Sydney Boys High School

MOORE PARK, SURRY HILLS

YEAR 10 ADVANCED MATHEMATICS

Yearly Examination 2018

General Instructions:

- All questions may be attempted.
- Write using black pen.
- Marks may be deducted for careless or badly arranged work.
- If you wish to rewrite an answer, draw a line through your faulty answer, and rewrite your answer on one of the blank pages of this booklet. Indicate you are doing this. Show the number and part of the answer being rewritten.
- All working and answers are to be written in this test booklet.
- Leave your answers in the simplest exact form, unless otherwise stated.
- Board approved calculators may be used.
- Clearly indicate your class by placing an **X** next to your class.

Time Allowed: 120 minutes

Reading Time: 5 minutes

Examiner: PSP

Name: _____

Class	Teacher	
10**A**	Mr A. Wang	
10**B**	Ms Ward	
10**C**	Ms Evans & Mr R. Wang	
10**P**	Mr Fuller	
10**L**	Ms Millar	
10**U**	Miss Chan	
10**S**	Mr Choy	

Section	Marks
A	/ 10
B	/ 15
C	/ 15
D	/ 15
E	/ 15
F	/ 15
G	/ 15
H	/ 16
Total	/ 116

Section A **Multiple Choice** **Circle the correct response**

1 Which of the following is the gradient of the line $y = 3x + 7$

(A) -7 (B) -3 (C) 3 (D) 5

2 The angle of elevation from point A to the top of a lighthouse is 30°.
What is the angle of depression from the top of the lighthouse to point A?

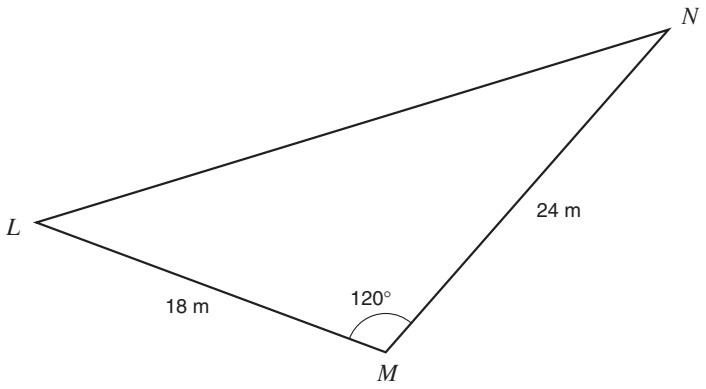
(A) 30° (B) 60° (C) 120° (D) 150°

3 The circle with radius 6 and centre (–3, 4) is shifted two units to the right and three units up.
Which of the following is the equation of the shifted circle?

(A) $(x-1)^2 + (y-7)^2 = 36$ (B) $(x+1)^2 + (y-7)^2 = 36$

(C) $(x+5)^2 + (y+1)^2 = 36$ (D) $(x+1)^2 + (y-1)^2 = 36$

4 In ΔLMN, what is the length of LN to the nearest tenth of a metre?



(A) 14.0 m (B) 21.6 m (C) 30.0 m (D) 36.5 m

5 What is the exact value of $\cos 210°$?

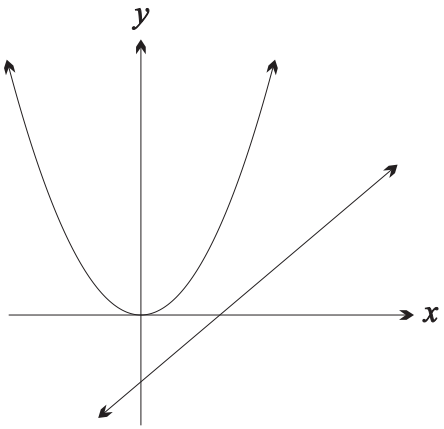
(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $-\frac{\sqrt{3}}{2}$

6 Which of the following statements is NOT true?

(A) $\log_3 15 - \log_3 5 = 1$ (B) $\log_4 2 + \log_4 8 = 1$

(C) $\log_5 \frac{1}{5} = -1$ (D) $\frac{\log_2 8}{\log_2 4} = \frac{3}{2}$

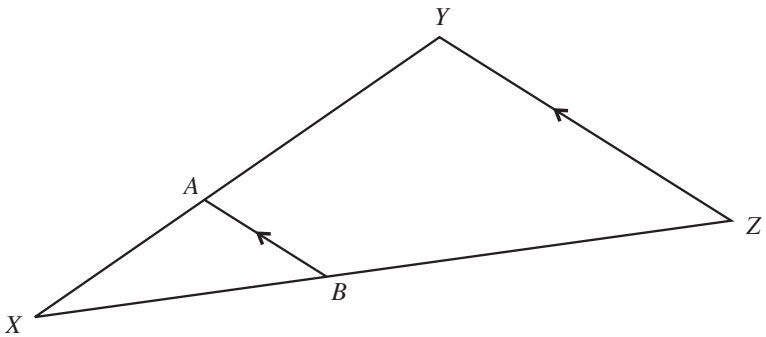
7 The diagram below shows the parabola $y = ax^2$ and the line $y = bx + c$.
Which of the following statements is true?



(A) $b^2 + 4ac < 0$ (B) $b^2 - 4ac < 0$

(C) $b^2 + 4ac > 0$ (D) $b^2 - 4ac > 0$

8



In the diagram above $\Delta XAB \,|||\, \Delta XYZ$, with $AB : YZ = 2 : 5$.
If the area of ΔXYZ is 200 cm^2, what is the area of ΔXAB?

(A) 32 (B) 80 (C) 120 (D) 168

9 What are the equations of the vertical and horizontal asymptotes of the graph whose equation is $y = \dfrac{2}{x-4} + 3$?

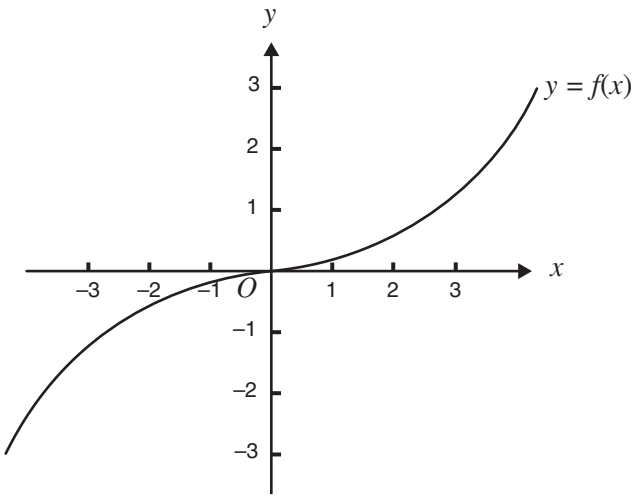
(A) $x = -4$, $y = -3$

(B) $x = 4$, $y = -3$

(C) $x = -4$, $y = 3$

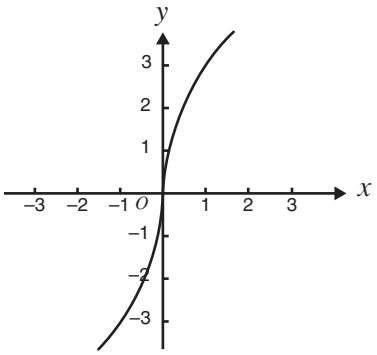
(D) $x = 4$, $y = 3$

10 The graph of $y = f(x)$ is drawn below.

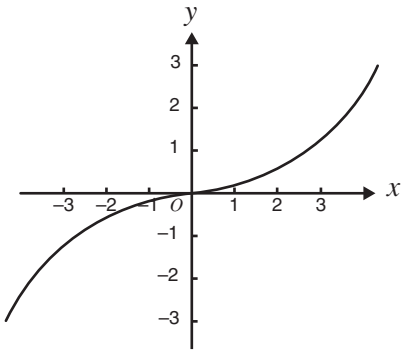


Which one of the following is most likely to be the graph of the inverse function?

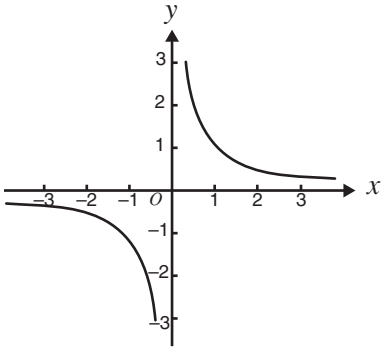
(A)



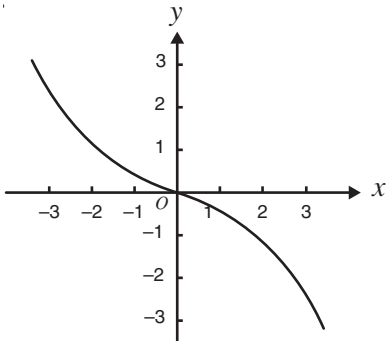
(B)



(C)



(D)



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Section B (15 marks)

(a) Write in expanded form: $(2a - 3)^2$ **1**

(b) Solve $3(x + 2)(2x + 1) = 0$ **2**

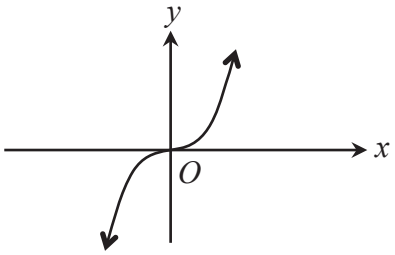
(c) Simplify $\sqrt{18} + \sqrt{32}$ **2**

(d) Factorise $2x^2 + 7x - 15$ **1**

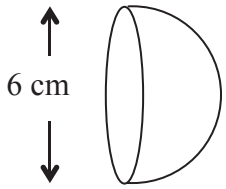
(e) Write $\frac{2}{\sqrt{3}}$ with a rational denominator **1**

(f) At a "25% off" sale, goods were sold for \$36. What was the price of the goods before the sale? **1**

(g) The graph of the polynomial $P(x) = x^3$ is illustrated. **2**
On the same axes, draw the graph of $y = P(x + 2)$. Indicate the intercepts.

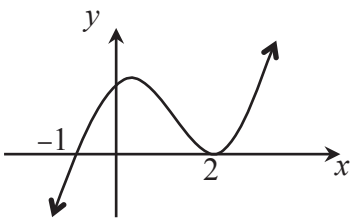


(h) Find the volume of this hemisphere, leaving your answer in terms of π. **1**



(i) If the probability of getting the measles as a teenager is 0.018, how many of 700 000 teenagers will *not* be expected to contract measles? **1**

(j) Write down in factored form, an equation which can be represented by this graph. **1**



(k) Solve $\log_2 x = 5$ **1**

(j) Solve $x = \log_3 \sqrt{3}$ **1**

Section C (15 marks)

(a) Solve the equation $3^{2-x} = 9^{x}$. **2**

(b) Human bones make up 18% of a person's total body weight. **1**
How many kilograms do the bones of a 75 kg person weigh?

(c) Find the y-intercept of the line with equation $3x + 4y = 24$. **1**

(d) If $\cos x = c$, write $\cos(180° + x)$ in terms of c. **1**

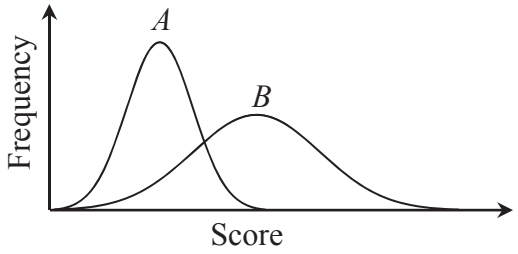
(e) Explain why $\sin 285° = \cos 195°$, without reference to a calculator. **2**

(g) Seymour buys a new telescope for \$4200. It depreciates in value by 10% in the 1st year then another 20% in the 2nd year. What is the telescope's value after 2 years? **2**

(h) By considering the differences, or otherwise, find how many terms there are in the following sequence. **2**

1.11, 1.12, 1.13, …, 9.98, 9.99?

(i) The graph shows the frequency curves for two sets of test results, A and B. **2**
Write a statement comparing the means and standard deviations of the sets of results.



(j) The mean of the heights of a large number of people is 155 cm and the standard deviation is 11.2 cm. A person is added to this group with a height of 170 cm. Explain what effect this will have on the standard deviation. **1**

(k) Circle the correct response.

For $x > y > 0$ let $A = \sqrt{x} - \sqrt{y}$ and $B = \sqrt{x - 2\sqrt{xy} + y}$

(A) $A > B$ (B) $A < B$ (C) $A = B$ (D) Cannot be determined.

Section D (15 marks)

(a) Find the quotient and remainder when $P(x) = 2x^2 + 3x - 4$ is divided by $x + 1$. **3**

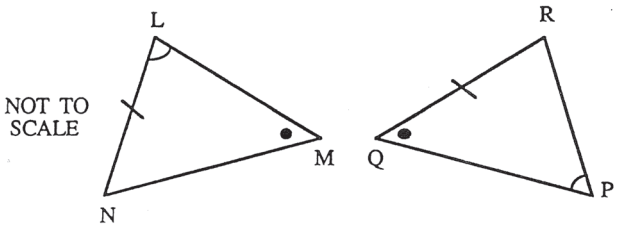
(b) (i) Draw invests \$320 for four years at a rate of 12% p.a., where the interest is compounded monthly. How much money does she have after 4 years? Give your answer correct to the nearest cent. **2**

(ii) How long will it take for her investment to triple? Give your answer to the nearest year. **2**

(c) For the diagram below, consider the following statements **1**

I $\Delta LMN \equiv \Delta PQR$

II $\Delta LMN \,|||\, \Delta PQR$



Which of the statements are always true?

(A) I only

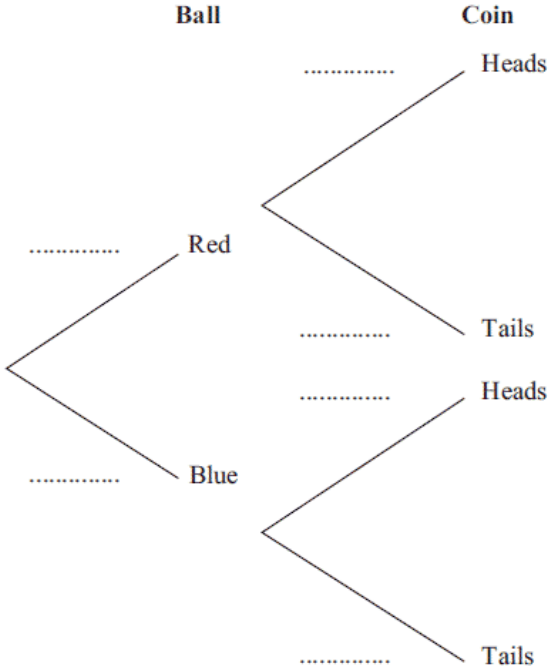
(B) II only

(C) Both I and II

(D) Neither I nor II

(d) Nahc conducts an experiment consists of selecting a ball from a bag and spinning a coin.
The bag contains five red balls and seven blue balls. A ball is selected at random from the bag, its colour is noted and then the ball is returned to the bag.
When a red ball is selected, a biased coin with probability $\frac{2}{3}$ of landing heads is spun.
When a blue ball is selected a fair coin is spun.

(i) Complete the probability tree diagram below **2**



(ii) Lledwod selects a ball and spins the appropriate coin. **2**
Find the probability that he obtains a head.

(iii) Now Llewop has selected a ball at random and obtained a head when she spun the appropriate coin. Find the probability that Llewop selected a red ball. **2**

(e) Circle the correct response. **1**

A sphere and a closed cylinder have the same radius. The height of the cylinder is four times the radius. What is the ratio of the volume of the cylinder to the volume of the sphere?

(A) 2 : 1 (B) 3 : 1 (C) 4 : 1 (D) 8 : 1

Section E (15 marks)

(a) The times, in seconds, taken by 20 people to solve a simple numerical puzzle were

17	19	22	26	28
31	34	36	38	39
41	42	43	47	50
51	53	55	57	58

(i) Calculate the mean and the standard deviation of these times correct to 2 dp. **2**

(ii) In fact, 23 people solved the puzzle. However, three of them failed to solve it within the allotted time of 60 seconds. **4**

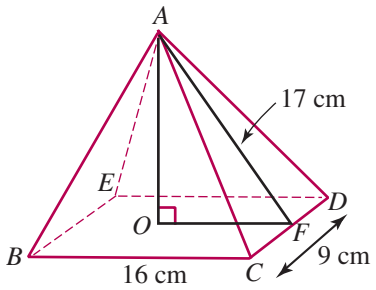
Calculate the median and the interquartile range of the times taken by all 23 people.

(iii) For the times taken by all 23 people, explain why:

(α) the mode is not an appropriate measure of central tendency; **1**

(β) the range is not an appropriate measure of spread. **1**

(b) In the right rectangular pyramid shown, $BC = 16$ cm, $CD = 9$ cm and $AF = 17$ cm.

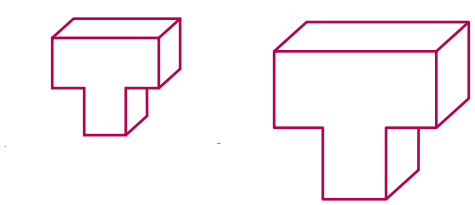


NOT TO SCALE

Find the volume of the pyramid. **2**

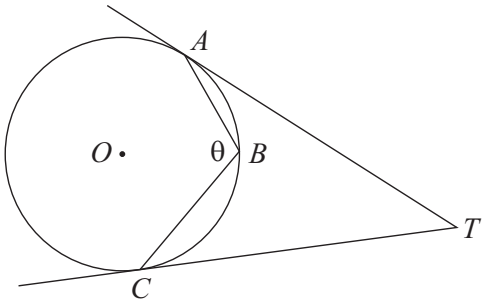
(c) A rectangle's perimeter is 16 and the length of a diagonal is 6. **3**
What is the area of the rectangle?

(d) The two similar solids below have volumes in the ratio 512 : 729. **2**
What is the ratio of the corresponding surface areas?



Section F (15 marks)

(a) The diagram shows a circle through A, B and C, with centre O. **3**
Tangents at A and C intersect at T, and $\angle ABC = \theta$.
What is the size of $\angle ATC$ in terms of θ? Give geometric reasons for your answer.



(b) Find the value of a if $x^3 + ax^2 + ax + 5$ gives the same remainder when it is divided by $x + 2$ or $x - 4$. **2**

(c) Given that $\log_m 2 = a$ and $\log_m 3 = b$, express $\log_m \frac{72}{m}$ in terms of a and b. **2**

(d) Consider the quadrilateral $ABCD$.

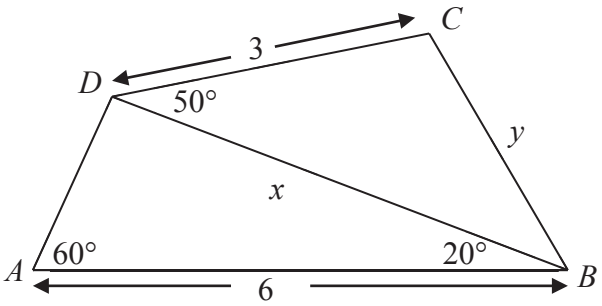


Figure not to scale

Lengths are in metres

Find correct to 2 significant figures:

(i) x **2**

(ii) y **2**

(e) (i) Find the radius and the coordinates of the centre of the circle with equation **2**

$$x^2 + y^2 + 4x - 8y - 5 = 0$$

(ii) The point $A(2, 1)$ lies on the circle described in part (i). **2**
Find the equation of the tangent to the circle at point A.

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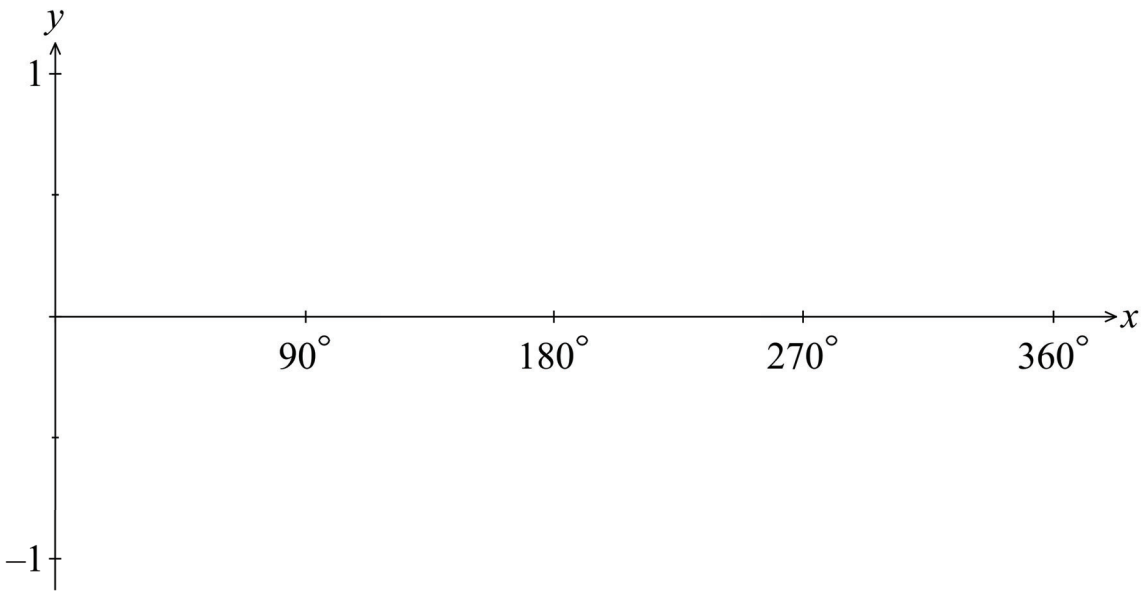
Section G (15 marks)

(a) (i) Sketch $y = \sin x$ for $0° \leq x \leq 360°$. **2**

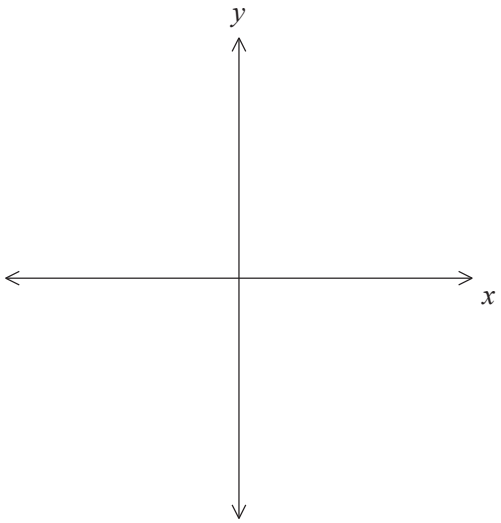


(ii) Hence, or otherwise, find out how many solutions there are to the equation **2**

$$\sin x = \frac{1}{2} - \frac{1}{180}x .$$

(iii) What is the domain of $y = \sqrt{1 - \sin x}$? **1**

(b) Sketch the graph of $y - 3 = 3^{x-3}$, including asymptotes and intercepts where possible. **2**

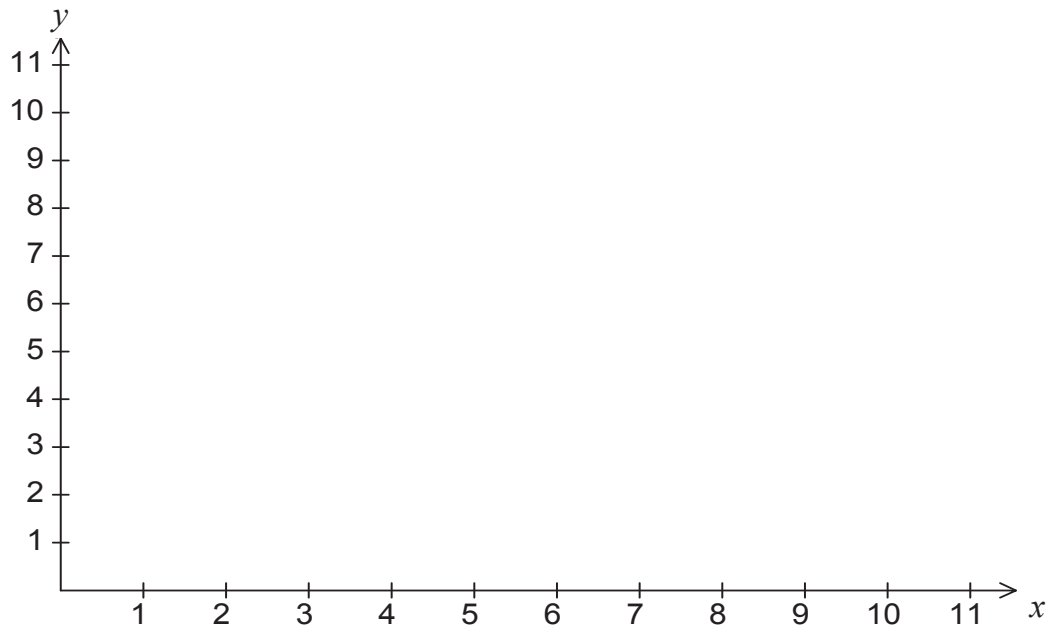


(c) (i) Show that $4x^3 - 21x + 10$ is divisible by $x - 2$. **1**

(ii) Hence solve $4x^3 - 21x + 10 = 0$ **3**

(d) Yak is a volleyball coach. He needs to take a squad of players to a tournament. **4**
He has two types of players in his squad, setters and spikers. The squad must contain at least 1 but no more than 3 setters and at least 4 spikers.
The squad must contain a total of at least 8 players, but no more than 10 players.
Let x be the number of setters in the squad and let y be the number of spikers in the squad.

By writing the constraints above as inequalities, indicate the region in the number plane that represents all of the constraints above.



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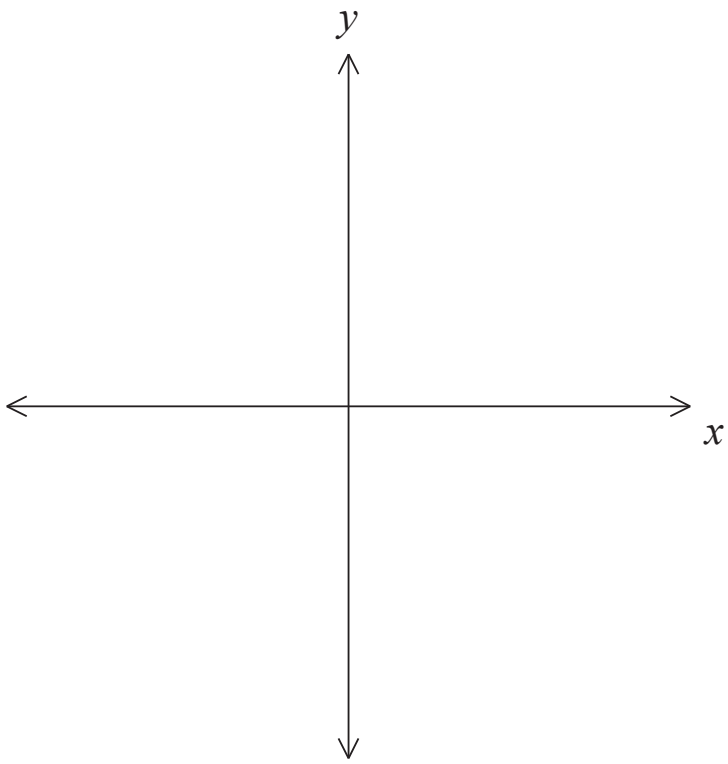
Section H (16 marks)

(a) Let $g(x) = x^2 - 9$ and let $f(x) = g(x)$ for $x \leq 0$.

(i) Draw a neat sketch of the function $y = f(x)$ clearly showing any intercepts **1**

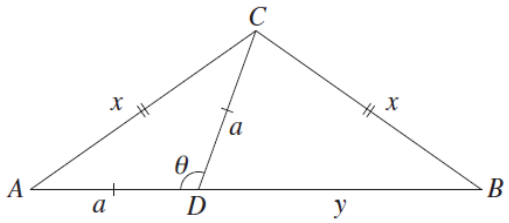


(ii) On the same diagram, sketch the graph of the inverse function $y = f^{-1}(x)$. **1**

(iii) What is the domain of the inverse function $y = f^{-1}(x)$. **1**

(iv) Evaluate $f^{-1}\big(g(2)\big)$, without finding a rule for $f^{-1}(x)$. **1**

(b) In the diagram ABC is an isosceles triangle with $AC = BC = x$. The point D on the interval AB is chosen so that $AD = CD$. Let $AD = a$, $DB = y$ and $\angle ADC = \theta$.



(i) If $\Delta ABC \,|||\, \Delta ACD$, show that $x^2 = a^2 + ay$. **2**

(ii) Show that $y = a(1 - 2\cos\theta)$. **2**

(iii) Deduce that $y \leq 3a$ **1**

(c) Solve $4 \times 3^{x-2} = 5 \times 2^{x+1}$. Express your answer correct to 2 decimal places. **2**

(d) Let $x = \frac{p}{q}$, where p and q are integers having no common divisors other than ± 1.

(i) Suppose that x is a root of the equation $ax^3 - 3x + b = 0$, where a and b are integers. Explain why p divides b and why q divides a. **3**

(ii) Hence deduce that $x^3 - 3x - 1 = 0$ has no rational root. **2**

End of paper

Sydney Boys High School

MOORE PARK, SURRY HILLS

YEAR 10 ADVANCED MATHEMATICS

Yearly Examination 2018

SAMPLE SOLUTIONS

MC Quick Answers

1. C
2. A
3. B
4. D
5. D
6. B
7. A
8. A
9. D
10. A

Section A Multiple Choice Solutions

1 Which of the following is the gradient of the line $y = 3x + 7$

(A) -7 (B) -3 **(C) 3** (D) 5

$y = mx + b$, where m = gradient

2 The angle of elevation from point A to the top of a lighthouse is 30°.
What is the angle of depression from the top of the lighthouse to point A?

(A) 30° (B) 60° (C) 120° (D) 150°

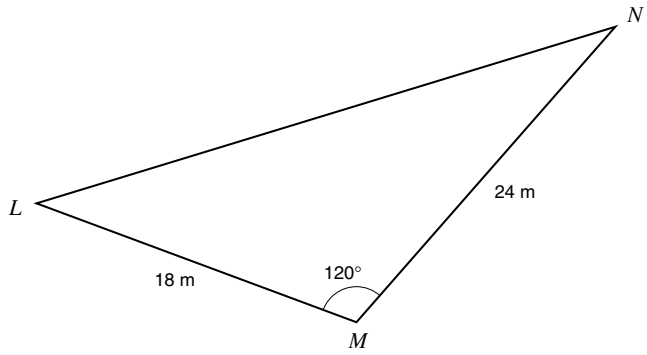
Angle of elevation is numerically equal to the angle of depression – alternate angles on parallel lines.

3 The circle with radius 6 and centre (–3, 4) is shifted two units to the right and three units up.
Which of the following is the equation of the shifted circle?

(A) $(x-1)^2 + (y-7)^2 = 36$ **(B)** $(x+1)^2 + (y-7)^2 = 36$
(C) $(x+5)^2 + (y+1)^2 = 36$ (D) $(x+1)^2 + (y-1)^2 = 36$

The new centre is $(-3+2, 4+3) = (-1, 7)$

4 In ΔLMN, what is the length of LN to the nearest tenth of a metre?



$$LN^2 = 18^2 + 24^2 - 2\times 18\times 24\times \cos 120°$$
$$= 1332$$
$$LN = 36.5 \text{ (1 dp)}$$

(A) 14.0 m (B) 21.6 m (C) 30.0 m **(D) 36.5 m**

5 What is the exact value of $\cos 210°$?

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) $\frac{\sqrt{3}}{2}$ **(D)** $-\frac{\sqrt{3}}{2}$

$$\cos 210° = \cos(180° + 30°)$$
$$= -\cos 30°$$
$$= -\frac{\sqrt{3}}{2}$$

6 Which of the following statements is NOT true?

(A) $\log_3 15 - \log_3 5 = 1$

(B) $\log_4 2 + \log_4 8 = 1$

(C) $\log_5 \frac{1}{5} = -1$

(D) $\frac{\log_2 8}{\log_2 4} = \frac{3}{2}$

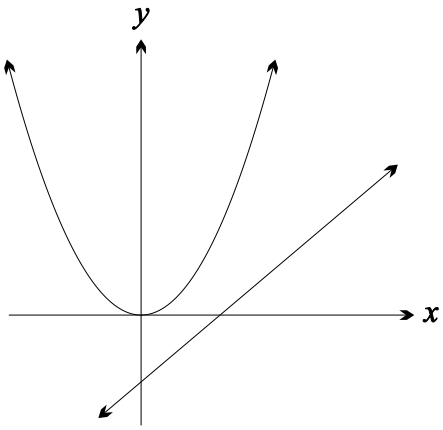
$\log_3 15 - \log_3 5 = \log_3 \frac{15}{5} = \log_3 3 = 1$

$\log_4 2 + \log_4 8 = \log_4 (2 \times 8) = \log_4 16 = 2$

$\log_5 \frac{1}{5} = \log_5 5^{-1} = -\log_5 5 = -1$

$\frac{\log_2 8}{\log_2 4} = \frac{\log_2 2^3}{\log_2 2^2} = \frac{3\log_2 2}{3\log_2 2} = \frac{3}{2}$

7 The diagram below shows the parabola $y = ax^2$ and the line $y = bx + c$.
Which of the following statements is true?



Solve simultaneously

$ax^2 = bx + c$

$\therefore ax^2 - bx - c = 0$

$\therefore \Delta = (-b)^2 - 4 \times a \times (-c) = b^2 + 4ac$

For no points of intersection $\Delta < 0$

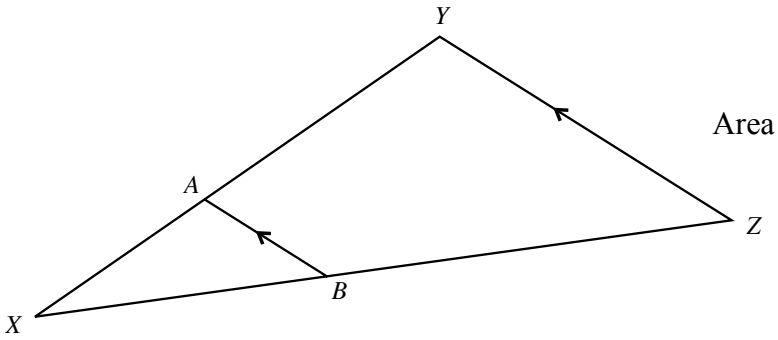
(A) $b^2 + 4ac < 0$

(B) $b^2 - 4ac < 0$

(C) $b^2 + 4ac > 0$

(D) $b^2 - 4ac > 0$

8



In the diagram above $\Delta XAB \,|||\, \Delta XYZ$, with $AB : YZ = 2 : 5$.
If the area of ΔXYZ is 200 cm^2, what is the area of ΔXAB?

Area ΔXAB : area $\Delta XYZ = 2^2 : 5^2$

$= 4 : 25$

Area $\Delta XAB = \frac{4}{25} \times 200 = 32$

(A) 32 (B) 80 (C) 120 (D) 168

9 What are the equations of the vertical and horizontal asymptotes of the graph whose equation is $y = \dfrac{2}{x-4} + 3$?

The domain is $x \neq 4$ and the range is $y \neq 3$

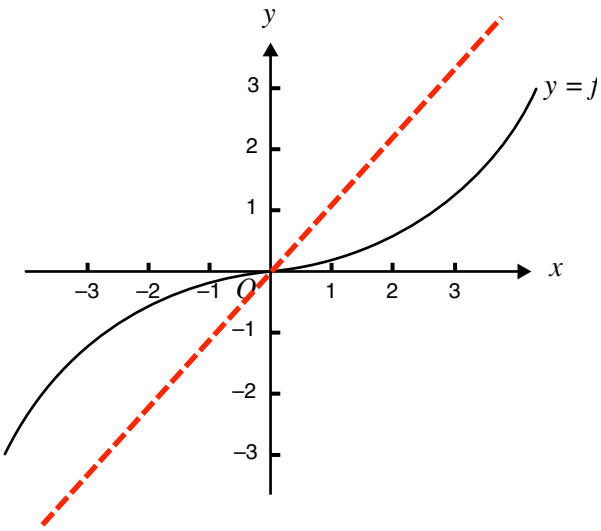
(A) $x = -4$, $y = -3$

(B) $x = 4$, $y = -3$

(C) $x = -4$, $y = 3$

(D) $x = 4$, $y = 3$

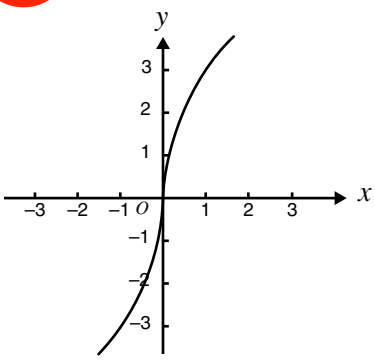
10 The graph of $y = f(x)$ is drawn below.



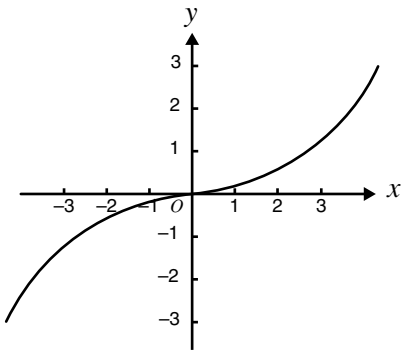
The inverse of a function is a reflection of the function in the line $y = x$.

Which one of the following is most likely to be the graph of the inverse function?

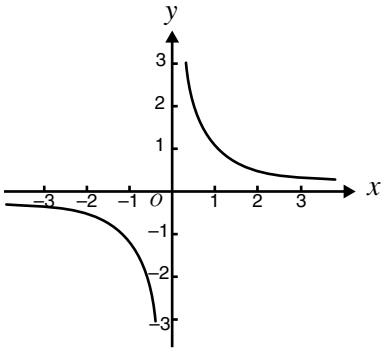
(A)



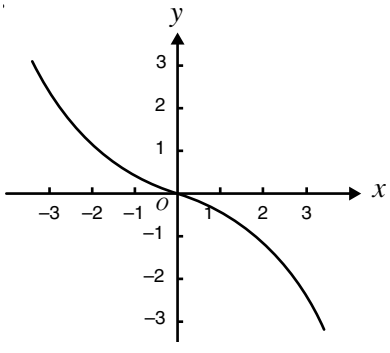
(B)



(C)



(D)



Section B (15 marks)

(a) Write in expanded form: $(2a-3)^2$ **1**

$4a^2 - 12a + 9$

(b) Solve $3(x+2)(2x+1)=0$ **2**

$x+2=0$ or $2x+1=0$

$x=-2$ $\quad x=-\frac{1}{2}$

(c) Simplify $\sqrt{18}+\sqrt{32}$ **2**

$= \sqrt{9\times 2} + \sqrt{16\times 2}$

$= 3\sqrt{2} + 4\sqrt{2}$

$= 7\sqrt{2}$

(d) Factorise $2x^2+7x-15$ **1**

$\times \left|\begin{matrix}-30\\ +7\end{matrix}\right.$

$= 2x^2 + 10x - 3x - 15$

$= 2x(x+5) - 3(x+5)$

$= (x+5)(2x-3)$

(e) Write $\frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ with a rational denominator **1**

$= \frac{2\sqrt{3}}{3}$

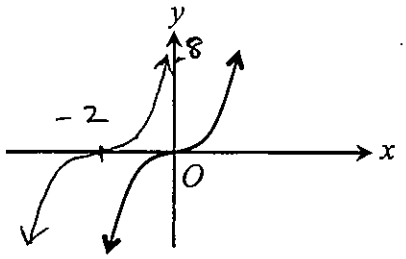
(f) At a "25% off" sale, goods were sold for \$36. What was the price of the goods before the sale? **1**

0.75 is \$36

0.25 is \$12

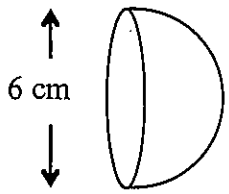
1.00 is \$48

(g) The graph of the polynomial $P(x) = x^3$ is illustrated. **2**
On the same axes, draw the graph of $y = P(x + 2)$. Indicate the intercepts.



*Students needed to show the correct shape [shifted horizontally 2 units to the left] & the x,y intercepts to gain full marks.

(h) Find the volume of this hemisphere, leaving your answer in terms of π. **1**

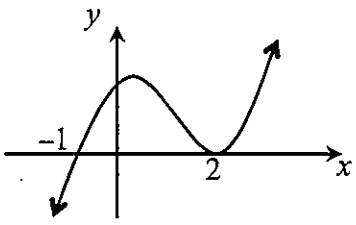


$V = \frac{1}{2} \times \frac{4}{3} \pi r^3$
$= \frac{2}{3} \pi (3)^3$
$= 18\pi \text{ cm}^3$

(i) If the probability of getting the measles as a teenager is 0.018, how many of 700 000 teenagers will *not* be expected to contract measles? **1**

$(1 - 0.018)\,700\,000$
$= 687400$

(j) Write down in factored form, an equation which can be represented by this graph. **1**



$y = a(x+1)(x-2)^2$ where $a > 0$

(k) Solve $\log_2 x = 5$ **1**

$x = 2^5$
$= 32$

(j) Solve $x = \log_3 \sqrt{3}$ **1**

$x = \log_3 3^{\frac{1}{2}}$
$= \frac{1}{2} \log_3 3$
$= \frac{1}{2}$

Section C

No Half Marks

(a) $3^{2-x} = 9^x$

$3^{2-x} = (3^2)^x$

$3^{2-x} = 3^{2x}$

$2 - x = 2x$

$2 = 3x$

$x = \frac{2}{3}$ [2]

(b) $75\text{ kg} \times \frac{18}{100} = 13.5\text{ kg}$ [1]

(c) y-int: $(x = 0)$ $3(0) + 4y = 24$

$4y = 24$

$y = 6$

$\therefore (0, 6)$ [1]

(d) $\cos x = c$, $\cos(180° + x) = -\cos x$

$= -c$ [1]

(e) $\sin 285° = \cos 195°$

S	A
T	C

$\sin 285° = \sin(360 - 75)°$

$= -\sin 75°$

$\cos 195 = \cos(180 + 15)$

$= -\cos 15$

Since $\sin\theta = \cos(90 - \theta)$

$-\sin 75 = -\cos(90 - 75)$

$= -\cos 15$

$\therefore \sin 285° = \cos 195°$ [2]

(g) $\$4200 \times (1-0.1) \times (1-0.2)$
$= 4200 \times 0.9 \times 0.8$
$= \$3024$ [2]

(h) 1.11, 1.12, 1.13, ..., 9.98, 9.99

difference $= 1.12 - 1.11$
$= 0.01$

$\therefore \left(\frac{9.99 - 1.11}{0.01}\right) + 1 = 889$ [2]

(i) mean A < mean B
standard deviation A < standard deviation B
[2]

(j) Standard deviation will increase slightly, [1]
as the score is larger than 1
standard deviation away from the mean

(k) A = B [1]

Section D (15 marks) **SOLUTIONS**

(a) Find the quotient and remainder when $P(x) = 2x^2 + 3x - 4$ is divided by $x + 1$. **3**

$$x+1 \overline{)2x^2+3x-4}$$

$2x + 1$

$2x^2 + 2x$

$x - 4$

$x + 1$

-5

Quotient $= 2x+1$

Remainder $= -5$

Generally well done

Silly mistakes such as :

→ $-4-+1 = -3$

→ ÷ by $x-1$

(b) (i) Draw invests \$320 for four years at a rate of 12% p.a., where the interest is compounded monthly. How much money does she have after 4 years? Give your answer correct to the nearest cent. **2**

$A = \$320(1+0.01)^{48}$

$= \$515.91$

AW for not compounding monthly

AW ½ for 9 or 10 yrs NO workin

(ii) How long will it take for her investment to triple? Give your answer to the nearest year.

AW1 for 9 or 10 years w/ working. Trial + error need to show A at 9 and 10 years.

$\$960 = 320(1+0.01)^n$

$3 = 1.01^n$

$n = \ln 3 / \ln 1.01$

$n = 110.41$ mths.

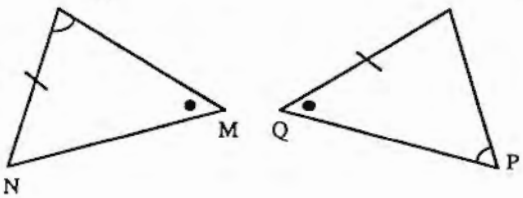
∴ It will take 10 years to triple.

AW 1½ for $\ln 3/\ln 1.01 = 9$yrs.

(c) For the diagram below, consider the following statements **1**

I $\Delta LMN \equiv \Delta PQR$ ✗ matching lengths & Ls not equal

II $\Delta LMN \,|||\, \Delta PQR$ ✓ equiangular



Which of the statements are always true?

(A) I only (B) II only

(C) Both I and II (D) Neither I nor II

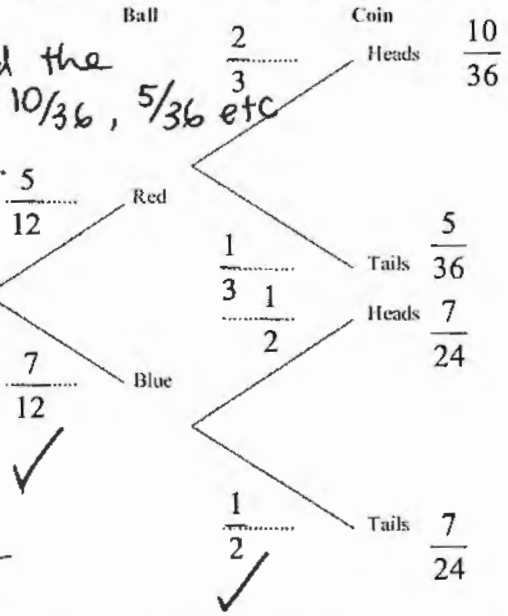
(d) Nahc conducts an experiment consists of selecting a ball from a bag and spinning a coin. The bag contains five red balls and seven blue balls. A ball is selected at random from the bag, its colour is noted and then the ball is returned to the bag.

When a red ball is selected, a biased coin with probability $\frac{2}{3}$ of landing heads is spun.

When a blue ball is selected a fair coin is spun.

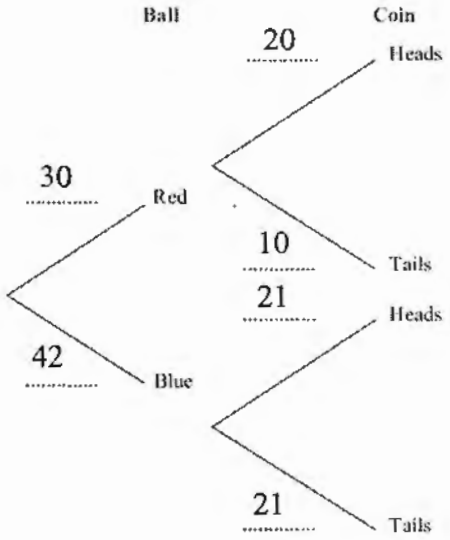
(i) Complete the probability tree diagram below **2**

A no. of Students had the probabilities 10/36, 5/36 etc for the coin.



1 mark for the ball

1 mark for the coin ✓



(ii) Lledwod selects a ball and spins the appropriate coin. **2**
Find the probability that he obtains a head.

$\frac{5}{12} \times \frac{2}{3} + \frac{7}{12} \times \frac{1}{2}$ ✓ 1mark

$= 41/72$ ✓ 1mark.

(iii) Now Llewop has selected a ball at random and obtained a head when she spun the appropriate coin. Find tbe probability that Llewop selected a red ball. **2**

red ball & head $5/18$ ✓
Out of total prob of obtaining a head. $= 5/18 \div 4/72$
$= 20/41$ ✓

Alternative red ball & head = 20
Total outcome for heads
$= 20+21 = 41$
$\therefore 20/41$

(e) Circle the correct response. **1**

A sphere and a closed cylinder have the same radius. The height of the cylinder is four times the radius. What is the ratio of the volume of the cylinder to the volume of the sphere?

(A) 2 : 1 (B) 3 : 1 (C) 4 : 1 (D) 8 : 1

If in doubt, substitute values for r and h eg r = 3 h = 12 and calculate volume.

Section E (15 marks)

(a) The times, in seconds, taken by 20 people to solve a simple numerical puzzle were

17	19	22	26	28
31	34	36	38	39
41	42	43	47	50
51	53	55	57	58

(i) Calculate the mean and the standard deviation of these times correct to 2 dp. **2**

$\bar{x} = 39.35$ $\sigma = 12.36$

(ii) In fact, 23 people solved the puzzle. However, three of them failed to solve it within the allotted time of 60 seconds. **4**

Calculate the median and the interquartile range of the times taken by all 23 people.

Median is the 12th score = 42

$Q_3 = 55$

$Q_1 = 31$

$IQR = 55 - 31 = 24$

(iii) For the times taken by all 23 people, explain why:

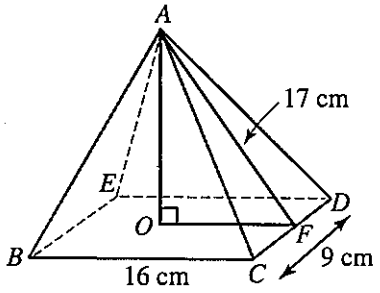
(α) the mode is not an appropriate measure of central tendency; **1**

All scores have equal frequency (no mode)

(β) the range is not an appropriate measure of **1**

Range is not known (3 unknowns).

(b) In the right rectangular pyramid shown, $BC = 16$ cm, $CD = 9$ cm and $AF = 17$ cm.

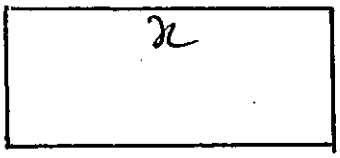


NOT TO SCALE

Find the volume of the pyramid. **2**

- Triad : 8, 15, 17 ✓
- $V = \frac{1}{3} A h$
  $= \frac{16 \times 9}{3} \times 15$
  $= 720 \text{ cm}^2$ ✓

(c) A rectangle's perimeter is 16 and the length of a diagonal is 6. **3**
What is the area of the triangle?

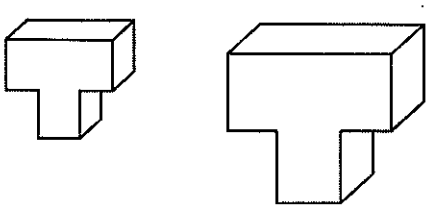


$A = x(8-x)$
$= 8x - x^2$ ✓ — (1)

Now, $x^2 + (8-x)^2 = 36$.
$\therefore x^2 - 8x = -14$ ✓
$\Rightarrow 8x - x^2 = 14$ — (2)

i.e. Area = 14 sq units. ✓

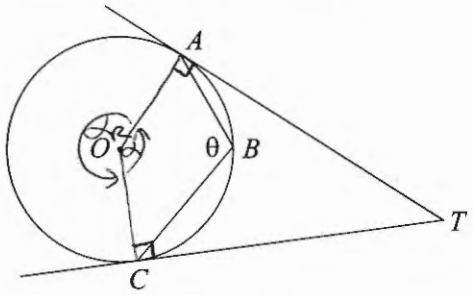
(d) The two similar solids below have volumes in the ratio 512 : 729. **2**
What is the ratio of the corresponding surface areas?



$\left(\frac{l_1}{l_2}\right)^2 = \left(\frac{512}{729}\right)^{2/3} = \frac{(2^9)^{2/3}}{(3^6)^{2/3}}$ ✓
$= \frac{2^6}{3^4} = \frac{64}{81}$ ✓
$\doteq 0.79$

Section F (15 marks)

(a) The diagram shows a circle through A, B and C, with centre O. 3
Tangents at A and C intersect at T, and $\angle ABC = \theta$.
What is the size of $\angle ATC$ in terms of θ? Give geometric reasons for your answer.



OA and OC are radii of circle.
$\angle OAT = \angle OCT = 90°$ (Tangent perpendicular to radius)
Let $\angle AOC$ be α and reflex angle $\angle AOC$ be α_R.
$\alpha_R = 2\theta$ (Angle subtended to centre is double angle subtended to circumference)
$\alpha + \alpha_R = 360°$ (Angle about a point)
$\therefore \alpha = 360° - 2\theta$
$\alpha + 90° + 90° + \angle ATC = 360°$ (Angle sum of quadrilateral)
$\therefore \angle ATC = 360° - (360° - 2\theta) - 90° - 90°$
$= 2\theta - 180°$

(b) Find the value of a if $x^3 + ax^2 + ax + 5$ gives the same remainder when it is divided by $x + 2$ or $x - 4$. 2

Let $P(x) = x^3 + ax^2 + ax + 5$
$P(-2) = (-2)^3 + (-2)^2 \times a + (-2) \times a + 5$
$= -8 + 4a - 2a + 5$
$= 2a - 3$
$P(4) = 4^3 + 4^2 \times a + 4a + 5$
$= 64 + 16a + 4a + 5$
$= 20a + 69$
$P(-2) = P(4)$
$\therefore 2a - 3 = 20a + 69$
$-18a = 72$
$a = -4$

2018 YR10 Yearly Exam Section F Marking Scheme and Feedback

Overview:

- Bottom corner is total marks of each page.
- **NO** remarking of pencil solutions.
- CTE: Carry-through error.
- WO: Working out.

Part A:

Marking scheme	Comments
(1): Tangents perpendicular to radius. (1): Angle subtended by arc to centre is double of angle subtended to circumference. (1): General knowledge of geometry and correct expression for $\angle ATC$. Alternate solution 2: 1. Produce tangents TA and TC to points R and S respectively. 2. Construct chord AC. 3. Derive expressions for $\angle ACS$ and $\angle CAR$ via angle in alternate segment ABC. 4. Derive expressions for $\angle ACT$ and $\angle CAT$ via angle on straight line. 5. Derive expression for $\angle ATC$ via angle sum of triangle ATC.	Key skills to demonstrate: • Tangent perpendicular to radius. • Angle subtended by arc to centre is double of angle subtended to circumference. • Geometry knowledge from junior years. Longer alternate solutions exist, but not highlighted. Common errors: • Mistaking $\angle OAT$ as $\angle OAB$ or $\angle CAT$ and similarly for $\angle OCT$. • Mistaking internal angle $\angle AOC$ as reflex angle $\angle AOC$. • Incorrectly dividing $360 - 2\theta$ by 2. • Constructing circle with radius BT and centre at B and T to incorrectly deduce $\angle ATC = \frac{\theta}{2}$ or $\angle ATC = 2\theta$ respectively. • Attempting to evaluate θ.

Part B:

Marking scheme	Comments
(1): Correct expression for $P(-2)$ and $P(4)$. (1): Indication of $P(-2) = P(4)$ and correct value of a.	Key skills to demonstrate: • Remainder theorem. • Same remainder implies $P(-2) = P(4)$. Solution via long division doable, but not for the faint of heart. Common errors: • Incorrectly equating $P(-2)$ or $P(4)$ to 0.

Section F (continued)

(c) Given that $\log_m 2 = a$ and $\log_m 3 = b$, express $\log_m \frac{72}{m}$ in terms of a and b. **2**

$\log_m\left(\frac{72}{m}\right) = \log_m 72 - \log_m m$

$= \log_m (2^3 \times 3^2) - 1$

$= \log_m (2^3) + \log_m (3^2) - 1$

$= 3\log_m 2 + 2\log_m 3 - 1$.

$= 3a + 2b - 1$

(d) Consider the quadrilateral $ABCD$.

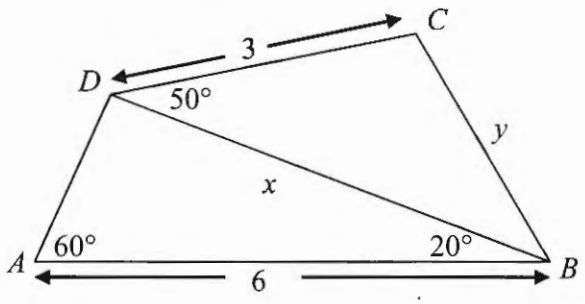


Figure not to scale

Lengths are in metres

Find correct to 2 significant figures:

(i) x **2**

$\angle ADB = 180 - 60 - 20$

$= 100°$

$\frac{x}{\sin 60°} = \frac{6}{\sin 100°}$

$x = \frac{6 \sin 60°}{\sin 100°}$

$= 5.3\text{m}$ (2 s.f.)

(ii) y **2**

$y^2 = 3^2 + 5.3^2 - 2 \times 3 \times 5.3 \times \cos 50°$

$y = 4.1\text{m}$ (2 s.f.)

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- WO: Working out.

Part C:

Marking scheme	Comments
(0.5) per correctly applied log identity.	Key skills to demonstrate: • Log identities: 4 needed to solve question. • Eliminating m from expression. Common errors: • Not realising that $\log_m \frac{1}{m} = -1$. • Not realising that $72 = 2^3 \times 3^2$. • Mistaking $(\log_m 2)^3 = \log_m(2^3)$ and vice versa. • Various other misuses of log identities.

Part D:

Marking scheme	Comments
Subpart I: (1): Correct value of $\angle ADB$. (1): Correct expression for sine rule and value of x. Subpart II: (1): Correct expression for cosine rule. (1): Correct value of y. 0.5 marks deducted for first instance of incorrect rounding in either subpart.	Key skills to demonstrate: • Sine and cosine rule. Common errors: • Forgetting that cosine rule uses y^2, not y. • Incorrect rounding via significant figures. • Attempting to deduce all angles.

(e) (i) Find the radius and the coordinates of the centre of the circle with equation **2**

$$x^2 + y^2 + 4x - 8y - 5 = 0$$

$x^2 + y^2 + 4x - 8y - 5 = 0$

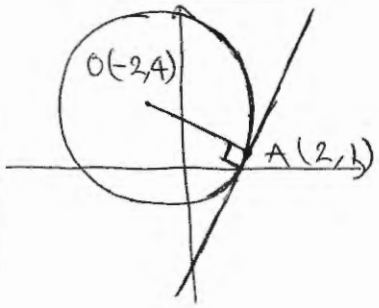
$(x^2 + 4x + 4) + (y^2 - 8y + 16) = 5 + 4 + 16$.

$(x + 2)^2 + (y - 4)^2 = 25$

Centre : $(-2, 4)$

Radius : 5 u.

(ii) The point $A(2, 1)$ lies on the circle described in part (i). **2**
Find the equation of the tangent to the circle at point A.



Tangent perpendicular to radius.

$\therefore m_{OA} \times m_A = -1$.

$m_{OA} = \frac{1-4}{2--2}$

$= \frac{-3}{4}$

$\therefore m_A = \frac{4}{3}$

$\frac{4}{3} = \frac{y-1}{x-2}$.

$3y - 3 = 4x - 8$

$y = \frac{4x-5}{3}$

2018 YR10 Yearly Exam Section F Marking Scheme and Feedback

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Part E:

Marking scheme	Comments
Subpart I: (1): Correctly completing the square. (1): Correct radius and centre.	Key skills to demonstrate: • Completing the square. Those who stumbled upon $r = 5$ with little evidence of knowing how to complete the square scored at most 0.5 marks. Common errors: • Factorising $x^2 + 4x$ or $y^2 - 8y$. • Forgetting to adjust RHS while forming perfect squares on LHS. • Forgetting that radius is expressed as r^2 in circle equation.
Subpart II: (1): Indication of radius perpendicular to tangent. (1): General knowledge of coordinate geometry and correct equation of tangent.	Key skills to demonstrate: • Tangent perpendicular to radius. • Linear equation via coordinate geometry. Not sure why some substituted point A into circle equation when question states that it lies on circle. Common errors: • Poor sketches of circles and lines leading to misconceptions about tangent equation.

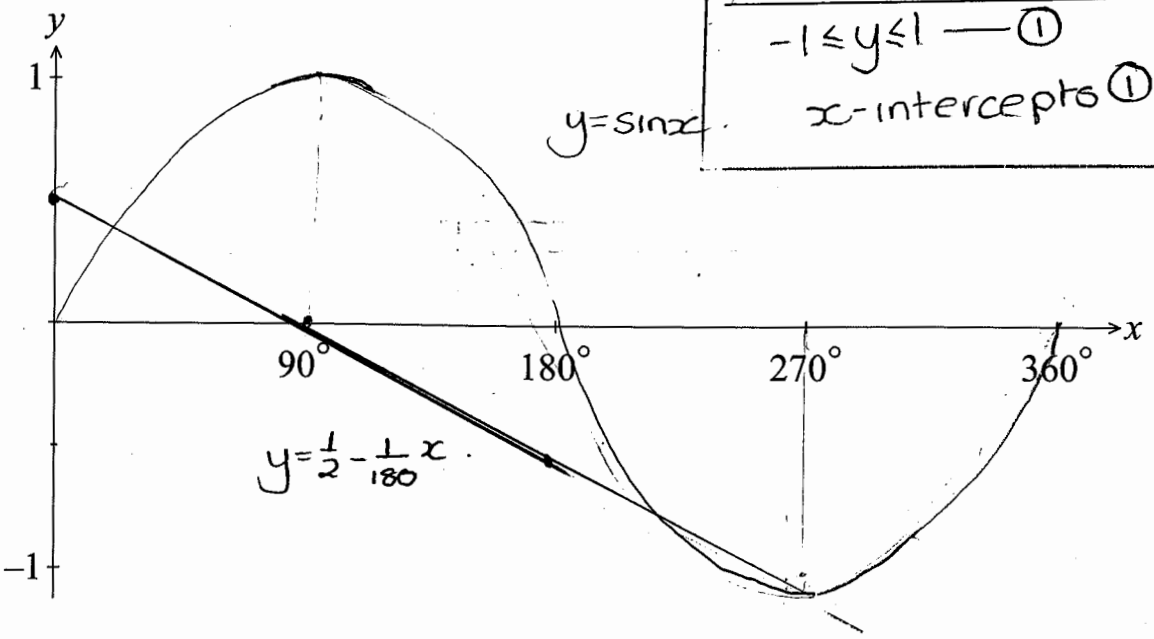
Section G (15 marks) NO HALF MARKS SECTION (a)

(a) (i) Sketch $y = \sin x$ for $0° \le x \le 360°$.

VERY WELL DONE 2

$-1 \le y \le 1$ — ①

x-intercepts ①



(ii) Hence, or otherwise, find out how many solutions there are to the equation **2**

$$\sin x = \frac{1}{2} - \frac{1}{180}x.$$

x	0	90	180	270	360
$\frac{1}{2} - \frac{x}{180}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-1\frac{1}{2}$

By inspection (3)

POORLY DONE

Correct graph ①

or Table of values ①

& Correct number of solutions ①

(iii) What is the domain of $y = \sqrt{1 - \sin x}$? **1**

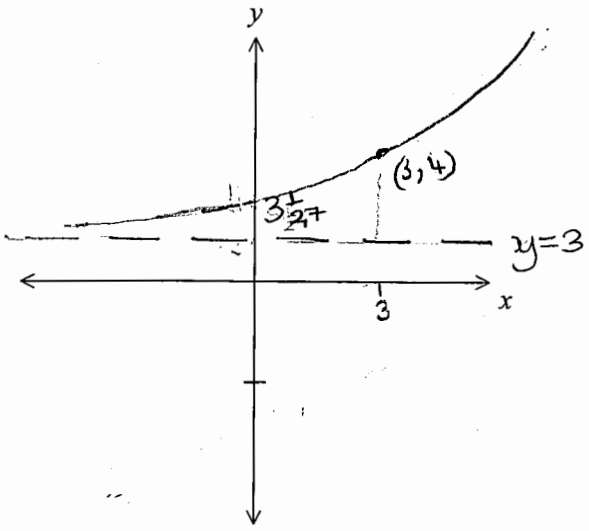
$x \in \mathbb{R}$ — ①

POORLY DONE

$0 \le x \le 360$ clearly stated ①

) Sketch the graph of $y-3=3^{x-3}$, including asymptotes and intercepts where possible. **2**

$y=3^{x-3}+3$



FAIRLY WELL DONE
no x intercept $(-\frac{1}{2})$
correct shape $(\frac{1}{2})$
no asymptote $(-\frac{1}{2})$

) (i) Show that $4x^3 - 21x + 10$ is divisible by $x - 2$. **1**

$P(2) = 4(2)^3 - 21(2) + 10$
$= 32 - 42 + 10$
$= 0$
Hence when divided by $(x-2)$ remainder is zero by factor thm $(x-2)$ is a factor and thus $4x^3-21x+10$ is divisble by $(x-2)$

POORLY DONE
No reference to remainder or the factor theorem $(-\frac{1}{2})$

(ii) Hence solve $4x^3 - 21x + 10 = 0$ **3**

$4x^2+8x-5$
$x-2 \overline{)\,4x^3+0x^2-21x+10}$
$4x^3-8x^2$
$8x^2-21x$
$8x^2-16x$
$-5x+10$
$-5x+10$

$P(x)=(x-2)(4x^2+8x-5)$
$=(x-2)(4x^2+10x-2x-5)$
$=(x-2)(2x(2x+5)-(2x+5))$
$=(x-2)(2x-1)(2x+5)$

Solutions are $x=2$ $x=\frac{1}{2}$ $x=-5/2$

REASONABLY WELL DONE
only factors (2)
only 2 solutions $(\frac{1}{2})$
correct to here $(\frac{1}{2})$

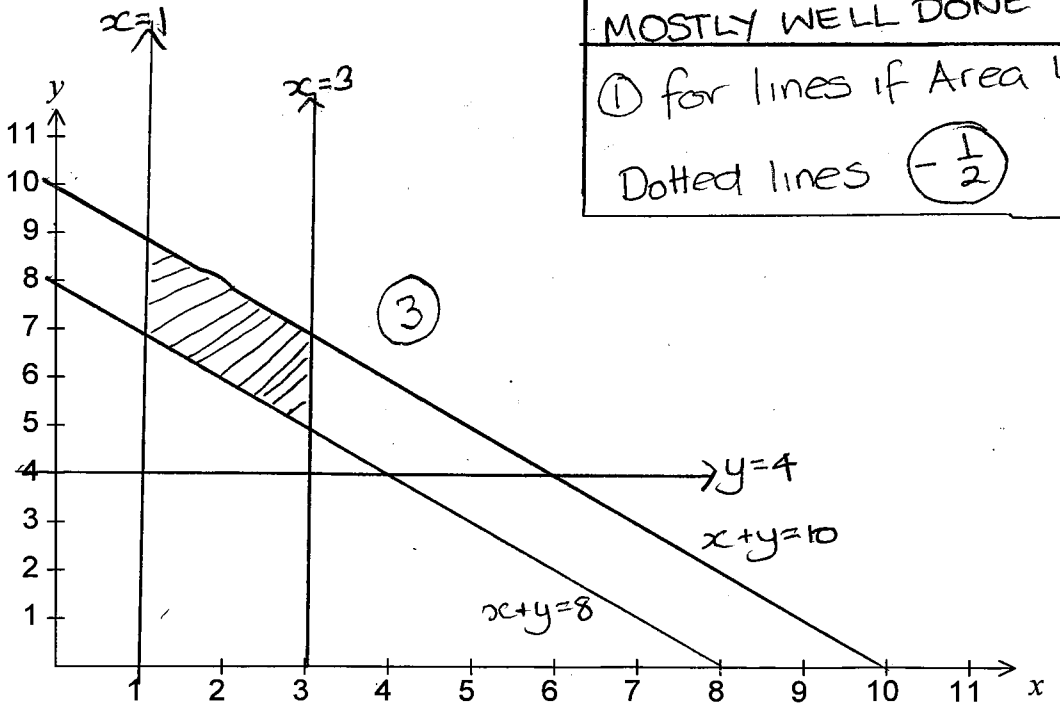
(d) Yak is a volleyball coach. He needs to take a squad of players to a tournament. **4**

He has two types of players in his squad, setters and spikers. The squad must contain at least 1 but no more than 3 setters and at least 4 spikers.
The squad must contain a total of at least 8 players, but no more than 10 players.
Let x be the number of setters in the squad and let y be the number of spikers in the squad.

By writing the constraints above as inequalities, indicate the region in the number plane that represents all of the constraints above.

$8 \leq x + y \leq 10$
$1 \leq x \leq 3$
$y \geqslant 4$ } ①

NOT WELL DONE
(-½) only 2 inequalities
(-½) < > symbols rather than ≤ ≥



③

MOSTLY WELL DONE
① for lines if Area Wrong
Dotted lines (-½)

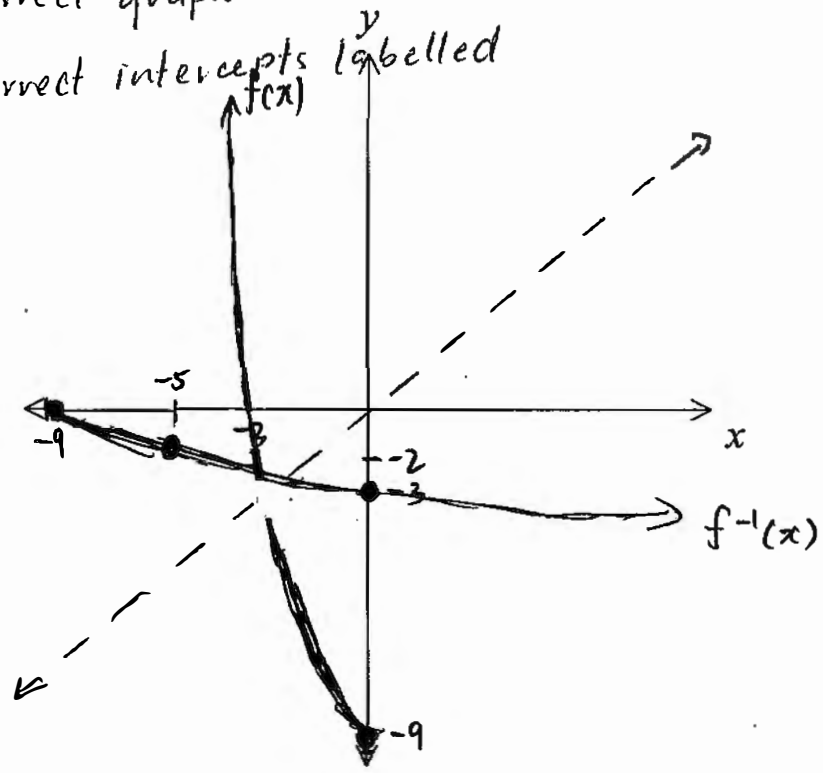
Section H (16 marks)

(a) Let $g(x) = x^2 - 9$ and let $f(x) = g(x)$ for $x \leq 0$.

(i) Draw a neat sketch of the function $y = f(x)$ clearly showing any intercepts **1**

0.5 correct graph

0.5 correct intercepts labelled



Marker's Comments:
Many students did not consider the domain *and* drew the whole parabola. Marks were deducted.

(ii) On the same diagram, sketch the graph of the inverse function $y = f^{-1}(x)$. **1**

0.5 mark for correct graph

0.5 mark for intercepts

(iii) What is the domain of the inverse function $y = f^{-1}(x)$. **1**

$x \geq -9$, $[-9, \infty)$

1 mark for correct domain

(iv) Evaluate $f^{-1}(g(2))$, without finding a rule for $f^{-1}(x)$. **1**

$g(2) = 4 - 9$

$= -5$

$f^{-1}(g(2)) = f^{-1}(-5)$

$= -2$

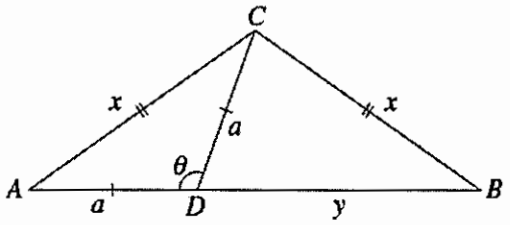
1 mark for correct answer.

Marker's comments:
Many students wrote 2 as part, or only answer and were penalized for it.
From the above graph, at x = -5 the solution is only valid at -2.

4

Section H (continued)

(b) In the diagram ABC is an isosceles triangle with $AC = BC = x$. The point D on the interval AB is chosen so that $AD = CD$. Let $AD = a$, $DB = y$ and $\angle ADC = \theta$.



Marker's comments:
Many students did not use similarity and therefore weren't successful in showing the statement is true. Students should note if the question state the two triangles are similar then there is no need (no marks given) in proving the triangles are similar.
Candidates need to give correct reasoning if they are using similarity.

(i) If $\Delta ABC \,|||\, \Delta ACD$, show that $x^2 = a^2 + ay$ **2**

$\frac{AB}{AC} = \frac{CB}{CD}$ (corresponding sides of similar triangles are in proportion) (1) mark for correct reasoning

$\frac{a+y}{x} = \frac{x}{a}$ (1) mark for ratio

$x^2 = a^2 + ay$

(ii) Show that $y = a(1 - 2\cos\theta)$ **2**

In ΔADC

$x^2 = a^2 + a^2 - 2a^2\cos\theta$ (Cosine Rule)

$x^2 = 2a^2 - 2a^2\cos\theta$ (1) mark

Sub in (i) [i.e $x^2 = a^2 + ay$]

$a^2 + ay = 2a^2 - 2a^2\cos\theta$ (1) mark.

$ay = a^2 - 2a^2\cos\theta$

$ay = a^2(1 - 2\cos\theta)$

$\therefore y = a(1 - 2\cos\theta)$

(iii) Deduce that $y \leq 3a$ **1**

Marker's comments
Poorly done by many students.

$-1 \leq \cos\theta \leq 1$

$2 \geq -2\cos\theta \geq -2$

$3 \geq -2\cos\theta + 1 \geq -1$ (0.5) mark

Since $a > 0$ as it is a length (0.5) mark.

$3a \geq a(-2\cos\theta + 1) \geq a$

$3a \geq y \geq a$

$\therefore y \leq 3a$.

5

Section H (continued)

(c) Solve $4\times3^{x-2}=5\times2^{x+1}$. Express your answer correct to 2 decimal places. **2**

Marker's comments
This was poorly done by many students. Students were more successful if they simplified the index as shown in the solution before using logarithm.

$\frac{4\times3^x}{3^2}=5\times2^x\times2$

$\frac{4}{9}\times3^x=10\times2^x$

$\frac{3^x}{2^x}=\frac{90}{4}$

$\left(\frac{3}{2}\right)^x=\frac{90}{4}$ (1 mark)

$x=\log_{\frac{3}{2}}\frac{90}{4}$

$=\frac{\log_{10}\left(\frac{90}{4}\right)}{\log_{10}\left(\frac{3}{2}\right)}$

$\simeq 7.68$ (2 d.p.) (1 mark)

(d) Let $x=\frac{p}{q}$, where p and q are integers having no common divisors other than ± 1.

(i) Suppose that x is a root of the equation $ax^3-3x+b=0$, where a and b are integers. Explain why p divides b and why q divides a. **3**

(1 mark for correct working)

If $x=\frac{p}{q}$ is a root of the equation

$a\left(\frac{p}{q}\right)^3-3\left(\frac{p}{q}\right)+b=0$

$\frac{ap^3}{q^3}-\frac{3p}{q}+b=0$

$ap^3-3pq^2+bq^3=0\ (*)$

$ap^3-3pq^2=-bq^3$

$p(ap^2-3q^2)=-bq^3$

Since $-bq^3$ has p as a factor and p and q have no common divisors other than ±1

$\therefore p$ divides b.

(1 mark for correct logical explanation of p divides b)

From $(*)$

$ap^3=q(3pq-bq^2)$

Since ap^3 has q as factor and p and q have no common divisors other than ±1

$\therefore q$ divides a

(1 mark for correct logical explanation of q divides a)

(ii) Hence deduce that $x^3-3x-1=0$ has no rational root. **2**

(1 mark why test $x=\pm1$)

(1 mark to prove the contradiction of having rational root)

If $x=\frac{p}{q}$ is a root of ax^3-3x+b, p divides b and q divides a

In $x^3-3x-1=0$

$a=1$, $b=-1$

$x=\frac{p}{q}$ $\therefore p$ must divide 1 and q divide -1

$\therefore p=\pm1$ and $q=\pm1$

$\therefore x=\pm1$

when $x=1$ LHS $=1^3-3-1$

$=-3$

$\neq0$

when $x=-1$ RHS $=(-1)^3-3(-1)+1$

$=1$

$\neq0$

$\therefore$ contradiction.

No rational root p/q.

End of paper