SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2006
YEAR 11 ACCELERATED
HALF YEARLY EXAM

Mathematics Accelerated

General Instructions

- Reading Time – 5 Minutes
- Working time – 90 Minutes
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators maybe used.
- Each Section is to be returned in a separate bundle.
- Marks may **NOT** be awarded for messy or badly arranged work.
- All necessary working should be shown in every question.

Total Marks – 89

- Attempt questions 1-6
- Hand up in 3 sections clearly marked A,B & C

Examiner: *P.Bigelow*

SECTION A

Question 1 (12 marks) **Marks**

a) Convert $\frac{13\pi}{6}$ to degrees **1**

b) If $f(x) = 3x^2 + 11x - 1$, evaluate $f(3) - f(-3)$ **2**

c) If $\sqrt{45} + \sqrt{80} = a\sqrt{5}$, find a **1**

d) How many significant zeros in 0.0040701? **1**

e) Find, without a calculator, $4.1\dot{3}$ as a fraction (in simplest form) **2**

f) Simplify $\log_7 98 - \log_7 2$ **2**

g) Write $\frac{1}{\sqrt[3]{x^4}}$ in index form **1**

h) Given the parabola $(x-4)^2 = 8(y+1)$ write down the co-ordinates of the focus and the equation of the directrix. **2**

Question 2 (15 Marks) **Marks**

a) Factorise: **3**

i) $49 - y^2$

ii) $6a^2 - a - 2$

iii) $8a^3 + 1$

b) Find: **2**

i) $\lim_{x \to 4} \frac{16 - x^2}{4 - x}$

ii) $\lim_{x \to \infty} \frac{4 + x - x^2}{3x^2 + 2x - 1}$

c) Solve, then graph, the solution on a number line: **4**

i) $x^2 \leq 4x$

ii) $|x - 5| > 9$

d) Sketch $y = \cos x$ for $0^\circ \leq x \leq 360^\circ$ **2**

e) Differentiate $f(x) = 1 - 2x + x^2$ from first principles **2**

f) Find, algebraically, the points of intersection of the curve $y = x^2$ and the line $x - y + 20 = 0$. **2**

SECTION B (start a new booklet)

Question 3 (15 marks) **Marks**

a) Differentiate the following: **3**

i) $y = 4 - 5x + 6x^3$

ii) $f(x) = (3 - 5x)^{10}$

iii) $y = 1 - \frac{1}{\sqrt{x}}$

b) Write down the exact value of sin 240° **1**

c) Simplify $a - \frac{1}{a}$ if $a = \sqrt{2} + 1$ **2**

d) Solve for *x*, $(0.2)^{x+1} = (0.008)^{x-1}$ **2**

e) Sketch: **3**

i) $y = 2^{-x}$

ii) $x^2 - 2x + y^2 + 4y - 4 = 0$

f) Between which two consecutive integers does $\log_7 100$ lie? **1**

g) State whether the following are ODD, EVEN or NEITHER (justify your answer with necessary working): **3**

i) $f(x) = \frac{1}{1+x^2}$

ii) $f(x) = \frac{1}{1-x^2}$

iii) $f(x) = \frac{-x}{1+x^2}$

(Question 4 on next page)

Question 4 (15 Marks) **Marks**

a) Show that the points (-4,-5), (2,7) and (5,13) are collinear **2**

b) Find, using the "k method", the equation of the line, passing through the intersection of the lines $2x - y + 1 = 0$ and $3x + y - 6 = 0$ and containing the point (-2,3) **3**

c) Show that the line $4x - 3y + 15 = 0$ is a tangent to the curve $x^2 + y^2 - 9 = 0$ **2**

d) Find the size of each internal angle in a regular 14 sided polygon **1**

e) Differentiate $y = \dfrac{x^2 + c}{x^2 - c}$ and hence find the value of c, if $\dfrac{dy}{dx} = 1$ at $x = -3$ **3**

f) A parabola $y = ax^2 + bx + c$ passes through A(-1,4), B(0,7) and C(1,8). Determine the values of *a, b,* and *c* **2**

g) Given the quadratic expression $x^2 + (k-3)x + k$, for what values of *k* is the expression positive for all values of *x*? **2**

SECTION C (start a new booklet)

Question 5 (17 marks) **Marks**

a) The points A, B and C are equally spaced on the circumference of a circle radius 6cm. **2**
Find the exact area of the triangle ABC.

b) Solve $5^x = 160$ (correct to 4 significant figures) **2**

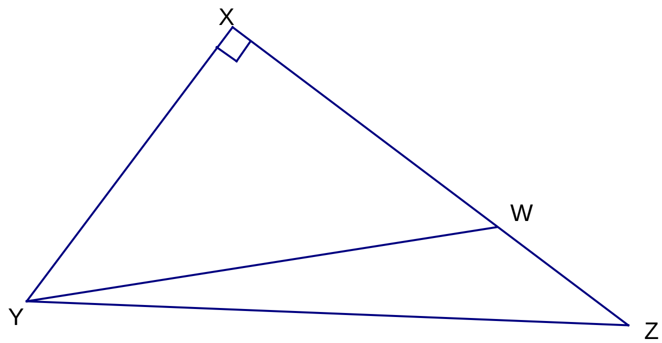
c) Find the point or points on the curve $f(x) = x^2 + \frac{1}{3}x^3$ where the tangent is inclined at 135° to the positive direction of the x-axis. **3**

d) Sketch the following regions: **4**

i) $y < \frac{1}{x}$

ii) $y \le \sqrt{4 - x^2}$

e) **3**



Triangle XYZ is a right-angled triangle at X.

W is a point on XZ such that XW = 2WZ.

Prove that $5WZ^2 = YZ^2 - YW^2$

f) Given that α and β are roots of $2x^2 - 6x + 1 = 0$ evaluate: **3**

i) $\frac{1}{\alpha} + \frac{1}{\beta}$

ii) $\alpha^2 + \beta^2$

iii) $\alpha^3 + \beta^3$

Question 6 (15 marks) **Marks**

a) State the domain and range of: **3**

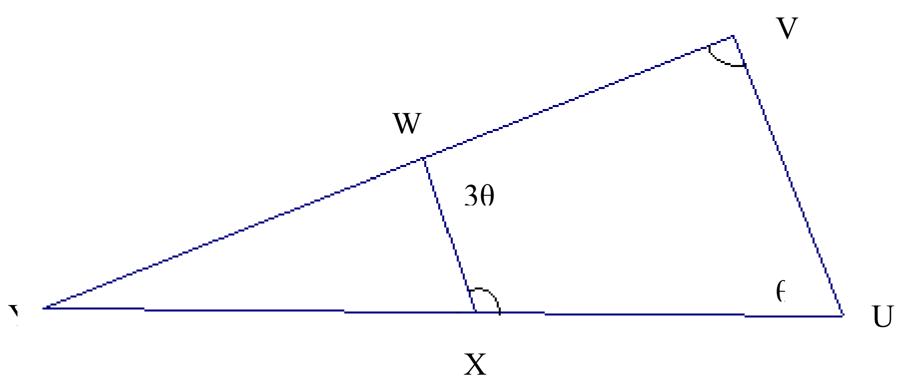
i) $y = 1 - 4x - 2x^2$

ii) $y = \sqrt{1 - 4x}$

b) Find the points on the curve $y = x + \frac{1}{x}$ where the normal is parallel **2**

to the line $2x + y - 13 = 0$

c) **3**



UVWX is a quadrilateral in which $U\hat{V}W = W\hat{X}U$ and $V\hat{W}X = 3V\hat{U}X$.

Prove that *UV* = *VY*.

d) In ΔABC, $\hat{A} = 38°21'$, $b = 11.6$cm and $a = 7.9$cm. **3**

Find the size of $\hat{B}$. (to the nearest degree)

e) The quadratic equation $x^2 + px + q = 0$, has one root twice the other. **4**

Prove:

i) $2p^2 = 9q$

ii) That the roots are rational whenever p is rational

This is the end of the paper.

YR 11 2006 Accelerated Half Yearly exam

Section A Q(1)

(a) $\frac{13\times180}{6} = 390^\circ$ (1)

(b) $f(x) = 3x^2 + 11x - 1$

$f(3) = 27 + 33 - 1 = 59$ (1)

$f(-3) = 27 - 33 - 1 = -7$

$f(3) - f(-3) = 59 - -7 = 66$ (1)

(c) $\sqrt{45} = \sqrt{9\times5} = 3\sqrt{5}$

$\sqrt{80} = \sqrt{16\times5} = 4\sqrt{5}$

$7\sqrt{5}$

$\Rightarrow a = 7$ (1)

(d) 2 (1)

(e) let $x = 4.13333\ldots$

$10x = 41.333\ldots$

$9x = 37.2$

$x = \frac{37.2}{9} = \frac{372}{90} = 4\frac{2}{15}$ (2)

(f) $\log_7\left(\frac{98}{2}\right) = \log_7 49 = x$

$7^x = 49$

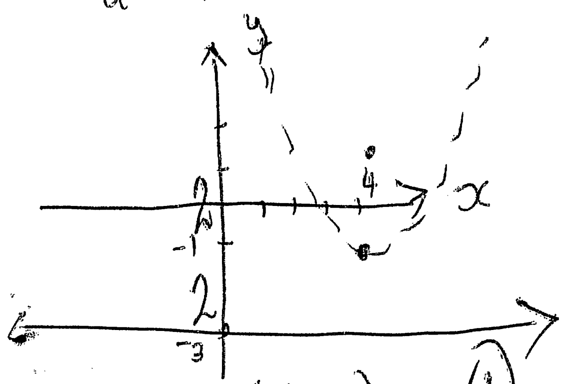
$x = 2$ (2)

(g) $x^{-\frac{4}{3}}$ (1)

(h) $(x-4)^2 = 4\times2(y+1)$

$V(4, -1)$

$a = 2$



Focus $(4, 1)$ (1)

$y = -3$ (1)

12/12

Question 2 (15 Marks) **Marks**

a) Factorise: 3

i) $49-y^2$ $= (7+y)(7-y)$

ii) $6a^2-a-2$ $= (3a-2)(2a+1)$

iii) $8a^3+1$ $= (2a+1)(4a^2-2a+1)$

b) Find: 2

i) $\lim\limits_{x\to 4}\dfrac{16-x^2}{4-x}$ $= \lim\limits_{x\to 4}(x+4) = 8$

ii) $\lim\limits_{x\to\infty}\dfrac{4+x-x^2}{3x^2+2x-1}$ $= \lim\limits_{x\to\infty}\dfrac{\frac{4}{x^2}+\frac{1}{x}-1}{3+\frac{2}{x}-\frac{1}{x^2}} = -\frac{1}{3}$

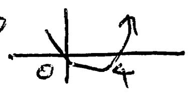
c) Solve, then graph, the solution on a number line: 4

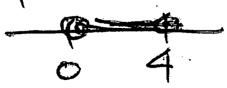
i) $x^2 \le 4x$

ii) $|x-5|>9$

$x^2-4x\le 0$ $\quad x(x-4)\le 0$

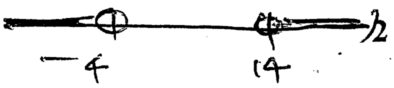
$0\le x\le 4$



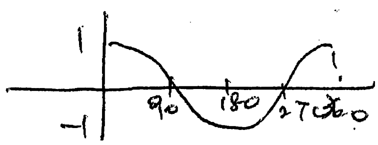


$x-5>9$ $\quad x-5<-9$

$x>14$ or $x<-4$



d) Sketch $y=\cos x$ for $0^\circ \le x \le 360^\circ$ 2



e) Differentiate $f(x)=1-2x+x^2$ from first principles 2

$\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0}\dfrac{(x+h)^2-2(x+h)+1-(x^2-2x+1)}{h}$

$= 2x-2$

f) Find, algebraically, the points of intersection of the curve $y=x^2$ and the line $x-y+20=0$. 2

$\begin{cases} y = x^2 \\ y = x+20 \end{cases}$

$x^2-x-20=0$

$(x-5)(x+4)=0$

$\therefore x=5 \quad x=-4$

$y=25 \quad y=16$

$(5,25)$ $(-4,16)$

QUESTION 3

(a)(i) $y' = -5 + 18x^2$

(ii) $f'(x) = 10(3-5x)^9 \times -5$

$= -50(3-5x)^9$

(iii) $y = -x^{-\frac{1}{2}}$

$y' = \frac{1}{2}x^{-\frac{3}{2}}$

$= \frac{1}{2\sqrt{x^3}}$

(b) $\sin 180° + 60° = -\sin 60°$

$= -\frac{\sqrt{3}}{2}$

(c) $\sqrt{2} + 1 - \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$

$= \sqrt{2} + 1 - \sqrt{2} + 1$

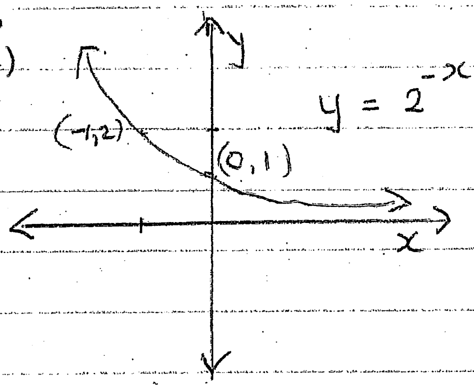
$= 2$

(d) $(0.2)^{x+1} = (0.2)^{3x-3}$

$x + 1 = 3x - 3$

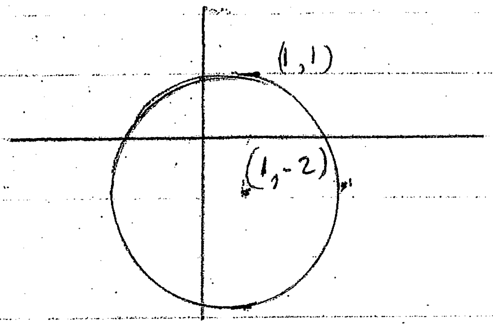
$x = 2$

(e)(i)



(ii) $(x-1)^2 + (y+2)^2 = 9$

circle, centre $(1,-2)$ $r = 3$



(f) $\frac{\ln 100}{\ln 7} = 2.367$

between 2 and 3.

(g)(i) $f(-x) = \frac{1}{1+(-x)^2} = \frac{1}{1+x^2}$

$= fx$

$\therefore$ EVEN

(ii) $f(-x) = \frac{1}{1-(-x)^2} = \frac{1}{1-x^2} = f(x)$

$\therefore$ EVEN

(iii) $f(-x) = \frac{x}{1+(-x)^2} = \frac{x}{1+x^2} = -f(x)$

$\therefore$ ODD.

Question 4

(a) $A(-4,-5)$ $B(2,7)$ $C(5,13)$

$\text{grad}_{AB}\text{ line} = \frac{7--5}{2--4} = 2$

Eq^n AB $\Rightarrow$ $y+5 = 2(x+4)$ (1)

$y = 2x+3$

$C(5,13) \Rightarrow 13 = 2(5)+3$ (1)

$\therefore$ C lies on line AB

OR

$M_{AB} = 2$ & $M_{BC} = 2$

and share common pt B

b) $(2x-y+1)+k(3x+y-6)=0$

$\therefore (-4-3+1)+k(-6+3-6)=0$

$\therefore -9k = 6 \Rightarrow \boxed{k=\frac{2}{3}}$ (3)

$\therefore$ Eq^n of line is $\boxed{y=3}$

~~$12x-y-9=0$~~

) Radius of circle = 3 units

⊥ Dist. from centre (0,0) to $4x-3y+15=0$

is $\frac{|4(0)-3(0)+15|}{\sqrt{4^2+3^2}}$ (1) $= \frac{15}{5} = 3$

Since this equals the radius length

$\Rightarrow$ line touches circle (once)

$\therefore$ tangent. (1)

(d) $\frac{(14-2)\times 180}{14} = 154°17'$ (1)

(e) $y = \frac{x^2+c}{x^2-c}$

$y' = \frac{(x^2-c).2x-(x^2+c).2x}{(x^2-c)^2}$

$y' = \frac{-4xc}{(x^2-c)^2}$ (3)

$1 = \frac{12c}{(9-c)^2}$

$(9-c)^2 = 12c$

$c^2-30c+81=0$

$(c-27)(c-3)=0$

$c=27$ or $c=3$

(f) $7 = a+b+c \Rightarrow \boxed{c=7}$

$4 = a-b+c$ } $a-b=-3$

$8 = a+b+c$ } $a+b=1$

$\boxed{a=-1}$ (2)

$\boxed{b=2}$

(f) quad >0 if (i) coeff $x^2>0$

and (ii) $\Delta<0$

(i) coeff of x^2 is 1 $\Rightarrow$ ∪

(ii) $\Delta = b^2-4ac$

$= (k-3)^2-4(1)(k)$ (1)

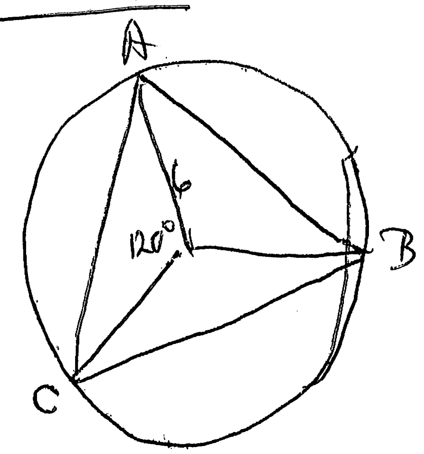
ie $\Delta = k^2-10k+9 < 0$

ie $(k-9)(k-1)<0$

$\therefore \boxed{1<k<9}$ (1)

SECTION C
Question 5

a)



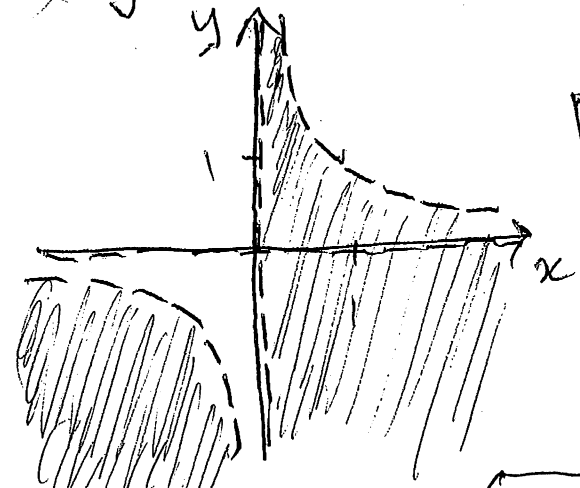
Area ABC $= 3 \times (\frac{1}{2} \times 6 \times 6 \sin 120°)$
$= 3 \times 18 \times \frac{\sqrt{3}}{2}$
$= 27\sqrt{3}$ [2]
$(\doteqdot 46.77)$

b) $5^x = 160$
$\log(5^x) = \log 160$
$x \log 5 =$
$x = \frac{\log 160}{\log 5}$ [2]
$\doteqdot 3.153$

c) $f(x) = x^2 + \frac{1}{3}x^3$
$f'(x) = 2x + x^2$ [1]
$m = \tan 135°$
$= -1$ [1]
$f'(x) = -1$ when
$x^2 + 2x = -1$
$x^2 + 2x + 1 = 0$
$(x+1)^2 = 0$
$x = -1$

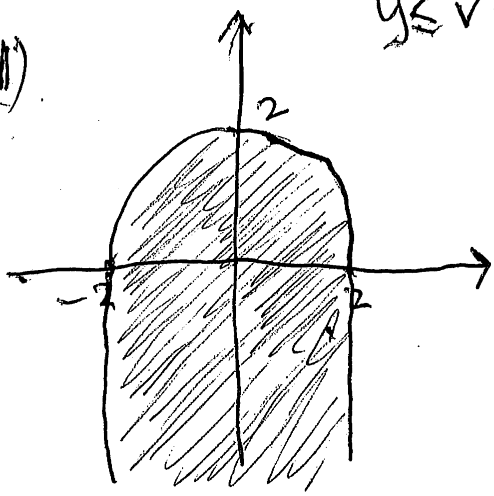
$\therefore$ Point is $(-1, \frac{2}{3})$ [1]

d) (i) $y \leq \frac{1}{x}$



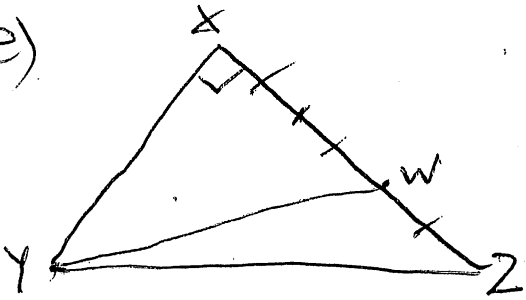
[2]

(ii) $y \leq \sqrt{4 - x^2}$



[2]

e)



① $YX^2 + 4ZW^2 = YW^2$ (Pythag.)
② $YX^2 + 9ZW^2 = YZ^2$ (")
② − ① $5ZW^2 = YZ^2 - YW^2$

[3]

Q5 (Continued)

f) $2x^2 - 6x + 1 = 0$

$\alpha + \beta = -\frac{(-6)}{2}$; $\alpha\beta = \frac{1}{2}$

$= 3$

(i) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$ [1]

$= \frac{3}{1/2}$

$= 6$

(ii) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (3)^2 - 2\left(\frac{1}{2}\right)$

$= 9 - 1$ [1]

$= 8$

(iii) $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$

$= (\alpha + \beta)((\alpha^2 + \beta^2) - \alpha\beta)$

$= 3\left(8 - \frac{1}{2}\right)$

$= 3 \times \frac{15}{2}$

$= \frac{45}{2} \left(= 22\frac{1}{2}\right)$

[1]

Question 6:

(a) (i) $y = 1 - 4x - 2x^2$

Domain $= \mathbb{R}$

Max value occurs at $x = -\frac{-4}{-2\times 2}$

$= -1$

$\therefore y = 1 + 4 - 2$

$= 3$ $1\frac{1}{2}$

$\therefore$ Range is $y \leq 3$

(ii) $y = \sqrt{1-4x}$

Domain $1 - 4x \geq 0$

$\therefore x \leq \frac{1}{4}$ $1\frac{1}{2}$

Range $y \geq 0$

(b) $y' = 1 - \frac{1}{x^2}$

For $2x + y - 13 = 0$, $m = -2$

$\therefore$ Gradient of normal $= -2$

$\therefore$ Gradient of tangent $= \frac{1}{2}$

$\therefore 1 - \frac{1}{x^2} = \frac{1}{2}$

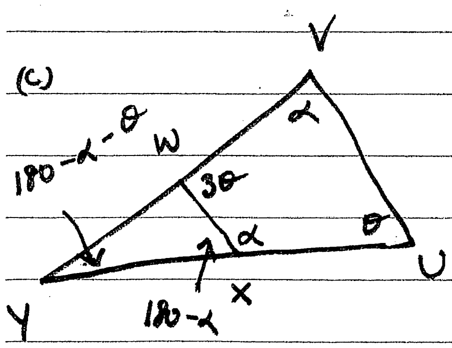
$\frac{1}{x^2} = \frac{1}{2}$

$x = \pm\sqrt{2}$ 2

$\therefore$ Points are $(\sqrt{2}, \sqrt{2} + \frac{1}{\sqrt{2}})$

and $(-\sqrt{2}, -\sqrt{2} - \frac{1}{\sqrt{2}})$

(c)



$\angle Y = 180 - \alpha - \theta$ ($\angle$ sum of Δ)

$(180 - \alpha - \theta) + (180 - \alpha) = 3\theta$ (ext $\angle$ of ΔYWX)

$\therefore 360 - 2\alpha = 4\theta$

$\therefore 180 - \alpha = 2\theta$

$\therefore \angle VYU = 180 - \alpha - \theta$

$= 2\theta - \theta$

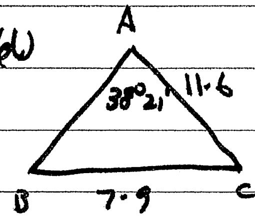
$= \theta$

$\therefore \angle VYU = \angle VUY$

$\therefore \Delta VYU$ is isosceles 3

$\therefore UV = VY$

(d)



$\frac{\sin B}{11.6} = \frac{\sin 38°21'}{7.9}$

$\therefore \sin B = \frac{11.6 \sin 38°21'}{7.9}$

$= 0.91106\ldots$

$\therefore B = 65.6523$ OR $114.34\ldots$ 3

$\approx 66°$ OR $114°$

(e) $x^2 + px + q = 0$

Roots are α and 2α

$\therefore \alpha + 2\alpha = -p$ i.e $3\alpha = -p$

$\alpha . 2\alpha = q$ i.e $2\alpha^2 = q$

(i) $2.\left(-\frac{p}{3}\right)^2 = q$

$\therefore \frac{2p^2}{9} = q$ 3

$\therefore 2p^2 = 9q$

(ii) $\alpha = -\frac{p}{3}$

If p is rational,

$p = \frac{a}{b}$ where a and b are integers

$\alpha = \frac{a}{-3b}$ 1

$= \frac{a}{c}$ where a and c are integers

$\therefore$ If p is rational α is rational, and 2α is rational

15