SYDNEY GIRLS HIGH SCHOOL

MATHEMATICS

YEAR 10

YEARLY EXAMINATION

2011

Time Allowed: 85 minutes + 5 minutes reading time

Topics: 20%: Quadratic Equations; Probability; Consumer Arithmetic; Number Plane Graphs; Surface Area and Volume
80%: Statistics; Similarity; Further Trigonometry; Further Algebra (including Variation); Logarithms (14:04); Logarithmic + Exponential Graphs (14:05)

Total: 90 marks

Instructions:

- There are FIVE (5) Questions of equal value.
- Attempt all questions.
- Show all necessary working. Marks may be deducted for badly arranged work or incomplete working.
- Start each Question on a new page.
- Write on one side of the paper only.
- Diagrams are NOT to scale.
- Board-approved calculators may be used.
- Write your name and Maths class clearly at the top of each question and clearly number each question.

Student's Name: ____ **Teacher's Name:** _____

QUESTION 1 (18 Marks)

MARKS

1. Calculate $\sqrt{\frac{5}{900\pi}}$ and express your answer in scientific notation, correct to 3 significant figures. 2

2. If a coin is tossed and a die is thrown, find the probability of getting:
   (a) a tail and a three. 1
   (b) a head and an odd number. 1
   (c) a tail and a number more than five. 1

3. Solve the following equation $2x(x+3)=0$ 1

4. What is the equation of the circle that has its centre at the origin and a diameter of 8 units? 1

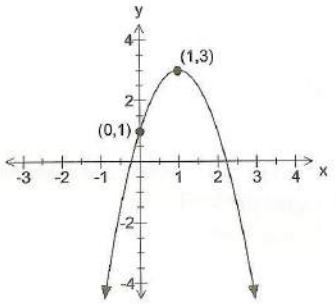
5. Find the volume of a sphere with a radius of 3.5cm, to 2 decimal places. 2

6. Find the curved surface area of a cone with a diameter of 6cm and slant height of 9cm, in terms of π. 2

7. Find the simple interest paid on \$3470 invested at 5.08% p.a for 7 months. 2

8. A new computer costs \$2599. If it depreciates by 15% per year, what will the computer be worth in five years? 2

9. Find the equation of the parabola below. Express the equation in the form $y=ax^2+bx+c$. 3

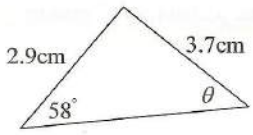


QUESTION 2 (18 Marks)

MARKS

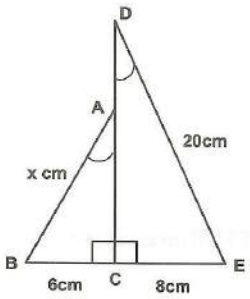
1. If $0^\circ < \theta < 180^\circ$, and $\tan\theta = -1.87$, find θ to the nearest minute. 2

2. Find the size of θ, to the nearest degree. 2



3. A model is made of a statue and the ratio of the volumes is 1:27.
   (a) Find the ratio of the heights of the statues. 1
   (b) Find the ratio of the surface areas of the statues. 1

4.



   (a) Prove that ΔABC is similar to ΔDEC, giving reasons. 3
   (b) Hence find the value of x. 2

5. Solve the following pair of simultaneous equations:
   $y = x^2 - x - 2$ and $y = 2x + 8$ 3

6. Simplify the following: $(x-5)^2 - (x+2)^2$ 2

7. Sketch the graph of $y = \log_{10} x$. Label the intercept and one other point on the graph. 2

QUESTION 3 (18 Marks)

MARKS

1. The back-to-back stem-and-leaf plot drawn below shows the number of days that both a group of boys and girls were absent from school over a three-year period.

Leaf	Stem	Leaf
Boys		Girls
	0	1 7
7 4 1 0	1	2 4 7 9 9
9 7 6 6 5 1 1 1 0	2	1 3 3 4 6 6
8 7 7 5 2	3	4 4 4 8
2	4	3 6
	5	4

(a) Calculate the median number of days absent for Boys only. 1

(b) Calculate the range for Boys only. 1

(c) Calculate the mode for Boys only. 1

(d) Calculate the mean for Girls only. 1

(e) Calculate the standard deviation, (correct to 2 decimal places), for Girls only. 1

(f) For the Girls only, find the: 4

i. Lower Quartile (Q_1)

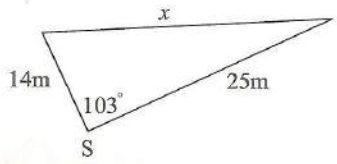
ii. Median (Q_2)

iii. Upper Quartile (Q_3)

iv. Inter-Quartile range

(g) Draw a box-and-whisker plot for the Girls only. 2

2. Selina (S) is 14m from one end of a bridge (x) and 25m from the other end of the bridge. If the angle opposite the bridge, between the lengths of 14m and 25m, is 103°, find the length of the bridge, to the nearest metre. The diagram below represents the information above. 2



3. The sun casts a shadow from a tree. Find the length of the shadow, to the nearest cm, if the height of the tree is 3.2m long and the angle of elevation of the sun is 78°. 2

4. Replace x with $-m$ in the expression $x^2+\frac{1}{x^3}-x$ and simplify. 2

5. Write $8^{\frac{1}{3}}=2$ in logarithmic form. 1

QUESTION 4 (18 Marks)

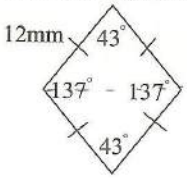
MARKS

1. A statue has a surface area of 1000 cm^2. A scale model is made with dimensions in the ratio 1:5. Find the surface area of the scale model. 2

2. Make y the subject of the equation $\frac{a}{y} = \frac{3y}{p+q}$ 3

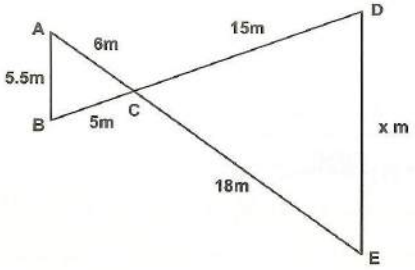
3. Rewrite $\log_{16} 8 = \frac{3}{4}$ in index form. 1

4. Find the area of the following figure, to 2 decimal places: 2



5. What values can x possibly take in the formula $y = \sqrt{6-3x}$? 2

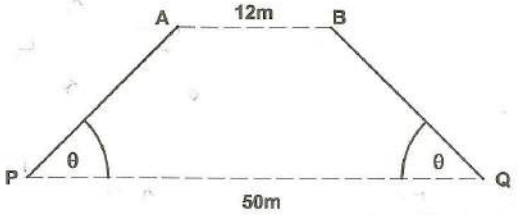
6.



(a) Prove that ΔABC is similar to ΔEDC, giving reasons. 3

(b) Hence find the value of x. 2

7.



The figure above shows the side view of a bridge opened to let boats pass underneath. When the equal arms of the bridge PA and QB are lowered, they meet exactly to form the straight roadway PQ, which is 50m long. When the arms PA and QB are raised through an angle θ as shown, the 'corridor' AB is 12m wide.

Calculate the size of angle θ, to the nearest degree. 3

QUESTION 5 (18 Marks)

MARKS

1. A grain silo is emptied into similar smaller shaped silos that have dimensions half of the original silo. How many smaller silos can be filled? 2

2. Solve the following equation: $(2^y)^2 - 12(2^y) + 11 = 0$ 3

3. M varies inversely as P. When P = 4, M = 30.
   (a) Find a formula relating M and P. 2
   (b) What is the value of P when M = 24. 1

4. Solve the following equations:
   (a) $\log_3 x = 4$ 1
   (b) $\log_9 (x-3) = -2$ 2

5. Annabelle recorded her marks for each test that she did in English and Maths throughout the year. The results are given in the table below.

Maths	83	79	81	92	85	76
English	72	88	89	76	90	72

   (a) Calculate the mean and standard deviation for each subject, correct to 2 decimal places. 2

   (b) Annabelle scores 88 in her next Maths test and 88 in her next English test. By using the results in part (a), in which subject has Annabelle performed better and why? 2

6. Two geologists on a large flat mining area drive 20 km from point A on a bearing of $S30^\circ E$ to a point B. They then drive on a bearing of $N20^\circ E$ to point C. If the distance AC is 31km, find the bearing of A from C. 3

End of Paper

Yr 10 Yearly Exam 2011

Question 1

1) $\frac{5}{\sqrt{900\pi}} \doteq 0.0421$ (to 3 sig. figs)

2) (a) $P(T3) = \frac{1}{12}$

(b) $P(H + \text{odd}) = \frac{3}{12} = \frac{1}{4}$

(c) $P(T + >5) = \frac{1}{12}$

3) $2x(x+3) = 0$

$x = 0$ or -3

4) $x^2 + y^2 = r^2$

$x^2 + y^2 = 4^2$

$x^2 + y^2 = 16$

5) $V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi (3.5)^3$

$\doteq 179.59 \text{ cm}^3$ (to 2 d.pl.)

6) Curved SA $= \pi rs$

$= \pi \times 3 \times 9$

$= 27\pi \text{ cm}^2$

7) $I = PRN$

$= 3470 \times 0.0508 \times \frac{7}{12}$

$= \$102.83$

8) $A = P(1-r)^n$

$= 2599 \times 0.85^5$

$= \$1153.19$

9) $y = -a(x-1)^2 + 3$

sub in $(0,1)$

$1 = -a + 3$

$a = 2$

$y = -2(x-1)^2 + 3$

$= -2(x^2 - 2x + 1) + 3$

$= -2x^2 + 4x - 2 + 3$

$\therefore y = -2x^2 + 4x + 1$

Question 2

1) $\tan\theta = -1.87$

If $\tan\theta = 1.87$

$\theta = 61°52'$

but $\tan\theta$ is neg. $\therefore \theta$ lies in 2nd Quad

$\theta = 180° - 61°52'$

$\therefore \theta \doteq 118°8'$ (To nearest min)

2) $\frac{\sin\theta}{2.9} = \frac{\sin 58°}{3.7}$

$\sin\theta = \frac{\sin 58°}{3.7} \times 2.9$

$\therefore \theta \doteq 42°$ (to nearest deg.)

3) $V_1 : V_2 = 1 : 27$

(a) $h_1 : h_2 = 1 : 3$

(b) $SA_1 : SA_2 = 1 : 9$

4) In $\triangle$s ABC and DEC:

1. $\angle BAC = \angle EDC$ (given)
2. $\angle ACB = \angle DCE$ (given)

$\therefore \triangle ABC \,|||\, \triangle DEC$ (equiangular)

Q2 cont.

4b) $\frac{x}{20} = \frac{6}{8}$ (corresp. sides of similar $\triangle$s)

$x = \frac{6}{8} \times 20$

$\therefore x = 15$

5) $y = x^2 - x - 2$ ..... (1)

$y = 2x + 8$ ..... (2)

(1) = (2): $x^2 - x - 2 = 2x + 8$

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$x = 5$ or $x = -2$

sub in (2): $y = 2(5) + 8$ $\quad y = 2(-2) + 8$

$= 18$ $\quad = 4$

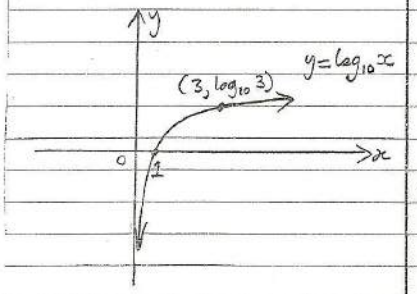
$\therefore x = 5, y = 18$ or $x = -2, y = 4$

6) $(x-5)^2 - (x+2)^2$

$= (x - 5 - (x+2))(x - 5 + x + 2)$

$= -7(2x - 3)$

7) $y = \log_{10} x$



Question 3

1a) Median = 10^{th} Score

$= 26$ days

b) Range $= 42 - 10$

$= 32$

c) Mode $= 21$

d) $\bar{x} = 25.75$

e) $\sigma \doteq 12.96$ (to 2 d.pl.)

f) i) Q_2 = Av. of $5^{th} + 6^{th}$ Scores

$= \frac{17+19}{2}$

$= 18$

ii) Q_2 = Median

= Av. of $10^{th} + 11^{th}$ Scores

$= \frac{23+24}{2}$

$= 23.5$

iii) Q_3 = Av. of $15^{th} + 16^{th}$ Scores

$= \frac{34+34}{2}$

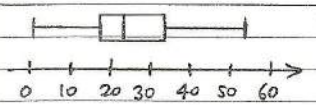
$= 34$

iv) I-Q Range $= Q_3 - Q_1$

$= 34 - 18$

$= 16$

g)



2) $a^2 = b^2 + c^2 - 2bc\cos A$

$x^2 = 14^2 + 25^2 - 2(14)(25)\cos 103°$

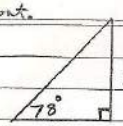
$x \doteq 31.28$

$\therefore$ Bridge Length $= 31$ m

(to nearest m)

cont.

3)



$\tan 78° = \frac{3.2}{x}$

$x = \frac{3.2}{\tan 78°}$

$\doteq 0.680$

$\therefore$ Length of shadow = 68 cm (to nearest cm)

4) $x^2 + \frac{1}{x^3} - x$

$= (-m)^2 + \frac{1}{(-m)^3} - (-m)$

$= m^2 - \frac{1}{m^3} + m$

5) $8^{\frac{1}{3}} = 2$

$\log_8 2 = \frac{1}{3}$

Question 4

1) $S_1 : S_2 = 1 : 5$

$SA_1 : SA_2 = 1 : 25$

$\frac{x}{1000} = \frac{1}{25}$

$x = \frac{1000}{25}$

$= 40$

$\therefore$ SA of model = 40 cm^2

2) $\frac{a}{y} = \frac{3y}{p+q}$

$3y^2 = a(p+q)$

$y^2 = \frac{ap + aq}{3}$

$y = \pm \sqrt{\frac{ap+aq}{3}}$

3) $\log_{16} 8 = \frac{3}{4}$ Note: $\frac{1}{2}$ mark deducted for no $\pm$.

$16^{\frac{3}{4}} = 8$

4) $A = 2 \times \frac{1}{2} ab \sin C$

$= 12 \times 12 \times \sin 43°$

$= 144 \sin 43°$

$\doteq 98.21$ mm^2 (to 2 dp)

5) $6 - 3x \geq 0$

$-3x \geq -6$

$\frac{-3}{-3} \quad \frac{-6}{-3}$

$x \leq 2$

6) a) In Δs ABC and EDC:

1. $\angle ACB = \angle ECD$ (vert. opp. $\angle$s)

2. $\frac{AC}{EC} = \frac{6}{18} = \frac{1}{3}$

$\frac{BC}{DC} = \frac{5}{15} = \frac{1}{3}$

$\therefore \Delta ABC \,|||\, \Delta EDC$ (2 pairs of sides in same ratio + ind. $\angle$ equal)

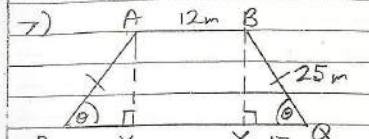
b) $\frac{x}{5.5} = \frac{3}{1}$ (corresp. sides of similar Δs)

$x = 3 \times 5.5$

$= 16.5$

Q4 cont.

7)



$YQ = (50 - 12) \div 2$

$= 19$ m

In ΔBQY: $\cos\theta = \frac{19}{25}$

$\theta \doteq 41°$ (to nearest deg.)

Question 5

1) $S_1 : S_2 = 1 : \frac{1}{2}$

$= 2 : 1$

$V_1 : V_2 = 2^3 : 1^3$

$= 8 : 1$

$\therefore$ 8 small silos can be filled

2) $(2^y)^2 - 12(2^y) + 11 = 0$

Let $x = 2^y$

$x^2 - 12x + 11 = 0$

$(x - 11)(x - 1) = 0$

$x = 11$ or $x = 1$

$2^y = 11$ $\quad 2^y = 1$

$y = \log_2 11$ or $y = 0$

3) $M \propto \frac{1}{P}$

(a) $M = \frac{k}{P}$

$30 = \frac{k}{4}$

$k = 120$

$M = \frac{120}{P}$

(b) $M = \frac{120}{P}$

$24 = \frac{120}{P}$

$P = \frac{120}{24}$

$= 5$

4)(a) $\log_3 x = 4$

$x = 3^4$

$\therefore x = 81$

(b) $\log_q (x - 3) = -2$

$q^{-2} = x - 3$

$x = 3\frac{1}{81}$

5)(a) Maths: $\bar{x} \doteq 82.67$

$\sigma \doteq 5.06$

English: $\bar{x} \doteq 81.17$

$\sigma \doteq 7.97$

(b) Maths: $z = \frac{x - \bar{x}}{\sigma}$

$= \frac{88 - 82.67}{5.06}$

$\doteq 1.05$ SD above mean

English: $z = \frac{88 - 81.17}{7.97}$

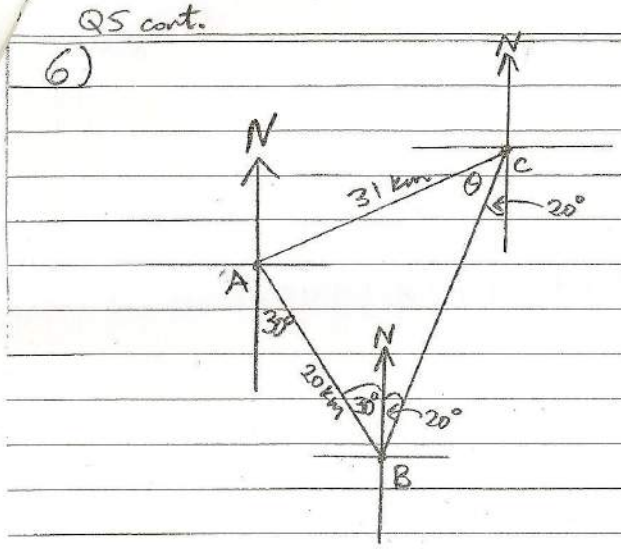
$\doteq 0.86$ SD above mean

$\therefore$ Maths mark is better as it has a higher Standard Deviation above Mean, than the English mark.

[OR Maths mark was more than 1 SD above mean, whereas English mark was less than 1 SD above mean $\therefore$ Maths mark is better]

Q5 cont.

6)



$$\frac{\sin \angle ACB}{20} = \frac{\sin 50^\circ}{31}$$

$$\sin \theta = \frac{20 \sin 50^\circ}{31}$$

$$\theta \doteq 30^\circ \text{ (to nearest deg.)}$$

$\therefore$ Bearing of A from C = S 50° W or 230°