 1$

(a) Sketch the graph, showing its important features (intercepts, vertex, axis of symmetry) **2**

(b) State whether this equation is a function or not, giving reasons. **1**

(c) Is the inverse relation of this equation a function? Why/Why not? **1**

4 Find the area of the triangle, to the nearest cm².



2

5 The marks shown in the table were obtained by 20 boys in a spelling test.

Score	2 correct	3 correct	4 correct	5 correct	6 correct	7 correct
Frequency	4	5	0	1	3	7

Find, from this table the:

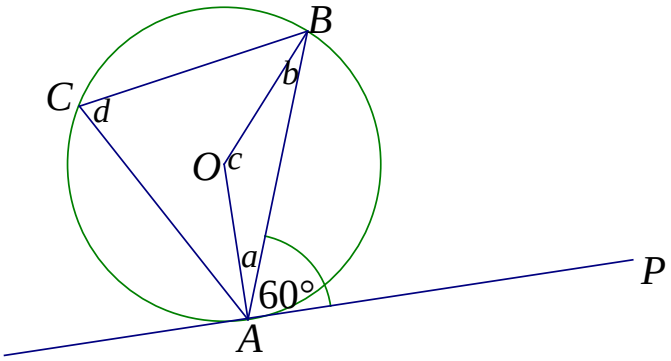
(a) range **1**

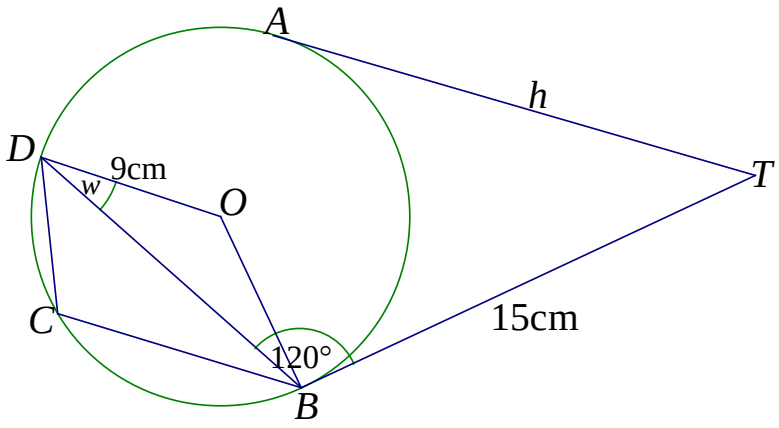
(b) median **1**

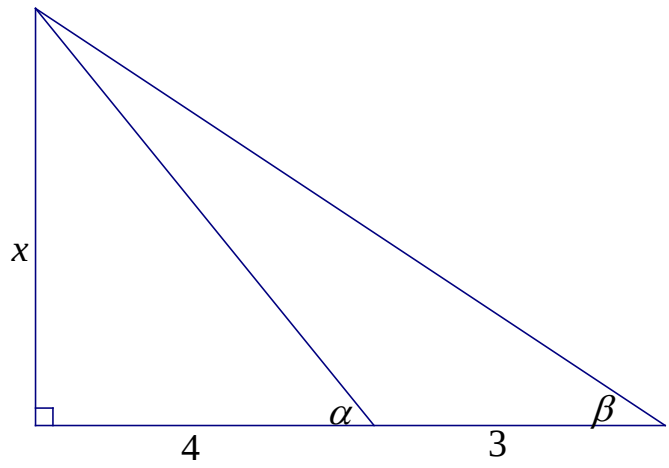
(c) mode **1**

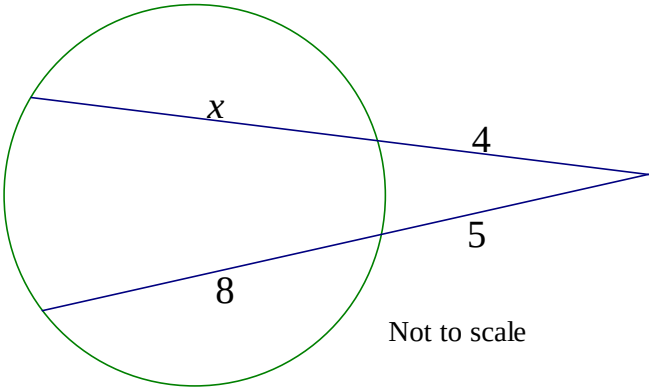
(d) mean correct to 2D.P. **2**

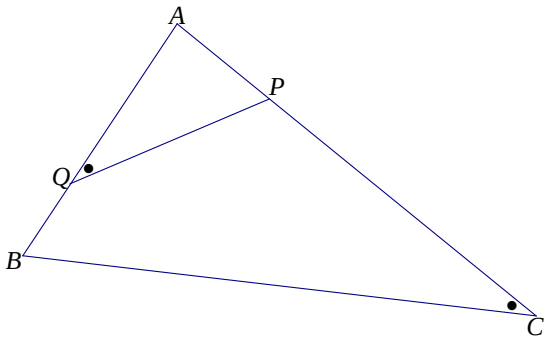
(e) standard deviation correct to 2D.P. **2**

SECTION E (18 Marks)		Marks
1	Simplify $\dfrac{a^{-\frac{1}{2}}b^{\frac{3}{2}} \times b^{-\frac{7}{8}}a^{3}}{a^{\frac{3}{2}}b^{-\frac{3}{2}}}$	**2**
2	Graph $y = 2^{x} - 5$ showing all the important features, given $-1 \le x \le 4$	**2**
3	 *PA* is a tangent to the circle, centre *O*. $\angle PAB = 60^{\circ}$. Find angles *a*, *b*, *c*, *d* giving a reason for each answer.	**6**

4	Sketch the graph of $(x-3)^2+(y+4)^2=25$ showing all the important features.	**2**
5	 *TA* and *TB* are tangent lines to the circle centre *O*. *OB* is a radius line and meets *TB* at *B*. Find *w* and *h*.	**2**

6	Sketch the graph of $y = \log_{10}(1+x)$ showing all of its important features.	**2**
7	 We are given that $\tan\alpha - \tan\beta = \frac{3}{14}$. Find x.	**2**

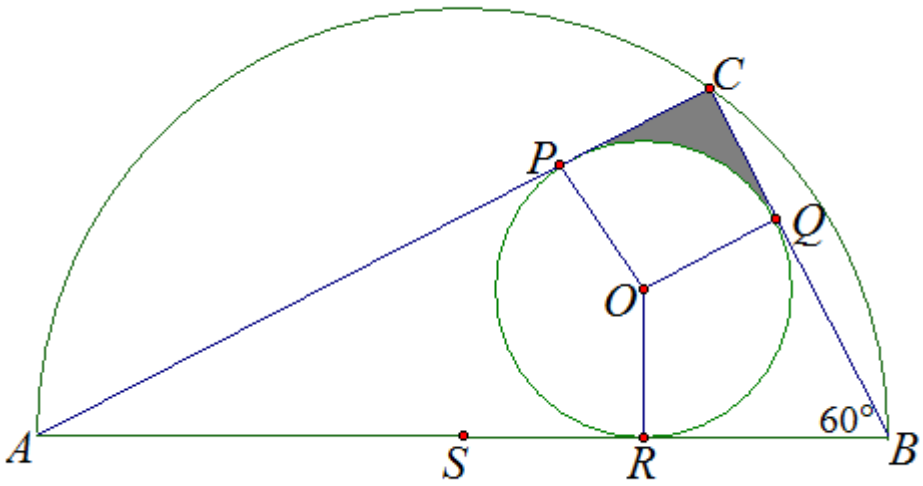
SECTION F (16 Marks)		Marks
1	(a) There are several definitions for $\lvert x \rvert$. One such definition is $\lvert x \rvert = \sqrt{x^2}$. By similarly defining $\lvert x-3 \rvert$, solve the following equation for x. $$\lvert x-3 \rvert = 2x-4$$	**2**
	(b) Investigate your solution and comment.	**1**
2	 Not to scale Calculate x.	**2**

3	The volumes of 2 similar square pyramids are in the ratio 8:27. Find the ratio of (a) their base edges (b) their areas	**1** **1**
4	 In the figure, $\angle AQP = \angle ACB$ (a) Prove ΔAPQ is similar to ΔABC. (b) If $AQ = 6,\ PQ = 4,\ AP = 4\frac{1}{2}$ and $BQ = 3$. Find the lengths of BC and PC.	**2** **3**

5	Over the last few years, NSW car number plates were made up of 3 letters followed by 3 digits eg ABC307. New registration plates now have 3 letters, 2 digits and then another letter eg PQR72X (a) How many plates were available under the old scheme?	**1**
	(b) How many plates are available under the new scheme?	**1**
6	 X Y Here are 2 similar boomerangs. *X* is 18cm long and has an area of 108cm². *Y* has an area of 48cm². What is the length of *Y*?	**2**

SECTION G (13 Marks)		Marks
1	Here are the % maths and English marks for 10 selected students. | Maths	72	63	87	94	55	46	66	81	62	84
---	---	---	---	---	---	---	---	---	---	---
English	61	39	52	45	79	59	51	63	71	75 | Ron scored 72% in maths and 71% in English. (a) Calculate the mean and standard deviation for each test. (b) Which is the better relative result for Ron? Give reasons.	**1** **1**
2	In ΔABC, AB = 15m, BC = 10m, $\angle BAC = 40°$. Find the value(s) of $\angle BCA$ to the nearest degree.	**3**

3



A semi-circle on diameter AB, passes through C. S is the midpoint of AB.
AC, BC, AB are tangents to the circle centre O at P, Q and R.
OR is 1 unit in length
$\angle CBA = 60^\circ$

(a) Find the size of $\angle ACB$ **1**

(b) Find the shaded area bounded by the arc PQ and the intervals PC and QC. **2**

4	(a) Expand $(x-y)^2$	**1**
	(b) If x and y are positive, prove that the sum of their squares is greater than or equal to twice their product.	**2**
	(c) Using 4(b) or otherwise, find the minimum value of $\frac{x}{y}+\frac{y}{x}$ when x and y are positive.	**2**

SECTION H (11 Marks)		Marks
1	*abc* and *cba* represent 2 separate 3 digit numbers with the order of their digits reversed. The first pronumeral stands for the hundreds. The second pronumeral stands for the tens. The third pronumeral stands for the units. Further, it is given that $0 < c < a < 10$ and *a*, *b*, *c* are all positive integers. (a) Prove that $abc - cba$ is a multiple of 99.	**2**
	(b) Since $abc - cba$ is a multiple of 99, it can be written in the form $99n$, where *n* is a positive integer. Find the value of *n*.	**2**

2 A father, in his will, left all of his money to his children in the following manner:

\$1000 to the first born and then $\frac{1}{10}$ of what then remains; \$2000 to the second born and $\frac{1}{10}$ of what then remains; then \$3000 to the third born and $\frac{1}{10}$ of what then remains and so on. When this is done, it was found that each child had the same amount.

(a) Let the total amount of money to be distributed be \$$P$. Write an equation to find how much the first child would receive. **1**

2

(b) Using 2(a) write down an equation to find out how much the second child would receive.

2

(c) By making your equation in 2(a) and 2(b) equal to each other, find
(i) \$$P$ **1**
(ii) How much do the first 2 children receive? **1**
(iii) How many children are in the family?

END OF EXAMINATION

EXTRA WORKING PAGE:

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MOORE PARK, SURRY HILLS

YEAR 10 ADVANCED MATHEMATICS

Yearly Examination 2016

Suggested Solutions

Sections	Marker
A	-
B	PSP
C	AW
D	AF
E	AYW
F	JM
G	EC/JD
H	BK

Multiple Choice Answers (Section A):

1. C	**3.** A	**5.** B	**7.** A	**9.** D
2. B	**4.** D	**6.** D	**8.** C	**10.** A

SECTION A: MULTIPLE CHOICE (10 MARKS)

Circle the correct letter *A, B, C, D* in these questions

1 A customer pays \$714 in cash for an article on which he has been given 16% discount. What is the selling price of the article? Solution: $(100-16)\% \times x = \$714$

$\therefore x = \$714 \div (84\times)$

$= \$714 \times \frac{100}{84}$

$= \$850$

A. \$599.76	B. \$615.50	C. \$850	D. \$865

2 The balance of an investment at compound interest on $\$P$ for 20 years at $r\%$ per annum payable half yearly is: Solution: The formula for compound interest when compounded half yearly is $A = P\left[1+\frac{r}{200}\right]^{2n}$. So, we have $n=20$, $\therefore A = P\left[1+\frac{r}{200}\right]^{40}$

A. $\$P\left(1+\frac{r}{100}\right)^{20}$	B. $\$P\left(1+\frac{r}{200}\right)^{40}$	C. $\$P\left(1+\frac{r}{200}\right)^{20}$	D. $\$P\left(1+\frac{40r}{100}\right)$

3 A cylinder and a cone have the same height. If the ratio of their base radii is 1 : 2, find the ratio of their volumes. Solution: $V(\text{cylinder}) = \pi r^2 h$ $V(\text{cone}) = \frac{1}{3}\pi r^2 h$.

Let $r_{\text{cylinder}} = 1$, $\therefore r_{\text{cone}} = 2$. Ratio of the volumes: $\pi(1)^2 h : \pi\frac{(2)^2 h}{3}$

$\pi h : \frac{4}{3}\pi h$

$1 : 4/3$

$3 : 4$ $\therefore$ ratio is 3 : 4

A. 3 : 4	B. 4 : 5	C. 1 : 2	D. 1 : 4

4 If $4P = 9Q$ then $\frac{4P^2}{9Q^2}$ is equal to Solution: Substitute $P = \frac{9Q}{4}$ into $\frac{4P^2}{9Q^2}$

$\Rightarrow \frac{4\left(\frac{9Q}{4}\right)^2}{9Q^2} = \frac{4\times 81Q^2}{16} \times \frac{1}{9Q^2}$

$= \frac{4\times 81\times Q^2}{16\times 9\times Q^2} = \frac{9}{4}$

A. $\frac{4}{9}$	B. $\frac{81}{16}$	C. 1	D. $\frac{9}{4}$

5 The square of $-2x+x^2$ is Solution: $(-2x+x^2)^2 = (-2x+x^2)(-2x+x^2)$

$= 4x^2 - 2x^3 - 2x^3 + x^4$

$= 4x^2 - 4x^3 + x^4$

A. $4x^2+x^4$	B. $4x^2-4x^3+x^4$	C. $4x^2+4x^3+x^4$	D. $4x-4x^2+x^3$

6 The solution to the equation $2x^2 = 7x$ is $x =$

Solution: $2x^2 = 7x$, immediately see 0 is a solution

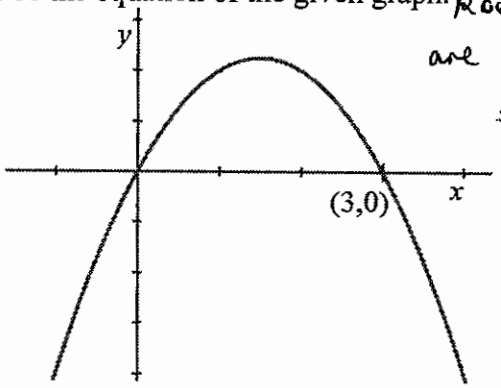
$2x = 7$

$x = 7/2$

$= 3\frac{1}{2}$. So $x = 3\frac{1}{2}$ or 0

A. $3\frac{1}{2}$ only	B. 0 or $-3\frac{1}{2}$	C. $3\frac{1}{2}$ or $-3\frac{1}{2}$	D. 0 or $3\frac{1}{2}$

7 Which of the following could be the equation of the given graph.



Roots of the function are $x = 3, 0$.

$\therefore f(x) = a(x-3)(x) = ax^2 - 3ax$

The graph is concave down, so a must be negative

$\therefore$ coefficient of x^2 must be negative, thus A is the only choice that fits the graph.

A. $y = 3x - x^2$	B. $y = 3x^2 - x$	C. $y = x - 3x^2$	D. $y = x^2 - 3x$

8 The solution to $4 - x \le \frac{2}{3}x - 3$ is:

Solution: $4 - x \le \frac{2}{3}x - 3$

$-\frac{2}{3}x - x \le -3 - 4$

$-\frac{2x}{3} - \frac{3x}{3} \le -7$

$-\frac{5x}{3} \le -7$

$-5x \le -21$

$x \ge \frac{21}{5}$

$\ge 4\frac{1}{5}$

A. $x \le 4\frac{1}{5}$	B. $x \le \frac{5}{21}$	C. $x \ge 4\frac{1}{5}$	D. $x \ge \frac{5}{21}$

9 $\log_3 15 + \log_3 18 - \log_3 10 =$

Solutions: $\log_3(15 \times 18) - \log_3 10 = \log_3(270 \div 10)$

$= \log_3(27)$

$= 3$

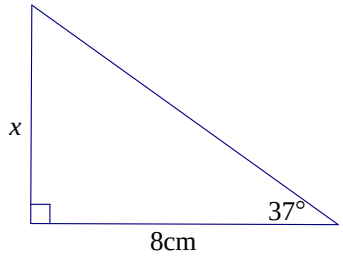
A. 1	B. 2	C. -2	D. 3

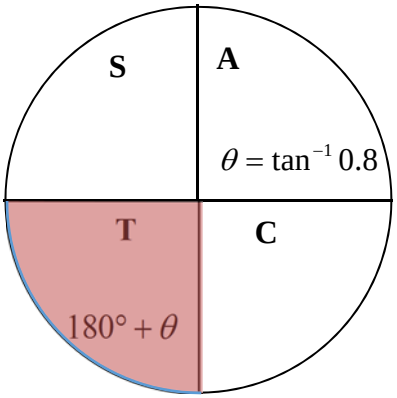
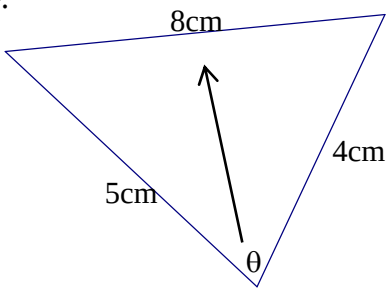
10 A fair coin is tossed 3 times. Find the probability that there is at least 1 tail.

Solution:

$P(\text{At least one tail}) = 1 - P(\text{No tails}) = 1 - \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = 1 - \frac{1}{8} = \frac{7}{8}$

A. $\frac{7}{8}$	B. $\frac{1}{2}$	C. $\frac{3}{8}$	D. $\frac{5}{8}$

SECTION B	SOLUTIONS	Marks
1	Find the cube root of $-64x^9$ $\sqrt[3]{-64x^9} = \left(-64x^9\right)^{\frac{1}{3}}$ $= -4x^{9\times\frac{1}{3}}$ $= -4x^3$ No marks were awarded if you only got the cube root of –64 correct.	**1**
2	Find the square root of $1\frac{7}{9}$ in exact form $\sqrt{1\frac{7}{9}} = \sqrt{\frac{16}{9}}$ $= \frac{\sqrt{16}}{\sqrt{9}}$ $= \frac{4}{3}$ **Note:** $\sqrt{16}$ means the (positive) square root of 16 i.e. $\sqrt{16} = 4$ However, the square roots of $1\frac{7}{9}$ are $\pm\frac{4}{3}$.	**1**
3	Find the exact value of $\cos 150°$. $\cos 150° = \cos(180-30)°$ [Second quadrant] $= -\cos 30°$ $= -\frac{\sqrt{3}}{2}$	**1**
4	Find the circumference of a circle with diameter 13.8 cm. Answer correct to 3 S.F. Circumference = $\pi \times$ diameter $= \pi \times 13.8$ = 43.35397862… = 43.4 (3 sig. fig.)	**2**
5	Find x, correct to 4 D.P.  $\tan 37° = \frac{x}{8}$ $\therefore x = 8\tan 37°$ $= 6.028432401...$ $= 6.0284$ (4 dp)	**2**
6	What is the supplement of 83º Supplement = 180° – 83° = 97°	**1**
7	What is 6 308 992 written in scientific notation correct to 2 S.F. 6 308 992 = $6.308\,992 \times 10^6$ $= 6.3 \times 10^6$ (2 sig. fig.)	**1**

SECTION B	SOLUTIONS	Marks
8	Given $\tan\theta = 0.8$ and $\cos\theta < 0$, find θ correct to the nearest degree. $0^\circ \le \theta \le 360^\circ$  $\tan\theta = 0.8 > 0$ $\therefore$ 1st and 3rd quadrants $\cos\theta < 0$ $\therefore$ 2nd and 3rd quadrants $\therefore\ \theta$ lies in the 3rd quadrant. $\therefore\ \theta = \tan^{-1} 0.8 + 180°$ $\fallingdotseq 219°$ **Note:** In general, $-90° < \tan^{-1}\theta < 90°$ Most students are misusing the notation. Students who presented two solutions could only get a maximum of 1 mark. Students who only presented the solution $\tan^{-1} 0.8$ or equivalent only got $\frac{1}{2}$ mark.	**2**
9	A number is picked at random from the set {1, 2, 3, 4, ……, 11}. Find the probability that the number picked is not prime. **Note:** 1 is NOT a prime number and 2 is a prime number Primes = {2, 3, 5, 7, 11} $\text{P(Not prime)} = 1 - \text{P(prime)}$ $= 1 - \frac{5}{11}$ $= \frac{6}{11}$	**1**
10	Make A the subject in $T = \sqrt{\frac{B}{A}}$ $T^2 = \frac{B}{A} \qquad \left(\Rightarrow AT^2 = B\right)$ $\therefore A = \frac{T^2}{B}$	**1**
11	Find θ, correct to the nearest minute.  Using the cosine rule: $\cos\theta = \frac{4^2 + 5^2 - 8^2}{2 \times 4 \times 5} = -\frac{23}{40}$ $\therefore \theta \fallingdotseq 125°6'$ The diagram is not to scale, so students putting down acute angles were penalised. Also students who wrote a reflex angle were penalised. The answer needs to be the nearest minute, otherwise students were penalised.	**2**

SECTION B	SOLUTIONS	Marks
12	A student invests \$200 into a savings account earning interest at 7% compounded annually.	
	(a) How much does he have in the account (to the nearest cent), after 2 years?	**1**
	After two years, the student has $\$200(1.07)^2 = \$228.98$	
	(b) After how many complete years will he first have more than \$3000?	**2**
	Let n be the number of years to get \$3000.	
	$3000 = 200(1.07)^n$ $\therefore 1.07^n = 15$ $\therefore n = \log_{1.07} 15$ $= \dfrac{\log_{10} 15}{\log_{10} 1.07}$ $\div 40.02518912$	
	Students who used trial and error or who showed no working got no marks if their answer was wrong.	
	So after 41 years the student will have more than \$3000.	

SECTION C (17 Marks)		Marks
1	In a game, 1 red die and 1 blue die are used. Both dice are unbiased but the faces of the red die are numbered 1, 1, 2, 3, 4, 5 and the faces of the blue die are numbered 1, 1, 2, 2, 4, 4. The 2 dice are thrown together. Find the probability that: (a) the number on the red die is odd. $P(\text{ODD}) = \frac{4}{6} = \boxed{\frac{2}{3}}$ (b) the number on the blue die is greater than the number on the red die $P(B > R) = \frac{12}{36}$ $= \boxed{\frac{1}{3}}$ (c) the numbers on the dice are identical. $P(R = B) = \frac{8}{36}$ $= \boxed{\frac{2}{9}}$ RED 1, 1, 2, 3, 4, 5 BLUE 1 ○ ○ 1 ○ ○ 2 ✓ ✓ ○ 2 ✓ ✓ ○ 4 ✓ ✓ ✓ ✓ ○ 4 ✓ ✓ ✓ ✓ ○	**1** **2** **1**
2	Given the points $A(-2,1)$, $B(5,-1)$ and $C(3,3)$. Find (a) the equation of the line through A, parallel to BC. (Answer in general form) $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m_{BC} = \frac{3 - -1}{3 - 5} = \frac{4}{-2} = -2$ $A(-2,1)$ $y - y_1 = m(x - x_1)$ $y - 1 = -2(x - -2)$ $y - 1 = -2x - 4$ $\boxed{2x + y + 3 = 0}$ (b) the equation of the perpendicular bisector of BC. (written in gradient/intercept form) $MP_1 = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ $\perp m_{BC} = \frac{-1}{m_{BC}} = +\frac{1}{2}$ $MP_{BC} = \left(\frac{8}{2}, \frac{2}{2}\right)$ $= (4, 1)$ $y - y_1 = m(x - x_1)$ $y - 1 = \frac{1}{2}(x - 4)$ $\boxed{y = \frac{1}{2}x - 1}$	**2** wrong constant ($\frac{1}{2}$) wrong sign ($-\frac{1}{2}$) not in general form ($-\frac{1}{2}$) **2** WRONG GRADIENT (−) sub wrong point (−1) $y = \frac{1}{2}x + b$ (1) incorrect algebra ($\frac{1}{2}$)

8

3 A glass sphere has a radius of 5.75cm.

(a) Calculate its volume correct to 2 D.P. **2**

$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi (5.75)^3$

WRONG FORMULA −1

$= 796.3282878 = \boxed{796.33\,cm^3\ (2dp)}$

(b) The glass sphere is tightly packed into a cylindrical gift box such that it just touches the curved surface of the box and the top and bottom of the box. Find the total surface area of the interior of the box correct to 2 D.P. **2**

(−1) WRONG FORMULA

(−1) WRONG RADIUS/HEIGHT

$S.A = 2\pi r^2 + 2\pi rh.$

$r = 5.75$

$h = 2r = 11.50\,cm$

$S.A. = 2\pi (5.75)^2 + 2\pi (5.75)(11.5) = 623.2134427$

$\boxed{= 623.21\,cm^2\ (2dp)}$

4 Find the exact value of $(10\,000\,000\,001)^2 - (9\,999\,999\,999)^2$ **2**

$[(10{,}000{,}000{,}001) + (9{,}999{,}999{,}999)][10{,}000{,}000{,}001 - 9{,}999{,}999{,}999]$

$(2 \times 10^{10})(2) = \boxed{4 \times 10^{10}}$

5 Given $\log_{10} Z = \log_{10} A + \log_{10} B - \log_{10} C$, express Z in terms of A, B and C. **1**

$\log_{10}(Z) = \log_{10}\left(\frac{AB}{C}\right)$

EQUATING

$\boxed{Z = \frac{AB}{C}}$

6 Solve for x, $\log_{10}(10x^2 + 12x - 3) = 1 + 2\log_{10} x$ **2**

$\log_{10}(10x^2 + 12x - 3) = \log_{10} 10 + \log_{10} x^2$

$= \log(10x^2)$

INCORRECT EXPANSION (−2)

EQUATING ∴ $\cancel{10x^2} + 12x - 3 = \cancel{10x^2}$

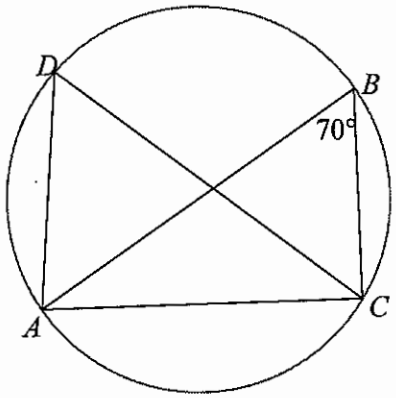
$12x = 3$

INCORRECT ALGEBRA (−½)

INCORRECT WORKING + CORRECT ANSWER (1)

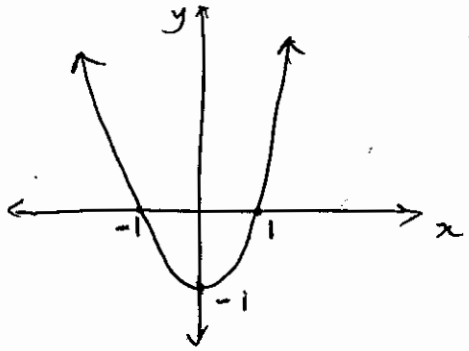
$\boxed{x = 1/4}$

9

SECTION D (22 Marks)		Marks
1	Given $f(x)=3-2x$, find	
	(a) $f(-1)$ $= 3-2(-1)$ $= 5$	1
	(b) $f^{-1}(x)$ $x = 3-2y$, $2y = 3-x$, $y = \frac{3-x}{2}$ $\therefore f^{-1}(x) = \frac{3-x}{2}$	1
	(c) $f^{-1}(2)$ $f^{-1}(2) = \frac{3-(2)}{2} = \frac{1}{2}$	1
	(d) a positive number q such that $f(q)=q^2$ $q^2 = 3-2q$, $q^2+2q-3=0$, $(q+3)(q-1)=0$, $q=1,-3$, since $q>0$ $q=1$	2
	(e) find r such that $f(2^r)<-5$ $3-2(2^r)<-5$, $-2^{r+1}<-8$, $2^{r+1}>2^3$ $\therefore r+1>3$, $r>2$	2
2	We are told that AB is a diameter and $\angle ABC = 70°$ 	
	Find: (a) $\angle BCA = 90°$ (angle in semi-circle)	1
	(b) $\angle ADC = 70°$ (angles in same segment)	1

3 For the equation $y = x^2 - 1$

(a) Sketch the graph, showing its important features (intercepts, vertex, axis of symmetry) **2**



Note: the axis of symmetry is the y-axis.

(b) State whether this equation is a function or not, giving reasons. **1**

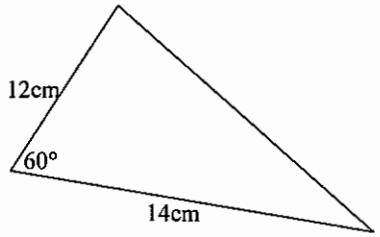
$y = x^2 - 1$ is a function as it satisfies the vertical line test. (a vertical line only cuts the curve once)

(c) Is the inverse relation of this equation a function? Why/Why not? **1**

No. The original function $y = x^2 - 1$ does not satisfy the horizontal line test.

Note: This is equivalent to testing whether the inverse relation satisfies the vertical line test.

4 Find the area of the triangle, to the nearest cm². **2**



$A = \frac{1}{2}(12)(14)\sin 60^\circ$

$\approx 73 \text{ cm}^2$

5 The marks shown in the table were obtained by 20 boys in a spelling test.

Score	2 correct	3 correct	4 correct	5 correct	6 correct	7 correct
Frequency	4	5	0	1	3	7

Find, from this table the:

(a) range $7 - 2 = 5$ **1**

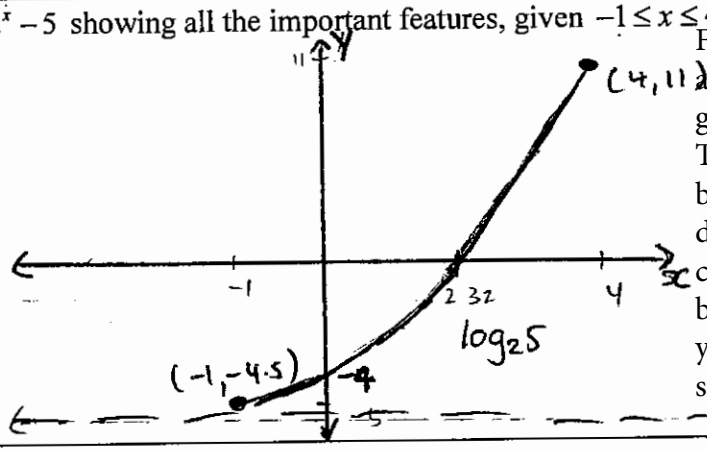
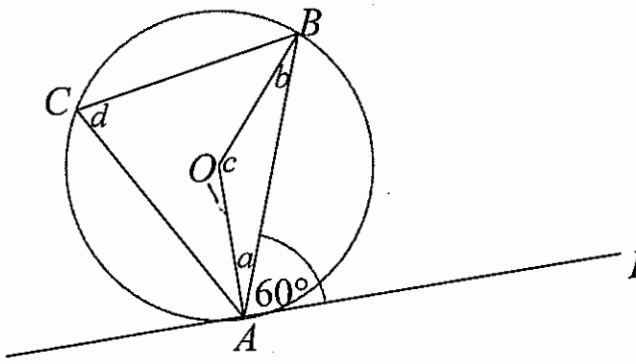
(b) median 5.5 (the average of the 10th & 11th score) **1**

(c) mode 7 **1**

(d) mean correct to 2D.P. 4.75 **2**

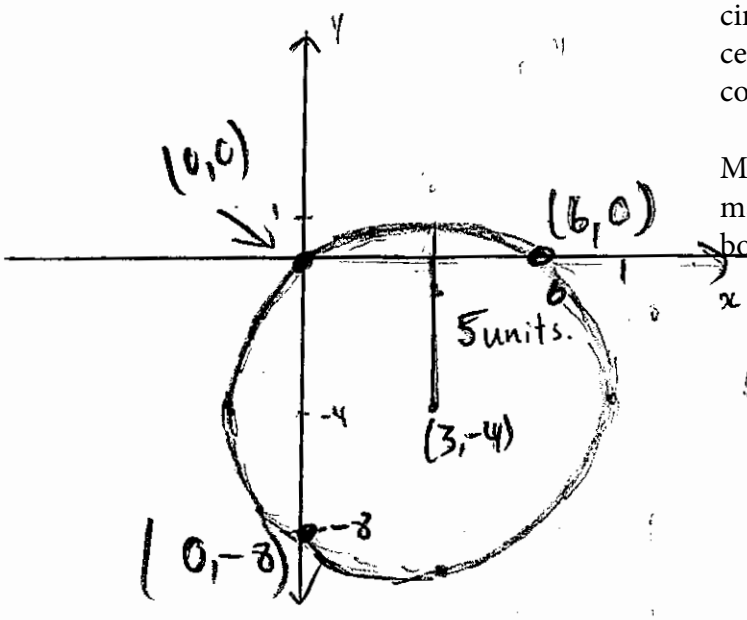
(e) standard deviation correct to 2D.P. 2.07 **2**

A half mark was deducted for the entire section E if the axes of any graph is not labelled.

SECTION E (18 Marks)		Marks
1	Simplify $\dfrac{a^{-\frac{1}{2}}b^{\frac{3}{2}} \times b^{-\frac{7}{8}}a^{3}}{a^{\frac{3}{2}}b^{-\frac{3}{2}}}$ $\dfrac{a^{-1/2}a^{3}b^{3/2}b^{-7/8}}{a^{3/2}b^{-3/2}}$ $\dfrac{a^{5/2}b^{5/8}}{a^{3/2}b^{-3/2}}$ 1 mark $a^{-\frac{1}{2}+3-\frac{3}{2}} = a$ $b^{3/2-7/8+3/2} = b^{17/8}$ $\therefore ab^{17/8}$ 1 mark Full marks awarded for correct working and correct answer. No marks will be awarded if students rewrote the question but in terms of surds.	2
2	Graph $y = 2^{x} - 5$ showing all the important features, given $-1 \le x \le 4$  Full marks awarded if all the information is given on the graph. This included both boundary points (as the domain is given), closed circle on the boundary points, x and y intercepts and correct shape graph.	2
3	 **3 marks** for correct values of a, b, c, d. **3 marks** for correct reasoning and working out i.e. appropriate abbreviation of the circle geometry must be given otherwise marks would be deducted. Also necessary working must shown if marks are to be awarded. *PA* is a tangent to the circle, centre *O*. $\angle PAB = 60°$. Find angles *a*, *b*, *c*, *d* giving a reason for each answer. $\angle OAP = 90°$ (Tangent perpendicular to radius $a = 90 - 60°$ (adjacent complementary angles) $= 30°$ $d = 60°$ (angle in alternate segment. $c = 120°$ (angles at the centre is twice the angle at circumference subtended by the same arc) $\therefore A\ b = 180 - 120 - 30$ (Angle sum of a Δ) $b = 30°$	6

4 Sketch the graph of $(x-3)^2+(y+4)^2=25$ showing all the important features. **2**

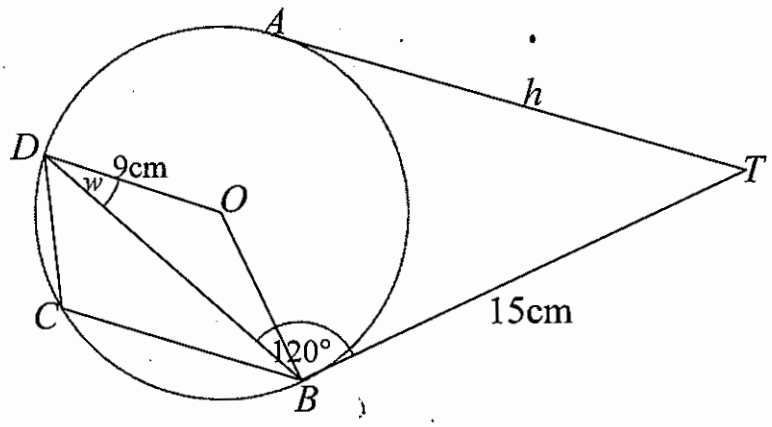
centre = (3, -4) radius = 5



Full marks awarded if a circle with the correct centre and radius is drawn correctly and labelled.

Many candidates lost half a mark due to not indicating both x and y intercepts.

5 **2**



TA and *TB* are tangent lines to the circle centre *O*. *OB* is a radius line and meets *TB* at *B*. Find *w* and *h*.

1 mark for h =15cm

h = 15cm (tangents from an external pt)

∠OBT = 90° (Tangent perpendicular to radius)

∴ ∠OBD = 120 − 90 = 30°

Since OB = OD (radii of circle)

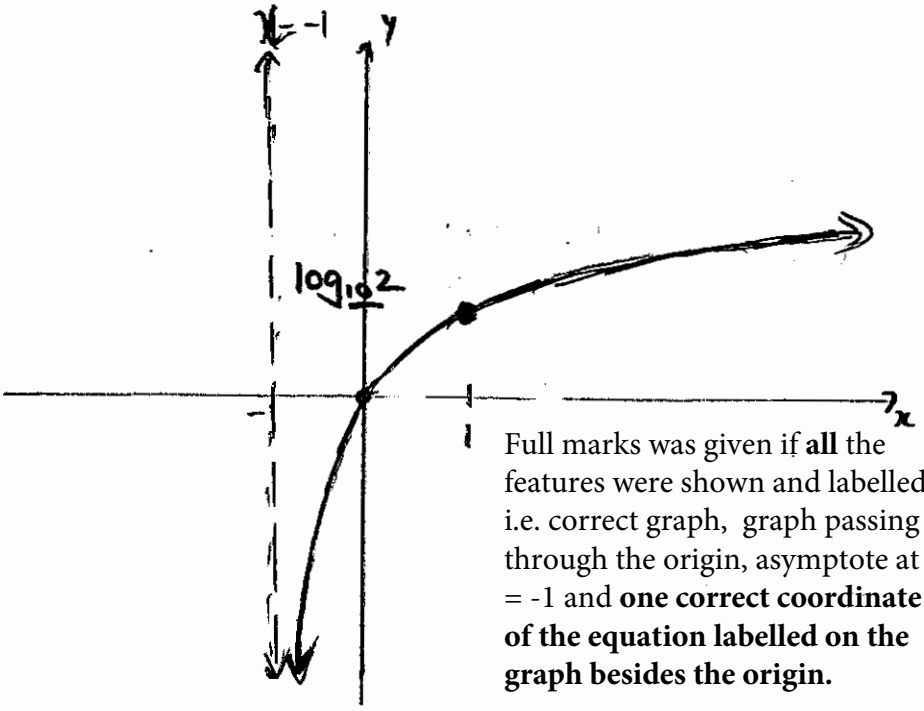
∴ △OBD is an isosceles △)

∴ ∠ODB = ∠OBD ∠Base ∠'s of an isosceles △)

∴ w = 30°

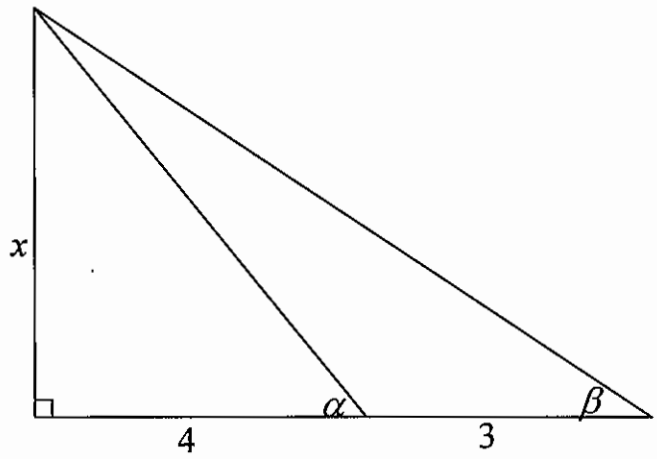
1 mark for w = 30 degrees

6 Sketch the graph of $y = \log_{10}(1+x)$ showing all of its important features. **2**



Full marks was given if **all** the features were shown and labelled i.e. correct graph, graph passing through the origin, asymptote at x = -1 and **one correct coordinate of the equation labelled on the graph besides the origin.**

7



We are given that $\tan\alpha - \tan\beta = \frac{3}{14}$. Find x. **2**

$\tan\alpha = \frac{x}{4}$

$\tan\beta = \frac{x}{7}$

$\tan\alpha - \tan\beta = \frac{x}{4} - \frac{x}{7} = \frac{7x-4x}{28} = \frac{3x}{28}$ 1 mark for correct working

$\therefore \frac{3}{14} = \frac{3x}{28}$

$\frac{6}{28} = \frac{3x}{28}$

$x = 2$. 1 mark for correct answer.

Section F (16 marks)

1. (a) $|x-3| = 2x-4$ [2]

Method 1: (square both sides).

$$(x-3)^2 = (2x-4)^2$$
$$x^2 - 6x + 9 = 4x^2 - 16x + 16$$
$$3x^2 - 10x + 7 = 0$$

P: 21, S: -10 $\}$ $(-7, -3)$

$$\therefore \frac{(3x-3)(3x-7)}{3} = 0$$
$$\frac{3(x-1)(3x-7)}{3} = 0$$
$$\therefore x - 1 = 0 \quad \text{or} \quad 3x - 7 = 0$$
$$x = 1 \qquad 3x = 7$$
$$x = \frac{7}{3}$$

Method 2: (take cases)

① inside the absolute value is positive

$$x - 3 = 2x - 4$$
$$x = 2x - 1$$
$$-x = -1$$
$$x = 1$$

② inside the absolute value is negative

$$-(x-3) = 2x - 4$$
$$-x + 3 = 2x - 4$$
$$-x = 2x - 7$$
$$-3x = -7$$
$$x = \frac{7}{3}$$

(b) Checking: [1]

When $x = \frac{7}{3}$: LHS $= \left|\frac{7}{3} - 3\right|$ RHS $= 2\left(\frac{7}{3}\right) - 4$

$= \left|-\frac{2}{3}\right|$ $= \frac{2}{3}$

$= \frac{2}{3}$ $= LHS$

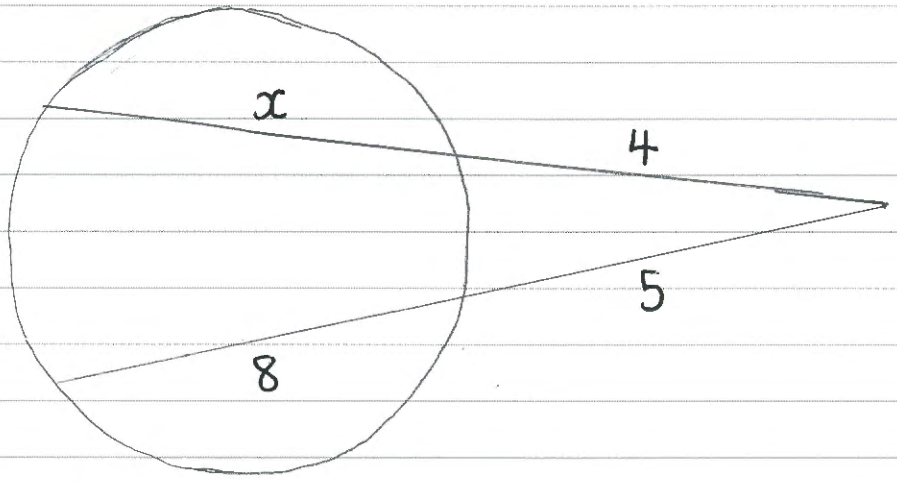
When $x = 1$: LHS $= |1 - 3|$ RHS $= 2(1) - 4$

$= |-2|$ $= -2$

$= 2$ $\neq LHS$

$\therefore$ $x = 1$, is not a solution.

2. [2]



$(x + 4) \times 4 = (8 + 5) \times 5$ (intercepts of intersecting secants)

$4x + 16 = 65$

$4x = 49$

$x = \frac{49}{4}$ or $\left(12\frac{1}{4}\right)$

3. $V_1 : V_2 = 8 : 27$

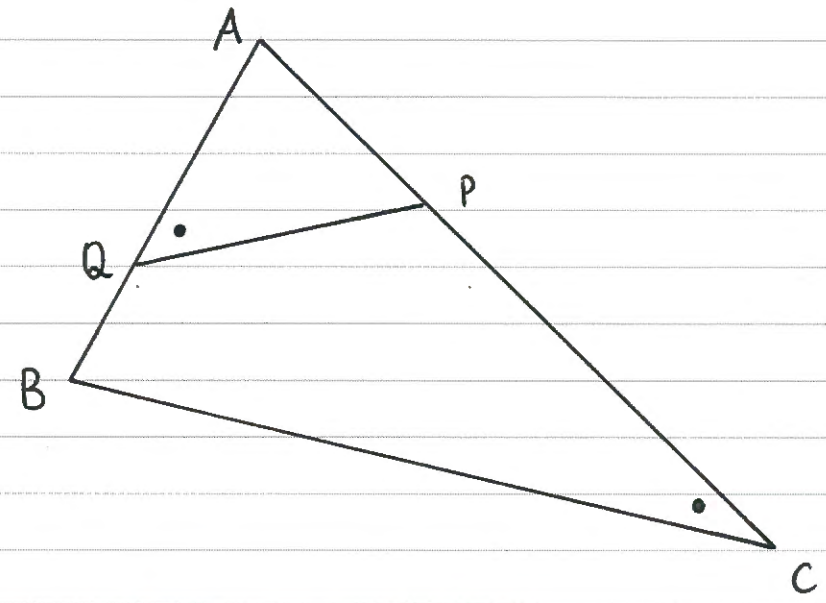
(a) Base edges: $\sqrt[3]{8} : \sqrt[3]{27}$ [1]

$= 2 : 3$

(b) Areas: $2^2 : 3^2$ [1]

$= 4 : 9$

4.



(NTS)

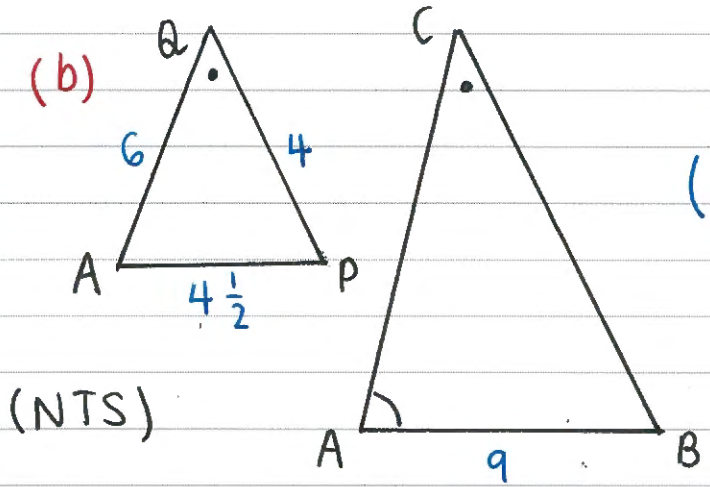
(a) In ΔAPQ and ΔABC

$\angle AQP = \angle ACB$ (given)

$\angle BAC$ is common

$\therefore \Delta APQ \,|||\, \Delta ABC$ (equiangular). [1] [1]

(b)



(NTS)

$\frac{AC}{AQ} = \frac{AB}{AP} = \frac{BC}{PQ}$

(matching sides of similar triangles). [1]

$\frac{BC}{4} = \frac{9}{4\frac{1}{2}}$

$BC = 2 \times 4$

$\therefore BC = 8$ [1]

$(6+3)$

$\frac{AC}{6} = \frac{9}{4\frac{1}{2}}$

$AC = 2 \times 6$

$\therefore AC = 12 \Rightarrow PC = AC - AP$

$= 12 - 4\frac{1}{2}$

$\therefore PC = 7\frac{1}{2}$ [1]

5.(a) $26 \times 26 \times 26 \times 10 \times 10 \times 10 = 17\,576\,000$ [1]

(b) $26 \times 26 \times 26 \times 10 \times 10 \times 26 = 45\,697\,600$ [1]

6. $A_x : A_y = 108 : 48 \Rightarrow l_x : l_y = \sqrt{108} : \sqrt{48}$

$= 6\sqrt{3} : 4\sqrt{3}$

$\therefore \frac{l_y}{l_x} \Rightarrow \frac{l_y}{18} = \frac{4}{6}$

$l_y = \frac{4}{6} \times 18$

$l_y = 12 \text{ cm}$

YEAR 10 ADVANCED MATHEMATICS YEARLY 2016

SECTION G

1) Calculator work, $\bar{x}$ = mean, σ_n = standard deviation for population.

σ_{n-1} = sample standard deviation $\sigma_{n-1} > \sigma_n$

$\bar{x}_{Maths} = 71$ $\sigma_{n\ Maths} = 14.51$

$\bar{x}_{English} = 59.5$ $\sigma_{n\ English} = 12.36$

[1 mark. Marked leniently]

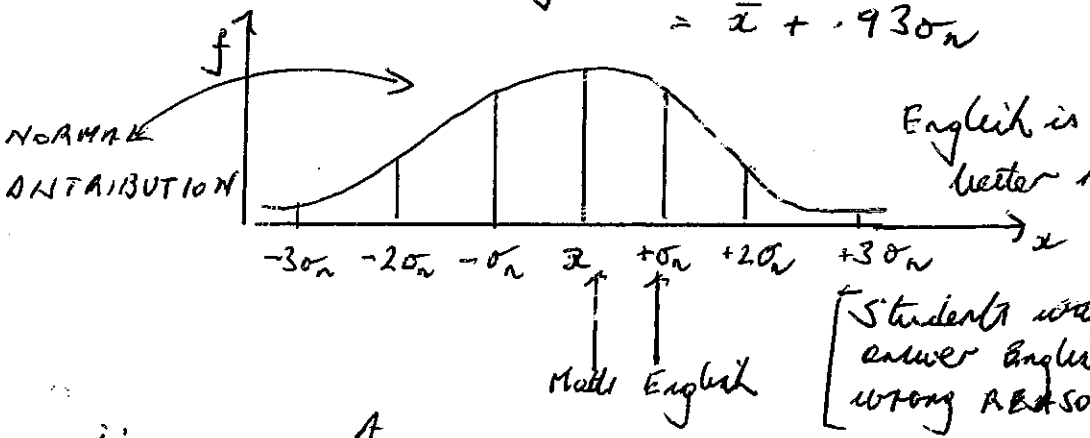
Almost all scores are within $\pm 3\sigma_n$ in a normal distribution. The more σ_n above mean, the better the score

RON 72% in Maths $= 71 + .069 \times 14.51$

$= \bar{x} + .069\sigma_n$

71% in English $= 59.5 + .93 \times 12.36$

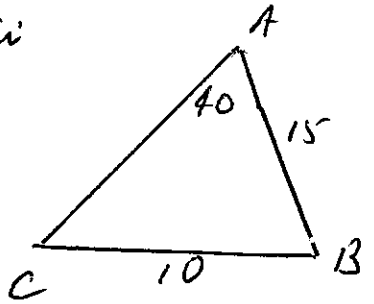
$= \bar{x} + .930_n$



English is the better score

[Students used the correct answer English but with the wrong REASON scored NIL]

ii



Triangle is NOT SAS

$\frac{\sin C}{15} = \frac{\sin 40}{10}$

$\sin C = \frac{15 \sin 40}{10}$

$= .96\ldots$

$C \doteq 75^\circ$ ← 2 marks/3

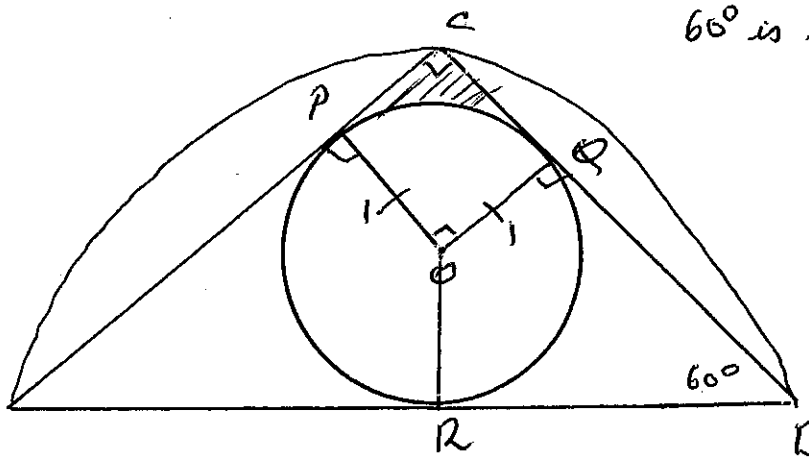
OR 105° (Ambiguous case) 1 extra mark

[Students with the wrong diagram but correct working scored 2/3]

Many students drew the wrong diagram and then used the cosine rule getting the wrong answer to this question but having the correct working for their diagram.

3)

NOTE
60° is superfluous



$A\hat{C}B = 90°$ (angle in semi circle)

OPCQ is a square side 1 unit $\Rightarrow$ Area $= 1u^2$

Sector POQ is a quadrant $\Rightarrow$ Area $= \frac{\pi}{4} \times 1^2$

Shaded Area $= 1 - \frac{\pi}{4}$ square units.

[Area of square 1 mark
Some progress ½, 1 mark
1 mark if answer only]

4) a $(x-y)^2 = x^2 - 2xy + y^2$ (one sign wrong ± scored ½)

b) $(x-y)^2 \geq 0$ ⇐ [Perfect Square (Need to show to score any marks) *** IMPORTANT]

$x^2 - 2xy + y^2 \geq 0$

$x^2 + y^2 \geq 2xy$

c) $x^2 + y^2 \geq 2xy$ ← MUST be shown to score full marks

Since $x, y > 0$

$\frac{x^2}{xy} + \frac{y^2}{xy} \geq 2$

$\frac{x}{y} + \frac{y}{x} \geq 2$

[1 mark if correct answer of 2 given with NO WORKING

$\frac{x}{y} + \frac{y}{x} = \frac{x^2+y^2}{xy}$

scores ½ mark]

Year 10 Yearly

Section H

1. (a) $abc = 100a + 10b + c$

$cba = 100c + 10b + a$

Then $abc - cba = 100(a-c) + (c-a)$ ✓

$= 100(a-c) - 1(a-c)$ ✓

$= 99(a-c)$

$= 99 \times k$ where $k \in \mathbb{Z}^+$ since $a > c$ ✓

$\therefore (abc - cba)$ is a multiple of 99.

Comments Students who used place values to express abc and cba as expanded numbers were able to do the question. Otherwise it was not done successfully. Proving sum of digits was a multiple of 9 tells us it is divisible by 9 but not by 11 and $\therefore$ by 99. ✓

(b) $abc - cba = 99n = 99(a-c)$

Since answer is at most a 3 digit no, then try values of n between 1 and 10

But largest value of $a = 9$

Smallest value of $c = 1$

$\therefore$ smallest $(a-c) = 8$

$\Rightarrow n = 8$ ✓ ✓ is largest value

n	99n	
1	99	✓
2	198	✓
3	297	✓
4	396	✓
5	495	✓
6	594	✓
7	693	✓
8	792	✓

$\therefore 1 \leq n \leq 8$

Comment:
ALL the values of *n* need to be shown to get full marks. Writing down 1 value scored 0 marks.

Section H

Q.2 (a) Let A_i be amount i^{th} child gets

Then $A_1 = \$1000 + \frac{P-1000}{10}$ ✓

(b) $A_2 = \$2000 + \frac{1}{10}(P - A_1 - 2000)$ ✓✓

$= 2000 + \frac{1}{10}\left(P - \left(1000 + \frac{P-1000}{10}\right) - 2000\right)$

$= 2000 + \frac{P}{10} - 100 - \frac{P-1000}{100} - 200$

$= 1700 + \frac{P}{10} - \frac{P}{100} + 10$

$A_2 = 1710 + \frac{9P}{100}$

2(c)(i) $A_1 = A_2$

$\Rightarrow 1000 + \frac{P-1000}{10} = 1710 + \frac{9P}{100}$

$\frac{10P - 10000}{100} = 710 + \frac{9P}{100}$

$\Rightarrow \frac{P}{100} - 100 = 710$

$\frac{P}{100} = 810$

$P = \$81000$

(ii) 1st child gets $1000 + \frac{80000}{10} = \$9000$

2nd child gets $2000 + 8100 - 900 - 200$

$= \$9000$

Each gets \$9000

Section H

Q 2(c) (iii) $81000 \div 9000 = 9$ children

Comments (a) Most students got the amount for the first child.

(b) If students had the correct expression prior to simplifying they got full marks. Only about $\frac{1}{4}$ of the cohort were able to correctly give the expression for the 2nd child's amount.

(c)(i) Since the question told students to equate (a) and (b) there were no marks for simpler incorrect expressions being worked out. ie. if their expressions resulted in a much easier equation no marks were awarded.

(ii) If students had incorrect answers from (i) and did not check amounts were equal for both children then no marks were given.

(iii) If a students incorrect answer from a previous part resulted in a non-positive integer then no marks were given as result was clearly wrong.