Sydney Boys High School

MOORE PARK, SURRY HILLS

YEAR 10 ADVANCED MATHEMATICS

Yearly Examination 2017

General Instructions:

- All questions may be attempted.
- Write using black pen.
- Marks may be deducted for careless or badly arranged work.
- All working and answers are to be written in this test booklet.
- If you wish to rewrite an answer, draw a line through your faulty answer and rewrite your answer on the back pages of this booklet. Show the number and part of the answer being rewritten
- Leave your answers in the simplest exact form, unless otherwise stated.
- Board approved calculators may be used.
- Clearly indicate your class by placing an **X** next to your class.

Time Allowed: 120 minutes

Reading Time: 5 minutes

Examiner: PSP

Name: _____

Class	Teacher	
10**A**	Ms Kilmore	
10**B**	Ms Chan	
10**C**	Ms Evans & Mr Elliott	
10**P**	Mr Dowdell & Mr Kwong	
10**L**	Mr Choy	
10**U**	Mr Wang	
10**S**	Mr Gainford	

Section	Marks
A	/ 10
B	/ 15
C	/ 15
D	/ 15
E	/ 15
F	/ 15
G	/ 15
H	/ 15
Total	/ 115

Section A **Multiple Choice** **Circle the correct response**

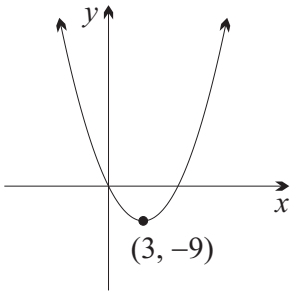
1 Which of the following is the solution of the quadratic equation $(7 - 2x)(3 + x) = 0$?

(A) $x = 3$ or $x = -\frac{7}{2}$ (B) $x = 3$ or $x = -\frac{2}{7}$

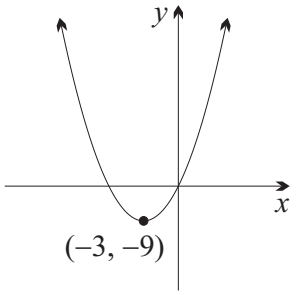
(C) $x = -3$ or $x = \frac{7}{2}$ (D) $x = -3$ or $x = \frac{2}{7}$

2 Which of the following could represent the graph of $y = (x - 3)^2 - 9$?

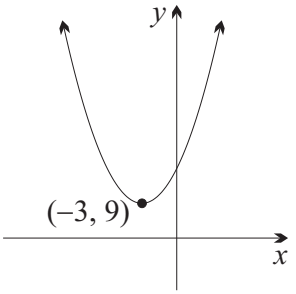
(A)



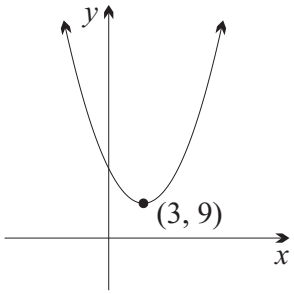
(B)



(C)



(D)



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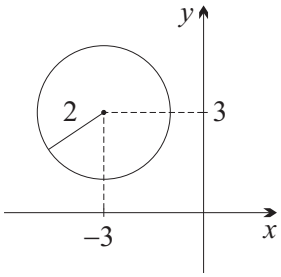
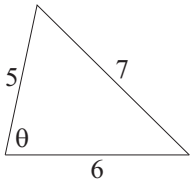


Which of the following is the equation of the circle graphed above?

(A) $(x - 3)^2 + (y + 3)^2 = 4$ (B) $(x - 3)^2 + (y + 3)^2 = 2$

(C) $(x + 3)^2 + (y - 3)^2 = 4$ (D) $(x + 3)^2 + (y - 3)^2 = 2$

4 In the diagram below, which of the following is a correct expression for $\cos\theta$?



(A) $\dfrac{5^2 + 6^2 - 7^2}{2 \times 5 \times 6}$ (B) $\dfrac{5^2 + 7^2 - 6^2}{2 \times 6 \times 7}$

(C) $\dfrac{5^2 + 6^2 + 7^2}{2 \times 5 \times 6}$ (D) $\dfrac{5^2 + 6^2 - 7^2}{2 \times 5 \times 7}$

5 Which of the following is the natural domain of $\frac{3}{\sqrt{2-x}}$?

(A) $x \leq 2$ (B) $x \geq 2$

(C) $x < 2$ (D) $x > 2$

6 Which of the following is the solution to the equation $\tan\theta = -1$ for $0° \leq \theta \leq 360°$?

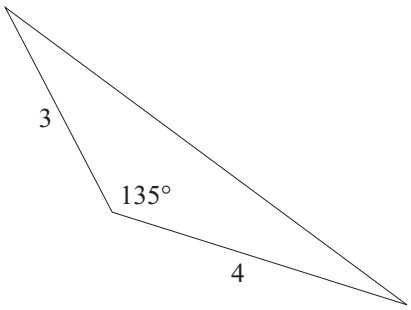
(A) $\theta = -45°$ or $\theta = 45°$ (B) $\theta = -45°$

(C) $\theta = 135°$ or $\theta = 225°$ (D) $\theta = 135°$ or $\theta = 315°$

7 Stewie Dent is collecting the following sets of data.
Which situation is a set of bivariate data?

(A) The time Stewie spends watching television each night.
(B) The height of each boy in Y10 and the length of his right foot.
(C) The best scores recorded by all Y10 students on a particular video game.
(D) The favourite TV show of each Y10 students.

8 Which of the following is the exact area of the triangle below?



(A) $6\sqrt{2}$ (B) $\frac{3}{\sqrt{2}}$

(C) $3\sqrt{2}$ (D) $\frac{12}{\sqrt{2}}$

9 Which of the following has the y-axis as an axis of symmetry?

(A) $y = \sin x$ (B) $y = x^2 - 2x$

(C) $y = \frac{1}{x+1}$ (D) $y = \sqrt{16 - x^2}$

10 If the discriminant of a quadratic equation is a square number, which of the following statements about the roots is true?

(A) Equal real roots (B) Real and rational roots

(C) No real roots (D) Real and irrational roots

Section B (15 marks)

(a) Expand and simplify $(2x - 1)^2 + 4x$. **2**

(b) Simplify $3\sqrt{2} + \sqrt{2}$. **1**

(c) Calculate the final value of an investment of \$5000 which has been earning interest of 4% pa, compounded annually for four years. **1**
Give your answer correct to the nearest cent.

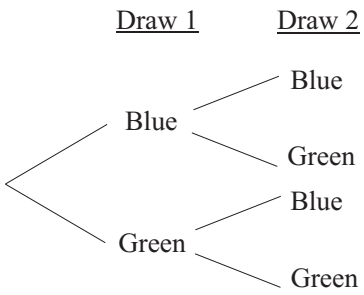
(d) Write down the equation of the vertical asymptote of $y = \frac{2}{3-x}$. **1**

(e) Solve $2^x = \frac{1}{2}$. **1**

(f) Express $y = \log_2 x$ in index form. **1**

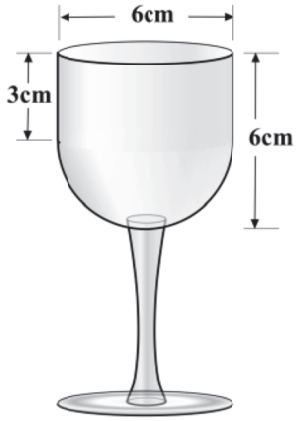
(g) A bag contains 2 blue marbles and 3 green marbles. Two marbles are drawn from the bag, one after the other, without replacement.

(i) Write the probability of each outcome on the branches below. **2**



(ii) Find the probability that one blue and one green marble are drawn from the bag. **2**

(h) The diagram below shows a glass. The bowl of the glass has a cylindrical top and a hemispherical base.



(i) Show that the volume of the bowl of the glass is 45π cm^3. **2**

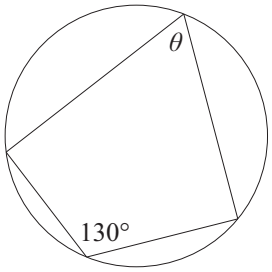
(ii) Find the height of liquid in the glass when it is half full. **2**

Section C (15 marks)

(a) Express $\frac{2}{\sqrt{5}}$ with a rational denominator. **1**

(b) Yohc paid \$420 as a deposit on a lap-top computer. **2**
Find the price of the computer if the deposit was 15% of the total cost.

(c) What is the value of angle θ in the diagram below? **2**
Give a geometric reason for your answer.

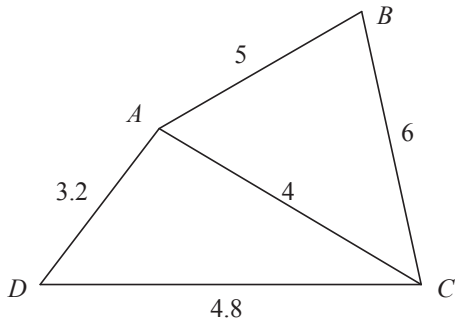


(d) Five years ago, Lledwod bought a car which has been depreciating at the rate of 15% pa. **2**
If it's current value is \$10 205, how much did he pay for the car? Answer to the nearest dollar.

(e) Given that k is a positive number specify the largest and the smallest of the following numbers: **2**

$$2^{-\frac{1}{2}k},\ 2^{\frac{1}{2}k},\ 2^{k},\ 2^{-k}$$

(f) Use the diagram below to answer the following.



(i) Prove $\Delta ACD \,|||\, \Delta ABC$ **2**

(ii) Hence show AC bisects $\angle DAB$. **1**

(g) If $3p^2 - 2 = 10$, find the value of p^4. **1**

(h) Determine whether the point (2, 3) lies inside, outside or on the circle **2**

$$(x - 1)^2 + (y + 1)^2 = 8.$$

Justify your answer.

Section D (15 marks)

(a) The results from a class test are displayed in the table below

x	f	cf
8	1	1
10	1	2
11	1	3
12	2	5
13	1	6
14	2	8
15	4	12
16	2	14
17	3	17
18	1	18
19	1	19

Use your calculator and the table to help answer the following questions.

(i) State the mode. **1**

(ii) Find $\bar{x}$ and σ_n. **2**
Express correct to one decimal place.

$\bar{x} =$

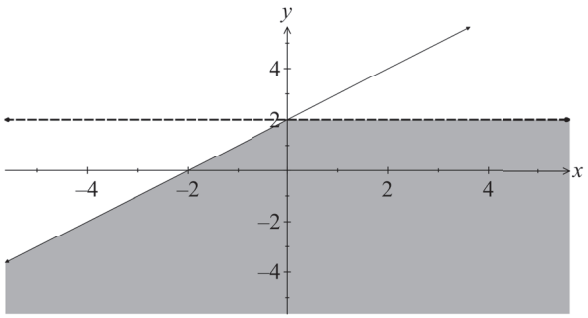
$\sigma_n =$

(iii) Rekrap awards a prize to those who get more than $\bar{x} + \sigma_n$ in the test. **1**
How many prizes are awarded?

(iv) State the median. **1**

(b) Solve $\log_3(x+2) = 2$. **2**

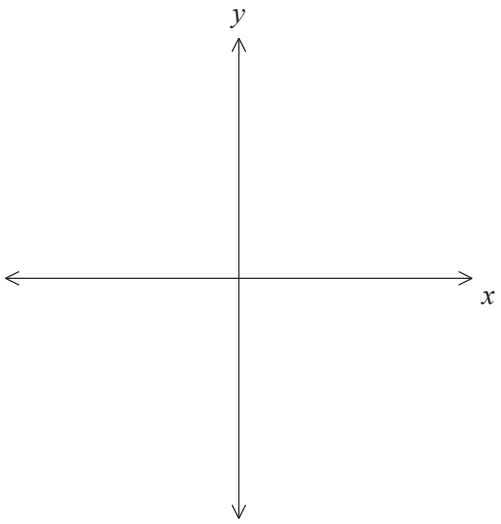
(c) Write down a pair of inequalities that define the shaded region below **2**



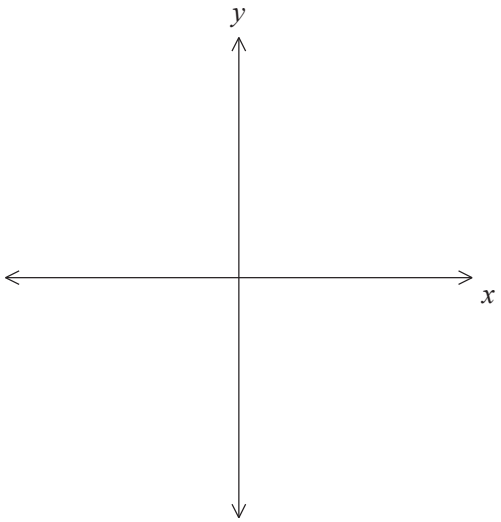
Note The shaded region is bounded by the lines $y = x + 2$ and $y = 2$.

(d) On separate diagrams, draw neat sketches of each of the following, showing intercepts, asymptotes and any other important features.

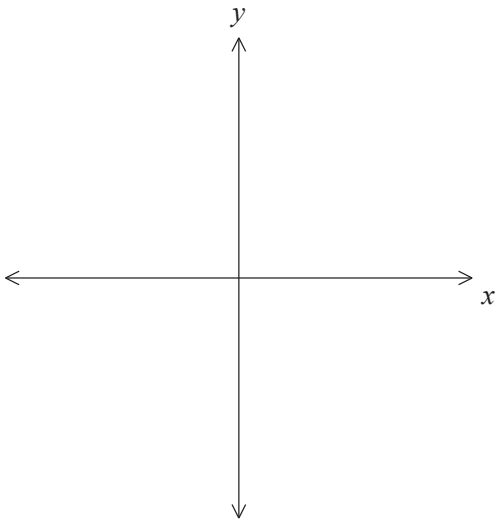
(i) $(x + 2)(y - 1) = -2$ **2**



(ii) $y = -(x + 1)^3 + 8$ **2**

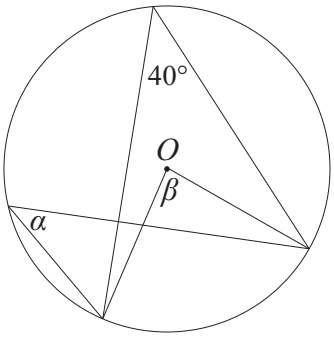


(iii) $x^2 + 2x + y^2 = 9$ **2**



Section E (15 marks)

(a) O is the centre of the circle below. What is the value of angles α and β in the diagram below? Give geometric reasons for your answer. **4**



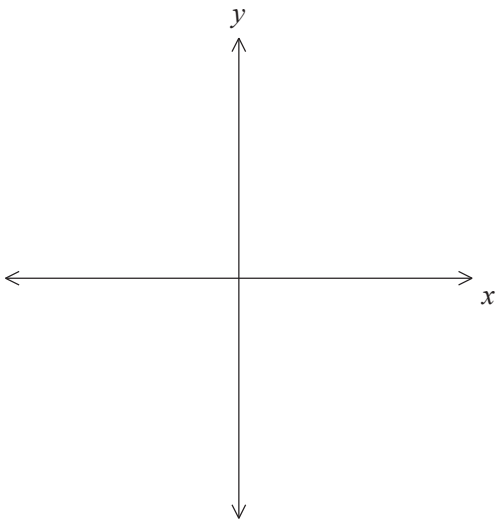
(b) A parabola has equation $y = x^2 + 4x + 5$

(i) Write down the y-intercept of the parabola **1**

(ii) By completing the square, express the parabola in the form $y = (x - h)^2 + k$. **1**

(iii) Write down the coordinates of the vertex. **1**

(iv) Sketch the parabola, using the information above. **2**

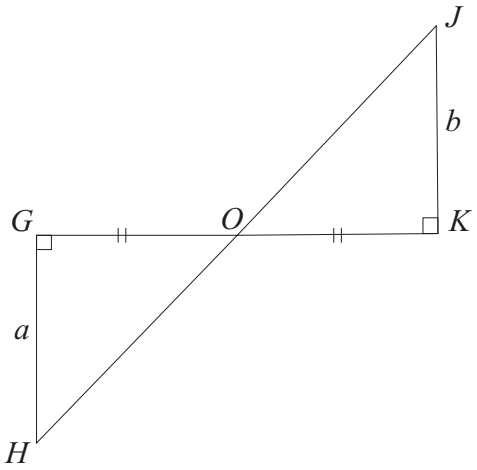


(c) Eromlik has just received her Yearly test result of 80%. She feels disappointed as her HY test result was 84%. Her teacher gave her the following details to show that she had improved.

	$\bar{x}$	σ	Eromlik's marks
HY	60	12	84
Yrly	65	6	80

Show, with calculations, why Eromlik has improved. **2**

(d) In the diagram below, $OG = OK$ and $\angle HGO = \angle JKO = 90°$.



(i) Prove $\Delta OGH \equiv \Delta OKJ$, giving clear geometric reasons **2**

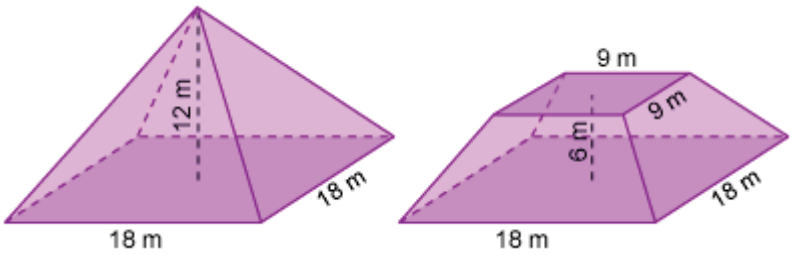
(ii) Hence give a reason why $a = b$. **1**

Section F (15 marks)

(a) State the number of solutions to the equation $\sin x = 0$ for $0° \leq x \leq 720°$? **1**

(b) Two lines have equations $kx + y = 2$ and $3x + 8y = 15$. **2**
Find the value of k if the lines do not intersect.

(c) A frustum is made by making a horizontal slice through the pyramid, and removing the small pyramid. **2**



By using similarity, or otherwise, find the volume of the frustum.

(d) (i) Factorise $x^2 + 4x + 4$. **1**

(ii) Hence factorise $x^4 - x^2 - 4x - 4$. **2**

(e) By first factorising $a^5 - a^4b - ab^4 + b^5$, show that $a^5 - a^4b \geq ab^4 - b^5$ for all positive numbers a and b. **2**

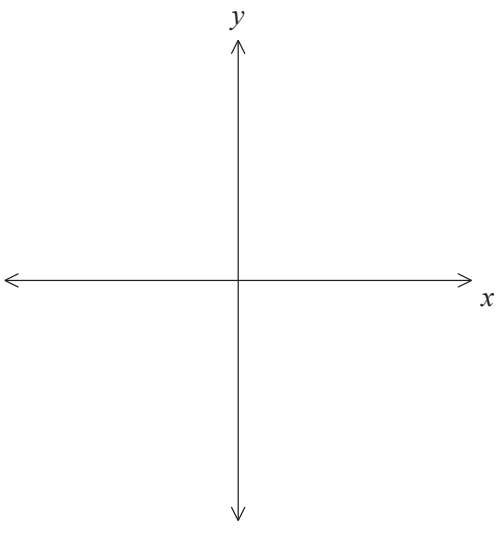
(f) Let $f(x) = \sqrt{9 - x^2}$

(i) Find $f(-1)$. **1**

(ii) Find $f(m^2)$. **1**

(iii) What is the domain of $f(x)$? **1**

(iv) Sketch $y = f(x+1) + 1$ **2**



Section G (15 marks)

(a) Given that $f(x) = 2 - \dfrac{1}{x-1}$

(i) Explain why $f^{-1}(x)$ exists. **1**

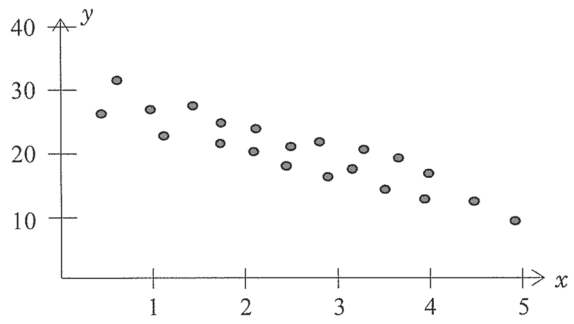
(ii) What is the domain and range of $f^{-1}(x)$. **2**

(iii) Find the equation of $f^{-1}(x)$. **2**

(b) For $x > 0$, which expression is NOT equivalent to $a^{\log_a x}$. **1**

(A) $\log_a(a^x)$ (B) $x^{\log_a a}$ (C) $(\log_a a)^x$ (D) $\dfrac{1}{a^{\log_a \frac{1}{x}}}$

(c) For the scatterplot below, which of the following is closest to the equation for the line of best fit? **1**



(A) $y = 30 - x$ (B) $y = 30 - 4x$ (C) $y = 30 - 10x$ (D) $y = 30 - 20x$

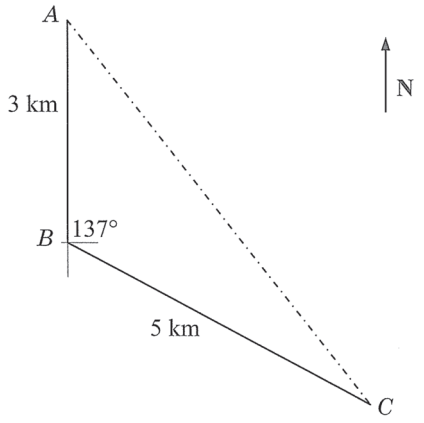
(d) By letting $u = x^2 + 2$, or otherwise, solve $(x^2 + 2)^2 - 4(x^2 + 2) + 3 = 0$ **2**

(e) Consider the formula: $t = \dfrac{6-r}{rx-x}$.

(i) Make x the subject of the formula. **2**

(ii) State any restrictions which may apply to the variables r and x. **1**

(f) Find the bearing of C from A, correct to the nearest degree. **3**

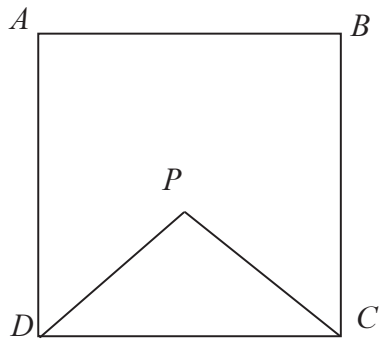


Section H (15 marks)

(a) If $0° < \phi < 90°$, simplify $\dfrac{\sin\phi\sin(180° - \phi)}{\cos(90° + \phi)}$ **2**

(b) Solve $2 \times 5^{x+1} = 17$. Express your answer correct to two decimal places. **2**

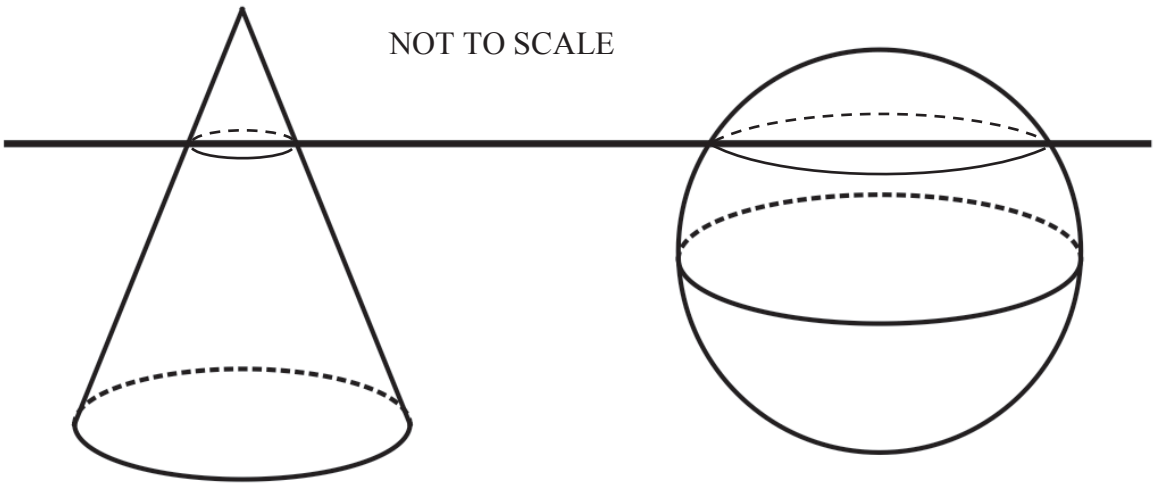
(c) A point P is chosen at random in a square $ABCD$. **2**



Not to scale

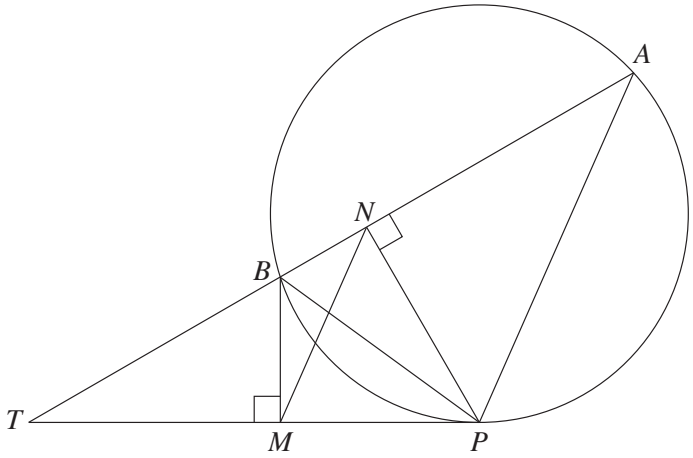
What is the probability that $\angle DPC$ is not acute. Justify your result.

(d) A sphere has radius 5 cm and a cone has height 10 cm and its base has radius 5 cm. **3**



The sphere and cone sit on a horizontal surface.
Find the height above the horizontal plane such that the circular cross section of the two solids have equal area.

(e)



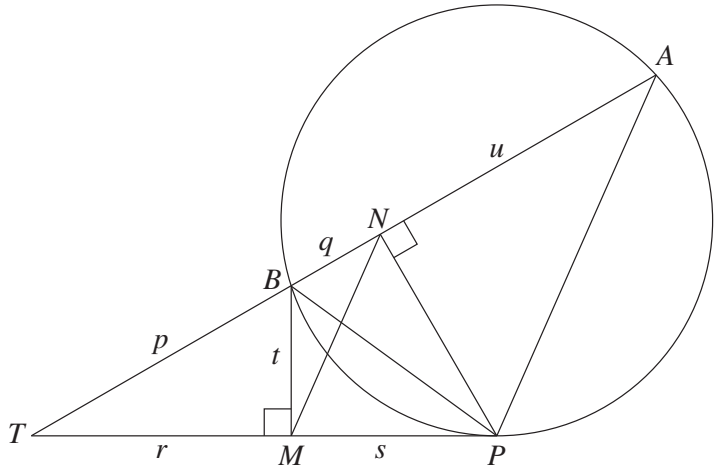
The points A, B and P lie on a circle.
The chord AB produced and the tangent at P intersect at the point T, as shown in the diagram. The point N is the foot of the perpendicular to AB through P, and the point M is the foot of the perpendicular to PT through B.

(i) Explain why $BNPM$ is a cyclic quadrilateral. **1**

(ii) Prove that MN is parallel to PA. **2**

Section H part (e) is continued on the following page

(e) Let $TB = p$, $BN = q$, $TM = r$, $MP = s$, $MB = t$ and $NA = u$.



(iii) Show that $\frac{s}{u} < \frac{r}{p}$. **2**

If needed, just state why two triangles are similar. Proof of similarity not needed.

(iv) Deduce that $s < u$. **1**

End of paper

SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2017

Year 10 Yearly

Advanced Mathematics

Suggested Solutions

Sections	Marker
A	**VL**
B	
C	**MK**
D	**AW**
E	**EC**
F	**TE**
G	**JC**
H	**BK**

Multiple Choice Answers:

1.	C	**4.**	A	**7.**	B	**10.**	B
2.	A	**5.**	C	**8.**	C		
3.	C	**6.**	D	**9.**	D		

Section A **Multiple Choice** **Circle the correct response**

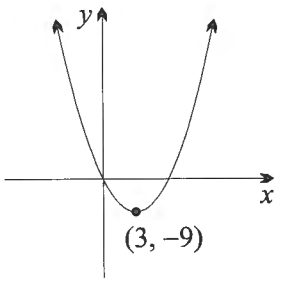
1 Which of the following is the solution of the quadratic equation $(7 - 2x)(3 + x) = 0$?

(A) $x = 3$ or $x = -\frac{7}{2}$ (B) $x = 3$ or $x = -\frac{2}{7}$

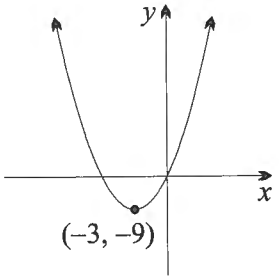
(C) $x = -3$ or $x = \frac{7}{2}$ (D) $x = -3$ or $x = \frac{2}{7}$

2 Which of the following could represent the graph of $y = (x - 3)^2 - 9$?

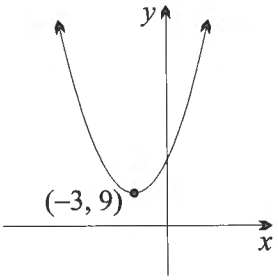
(A)



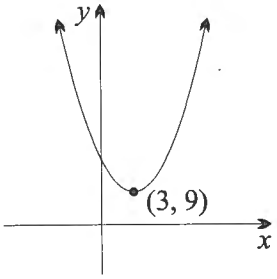
(B)



(C)



(D)



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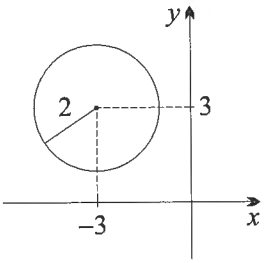
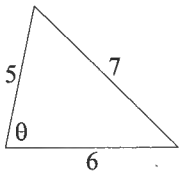


Which of the following is the equation of the circle graphed above?

(A) $(x - 3)^2 + (y + 3)^2 = 4$ (B) $(x - 3)^2 + (y + 3)^2 = 2$

(C) $(x + 3)^2 + (y - 3)^2 = 4$ (D) $(x + 3)^2 + (y - 3)^2 = 2$

4 In the diagram below, which of the following is a correct expression for $\cos\theta$?



(A) $\frac{5^2 + 6^2 - 7^2}{2 \times 5 \times 6}$ (B) $\frac{5^2 + 7^2 - 6^2}{2 \times 6 \times 7}$

(C) $\frac{5^2 + 6^2 + 7^2}{2 \times 5 \times 6}$ (D) $\frac{5^2 + 6^2 - 7^2}{2 \times 5 \times 7}$

5 Which of the following is the natural domain of $\frac{3}{\sqrt{2-x}}$?

(A) $x \leq 2$ (B) $x \geq 2$

(C) $x < 2$ (D) $x > 2$

6 Which of the following is the solution to the equation $\tan\theta = -1$ for $0° \leq \theta \leq 360°$?

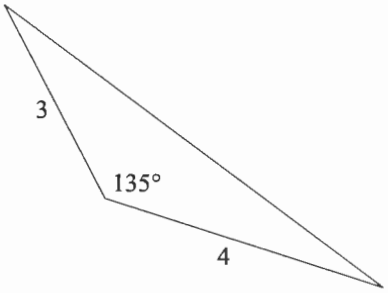
(A) $\theta = -45°$ or $\theta = 45°$ (B) $\theta = -45°$

(C) $\theta = 135°$ or $\theta = 225°$ (D) $\theta = 135°$ or $\theta = 315°$

7 Stewie Dent is collecting the following sets of data.
Which situation is a set of bivariate data?

(A) The time Stewie spends watching television each night.
(B) The height of each boy in Y10 and the length of his right foot.
(C) The best scores recorded by all Y10 students on a particular video game.
(D) The favourite TV show of each Y10 students.

8 Which of the following is the exact area of the triangle below?



(A) $6\sqrt{2}$ (B) $\frac{3}{\sqrt{2}}$

(C) $3\sqrt{2}$ (D) $\frac{12}{\sqrt{2}}$

9 Which of the following has the y-axis as an axis of symmetry?

(A) $y = \sin x$ (B) $y = x^2 - 2x$

(C) $y = \frac{1}{x+1}$ (D) $y = \sqrt{16 - x^2}$

10 If the discriminant of a quadratic equation is a square number, which of the following statements about the roots is true?

(A) Equal real roots (B) Real and rational roots

(C) No real roots (D) Real and irrational roots

Section B (15 marks)

(a) Expand and simplify $(2x-1)^2+4x$. **2**

$4x^2 - 4x + 1 + 4x$ ✓1 mark

$= 4x^2 + 1$ ✓1 mark

(b) Simplify $3\sqrt{2}+\sqrt{2}$. **1**

$4\sqrt{2}$ ✓1 mark

(c) Calculate the final value of an investment of \$5000 which has been earning interest of 4% pa, compounded annually for four years. Give your answer correct to the nearest cent. **1**

$5000(1.04)^4$ $\frac{1}{2}$ mark

final value is \$5849.29 $\frac{1}{2}$ mark

$\frac{1}{2}$ mark deducted if the answer was only "3"

(d) Write down the equation of the vertical asymptote of $y=\frac{2}{3-x}$. **1**

$3-x \neq 0$ vertical asymptote : $x=3$ ✓1 mark

(e) Solve $2^x=\frac{1}{2}$. **1**

$2^x = 2^{-1}$ $\frac{1}{2}$ mark

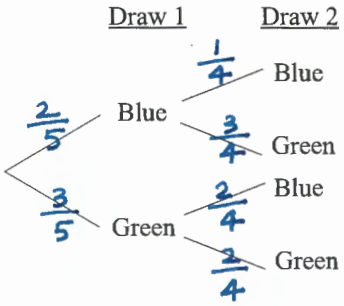
$\therefore x = -1$ $\frac{1}{2}$ mark

(f) Express $y=\log_2 x$ in index form. **1**

$x = 2^y$ ✓1 mark

(g) A bag contains 2 blue marbles and 3 green marbles. Two marbles are drawn from the bag, one after the other, without replacement.

(i) Write the probability of each outcome on the branches below. **2**



$\frac{1}{2}$ mark deducted for every error

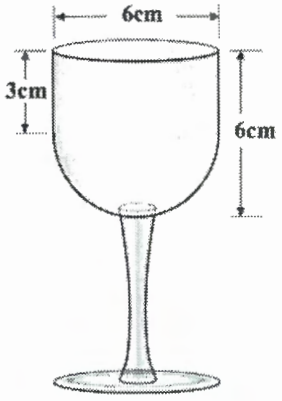
(ii) Find the probability that one blue and one green marble are drawn from the bag. **2**

P(blue & green) + P(green & blue)

$= \frac{2}{5} \times \frac{3}{4} + \frac{3}{5} \times \frac{2}{4}$ ✓ 1 mark

$= \frac{3}{5}$ ✓ 1 mark

(h) The diagram below shows a glass. The bowl of the glass has a cylindrical top and a hemispherical base.



$\frac{1}{2}$ mark deducted if final answer was not simplified

(i) Show that the volume of the bowl of the glass is $45\pi\ \text{cm}^3$. **2**

V of bowl = V of cylindrical top + V of hemispherical base

$= \pi r^2 h + \frac{1}{2} \times \frac{4}{3} \pi r^3$ ✓ 1 mark

$= (27\pi + 18\pi)\ \text{cm}^3$ ✓ 1 mark

$= 45\pi\ \text{cm}^3$

(ii) Find the height of liquid in the glass when it is half full. **2**

V when half full $= \frac{45\pi}{2}$ → $\frac{1}{2}$ mark

$\frac{45\pi}{2} = 18\pi + x$] $\frac{1}{2}$ mark

$x = 4.5\pi$

$4.5\pi = \pi(3)^2 h$

$h = 0.5$ → $\frac{1}{2}$ mark

height of liquid $= (0.5 + 3)\ \text{cm} = 3.5\ \text{cm}$ $\frac{1}{2}$ mark

Solutions

Section C (15 marks)

(a) Express $\frac{2}{\sqrt{5}}$ with a rational denominator. **1**

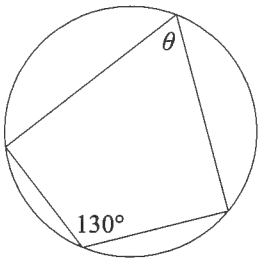
$\frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$ ✓

(b) Yohc paid \$420 as a deposit on a lap-top computer. **2**
Find the price of the computer if the deposit was 15% of the total cost.

$420 = 15\%$ ✓
$28 = 1\%$
$\therefore 2800 = 100\%$

$\therefore$ Price of computer is \$2800 ✓

(c) What is the value of angle θ in the diagram below? **2**
Give a geometric reason for your answer.



$\theta = 50°$ ✓ (opposite $\angle$s are supplementary for a cyclic quadrilateral) ✓

Must have reason for full marks

(d) Five years ago, Lledwod bought a car which has been depreciating at the rate of 15% pa. **2**
If it's current value is \$10 205, how much did he pay for the car? Answer to the nearest dollar.

$A = P(1-r)^n$
$10205 = P(1-15\%)^5$ ✓
$\therefore P = 22999.49925$
$= \$22999$ (nearest dollar) ✓

(e) Given that k is a positive number specify the largest and the smallest of the following numbers: **2**

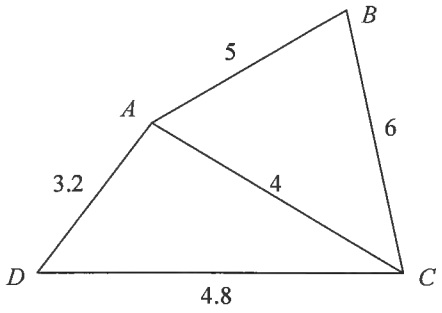
$$2^{-\frac{1}{2}k},\ 2^{\frac{1}{2}k},\ 2^{k},\ 2^{-k}$$

Largest: 2^k ✓

Smallest: $\frac{1}{2^k} = 2^{-k}$ ✓

If clearly shown ascending/descending order, you may get marks

(f) Use the diagram below to answer the following.



(i) Prove $\Delta ACD \,|||\, \Delta ABC$ **2**

$\frac{AD}{AC} = \frac{AC}{AB} = \frac{DC}{BC} = 0.8 = \frac{4}{5}$ ✓ (matching)

Since all sides are in proportion,

$\Delta ACD|||\Delta ABC$

Must have correct reason and show value of ratio.

(ii) Hence show AC bisects $\angle DAB$. **1**

Since $\Delta ACD|||\Delta ABC$,

$\angle DAC = \angle BAC$ (matching Ls of similar Δs are equal) ✓

$\therefore AC$ bisects $\angle DAB$

Must have correct reasoning since we already know that AC bisects ∠DAB.

For correct reasoning and abbreviation, please refer to the 'Reasoning in Geometry' booklet on R-drive

(g) If $3p^2 - 2 = 10$, find the value of p^4. **1**

$3p^2 - 2 = 10$

$3p^2 = 12$

$p^2 = 4$

$\therefore p^4 = 16$ ✓

Note that $p^4 \neq \pm 16$ since p^4 is ALWAYS positive

If you use Trig and are correct, only partial marks since & states 'Hence'

(h) Determine whether the point (2, 3) lies inside, outside or on the circle **2**

$$(x-1)^2 + (y+1)^2 = 8.$$

Justify your answer.

Sub (2,3):

$1^2 + 4^2 = 17$

$17 > 8$ ✓

∴ lies outside the circle. Since it is larger than the radius ✓

Section D (15 marks)

(a) The results from a class test are displayed in the table below

x	f	cf
8	1	1
10	1	2
11	1	3
12	2	5
13	1	6
14	2	8
15	4	12
16	2	14
17	3	17
18	1	18
19	1	19

Use your calculator and the table to help answer the following questions.

(i) State the mode. **1**

15

(ii) Find $\bar{x}$ and σ_n. **2**
Express correct to one decimal place.

$\bar{x} =$ 14.4

$\sigma_n =$ 2.8

No half marks given.

(iii) Rekrap awards a prize to those who get more than $\bar{x} + \sigma_n$ in the test. **1**
How many prizes are awarded?

2

(iv) State the median. **1**

15

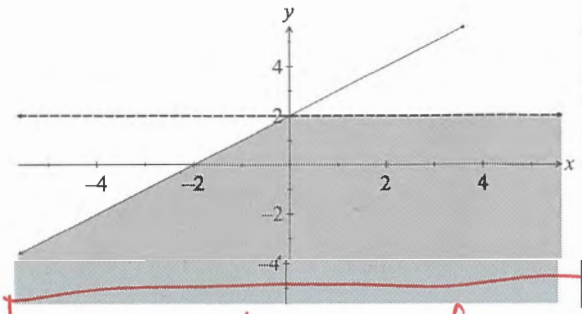
(b) Solve $\log_3(x+2) = 2$. **2**

$x + 2 = 3^2$ ① mark

$x + 2 = 9$

$x = 7$ ① mark.

(c) Write down a pair of inequalities that define the shaded region below **2**



Note The shaded region is bounded by the lines $y = x + 2$ and $y = 2$.

$y \leq x + 2$

$y < 2$

* Marks only given for correct inequality & each.

9

Section D (continued)

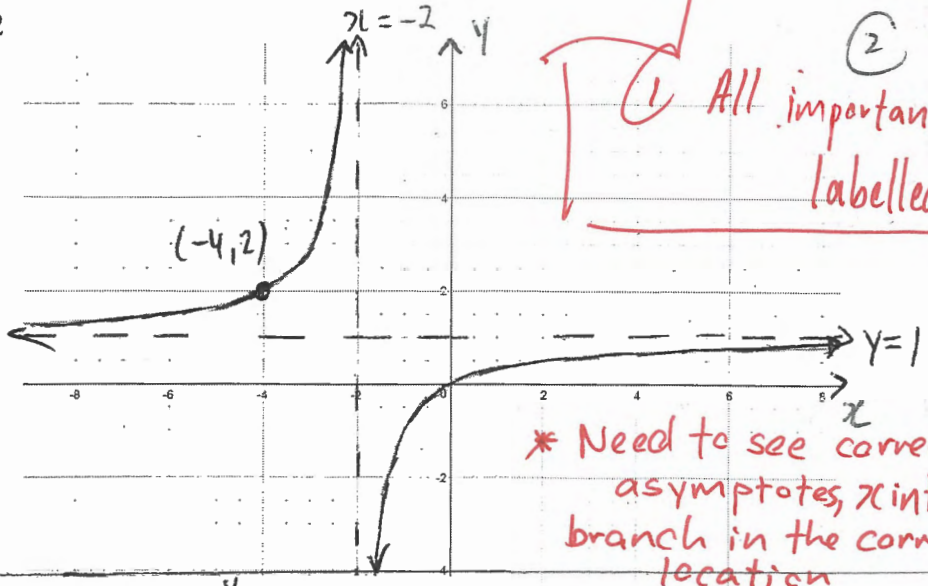
(d) On separate diagrams, draw neat sketches of each of the following, showing intercepts, asymptotes and any other important features.

(i) $(x+2)(y-1)=-2$

$y-1=\frac{-2}{x+2}$

$y=\frac{-2}{(x+2)}+1$

Asymptotes:
$y=1$
$x=-2$
x-intercept $=(0,0)$



1 mark for correct graph at correct location on number plane. (2)

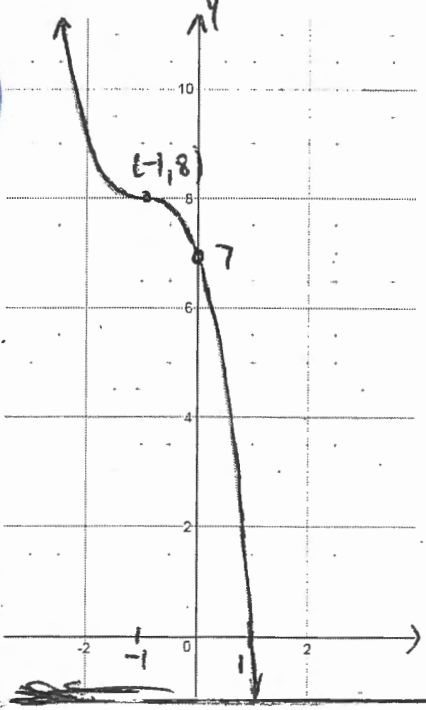
1 All important feature labelled.

* Need to see correct asymptotes, x intercept, branch in the correct location

2

(ii) $y=-(x+1)^3+8$

y-intercept: (0,7)
x-intercept: (1,0)



* Need to see correct:
→ x-intercept
→ y-intercept
→ inflexion at (-1,8)
→ Correct shape.

2

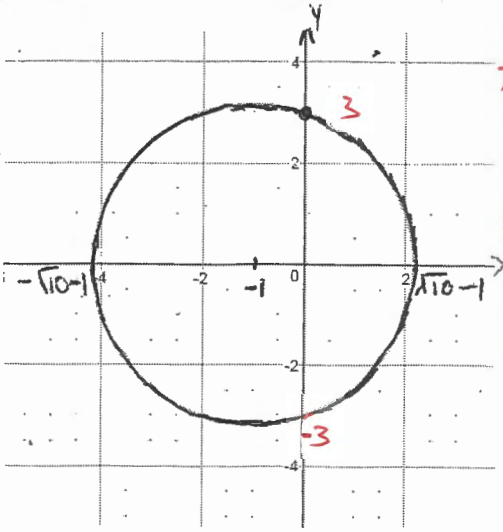
(iii) $x^2+2x+y^2=9$

$x^2+2x+1+y^2=9+1$

$(x+1)^2+y^2=10$

Centre $=(-1,0)$

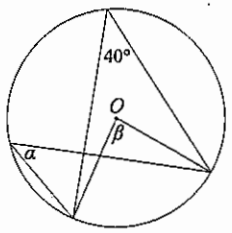
radius $=\sqrt{10}$



* Need to see correct:
→ intercepts LABELLED
→ Centre LABELLED
→ Correct Shape.

Section E (15 marks)

(a) O is the centre of the circle below. What is the value of angles α and β in the diagram below? Give geometric reasons for your answer. [4]



$\alpha = 40$ Angles in the same segment

$\beta = 80$

(Angle at the centre is twice the angle at the circumference subtended by the same arc)

(b) A parabola has equation $y = x^2 + 4x + 5$

(i) Write down the y-intercept of the parabola [1]

5

(ii) By completing the square, express the parabola in the form $y = (x-h)^2 + k$. [1]

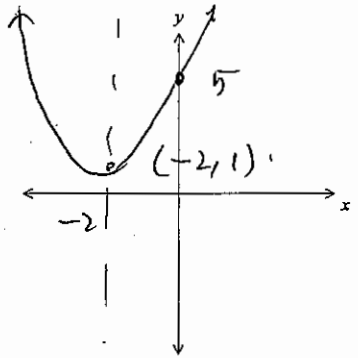
$y = (x+2)^2 + 1$

$y = (x+2)^2 + 1$

(iii) Write down the coordinates of the vertex. [1]

$(-2, 1)$

(iv) Sketch the parabola, using the information above. [2]



- Vertex
- axis of parabola
- y-intercept
- Smoothness

(c) Eromlik has just received her Yearly test result of 80%. She feels disappointed as her HY test result was 84%. Her teacher gave her the following details to show that she had improved.

	$\bar{x}$	σ	Eromlik's marks
HY	60	12	84
Yrly	65	6	80

24 (3)

15 (2.5)

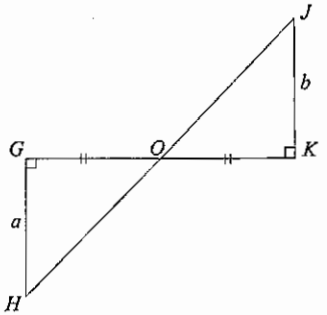
Show, with calculations, why Eromlik has improved. [2]

$84 - 60 = 24$, $24 \div 12 = 2$ SD above $\bar{x}$

$80 - 65 = 15$, $15 \div 6 = 2.5$ SD above $\bar{x}$

$\therefore$ improved in Yearly $\because 2.5 > 2$

(d) In the diagram below, $OG = OK$ and $\angle HGO = \angle JKO = 90°$.



(i) Prove $\Delta OGH \equiv \Delta OKJ$, giving clear geometric reasons [2]

$\angle HOG = \angle JOK$ (vertically opp $\angle$'s are equal)

$\angle OGH = \angle JKO = 90$ (given)

$OG = OK$ given

$\therefore \Delta GOH \equiv \Delta KOJ$ (AAS)

(ii) Hence give a reason why $a = b$. [1]

$a = b$ (corresponding sides of congruent Δ's are equal) or matching

Section F (15 marks)

(a) State the number of solutions to the equation $\sin x = 0$ for $0° \le x \le 720°$? 1

$\sin 0°, 180°, 360°, 540°, 720° = 0$ 5

right or wrong – no ½ mark

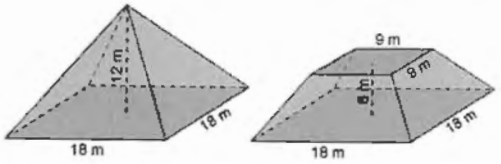
(b) Two lines have equations $kx + y = 2$ and $3x + 8y = 15$. 2
Find the value of k if the lines do not intersect.

GRADIENT OF LINES ARE $-K$ AND $-\frac{3}{8}$

$\therefore -K = -\frac{3}{8}$ (PARALLE LINES DO NOT INTERSECT)

$\therefore K = \frac{3}{8}$

(c) A frustum is made by making a horizontal slice through the pyramid, and removing the small pyramid. 2



By using similarity, or otherwise, find the volume of the frustum.

VOL OF LARGE PYRAMID $= \frac{1}{3} \cdot 18^2 \cdot 12$
$= 1296 \text{ m}^3$

VOL. OF SMALL PYRAMID $= (\frac{1}{2})^3 \times 1296$
(ALL SIDES $\frac{1}{2}$ OF LARGE) $= 162 \text{ m}^3$

$\therefore$ AREA OF FRUSTRUM $= 1134 \text{ m}^3$

1 for either of these. 1 for the other 2.

(d) (i) Factorise $x^2 + 4x + 4$. 1

$(x+2)^2$

(ii) Hence factorise $x^4 - x^2 - 4x - 4$. 2

$x^4 - x^2 - 4x - 4 = x^4 - (x+2)^2$
$= (x^2 - (x+2))(x^2 + (x+2))$
$= (x^2 - x - 2)(x^2 + x + 2)$

1½ for $(x^2 - (x+2))(x^2 + (x+2))$

Section F (continued)

(e) By first factorising $a^5 - a^4b - ab^4 + b^5$, show that $a^5 - a^4b \ge ab^4 - b^5$ 2
for all positive numbers a and b.

$a^5 - a^4b \ge ab^4 - b^5$
$a^5 - a^4b - ab^4 + b^5 \ge 0$
$a^4(a-b) - b^4(a-b) \ge 0$
$(a^4 - b^4)(a-b) \ge 0$
$(a^2+b^2)(a^2-b^2)(a-b) \ge 0$
$(a^2+b^2)(a-b)^2(a+b) \ge 0$
Each of these 3 terms is ≥ 0 for all +ve a, b
$\therefore$ RED

(f) Let $f(x) = \sqrt{9 - x^2}$

(i) Find $f(-1)$. 1

$f(-1) = \sqrt{9-(-1)^2} = \sqrt{8} = 2\sqrt{2}$

½ if in decimal or $\sqrt{8}$ form.

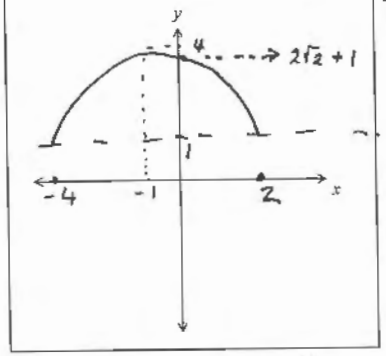
(ii) Find $f(m^2)$. 1

$f(m^2) = \sqrt{9 - m^4}$

(iii) What is the domain of $f(x)$? 1

$9 - x^2 \ge 0$
$-3 \le x \le 3$

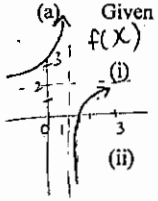
(iv) Sketch $y = f(x+1) + 1$ 2



no marks for st line graphs.

Section G (15 marks)

(a) Given that $f(x)=2-\frac{1}{x-1}$



(i) Explain why $f^{-1}(x)$ exists. 1

horizontal line test (1/2) hort. line test w/ graph (1)

for every y value there is a unique x-value ✓

(ii) What is the domain and range of $f^{-1}(x)$. 2

D: $x \in \mathbb{R}, x \neq 2$ ✓

R: $y \in \mathbb{R}, y \neq 1$ ✓

AN1 for D+R of f(x)

(iii) Find the equation of $f^{-1}(x)$. 2

$x = 2 - \frac{1}{y-1}$

$\frac{1}{y-1} = 2-x$ ✓

$\frac{1}{2-x} = y-1$ ✓

$y = 1 + \frac{1}{2-x}$ ✓

½ for each soln.

f(x) D: $x \in \mathbb{R}, x \neq 1$

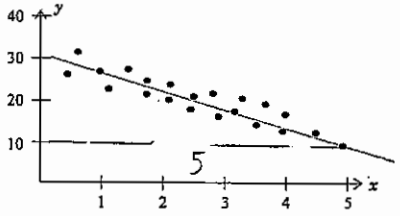
R: $y \in \mathbb{R}, y \neq 2$

$\frac{2-x}{2-x} + \frac{1}{2-x} = \frac{3-x}{2-x} = \frac{-(x-3)}{-(x-2)} = \frac{(x-3)}{(x-2)}$

(b) For $x > 0$, which expression is NOT equivalent to $a^{\log_a x}$. 1

(A) $\log_a(a^x)$ ✓ (B) $x^{\log_a a}$ ✓ (C) $(\log_a a)^x$ (D) $\frac{1}{a^{\log_a \frac{1}{x}}}$ ✓

(c) For the scatterplot below, which of the following is closest to the equation for the line of best fit? 1



20 5

$m = \frac{-20}{5} = -4$

$b = 30$

$y = -4x + 30$

(A) $y = 30 - x$ (B) $y = 30 - 4x$ ✓ (C) $y = 30 - 10x$ (D) $y = 30 - 20x$

(d) By letting $u = x^2 + 2$, or otherwise, solve $(x^2+2)^2 - 4(x^2+2) + 3 = 0$ 2

$u^2 - 4u + 3 = 0$

$(u-3)(u-1) = 0$

$u = 3, \quad u = 1$ ✓

$x^2 + 2 = 3$ $\quad$ $x^2 + 2 = 1$

$x^2 = 1$ $\quad$ $x^2 = -1$

$x = \pm 1$ $\quad$ no soln

minus ½ mark if students ignore $x^2 = -1$

$\therefore x = \pm 1$ ✓

9

(e) Consider the formula: $t = \frac{6-r}{rx-x}$.

$rx - x \neq 0$ $\quad x \neq 0$

$x(r-1) \neq 0$ $\quad r \neq 1$

(i) Make x the subject of the formula. 2

$t = \frac{6-r}{x(r-1)}$

$tx = \frac{6-r}{r-1}$ ✓

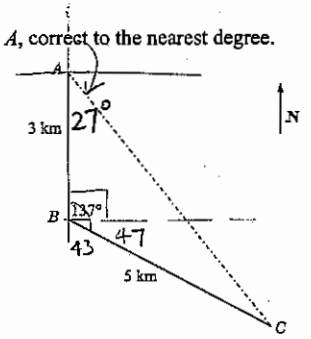
$x = \frac{6-r}{t(r-1)}$ ✓

(ii) State any restrictions which may apply to the variables r and x. 1

$r \neq 1$ ✓ ½ each.

$x \neq 0$

(f) Find the bearing of C from A, correct to the nearest degree. 3



0.2735526855

$AC = \sqrt{3^2 + 5^2 - 2 \times 3 \times 5 \times \cos 137°}$

$AC = \sqrt{55.94061105}$

$AC = 7.47934563$ km ✓

$\frac{\sin \angle BAC}{5} = \frac{\sin 137}{7.47934563}$

$= 27°$ (nearest degree) ✓

Bearing C from A = 153°

6

Section H (15 marks)

✓ = ½ mark

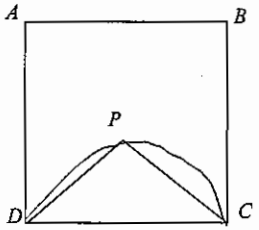
(a) If $0° < \phi < 90°$, simplify $\dfrac{\sin\phi\sin(180°-\phi)}{\cos(90°+\phi)}$ 2

$$\frac{\sin^2\phi}{-\sin\phi} = -\sin\phi$$

(b) Solve $2 \times 5^{x+1} = 17$. Express your answer correct to two decimal places. 2

$$5^{x+1} = 8.5$$
$$(x+1)\log 5 = \log 8.5$$
$$x = \frac{\log 8.5}{\log 5} - 1 = 0.32969$$
$$= 0.33 \text{ (2dp)}$$

(c) A point P is chosen at random in a square $ABCD$. 2



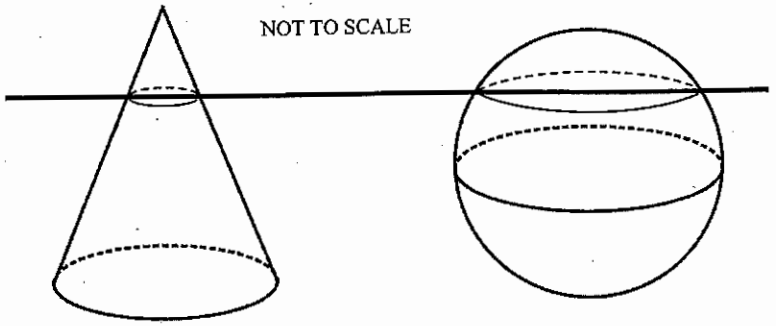
Not to scale

Let side of square = r
$\Rightarrow$ radius of semi-circle = $r/2$
$\Rightarrow$ Area semi-circle = $\frac{1}{2}\pi\left(\frac{r}{2}\right)^2$

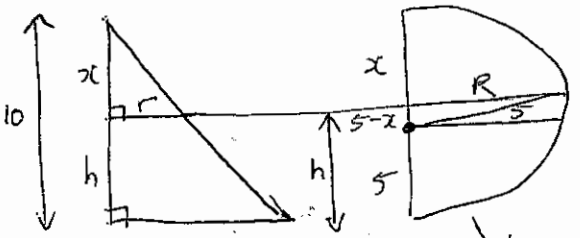
What is the probability that $\angle DPC$ is not acute. Justify your result.

Consider semi-circle DPC
If P lies on semi-circle $\Rightarrow \angle DPC = 90°$
$\therefore$ Inside Area of semi-circle, $\angle DPC$ is obtuse.
$\therefore P(\text{not acute}) = \dfrac{\text{Area semi-circle}}{\text{Area Square}} = \dfrac{\frac{1}{8}\pi r^2}{r^2}$
$= \dfrac{\pi}{8}$

(d) A sphere has radius 5 cm and a cone has height 10 cm and its base has radius 5 cm. 3



The sphere and cone sit on a horizontal surface.
Find the height above the horizontal plane such that the circular cross section of the two solids have equal area.



Circle at height h

$$\frac{r}{5} = \frac{x}{10}$$
$$5x = 10r$$
$$x = 2r$$
$$\Rightarrow r = x/2$$
$$A_{circle} = \pi r^2 = \pi\frac{x^2}{4} \quad (1)$$

Circle at height, h

$$R^2 + (5-x)^2 = 5^2 \text{ (Pythag.)}$$
$$R^2 = 25 - (25 - 10x + x^2)$$
$$R^2 = 10x - x^2$$
$$A_{circle} = \pi R^2$$
$$= \pi(10x - x^2) \quad (2)$$

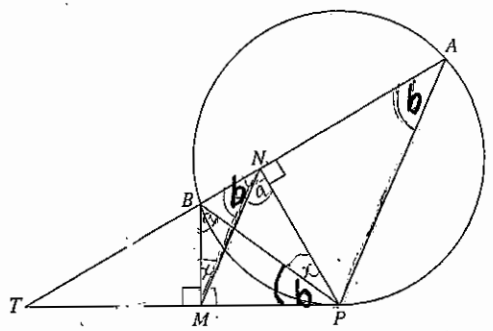
Then $(1) = (2) \Rightarrow \dfrac{x^2}{4} = 10 - x^2$
$$x^2 = 40 - 4x^2$$
$$5x^2 - 40x = 0$$
$$5x(x-8) = 0 \Rightarrow x = 0 \text{ or } x = 8$$

$x = 0$: no circle formed

If $x = 8$, $h = 2$cm $\therefore$ 2cm above plane

Section H (continued)

(e)



$TA \times TB = TP^2$

The points A, B and P lie on a circle.
The chord AB produced and the tangent at P intersect at the point T, as shown in the diagram. The point N is the foot of the perpendicular to AB through P, and the point M is the foot of the perpendicular to PT through B.

(i) Explain why $BNPM$ is a cyclic quadrilateral. **1**

$\angle PNB = 90°$ (Angles on st. line) ✓
$\angle BMP = 90°$ (Same reason)
$\therefore$ opp $\angle$'s in quad. are supplementary
$\Rightarrow BNPM$ is cyclic. ✓

$\checkmark = \frac{1}{2}M$

(ii) Prove that MN is parallel to PA. **2**

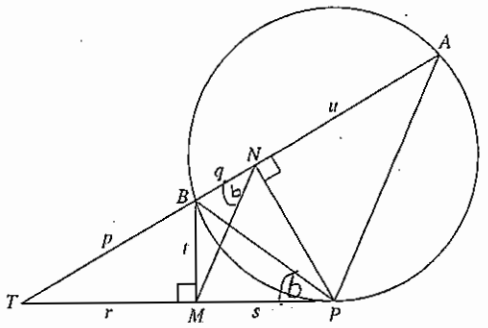
Let $\angle BNM = b$
Then $\angle BPM = b$ (Angles in same segment) ✓
Also $\angle BPM = \angle PAB$ (Alt Segment Theorem) ✓
$= b$
Then $\angle BNM = \angle PAB = b$ ✓
$\Rightarrow MN \parallel PA$ (corresponding $\angle$s are equal) ✓

$\checkmark = \frac{1}{2}M$

Section H part (e) is continued on the following page

Section H (continued)

(e) Let $TB = p$, $BN = q$, $TM = r$, $MP = s$, $MB = t$ and $NA = u$.



(iii) Show that $\frac{s}{u} < \frac{r}{p}$. **2**

If needed, just state why two triangles are similar. Proof of similarity not needed.

$\triangle TMB \parallel\parallel \triangle TNP$ (Equiangular) ✓
$\Rightarrow \frac{r}{p+q} = \frac{t}{NP} = \frac{s}{t+s}$ (1)
$\triangle MBP \parallel\parallel \triangle NPA \Rightarrow \frac{t}{NP} = \frac{BP}{PA} = \frac{s}{u}$ (2) ✓
From (1)(2) $\Rightarrow \frac{r}{p+q} = \frac{p}{s+r} = \frac{s}{u}$
Now $\frac{s}{u} = \frac{r}{p+q} < \frac{r}{p}$ since a smaller denominator gives a larger fraction. ✓

(iv) Deduce that $s < u$. **1**

Then $\frac{TM}{BT} < 1 \Rightarrow \frac{r}{p} < 1$ since hypotenuse (p) is largest side in $\triangle BTM$
Since $\frac{s}{u} < \frac{r}{p} \Rightarrow \frac{s}{u} < 1$
$\therefore s < u$

End of paper