

File

Name: \_\_\_\_\_

Teacher: \_\_\_\_\_

## SYDNEY TECHNICAL HIGH SCHOOL



### YR 10 YEARLY EXAM

## MATHEMATICS

2013

**Time Allowed:** 2 hours

Total Marks: 90 minutes

Section I: Q1-15 multiple choice (15 Marks)  
Allow approximately 20 mins for Section I

Section II: Q16-20 (75 marks)  
Allow approximately 100 mins for Section II

#### General Instruction:

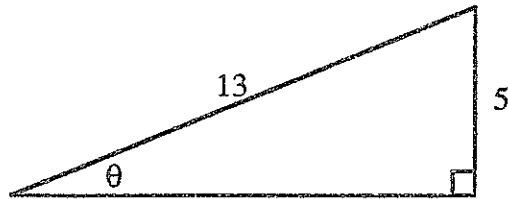
- There is a multiple choice answer sheet in your answer book for Q1-15. Shade the correct response.
- All answers to Q16-20 must be written in your answer book, with complete and organised setting out and working.
- Marks will not be given if required working is not shown.
- Marks are indicated for each question but may be changed.
- Use blue or black pen only.
- Calculators may be used
- **START EACH QUESTION ON A NEW PAGE**

**Section I**

Questions 1-15 (multiple choice)

(1 mark each) – answers on answer sheet in your answer book

1.



Correct to 2 decimal places,  $\tan \theta =$

- (A) 0.38
- (B) 0.42
- (C) 2.40
- (D) 22.62

2.

A car is travelling at a speed of  $v$  kilometres per hour. This speed can be converted to metres per second by using the expression

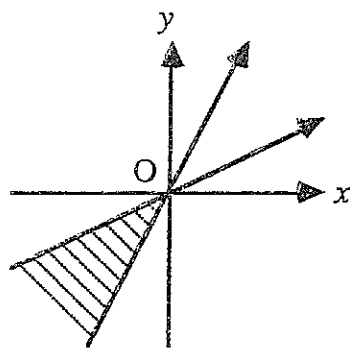
(A)  $\frac{1000 \times v}{60 \times 60}$

(B)  $\frac{60 \times 60 \times v}{1000}$

(C)  $1000 \times 60 \times 60 \times v$

(D)  $\frac{v}{1000 \times 60 \times 60}$

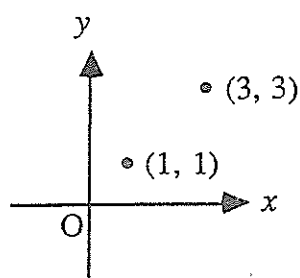
3.



Which pair of inequalities could represent the shaded region?

- (A)  $y \leq \frac{1}{3}x, y \leq 3x$
- (B)  $y \leq \frac{1}{3}x, y \geq 3x$
- (C)  $y \geq \frac{1}{3}x, y \leq 3x$
- (D)  $y \geq \frac{1}{3}x, y \geq 3x$

4.



The points which are equidistant from (1, 1) and (3, 3) lie on the line with equation

- (A)  $x = 2$
- (B)  $y = 2$
- (C)  $y = x$
- (D)  $x + y = 4$

5.

The local sports store advertised a 45% discount on all tennis racquets. Chris bought a racquet and paid \$88 after discount.

How much did Chris save?

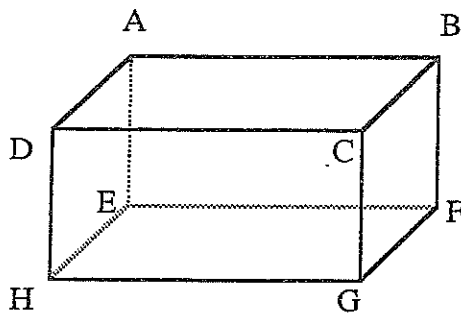
- (A) \$16.00                      (B) \$39.60                      (C) \$48.40                      (D) \$72.00

6.

The gradient of the line  $2x - 5y + 1 = 0$  is

- (A)  $-2$                       (B)  $-\frac{2}{5}$                       (C)  $\frac{2}{5}$                       (D)  $2$

7.



$AB = 4 \text{ m}$

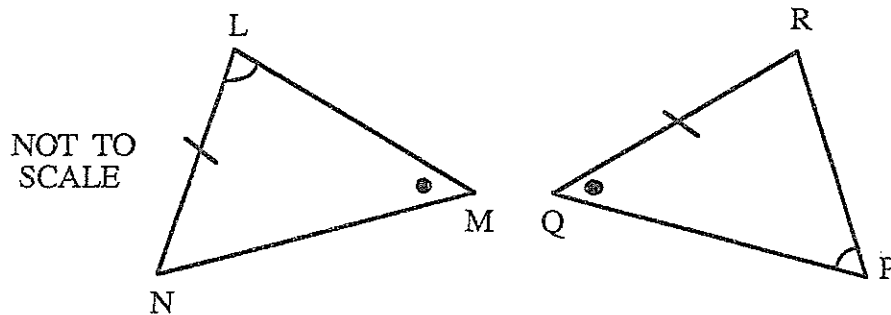
$BF = 2 \text{ m}$

$FG = 3 \text{ m}$

The size of angle BHF to the nearest minute is

- (A)  $21^\circ 48'$                       (B)  $23^\circ 35'$                       (C)  $26^\circ 34'$                       (D)  $36^\circ 52'$

8.



Consider these statements:

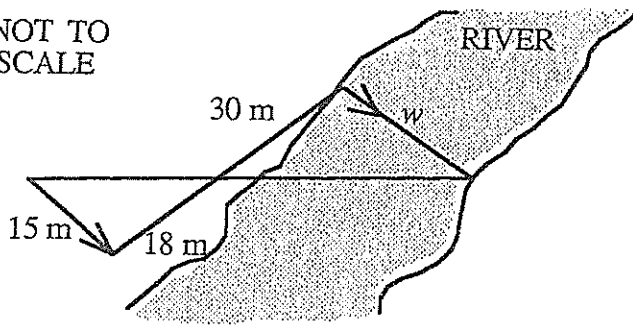
- I. Triangles LMN and PQR are congruent.  
II. Triangles LMN and PQR are similar.

Which is always correct?

- (A) I only                      (B) II only                      (C) Both I and II                      (D) Neither I nor II

9.

NOT TO SCALE



A student uses the diagram to find the width of the river,  $w$ .

The value of  $w$  is

- (A) 18
- (B) 25
- (C) 27
- (D) 36

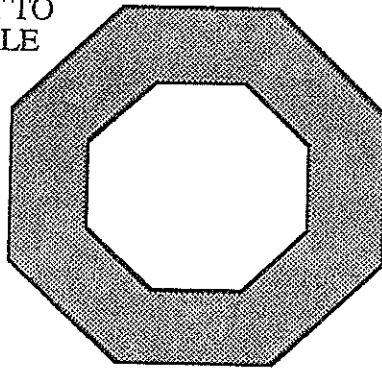
10.

If  $x$  is a number between 0 and 1, which of the following has the greatest (maximum) value?

- (A)  $x^{-1}$
- (B)  $x^0$
- (C)  $x^{1/2}$
- (D)  $x^2$

11.

NOT TO SCALE



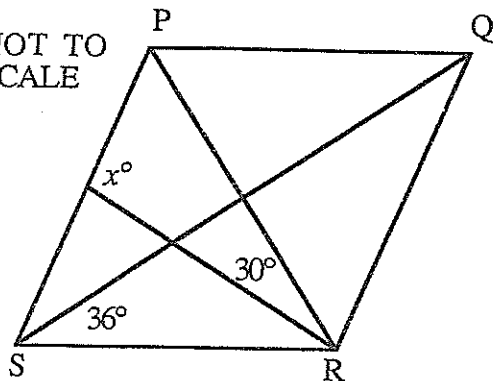
This shape is made by cutting a small regular octagon from the centre of a larger similar regular octagon. Corresponding sides are in the ratio 3 : 4.

What fraction of the larger octagon has been cut out?

- (A)  $\frac{1}{4}$
- (B)  $\frac{3}{4}$
- (C)  $\frac{7}{16}$
- (D)  $\frac{9}{16}$

12.

NOT TO SCALE



PQRS is a rhombus. Find the value of  $x$ .

- (A) 90
- (B) 96
- (C) 102
- (D) 108

13.

If  $y = 2\sqrt{x}$ , then  $x =$

(A)  $\frac{y}{2}$

(B)  $\frac{y}{4}$

(C)  $\frac{y^2}{2}$

(D)  $\frac{y^2}{4}$

14.

For the two sets of scores,

Set A : 4, 6, 7, 17, 18, 20

Set B : 2, 8, 9, 15, 16, 22

consider these statements:

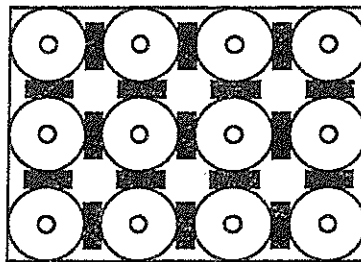
- I. Both sets have the same mean.  
 II. Both sets have the same standard deviation.

Which is correct?

- (A) I only      (B) II only      (C) Both I and II      (D) Neither I nor II

15.

Bottles are packed into a box with a piece of packing between each bottle, as shown below :



How many pieces of packing are needed to pack a rectangular box fully, if the box has  $n$  bottles along the length and  $k$  bottles across the breadth?

- (A)  $nk$   
 (B)  $(n - 1)(k - 1)$   
 (C)  $nk + (n - 1) + (k - 1)$   
 (D)  $n(k - 1) + k(n - 1)$

Section II Question 16-20

Question 16 (15 marks)

a) If  $m = \frac{1.6 \times 10^{-19}}{1.76 \times 10^{11}}$  write  $m$  in scientific notation correct to 4 significant figures (2)

b) Solve for  $x$ ,  $x + 1 = \frac{x+2}{3}$  (2)

c) Solve for  $x$  and  $y$   
 $x - y = -2$  (2)  
 $2x + 3y = 41$

d) Write as a single fraction  
 $\frac{m+n}{2} - \frac{m-n}{3}$  (2)

e) Factorise fully  
i)  $15x^2 - x - 2$  (1)  
ii)  $x^3 - 3x^2 + x - 3$  (1)  
iii)  $2x^2 - 72$  (2)

f) Simplify  $\frac{x^2-81}{x^2-10x+9}$  (2)

g) If the probability of an event occurring is  $\frac{1}{x}$ , what is the probability of the event not occurring. (1)

**Question 17 (15 marks) (start a new page)**

- a) Plot the points A (1,7), B (-1,5) and C (4, -1) on a number plane
- i) Find the exact length of AC (1)
  - ii) Find the gradient of AC (1)
  - iii) Find the equation of AC in general form (2)
  - iv) If ABCD is a parallogram, find the co-ordinates of D (1)

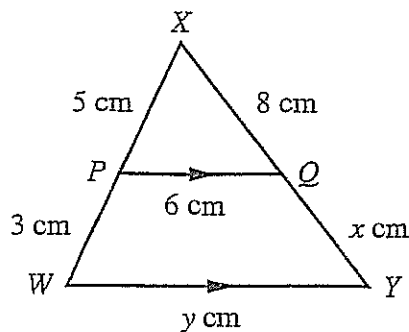
b) Expand and simplify  $(3 - 2\sqrt{3})^2$  (2)

c) Solve for  $x$ ,  $3^x = \frac{1}{81}$  (2)

- d) Solve for  $t$ , leaving your answer in simplest exact form

$$t^2 - 4t - 2 = 0 \quad (2)$$

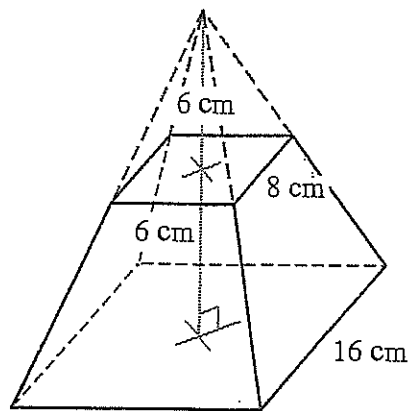
- e) i) Redraw the diagram below onto your answer sheets
- ii) Prove that  $\Delta XPQ$  is similar to  $\Delta XWY$  and find  $x$  and  $y$  (reasons required) (4)



**Question 18 (15 marks) (start a new page)**

a)

(2)

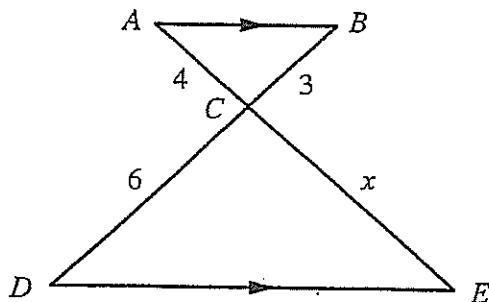


A square pyramid with base edge 16cm and height 12 cm, has another square pyramid with base edge 8cm and height 6cm removed from its top. Find the volume of the frustum that is formed.

b) A lead cube of sides 8cm is melted down to make lead spheres, each with radius 7mm. How many complete spheres can be made. (2)

c) If  $\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}} = a + b\sqrt{6}$  find  $a$  and  $b$  (2)

d)



Find  $x$  (give a reason for your answer) (2)

e) Simplify  $\log(x^2 + 6x + 9) - \log(x + 3)$  and choose the correct answer. (1)

A)  $\log(x^2 + 5x + 6)$       B)  $\log(x + 3)$

C)  $\log x + \log 3$       D) 2



f) Given  $\log_b 2 = 0.43$  and  $\log_b 3 = 0.68$  evaluate

i)  $\log_b 6$  (1)

ii)  $\log_b 2\sqrt{3}$  (1)

g) Rearrange the equation below into the form  $ax^2+bx+c=0$  and hence solve for  $x$ , correct to 2 decimal places.

$$x = \frac{5x-3}{x} \quad (2)$$

h) In a class of 26 students, all of whom take at least one of History or Geography, there are 20 who take History and 17 who take Geography.

i) Draw a Venn diagram to show this information (1)

ii) Find the probability that a student chosen at random will study History but not Geography. (1)

**Question 19 (15 marks) (start a new page)**

a) The ratio of the heights of two similar cylinders is 2:3. If 4 kg of gas fills the smaller cylinder, how much gas is needed to fill the larger one? (2)

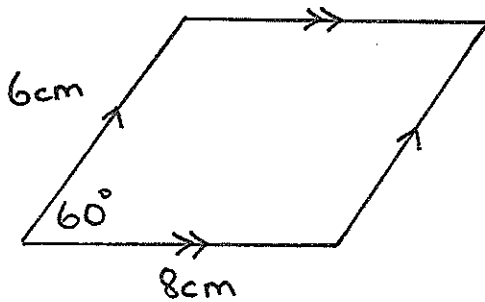
b) Make neat sketches of the following on separate number planes. Show where each curve cuts the  $x$  and  $y$  axes.

i)  $x^2 + y^2 = 4$  (2)

ii)  $y = x^2 - x$  (2)

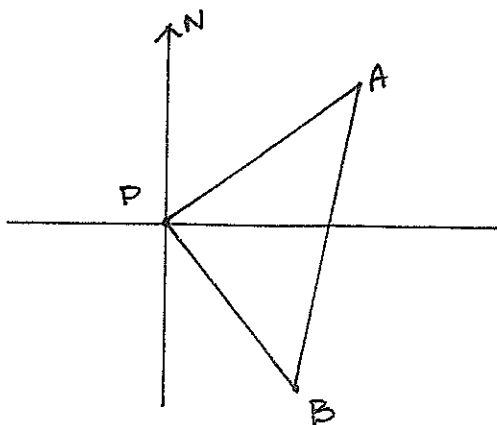
c) For the curve  $y = \frac{-2}{x}$  find the equation of the vertical asymptote (1)

- d) For the curve  $y = 4 - (x + 1)^2$  find
- i) the vertex (1)
  - ii) the y intercept (1)
  - iii) the x intercepts (1)
- e) Evaluate  $\log_2\left(\frac{1}{8}\right)$  (1)
- f) Simplify fully  $\log_7 49a - \log_7 a$ . (2)
- g) Find the exact area of the parallelogram. (2)



Question 20 (15 marks) (start a new page)

a)



In the diagram above boat A sails 9 km from port P on a bearing of 035°.

Boat B sails 7 km from port P on a bearing of 160° .

Redraw the diagram in your answer book and show all information given.

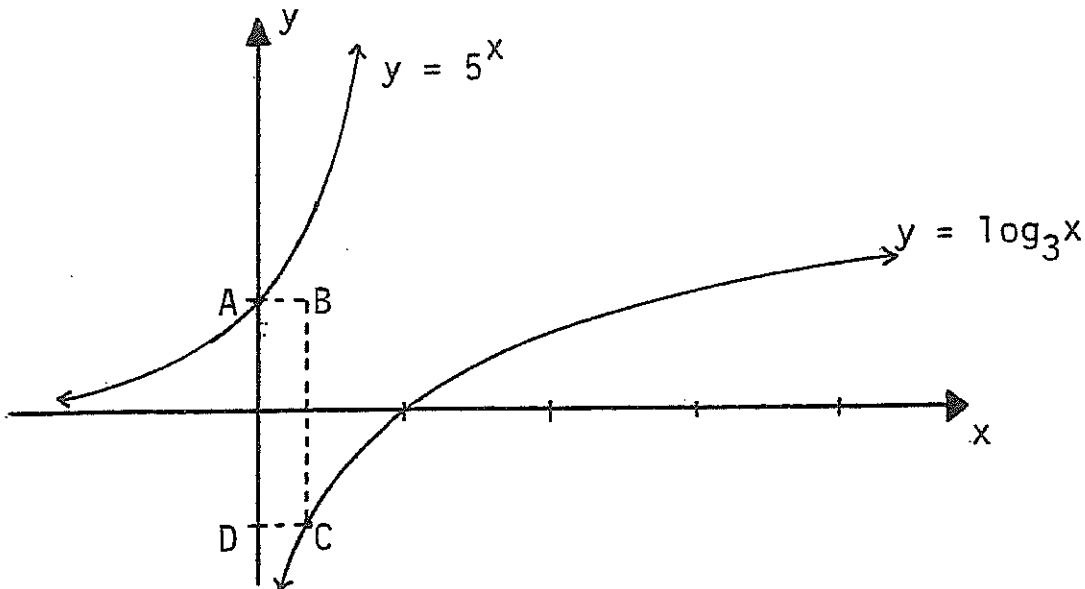
i) Find the distance AB, correct to 1 decimal place (2)

ii) Find the angle PAB to nearest degree (2)

iii) Hence, find the bearing of B from A to the nearest degree. (1)

b) If  $4^t = 100$  find  $t$  correct to 2 decimal place. (2)

c)



In the diagram above, ABCD is a rectangle.

The coordinates of D are  $(0, -1)$

i) Find the coordinates of C (1)

ii) Find the area of rectangle ABCD. (1)

d) Three coins are tossed simultaneously

i) Draw a tree diagram to show all possible outcomes (1)

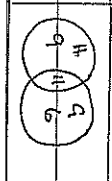
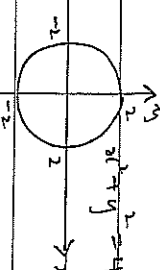
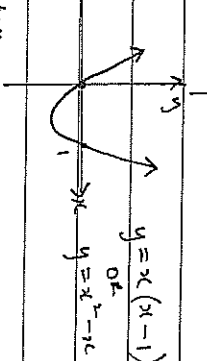
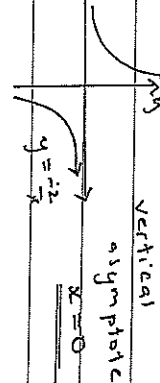
ii) Find the probability that at most two heads are thrown (1)

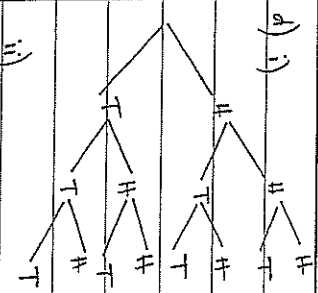
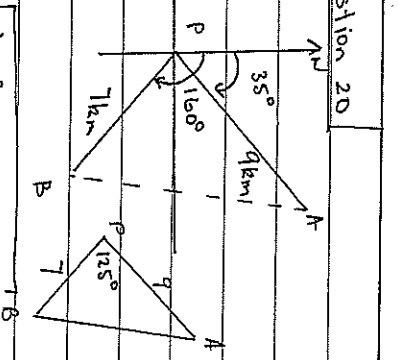
e) Solve for  $x$

$$\log_2(x + 1) - \log_2(x - 1) = 3 \quad (2)$$

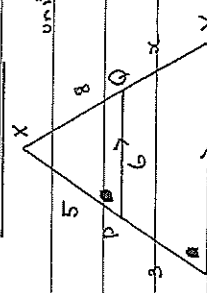
f) Let  $x = \log_b a$  and  $y = \log_a b$  and hence prove

$$\log_b a \cdot \log_a b = 1 \quad (2)$$

<p>f) i) <math>\log_b 6 = \log_b(3 \times 2)</math>  <math>= \log_b 3 + \log_b 2</math>  <math>= 0.43 + 0.68</math>  <math>= 1.11</math></p> <p>ii) <math>\log_2 2(3) = \log_2 2 + \frac{1}{2} \log_2 3</math>  <math>= 0.43 + 0.34</math>  <math>= 0.77</math></p> <p>g) <math>x = 5x - 3</math>  <math>x^2 = 5x - 3</math>  <math>x^2 - 5x + 3 = 0</math>  <math>x = \frac{5 \pm \sqrt{25 - 4 \cdot 1 \cdot 3}}{2}</math>  <math>x = \frac{5 \pm \sqrt{13}}{2}</math>  <math>x = 4.30</math> or <math>0.70</math>  <math>\Sigma = 26</math></p> <p>ii) </p> <p>ii) <math>\frac{9}{26}</math></p>	<p>b) i)   <math>x^2 + y^2 = 4</math></p> <p>ii)   <math>y = x(x-1)</math>          or <math>y = x^2 - x</math></p> <p>c)           vertical asymptote <math>x=0</math>  <math>y = -x^2</math></p>
<p>d) <math>y = 4 - (x+1)^2</math>          i) <math>V(-1, 4)</math>          ii) <math>x=0 \therefore y=3</math>          iii) <math>y=0 \Rightarrow 0 = 4 - (x+1)^2</math>  <math>(x+1)^2 = 4</math>  <math>x+1 = \pm 2</math>  <math>x = 1, -3</math></p>	<p>e) <math>\log_2 \left(\frac{1}{8}\right) = x</math>  <math>\therefore 2^x = \frac{1}{8}</math>  <math>2^x = 2^{-3}</math>  <math>x = -3</math></p>
<p>Question 19</p> <p>a) 2:3 ratio riders  <math>2^3 : 3^3</math> ratio volumes  <math>\frac{2^3}{3^3} = \frac{4}{27}</math>  <math>27 \times 4 = 8V</math>  <math>\therefore V = 13\frac{1}{2}</math> kg larger bottle</p>	
<p>f) <math>\log_1 49a - \log_1 a</math>  <math>\log_1 7^2 \cdot a - \log_1 a</math>  <math>2 \log_1 7 + \log_1 a - \log_1 a = 2</math></p>	<p>f) <math>\log_1 49a - \log_1 a</math>  <math>\log_1 7^2 \cdot a - \log_1 a</math>  <math>2 \log_1 7 + \log_1 a - \log_1 a = 2</math></p>

<p>g) <math>A = 2x \left(\frac{1}{2} \cdot 6 \cdot 8 \cdot \sin 60^\circ\right)</math>  <math>= 48 \cdot \frac{\sqrt{3}}{2}</math>  <math>= 24\sqrt{3} \text{ cm}^2</math></p>	<p>d) i)           ii) <math>P(\text{at most 2 H}) = 1 - P(3H)</math>  <math>= 1 - \frac{1}{8}</math>  <math>= \frac{7}{8}</math></p>
<p>Question 20</p> <p>a) </p> <p>i) <math>AB = \sqrt{9^2 + 7^2 - 2 \cdot 9 \cdot 7 \cos 125^\circ}</math>  <math>= 14.2 \text{ km}</math>  <math>\sin A = \frac{\sin 125^\circ}{14.2}</math>  <math>A = 24^\circ</math>          Bearing 191°</p>	
<p>ii) <math>\sin A = \frac{\sin 125^\circ}{14.2}</math>  <math>A = 24^\circ</math></p>	<p>e) <math>\log_2 (x+1) - \log_2 (x-1) = 3</math>  <math>\log_2 \left(\frac{x+1}{x-1}\right) = 3</math>  <math>2^3 = \frac{x+1}{x-1}</math>  <math>8(x-1) = x+1</math>  <math>8x - 8 = x + 1</math>  <math>7x = 9</math>  <math>x = \frac{9}{7}</math></p>
<p>b) <math>A^t = 100</math>  <math>t \log_{10} 4 = \log_{10} 100</math>  <math>t = \frac{\log_{10} 100}{\log_{10} 4}</math>  <math>t = 3.32</math></p>	<p>f) <math>x = \log_b a \quad y = \log_a b</math>  <math>b^x = a \quad a^y = b</math>          substitute  <math>(a^y)^x = a</math>  <math>\therefore a^{xy} = a</math></p>
<p>c) i) <math>C\left(\frac{1}{3}, -1\right)</math>          ii) <math>\frac{2}{3}</math> units</p>	<p><math>\log_a a \cdot \log_a b = 1</math></p>

YEAR 10	YEARLY EXAM	NOVEMBER (SOLUTIONS)
Section I (m.c)		
Q1 B	Q6 C	Q11 D
2 A	7 A	12 B
3 B	8 B	13 D
4 D	9 B	14 C
5 D	10 A	15 <del>D</del>
Question 16		
a)	$9.091 \times 10^{-31}$	
b)	$x+1 = \frac{x+2}{3}$	
	$3(x+1) = x+2$	
	$3x+3 = x+2$	
	$2x = -1$	
	$x = -\frac{1}{2}$	
Question 17		
a)	$x - y = -2$ $x(2)$	
	$2x + 3y = 4$	
	$2x - 2y = -4$	
	$5y = 4 - 5$	
	$y = 9$ ?	
	sub into (1) $x = 7$	
d)	$\frac{m+n}{2} = \frac{m-n}{3}$	
	$3(m+n) - 2(m-n) = 2$	
	$3m + 3n - 2m + 2n = 2$	
	$m + 5n = 2$	
	$m = \frac{2-5n}{1}$	
	$m = 2 - 5n$	
	$2 - 5n = 2 - 5n$	
	$0 = 0$	
	$AC = \sqrt{(4-1)^2 + (-1-7)^2}$	
	$= \sqrt{9 + 64}$	
	$= \sqrt{73}$ units	
ii)	$m = \frac{7-1}{1-4} = -\frac{8}{3}$	

iii)	$y-7 = -\frac{8}{3}(x-1)$	$\therefore \frac{5}{8} = \frac{8}{x+8} = \frac{6}{y}$ (corresponding sides in similar triangles)
	$3y - 21 = -8x + 8$	
AC:	$8x + 3y - 29 = 0$	$5(x+8) = 64$
iv)	D (6, 1)	$5x + 40 = 64$
b)	$(3 - 2\sqrt{3})(3 - 2\sqrt{3})$	$5x = 24$
	$= 9 - 12\sqrt{3} + 12$	$x = \frac{24}{5}$
	$= 21 - 12\sqrt{3}$	
Question 18		
a)	$3x = \frac{1}{81}$	$V(\text{large pyramid}) = \frac{1}{3} \times 16^2 \times 12$
	$3x = 3^{-4}$	$V(\text{small pyramid}) = \frac{1}{3} \times 8^2 \times 6$
	$\therefore x = -4$	$\therefore V(\text{frustum}) = 896 \text{ cm}^3$
d)	$x = 4 \pm \frac{\sqrt{16 - 4 \cdot 1 \cdot -2}}{2}$	b) $V_{\text{cube}} = 8^3 \text{ cm}^3 = 512 \text{ cm}^3$
	$x = 4 \pm \sqrt{24}$	$V_{\text{sphere}} = \frac{4}{3} \pi \times 0.7^3 = 1.4316 \dots \text{ cm}^3$
	$x = 4 \pm 2\sqrt{6}$	(355 if rounded off V of sphere)
	$x = 2(2 \pm \sqrt{6})$	$\therefore$ 356 complete spheres formed
	$\therefore x = 2 \pm \sqrt{6}$	
e)	$\frac{\sqrt{2-\sqrt{3}}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2-\sqrt{6}}{2}$	
	$= 1 - \frac{1}{2}\sqrt{6}$	
	$\therefore a = 1$ $b = -\frac{1}{2}$	
e) i)		d) $\frac{3}{6} = \frac{4}{x}$ (corresponding sides in similar triangles)
	In $\Delta$ s XPQ and XNY	$3x = 24$
	$x$ is common	$x = \frac{24}{3}$
	$\hat{X}PQ = \hat{X}NY$ (corresponding angles)	$x = 8$
	$\hat{P}XQ = \hat{Q}XN$ (vertically opposite angles)	
	$\therefore \Delta XPQ \sim \Delta XNY$ (equiangular)	e) <u>B</u>

	Trig	Logs/ Indices	Similarity Geom	No. Plane Graphs	Alg/ Equations	Misc.
Q1	/1					
2						/1
3				/1		
4				/1		
5						/1
6				/1		
7	/1					
8			/1			
9			/1			
10					/1	
11			/1			
12			/1			
13					/1	
14						/1
15					/1	
Q16 a)					/1	/2
b)					/2	
c)					/2	
d)					/2	
e) i)					/1	
ii)					/1	
iii)					/2	
f)					/2	
g)						/1

	Trig	Logs/ Indices	Sim. Geom	No. Plane Graphs	Alg + Equations	Misc.
Q17 a) i)				/1		
ii)				/1		
iii)				/2		
iv)				/1		
b)						/2
c)		/2				
d)					/2	
e) i)			/4			
ii)						/2
Q18 a)						/2
b)						/2
c)					/2	
d)			/2			
e)		/1				
f) i)		/1				
ii)		/1				
g)					/2	
h) i)						/1
ii)						/1
Q19 a)			/2			/1
b) i)				/2		
ii)				/2		
c)				/1		
d) i)				/1		
ii)				/1		
iii)				/1		

